# Op Amp

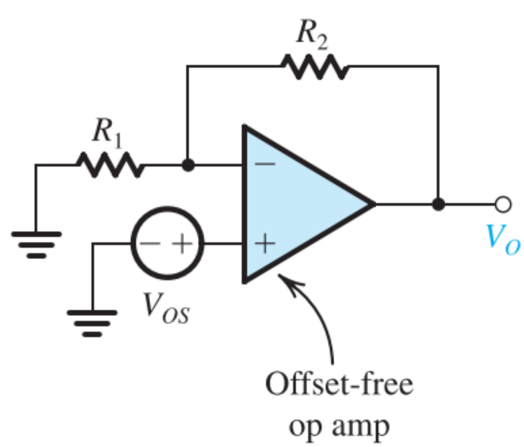
## Analysis Method

There are tree rules to analyze the op amp circuit

- **virtual short**: $v_+ = v_-$
- **virtual open**: $i_+ = 0, i_- = 0$
- **zero output resistance**: $R_o = 0$

## DC Imperfections

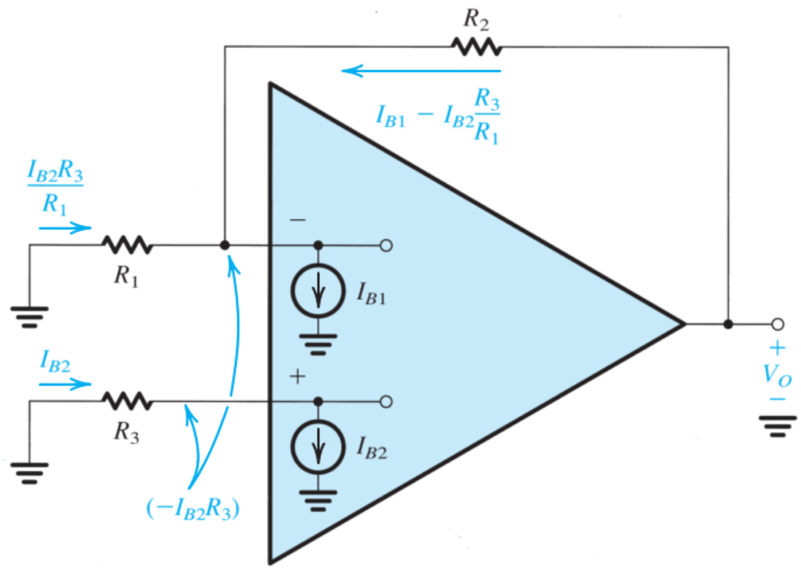
### Offset Voltage




The output voltage due to $V_{OS}$ is found to be

$$V_O = V_{OS}\left[1 + \frac{R_2}{R_1}\right]$$

### Bias Currents




The average value $I_B$ is called the **input bias current**

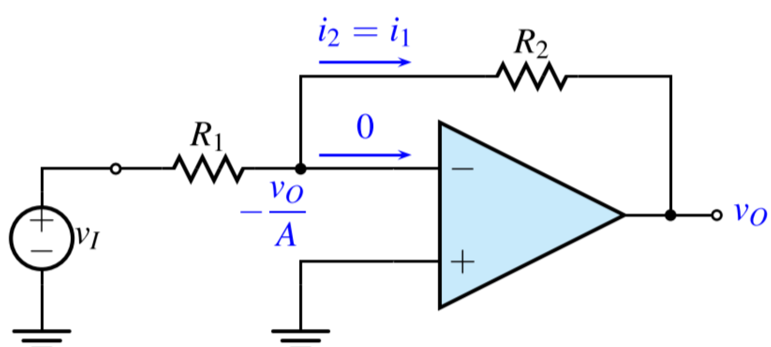
$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

