Op Amp

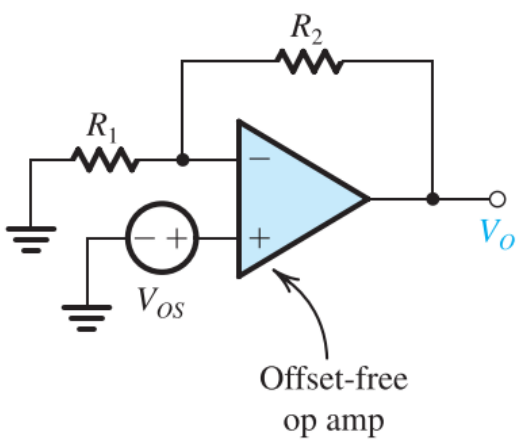
Analysis Method

There are tree rules to analyze the op amp circuit

- **virtual short**: $v_+ = v_-$
- **virtual open**: $i_+ = 0, i_- = 0$
- **zero output resistance**: $R_o = 0$

DC Imperfections

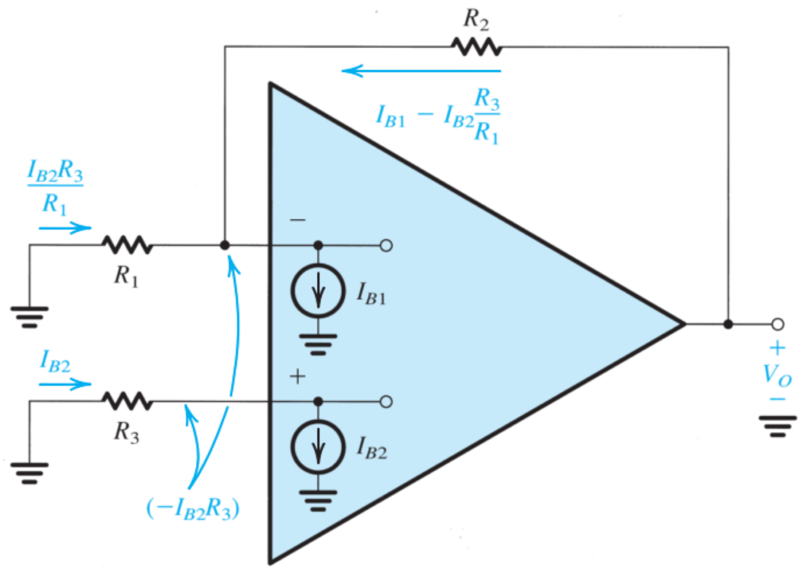
Offset Voltage



The output voltage due to V_{OS} is found to be

$$V_O = V_{OS}\left[1 + \frac{R_2}{R_1}\right]$$

Bias Currents



The average value I_B is called the **input bias current**

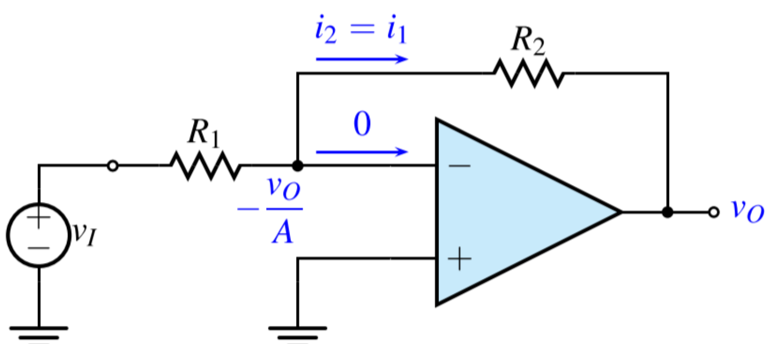
$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

and the difference is called the **input offset current** and is given by

$$I_{OS} = |I_{B1} - I_{B2}|$$

- DC: $R_3 = R_1 \| R_2$
- AC: $R_3 = R_2$

Finite Loop Gain

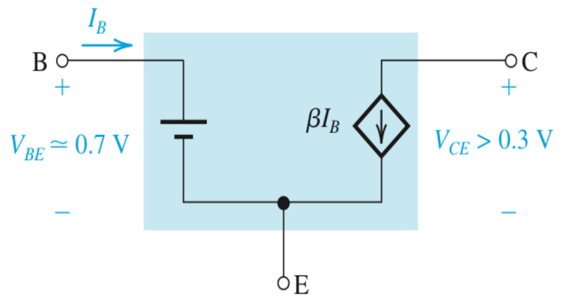
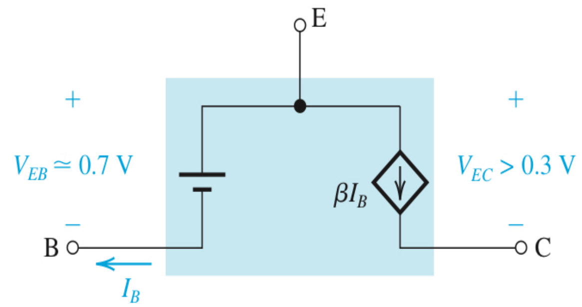
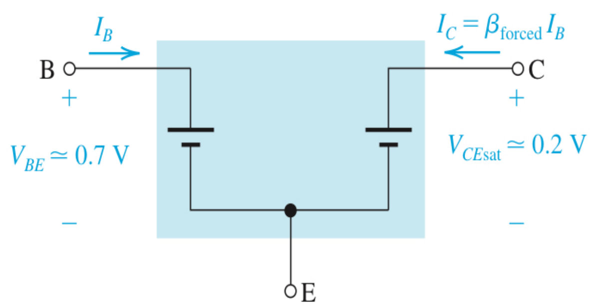
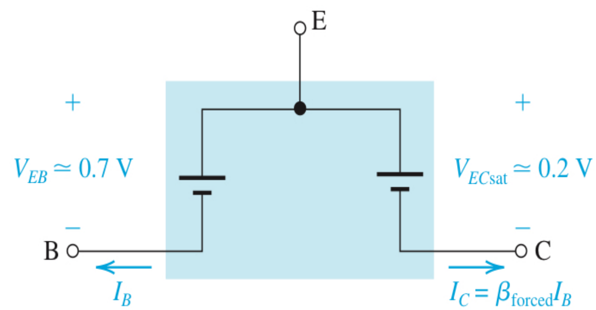


$$v_+ - v_- = \frac{v_O}{A} \Rightarrow v_- = -\frac{v_O}{A}$$

BJT

DC Analysis

Table 6.3 Simplified Models for the Operation of the BJT in DC Circuits

	npn	*pnp*
Active EBJ: Forward Biased CBJ: Reverse Biased		
Saturation EBJ: Forward Biased CBJ: Forward Biased		

> **cut off** mode: $\|V_{BE}\| < 0.5\ V$

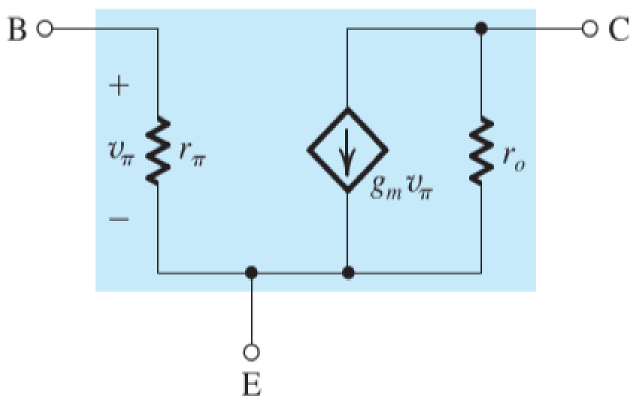
Small Signal Analysis

1. DC analysis, cut all capacitors and sig.
2. find I_C
3. cal g_m, r_e and r_π and draw the ac circuit
4. find V_o/V_i
5. find input and output resistance

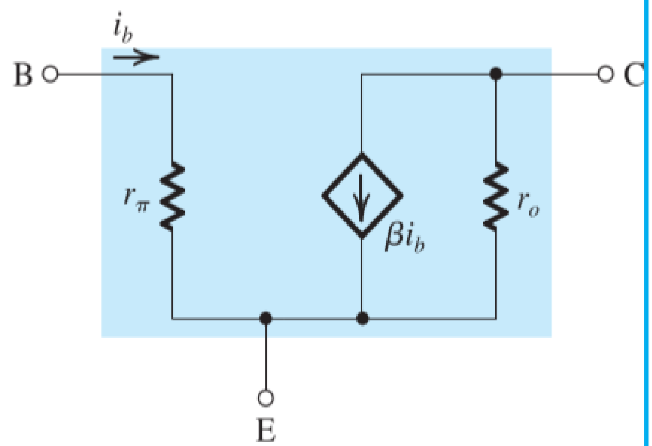
Table 7.3 Small-Signal Models of the BJT