and the difference is called the **input offset current** and is given by

$$I_{OS} = |I_{B1} - I_{B2}|$$

- DC: $R_3 = R_1 \| R_2$
- AC: $R_3 = R_2$

### Finite Loop Gain

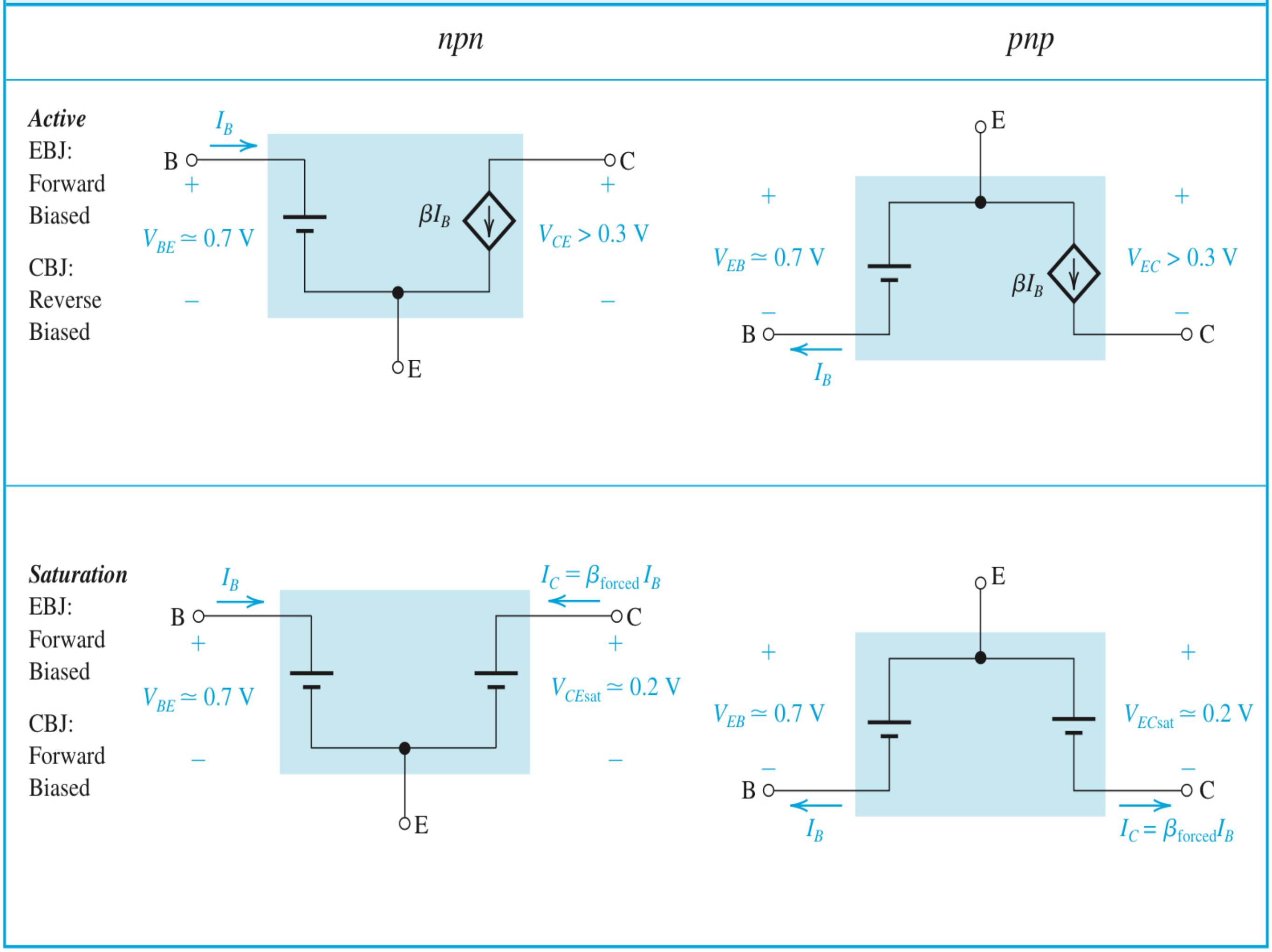



$$v_+ - v_- = \frac{v_O}{A} \Rightarrow v_- = -\frac{v_O}{A}$$

# BJT

## DC Analysis

**Table 6.3** Simplified Models for the Operation of the BJT in DC Circuits




> **cut off** mode: $||V_{BE}|| < 0.5\ V$

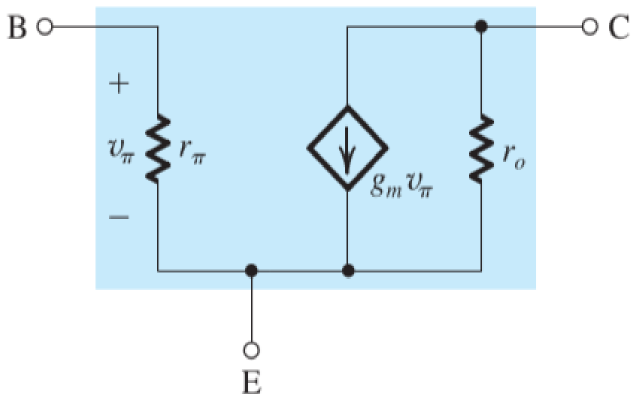
## Small Signal Analysis

1. DC analysis, cut all capacitors and sig.
2. find $I_C$
3. cal $g_m$, $r_e$ and $r_\pi$ and draw the ac circuit
4. find $V_o/V_i$
5. find input and output resistance

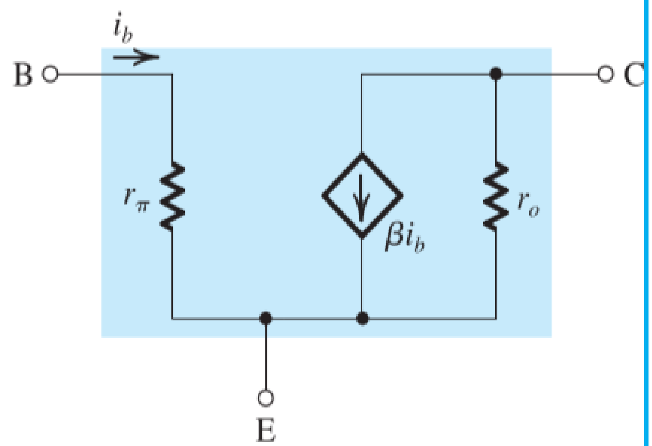
**Table 7.3** Small-Signal Models of the BJT

Hybrid-$\pi$ Model

- $(g_m v_\pi)$ Version

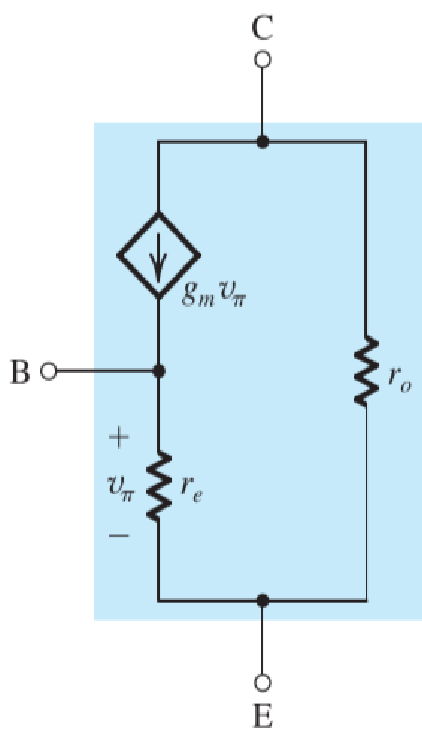



- $(\beta i_b)$ Version

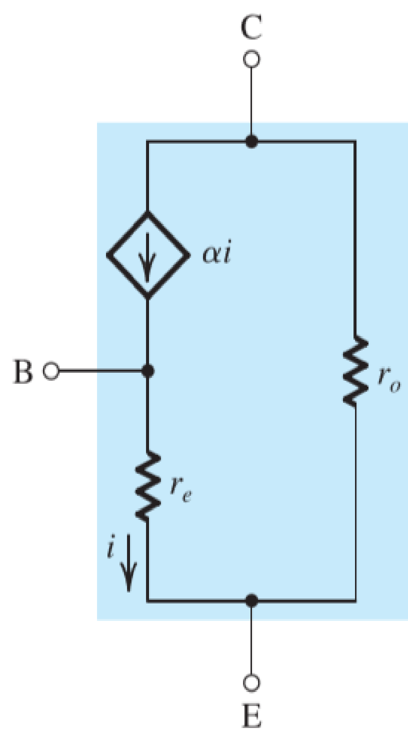



T Model

- $(g_m v_\pi)$ Version




- $(\alpha i)$ Version

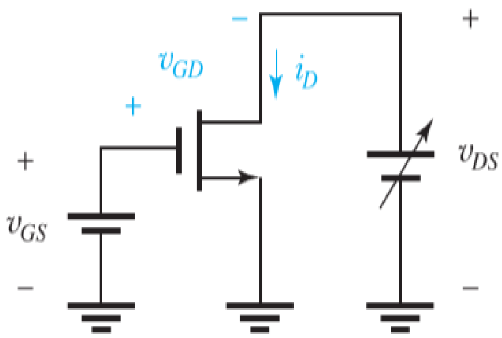


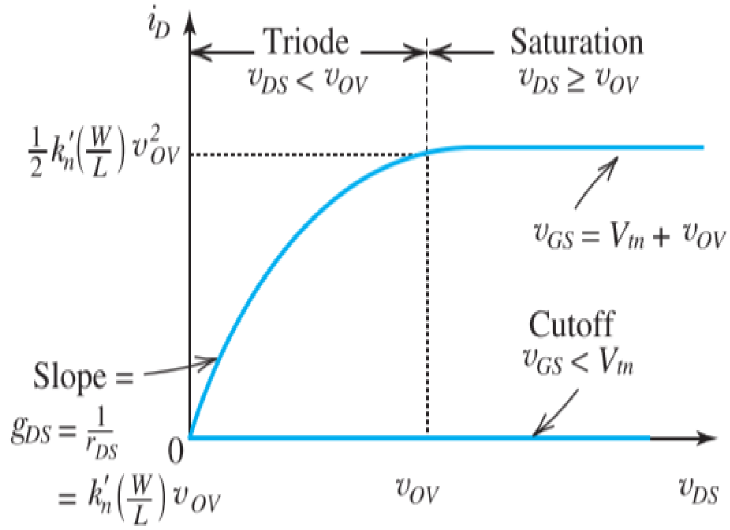
| | | | |
|---|---|---|---|
| Model Parameters in Terms of DC Bias Currents | | | |
| $g_m = \dfrac{I_C}{V_T}$ | $r_e = \dfrac{V_T}{I_E} = \alpha \dfrac{V_T}{I_C}$ | $r_\pi = \dfrac{V_T}{I_B} = \beta \dfrac{V_T}{I_C}$ | $r_o = \dfrac{\lvert V_A \rvert}{I_C}$ |
| In Terms of $g_m$ | | | |
| $r_e = \dfrac{\alpha}{g_m}$ | $r_\pi = \dfrac{\beta}{g_m}$ | | |
| In Terms of $r_e$ | | | |
| $g_m = \dfrac{\alpha}{r_e}$ | $r_\pi = (\beta + 1) r_e$ | $g_m + \dfrac{1}{r_\pi} = \dfrac{1}{r_e}$ | |
| Relationships between $\alpha$ and $\beta$ | | | |
| $\beta = \dfrac{\alpha}{1-\alpha}$ | $\alpha = \dfrac{\beta}{\beta+1}$ | $\beta + 1 = \dfrac{1}{1-\alpha}$ | |