Hybrid-π Model

■ ($g_m v_\pi$) Version

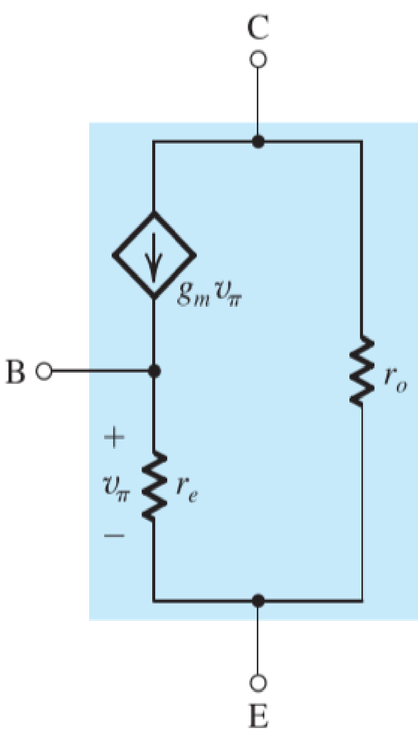


■ (βi_b) Version

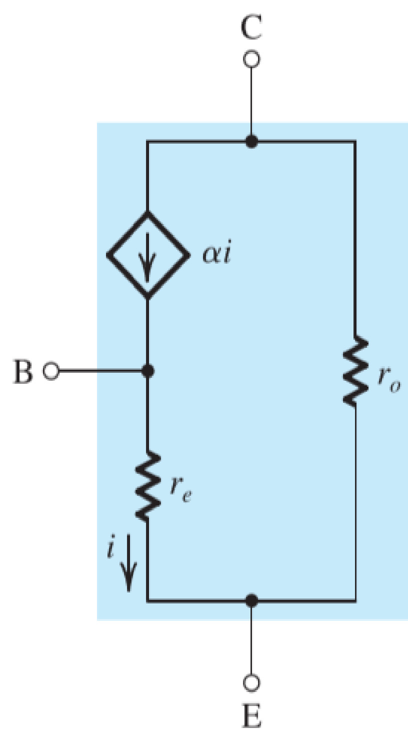


T Model

■ ($g_m v_\pi$) Version



■ (αi) Version

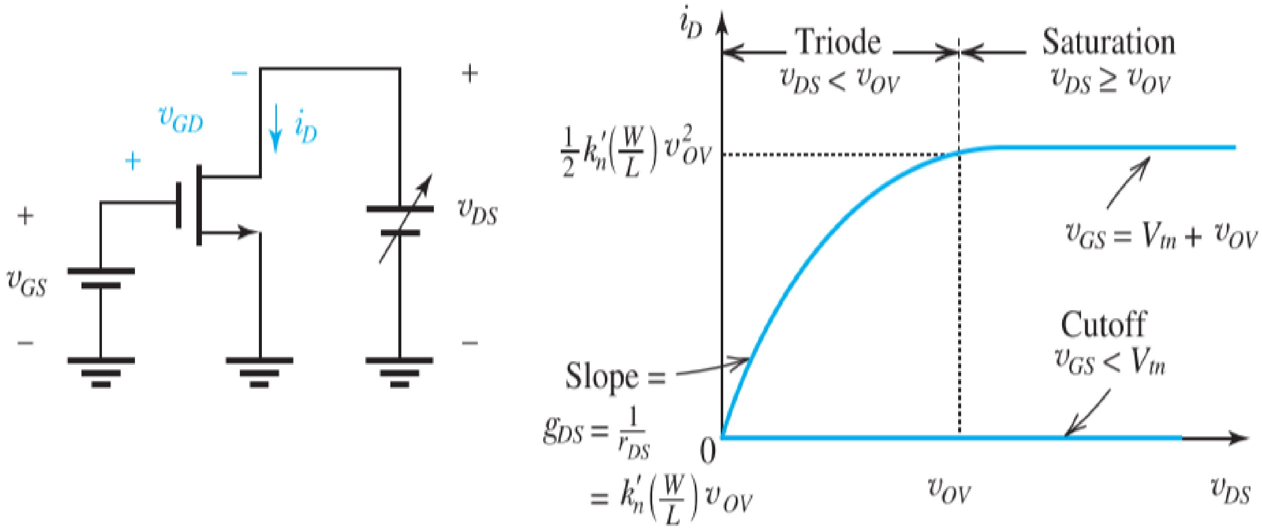


Model Parameters in Terms of DC Bias Currents			
$g_m = \frac{I_C}{V_T}$	$r_e = \frac{V_T}{I_E} = \alpha \frac{V_T}{I_C}$	$r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C}$	$r_o = \frac{\lvert V_A \rvert}{I_C}$
In Terms of g_m			
$r_e = \frac{\alpha}{g_m}$	$r_\pi = \frac{\beta}{g_m}$		
In Terms of r_e			
$g_m = \frac{\alpha}{r_e}$	$r_\pi = (\beta + 1) r_e$	$g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$	
Relationships between α and β			
$\beta = \frac{\alpha}{1 - \alpha}$	$\alpha = \frac{\beta}{\beta + 1}$	$\beta + 1 = \frac{1}{1 - \alpha}$	

MOSFET

DC Analysis

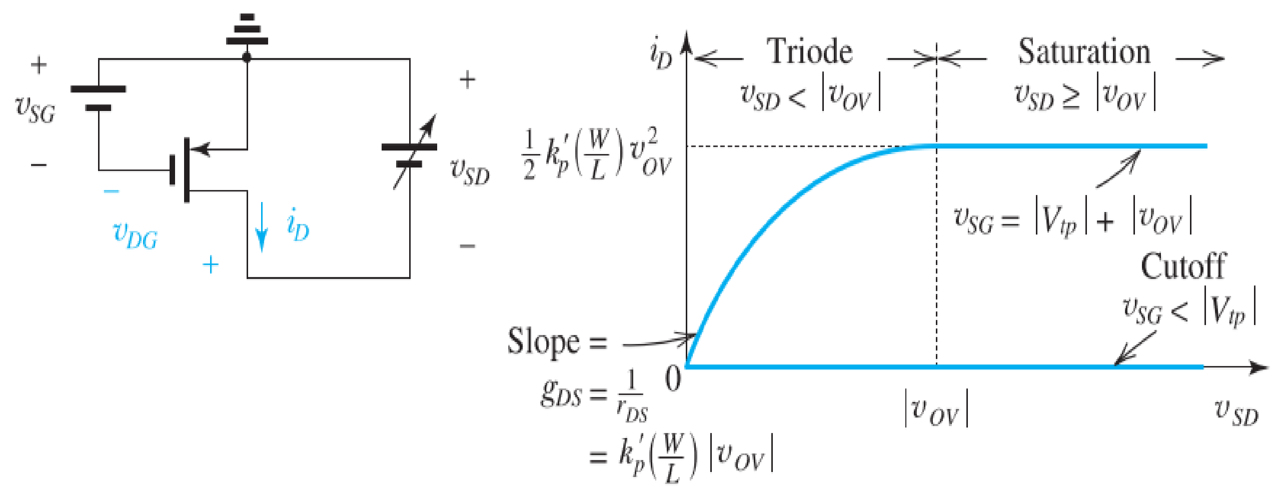
Table 5.1 Regions of Operation of the Enhancement NMOS Transistor



- $v_{GS} < V_{tn}$: no channel; transistor in cutoff; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$: a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched off at the drain end;

Triode Region	*Saturation Region*
Continuous channel, obtained by: $v_{GD} > V_{tn}$	Pinched-off channel, obtained by: $v_{GD} \leq V_{tn}$
or equivalently: $v_{DS} < v_{OV}$	or equivalently: $v_{DS} \geq v_{OV}$
Then, $i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$	Then $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$
or equivalently, $i_D = k_n'\left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$	or equivalently, $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)v_{OV}^2$

Table 5.2 Regions of Operation of the Enhancement PMOS Transistor



- $v_{SG} < |V_{tp}|$: no channel; transistor in cutoff; $i_D = 0$
- $v_{SG} = |V_{tp}| + |v_{OV}|$: a channel is induced; transistor operates in the triode region or in the saturation region depending on whether the channel is continuous or pinched off at the drain end;

Triode Region

Continuous channel, obtained by:

$$v_{DG} > |V_{tp}|$$

or equivalently

$$v_{SD} < |v_{OV}|$$

Then

$$i_D = k_p'\left(\frac{W}{L}\right)\left[(v_{SG} - |V_{tp}|)v_{SD} - \frac{1}{2}v_{SD}^2\right]$$

or equivalently

$$i_D = k_p'\left(\frac{W}{L}\right)\left(|v_{OV}| - \frac{1}{2}v_{SD}\right)v_{SD}$$

Saturation Region

Pinched-off channel, obtained by:

$$v_{DG} \leq |V_{tp}|$$

or equivalently

$$v_{SD} \geq |v_{OV}|$$

Then

$$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)(v_{SG} - |V_{tp}|)^2$$

or equivalently

$$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)v_{OV}^2$$

Small Signal Analysis

1. DC analysis, cut all capacitors and sig.
2. find I_D and V_{OV}
3. cal g_m and draw the ac circuit

4. find V_o/V_i
5. find input and output resistance

Table 7.2 Small-Signal Models of the MOSFET

Small-Signal Parameters

NMOS transistors

- **Transconductance:**

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{OV}}$$

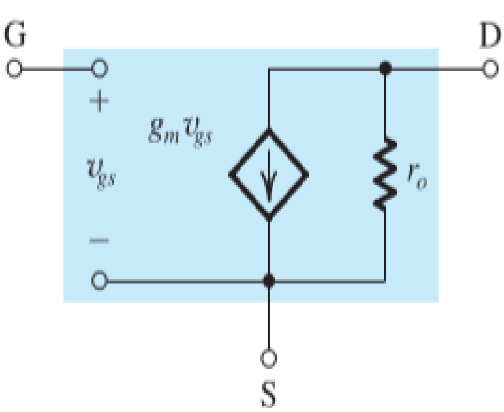
- Output resistance:

$$r_o = V_A/I_D = 1/\lambda I_D$$

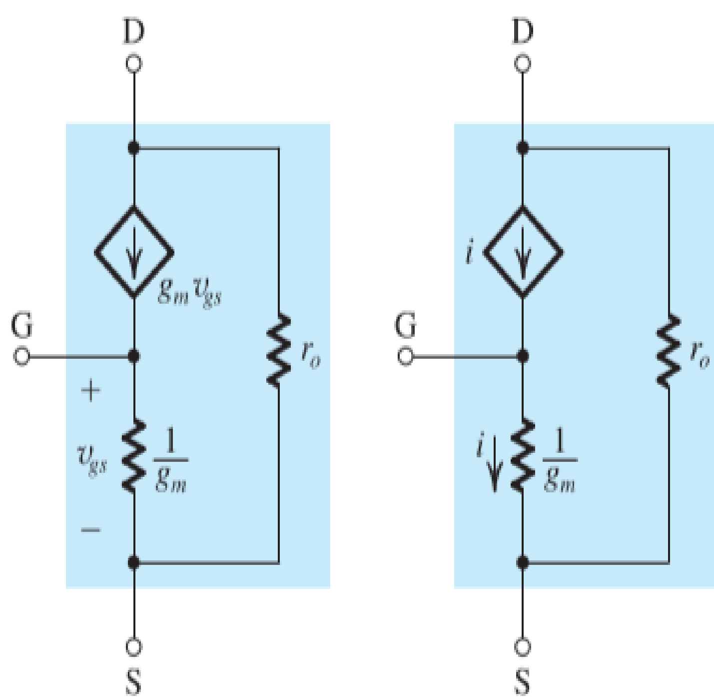
PMOS transistors

Same formulas as for NMOS *except* using $|V_{OV}|$, $|V_A|$, $|\lambda|$ and replacing μ_n with μ_p.

Small-Signal, Equivalent-Circuit Models



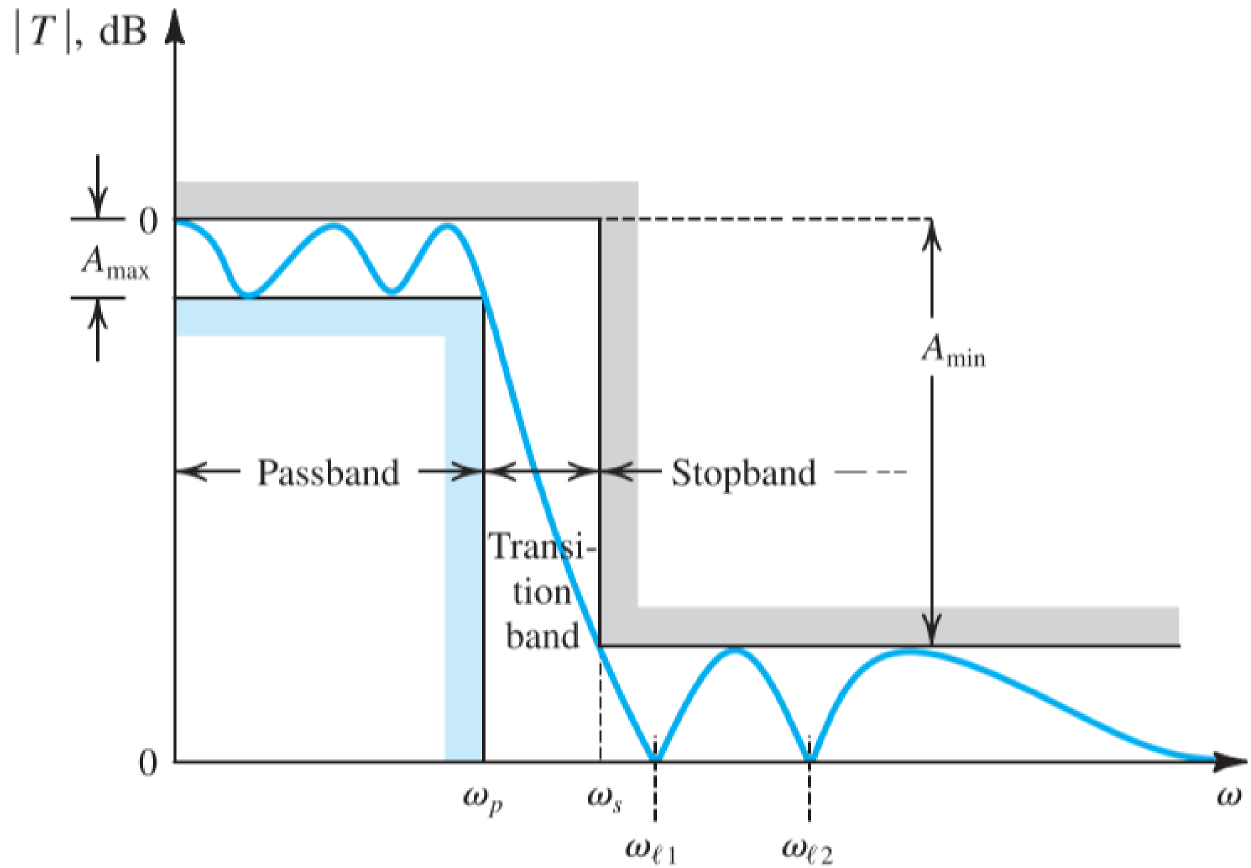
Hybrid-π model



T models

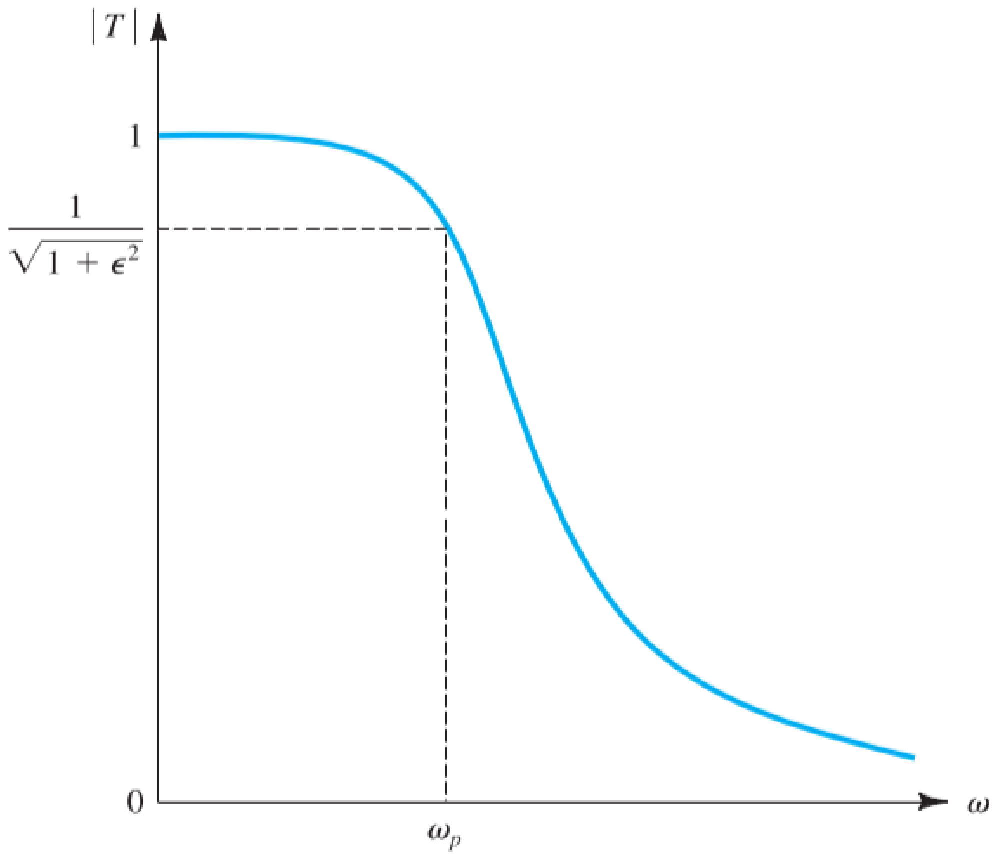
Filter

Filter Specification



- ω_p: the passband edge
- A_{max}: the maximum allowed variation in passband transmission
- ω_s: the stopband edge
- A_{min}: the minimum required stopband attenuation