# MOSFET

## DC Analysis

**Table 5.1** Regions of Operation of the Enhancement NMOS Transistor

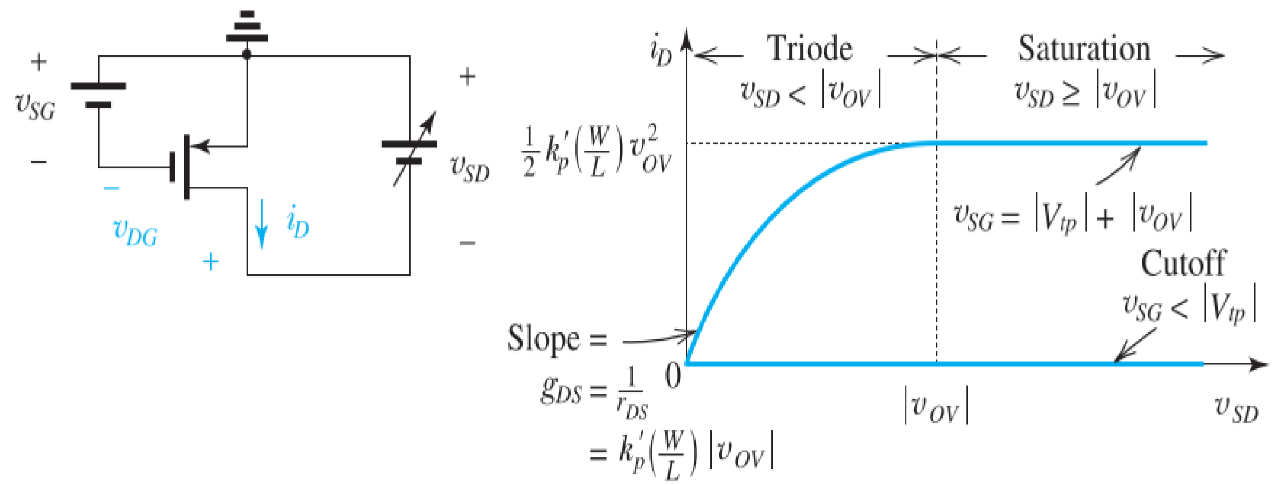



- $v_{GS} < V_{tn}$ : no channel; transistor in cutoff; $i_D = 0$
- $v_{GS} = V_{tn} + v_{OV}$ : a channel is induced; transistor operates in the triode region or the saturation region depending on whether the channel is continuous or pinched off at the drain end;

| *Triode Region* | *Saturation Region* |
|---|---|
| Continuous channel, obtained by: $v_{GD} > V_{tn}$ | Pinched-off channel, obtained by: $v_{GD} \leq V_{tn}$ |
| or equivalently: $v_{DS} < v_{OV}$ | or equivalently: $v_{DS} \geq v_{OV}$ |
| Then, $i_D = k_n'\left(\frac{W}{L}\right)\left[(v_{GS} - V_{tn})v_{DS} - \frac{1}{2}v_{DS}^2\right]$ | Then $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)(v_{GS} - V_{tn})^2$ |
| or equivalently, $i_D = k_n'\left(\frac{W}{L}\right)\left(v_{OV} - \frac{1}{2}v_{DS}\right)v_{DS}$ | or equivalently, $i_D = \frac{1}{2}k_n'\left(\frac{W}{L}\right)v_{OV}^2$ |

**Table 5.2** Regions of Operation of the Enhancement PMOS Transistor




- $v_{SG} < |V_{tp}|$: no channel; transistor in cutoff; $i_D = 0$
- $v_{SG} = |V_{tp}| + |v_{OV}|$ : a channel is induced; transistor operates in the triode region or in the saturation region depending on whether the channel is continuous or pinched off at the drain end;

*Triode Region*

Continuous channel, obtained by:

$$v_{DG} > |V_{tp}|$$

or equivalently

$$v_{SD} < |v_{OV}|$$

Then

$$i_D = k_p'\left(\frac{W}{L}\right)\left[(v_{SG} - |V_{tp}|)v_{SD} - \frac{1}{2}v_{SD}^2\right]$$

or equivalently

$$i_D = k_p'\left(\frac{W}{L}\right)\left(|v_{OV}| - \frac{1}{2}v_{SD}\right)v_{SD}$$

*Saturation Region*

Pinched-off channel, obtained by:

$$v_{DG} \leq |V_{tp}|$$

or equivalently

$$v_{SD} \geq |v_{OV}|$$

Then

$$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)(v_{SG} - |V_{tp}|)^2$$

or equivalently

$$i_D = \frac{1}{2}k_p'\left(\frac{W}{L}\right)v_{OV}^2$$

# Small Signal Analysis

1. DC analysis, cut all capacitors and sig.
2. find $I_D$ and $V_{OV}$
3. cal $g_m$ and draw the ac circuit

4. find $V_o/V_i$
5. find input and output resistance

**Table 7.2** Small-Signal Models of the MOSFET

*Small-Signal Parameters*

**NMOS transistors**

- **Transconductance:**

$$g_m = \mu_n C_{ox} \frac{W}{L} V_{OV} = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{OV}}$$