Butterworth Filter



$$\|T(j\omega)\| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^2}}$$

At the frequency of ω_p

$$\|T(j\omega)\| = \frac{1}{\sqrt{1+\epsilon^2}}$$

Then the parameter ϵ determines A_{max}

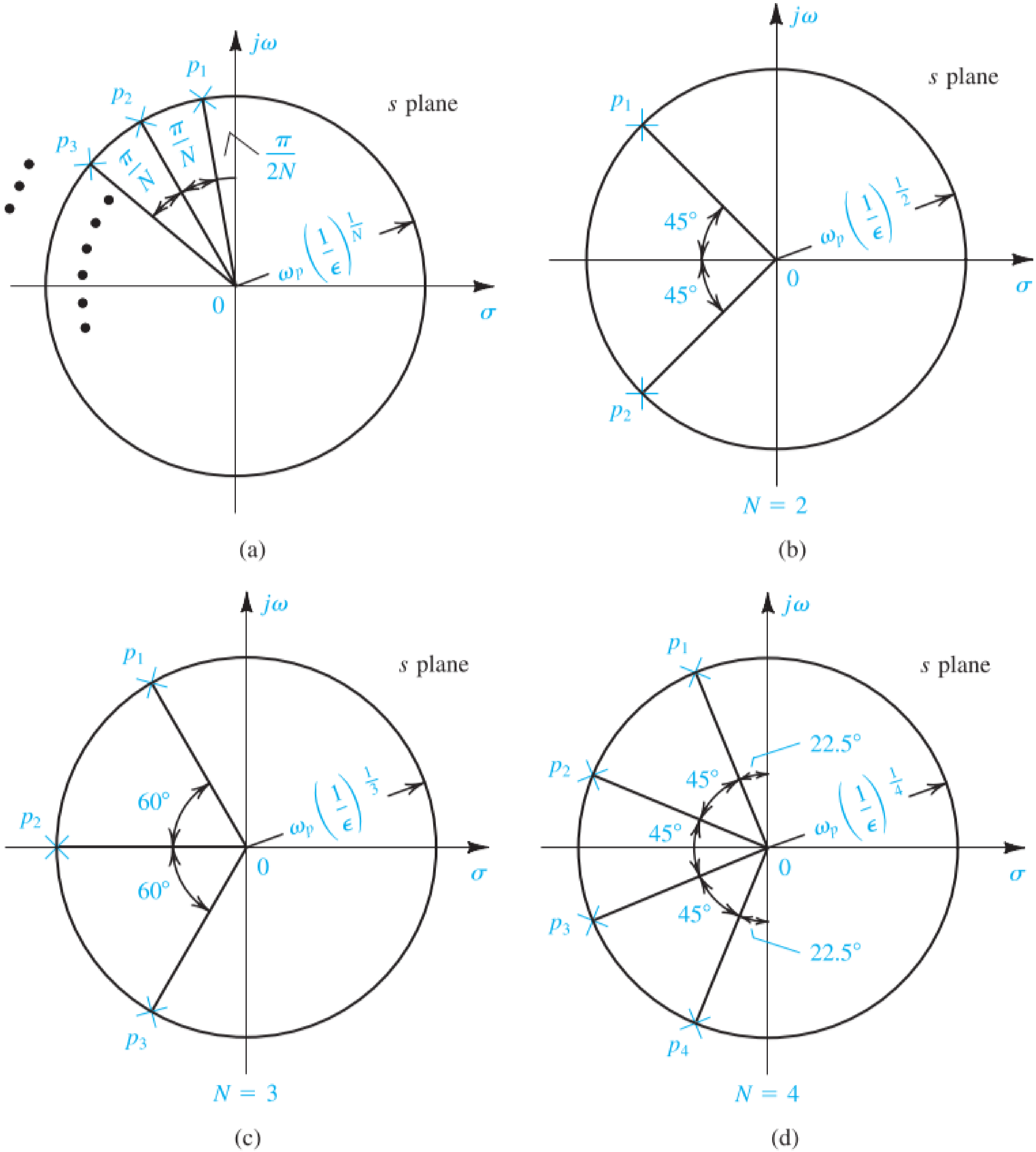
$$A_{max} = 10\log\left(1+\epsilon^2\right)$$

$$\epsilon = \sqrt{10^{A_{max}/10}-1}$$

At the edge of stopband $\omega = \omega_s$, check the attenuation of the filter

$$A(\omega_s) = 10\log\left[1+\epsilon^2(\omega_s/\omega_p)^{2N}\right]$$

Find the lowest filter order N such that $A(\omega_s) > A_{min}$



Then the transfer function could be written as

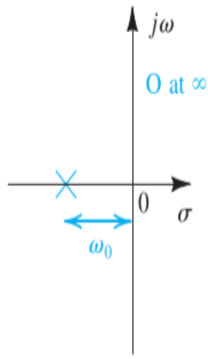
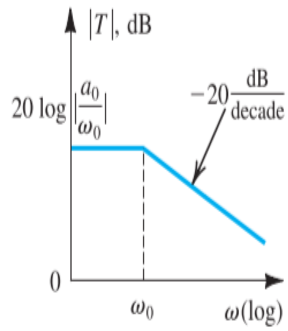
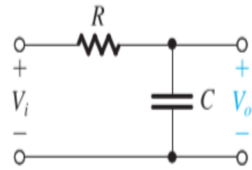
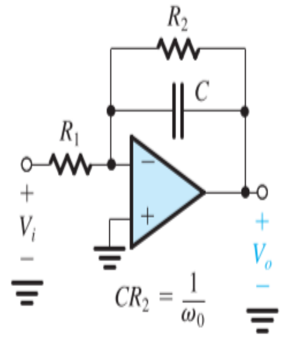
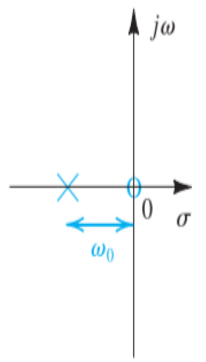
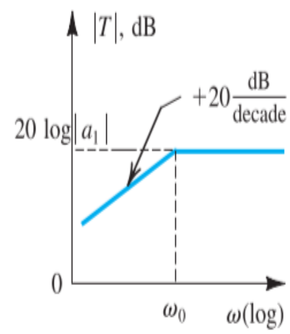
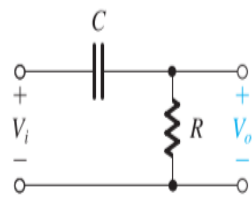
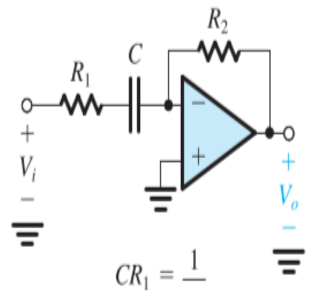
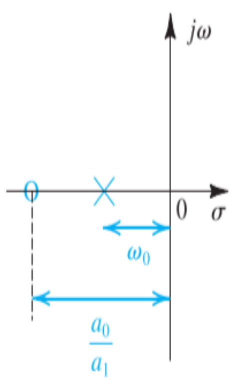
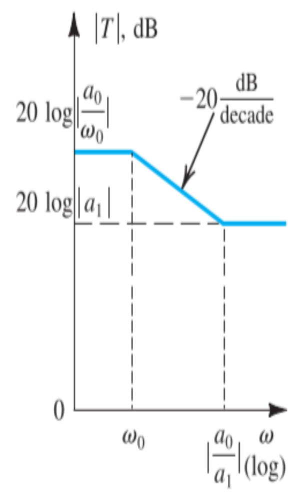
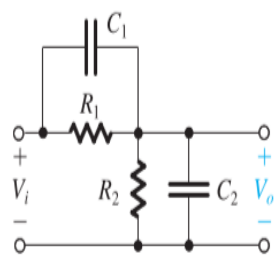
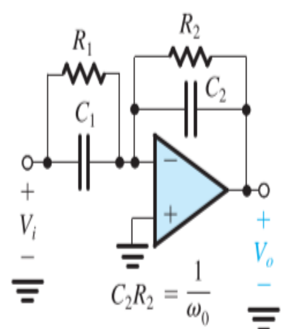
$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