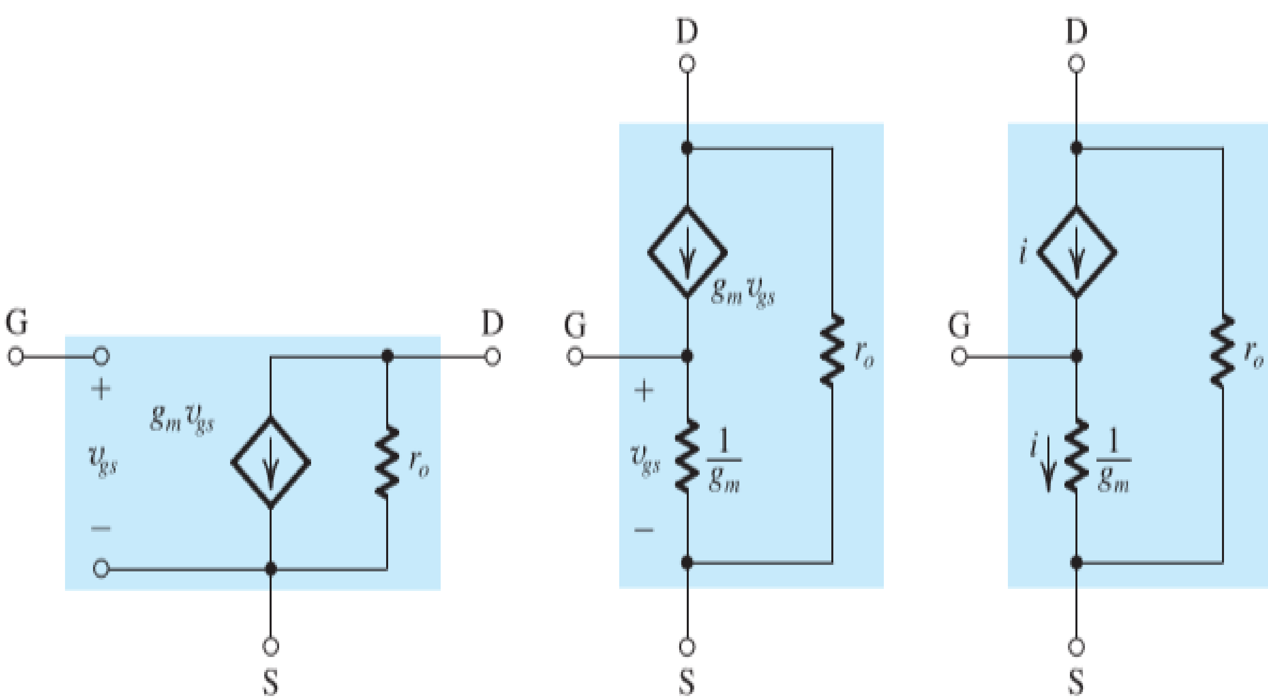
- Output resistance:

$$r_o = V_A / I_D = 1/\lambda I_D$$

**PMOS transistors**

Same formulas as for NMOS *except* using $|V_{OV}|$, $|V_A|$, $|\lambda|$ and replacing $\mu_n$ with $\mu_p$.

***Small-Signal, Equivalent-Circuit Models***

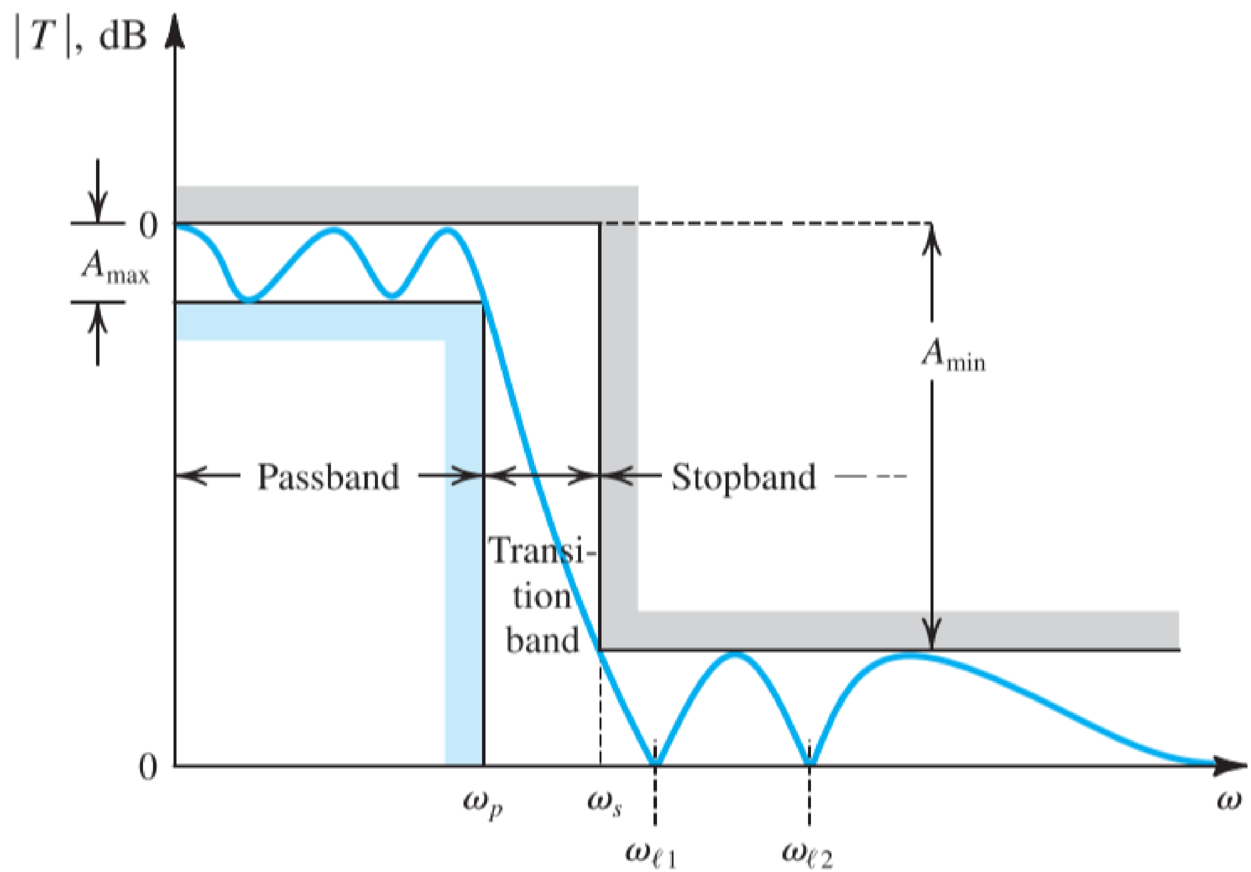



Hybrid-$\pi$ model

T models

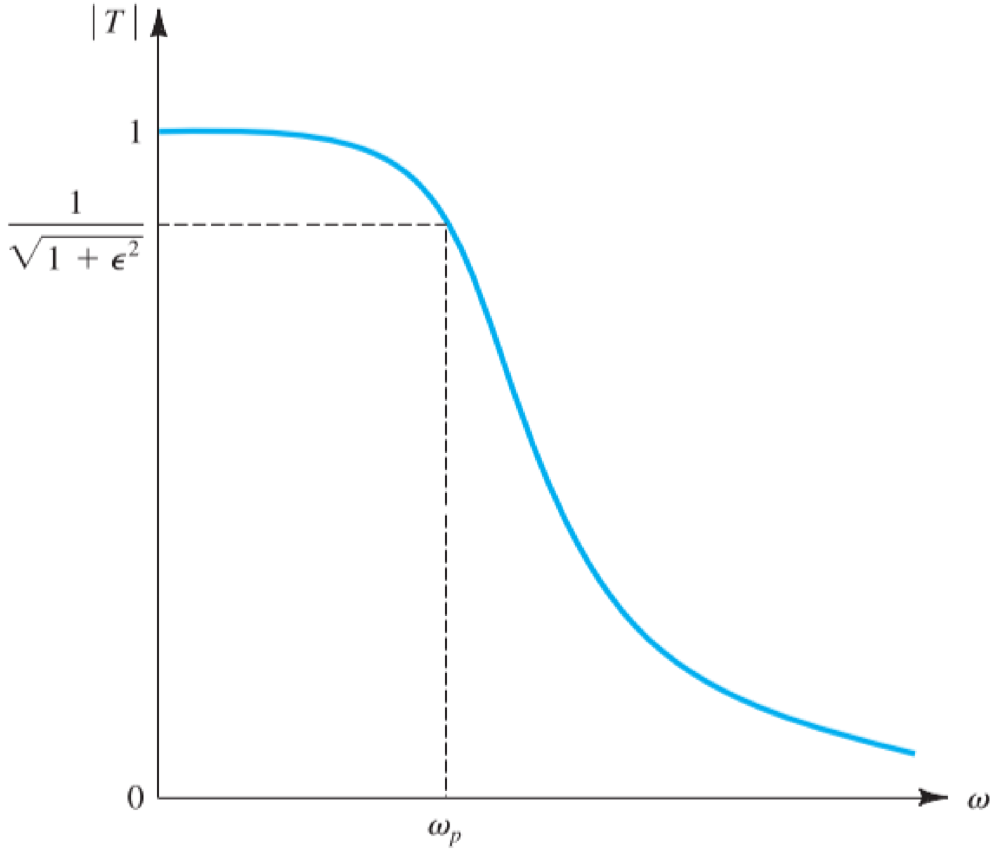
# Filter

## Filter Specification




- $\omega_p$: the passband edge
- $A_{max}$: the maximum allowed variation in passband transmission
- $\omega_s$: the stopband edge
- $A_{min}$: the minimum required stopband attenuation

## Butterworth Filter




$$\|T(j\omega)\| = \frac{1}{\sqrt{1+\epsilon^2\left(\frac{\omega}{\omega_p}\right)^2}}$$