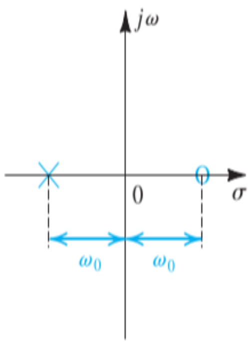
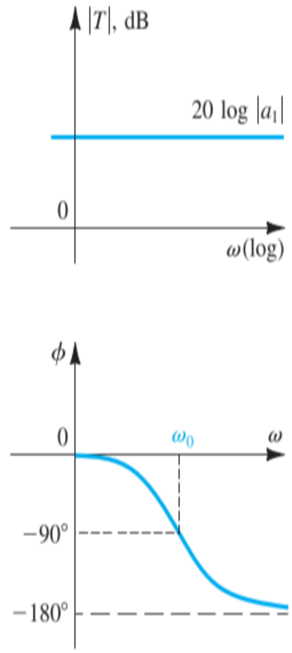
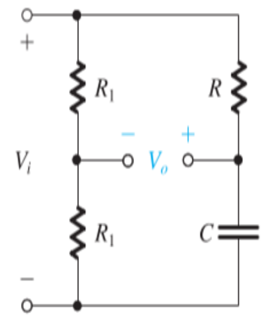
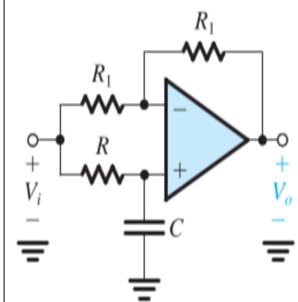
- $\omega_0 = \omega_p(1/\epsilon)^{1/N}$

First Order Filters

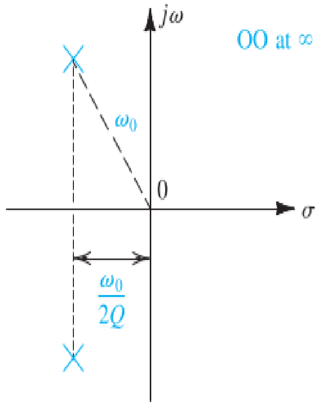
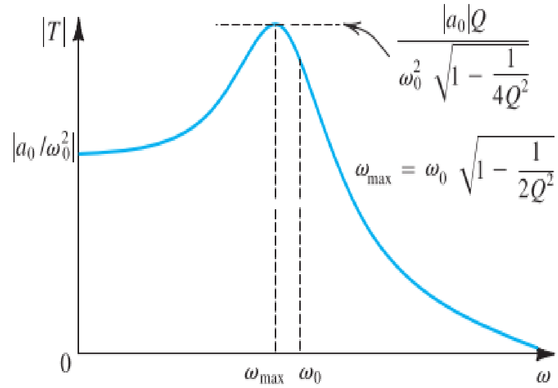
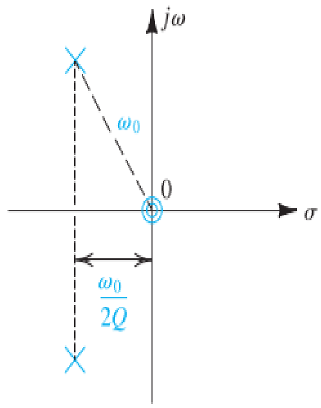
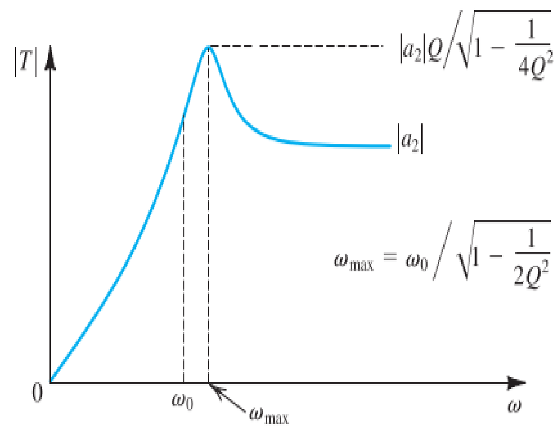
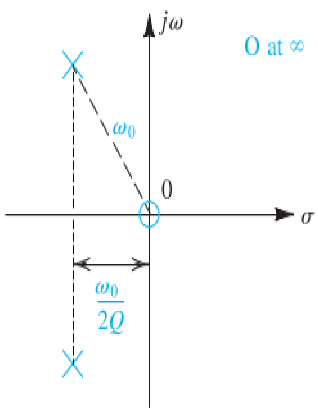
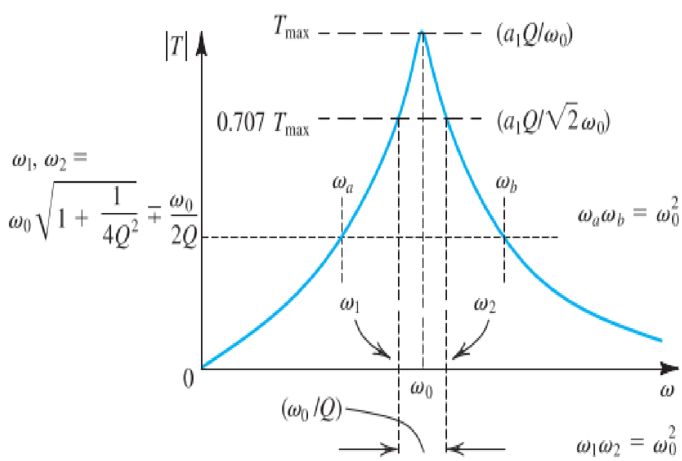
The general first-order transfer function is given by

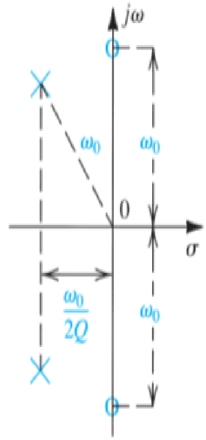
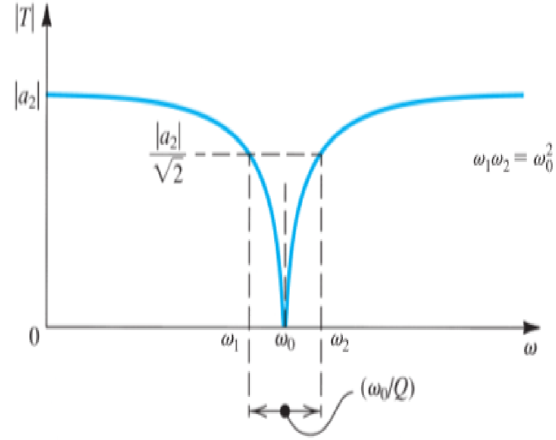
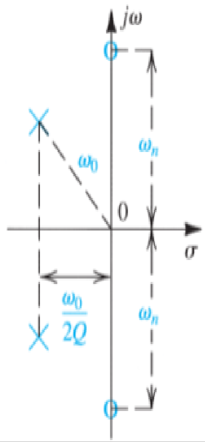
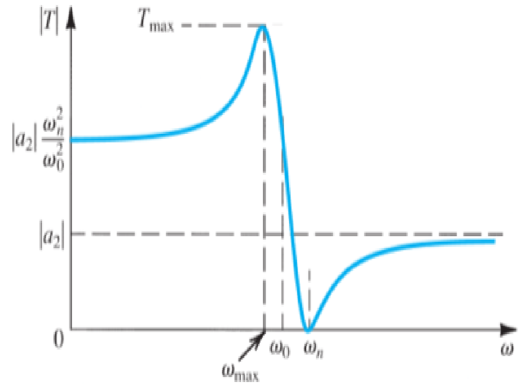
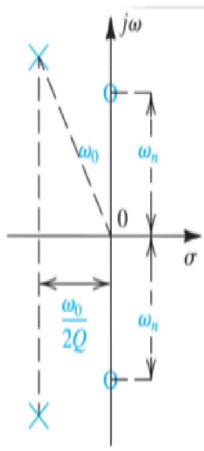
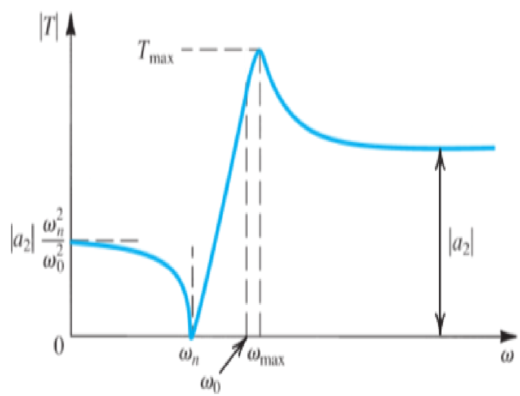
$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