At the frequency of $\omega_p$

$$\|T(j\omega)\| = \frac{1}{\sqrt{1+\epsilon^2}}$$

Then the parameter $\epsilon$ determines $A_{max}$

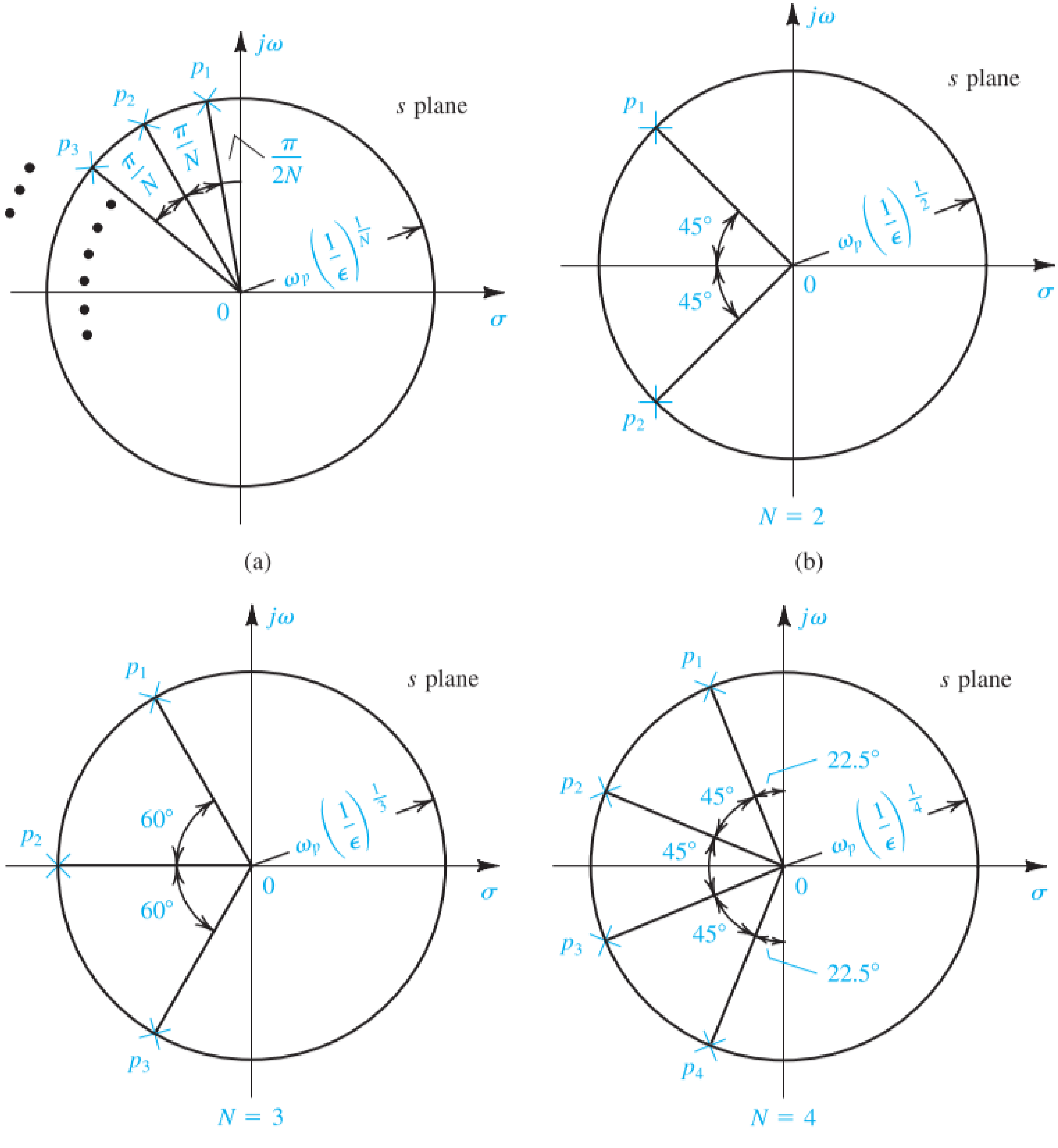
$$A_{max} = 10\log\left(1+\epsilon^2\right)$$

$$\epsilon = \sqrt{10^{A_{max}/10} - 1}$$

At the edge of stopband $\omega = \omega_s$, check the attenuation of the filter

$$A(\omega_s) = 10\log\left[1+\epsilon^2(\omega_s/\omega_p)^{2N}\right]$$

Find the lowest filter order $N$ such that $A(\omega_s) > A_{min}$



(a) (b) (c) (d)