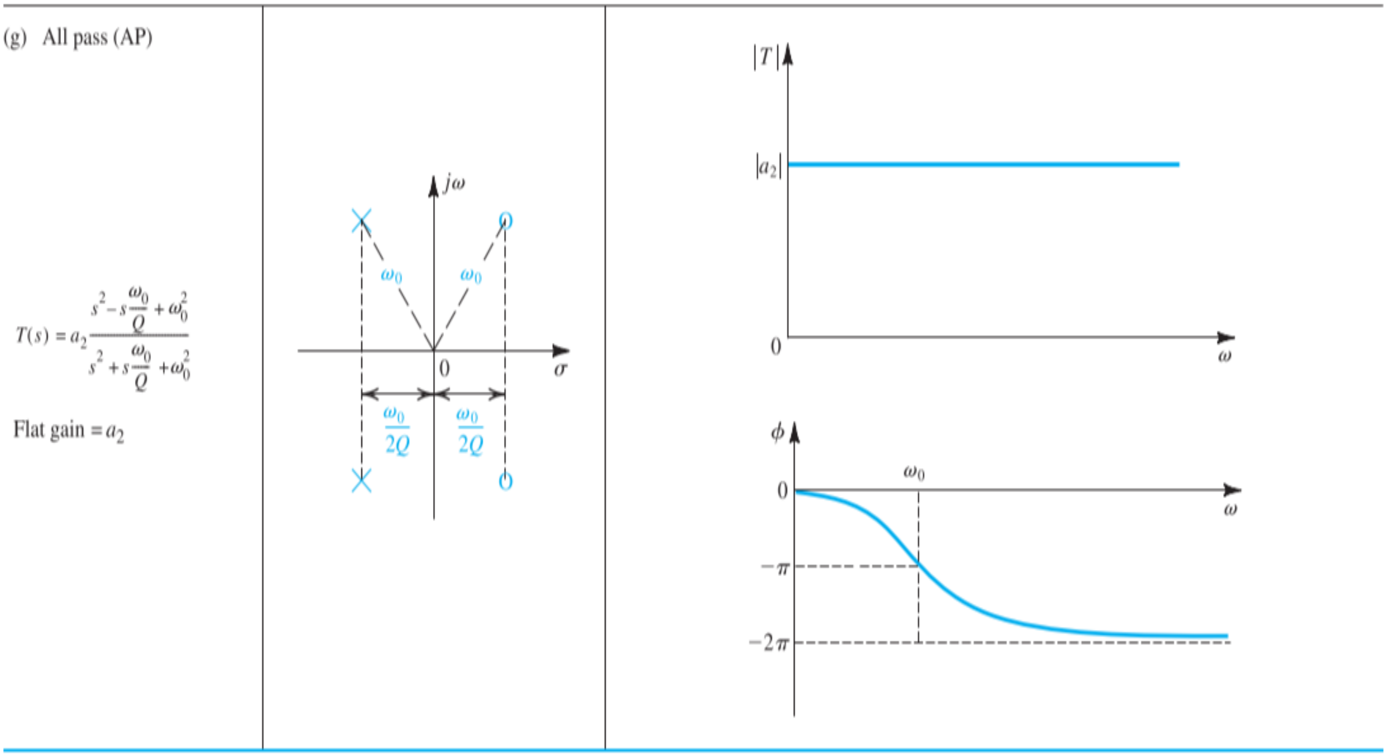
Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\lvert T \rvert$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ DC gain = 1	 $CR_2 = \dfrac{1}{\omega_0}$ DC gain $= -\dfrac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \dfrac{a_1 s}{s + \omega_0}$			 $CR = \dfrac{1}{\omega_0}$ High-frequency gain = 1	 $CR_1 = \dfrac{1}{\omega_0}$ High-frequency gain $= -\dfrac{R_2}{R_1}$
(c) General $T(s) = \dfrac{a_1 s + a_0}{s + \omega_0}$			 $(C_1 + C_2)(R_1 /\!/ R_2) = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ DC gain $= \dfrac{R_2}{R_1 + R_2}$ HF gain $= \dfrac{C_1}{C_1 + C_2}$	 $C_2 R_2 = \dfrac{1}{\omega_0}$ $C_1 R_1 = \dfrac{a_1}{a_0}$ DC gain $= -\dfrac{R_2}{R_1}$ HF gain $= -\dfrac{C_1}{C_2}$

$T(s)$	Singularities	$\|T\|$ and ϕ	Passive Realization	Op Amp–RC Realization
All pass (AP) $T(s) = -a_1 \dfrac{s-\omega_0}{s+\omega_0}$ $a_1 > 0$			 $CR = 1/\omega_0$ Flat gain $(a_1) = 0.5$	 $CR = 1/\omega_0$ Flat gain $(a_1) = 1$ $\left\|\dfrac{V_o}{V_i}\right\| = 1$ $\phi(\omega) = -2\tan^{-1}(\omega CR)$

Second Order Transfer Functions

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T\rvert$
(a) Low pass (LP) $T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain $= \dfrac{a_0}{\omega_0^2}$		
(b) High pass (HP) $T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ High-frequency gain $= a_2$		
(c) Bandpass (BP) $T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$		

Filter Type and $T(s)$	s-Plane Singularities	$\lvert T \rvert$
(d) Notch $T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ DC gain = High-frequency gain $= a_2$		 $\omega_1\omega_2 = \omega_0^2$
(e) Low-pass notch (LPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \geq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $\omega_{max} = \omega_0 \sqrt{\dfrac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\dfrac{\omega_n^2}{\omega_0^2} + \dfrac{1}{2Q^2} - 1}}$
(f) High-pass notch (HPN) $T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$ $\omega_n \leq \omega_0$ DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$ High-frequency gain $= a_2$		 $T_{max} = \dfrac{\lvert a_2 \rvert \; \lvert \omega_n^2 - \omega_{max}^2 \rvert}{\sqrt{(\omega_0^2 - \omega_{max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{max}^2}}$

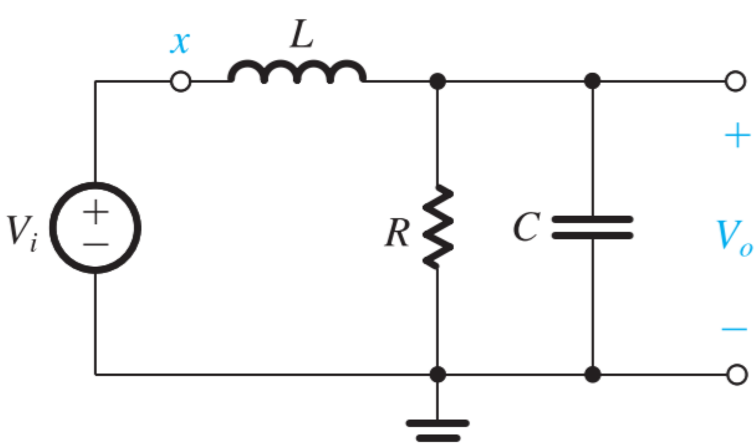


Second-Order LCR Resonator

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 CR$$

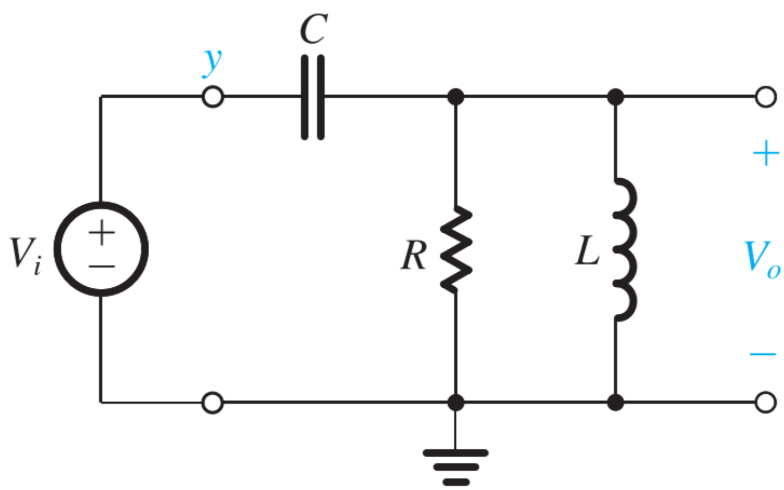
Low-Pass Function



$$T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order low-pass filters have **2 zeros** at ∞

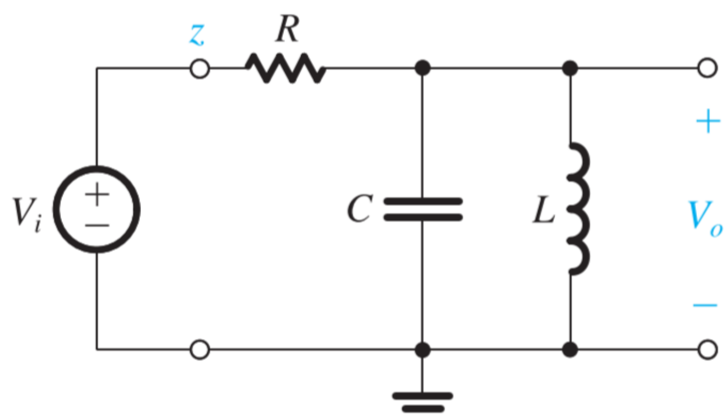
High-Pass Function



$$T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{s^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order high-pass filters have **2 zeros** at $s = 0$

Band-Pass Function

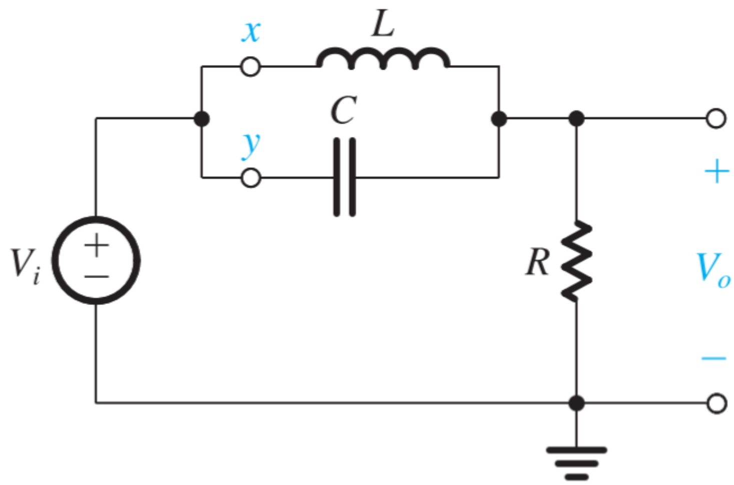


(d) BP

$$T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{(\omega_0/Q)s}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order band-pass filters have **1 zero** at ∞ and **1 zero** at $s = 0$

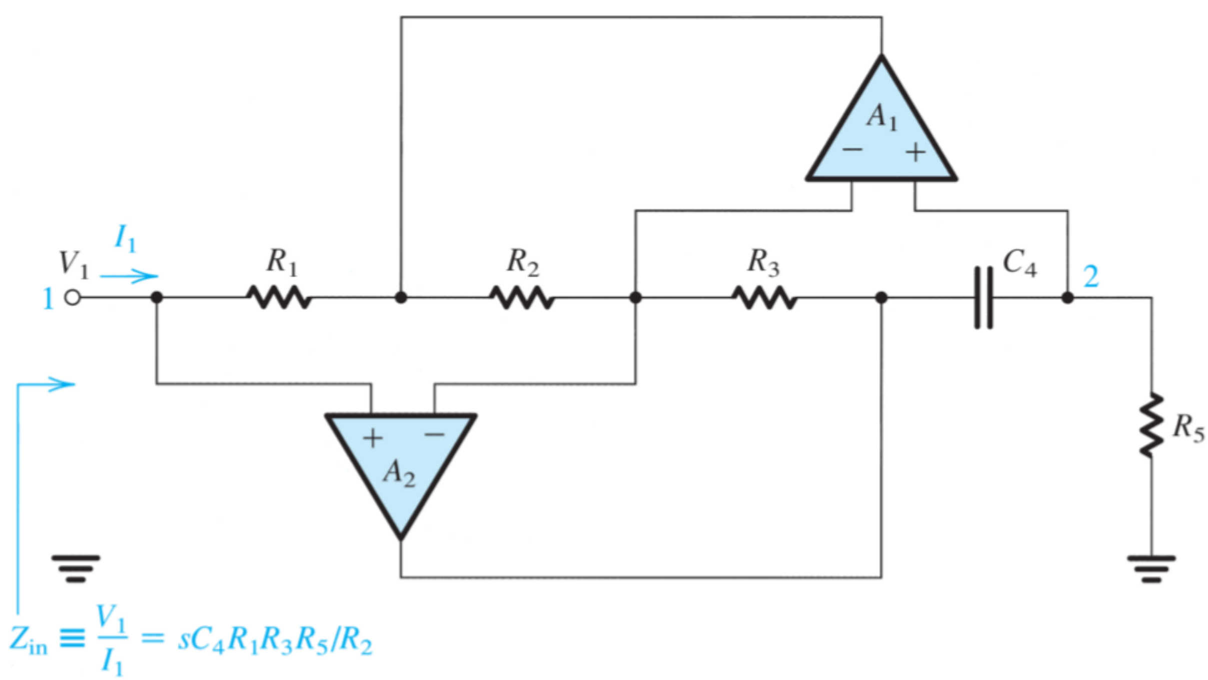
Notch Function



$$T(s) = \frac{Z_2}{Z_1 + Z_2} = a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order notch(bandstop) filters have **2 zeros** at $j\omega$ axis

Op Amp-RC Resonator

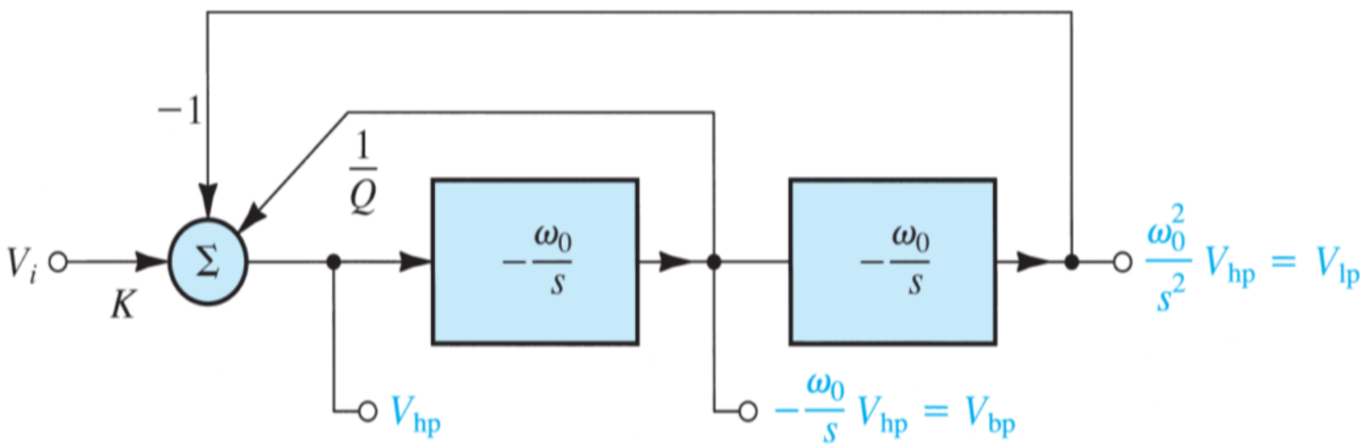


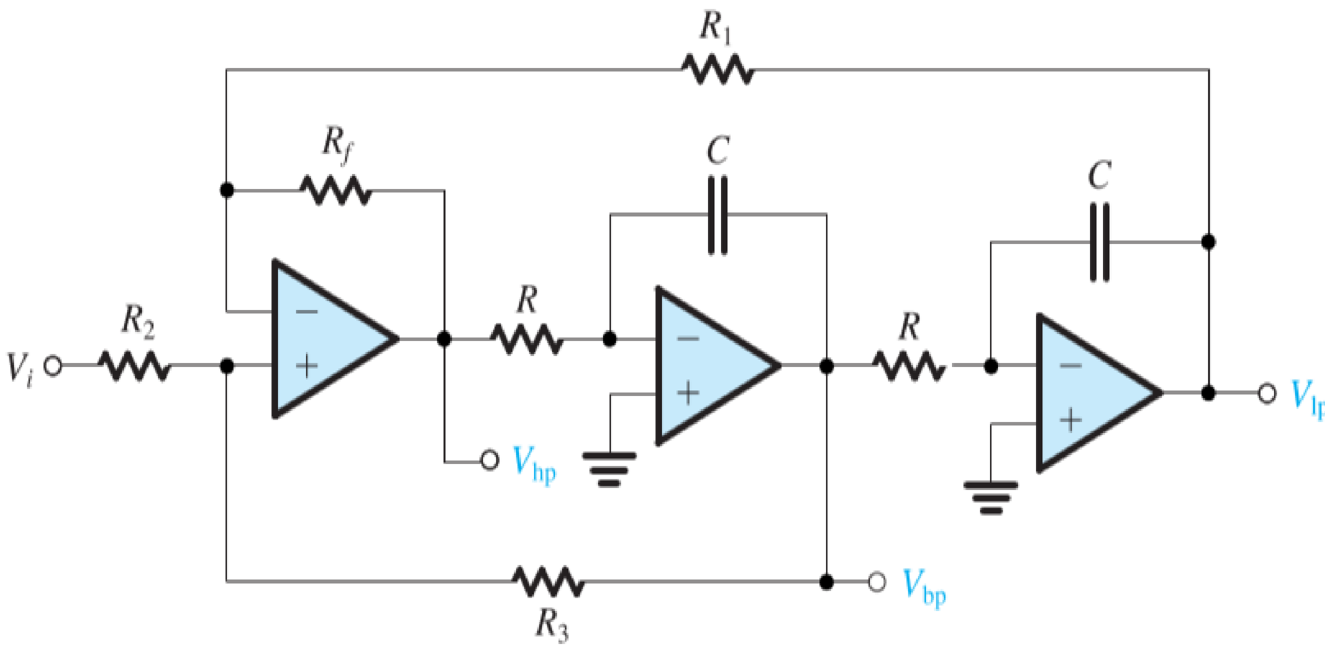
$$L = \frac{C_4 R_1 R_3 R_5}{R_2}$$

The design of the circuit is usually selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C$, which leads to $L = CR^2$

KHN Biquad

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

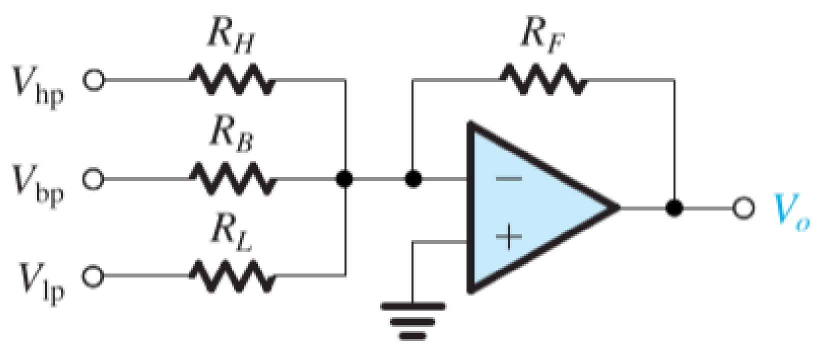




$$V_{hp} = V_i\frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right) + V_{bp}\frac{R_2}{R_2+R3}\left(1+\frac{R_f}{R_1}\right) - V_{lp}\frac{R_f}{R_1}$$

$$= V_i\frac{R_3}{R_2+R_3}\left(1+\frac{R_f}{R_1}\right) + \left(-\frac{\omega_0}{s}V_{hp}\right)\frac{R_2}{R_2+R3}\left(1+\frac{R_f}{R_1}\right) - \left(\frac{\omega_0^2}{s^2}V_{hp}\right)\frac{R_f}{R_1}$$

- $CR = 1/\omega_0$
- $R_f = R_1$
- $R_3/R_2 = 2Q-1$
- $K = 2-(1/Q)$

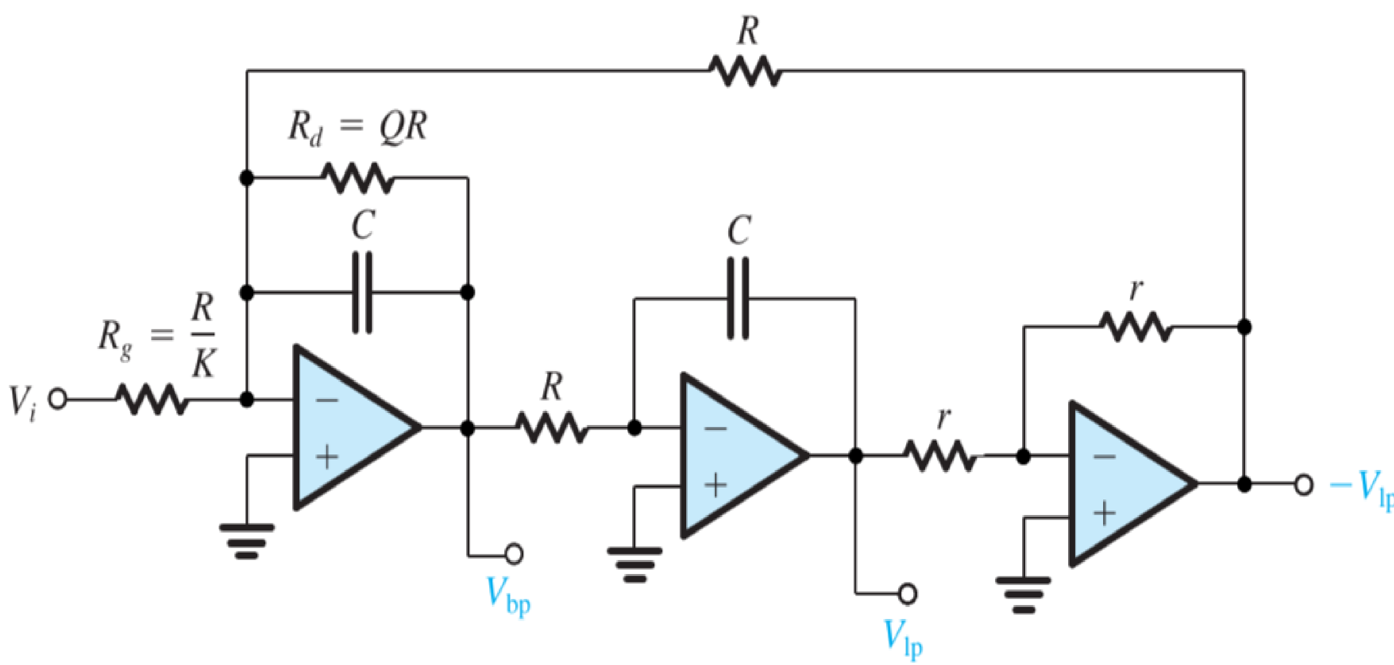


$$V_o = -(\frac{R_F}{R_H}V_{hp} + \frac{R_F}{R_B}V_{bp} + \frac{R_F}{R_L}V_{lp})$$

$$= -V_i(\frac{R_F}{R_H}T_{hp} + \frac{R_F}{R_B}T_{bp} + \frac{R_F}{R_L}T_{lp})$$

$$\frac{V_o}{V_i} = -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Tow–Thomas Biquad



$$\frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}}$$

- $r =$ arbitrary
- $C =$ arbitrary
- $R = 1/\omega_0$

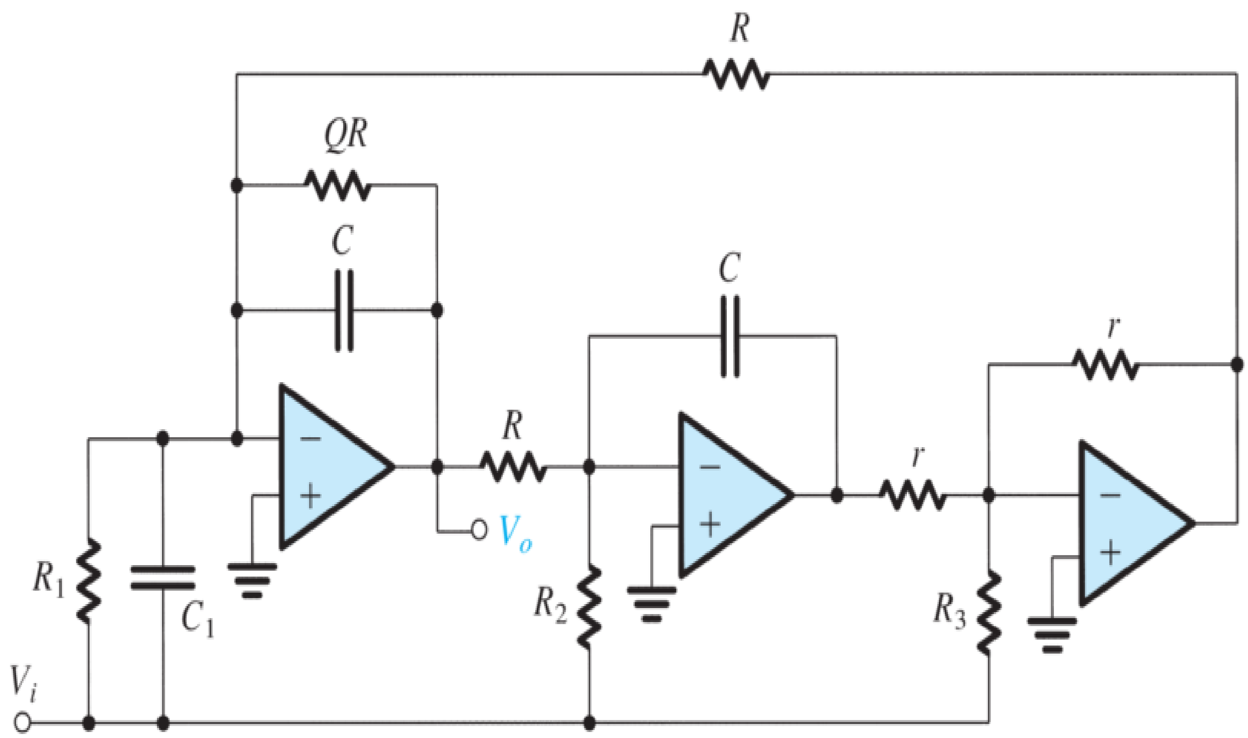


Figure 17.26 The Tow–Thomas biquad with feedforward. The transfer function of Eq. (17.68) is realized by feeding the input signal through appropriate components to the inputs of the three op amps. This circuit can realize all special second-order functions. The design equations are given in Table 17.2.

Table 17.2 Design Data for the Circuit in Fig. 17.26

All cases	$C =$ arbitrary, $R = 1/\omega_0 C$, $r =$ arbitrary
LP	$C_1 = 0$, $R_1 = \infty$, $R_2 = R/$dc gain, $R_3 = \infty$
Positive BP	$C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr/$center-frequency gain
Negative BP	$C_1 = 0$, $R_1 = QR/$center-frequency gain, $R_2 = \infty$, $R_3 = \infty$
HP	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$
Notch (all types)	$C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2/$high-frequency gain, $R_3 = \infty$
AP	$C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R/$gain, $R_3 = Qr/$gain