Then the transfer function could be written as

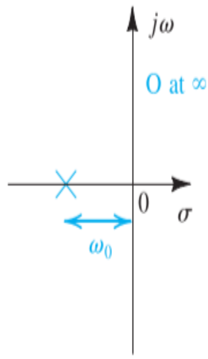
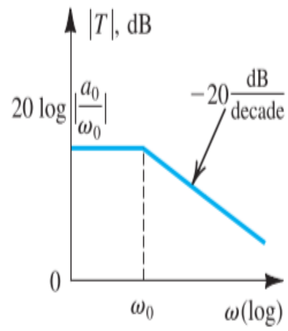
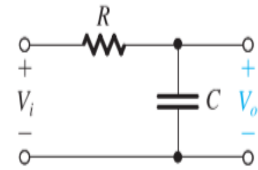
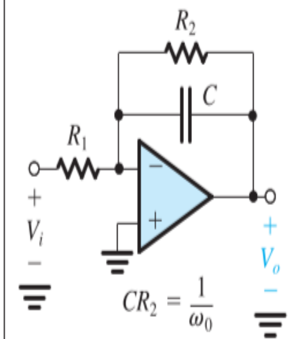
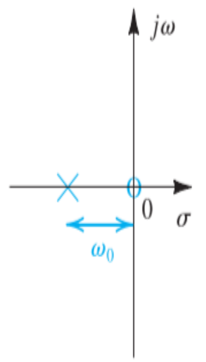
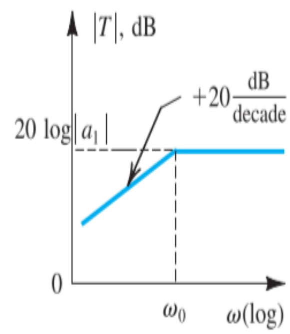
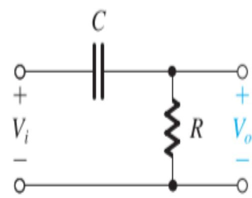
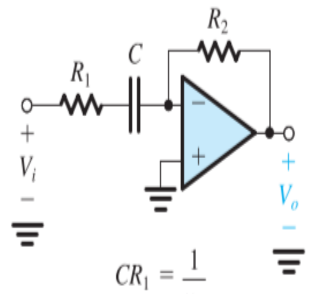
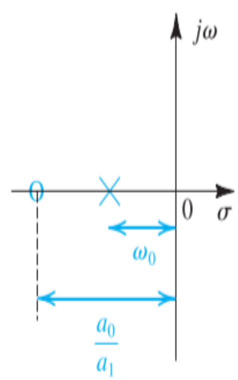
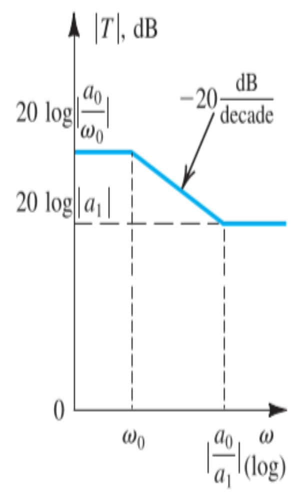
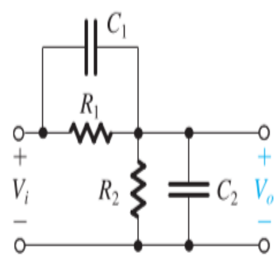
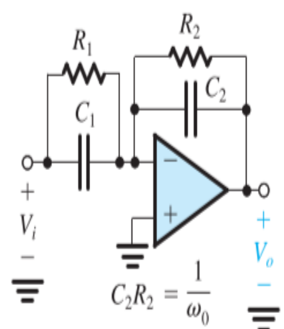
$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\cdots(s-p_N)}$$

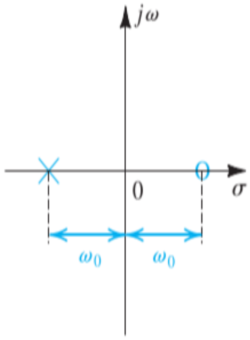
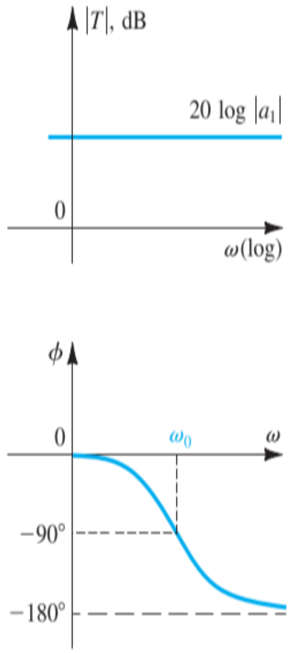
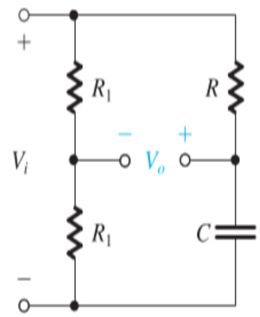
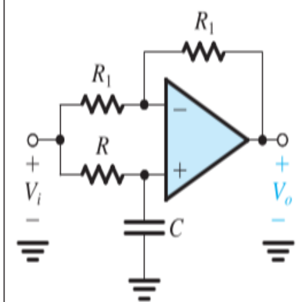
- $\omega_0 = \omega_p(1/\epsilon)^{1/N}$

# First Order Filters

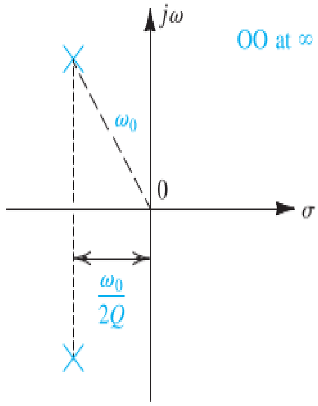
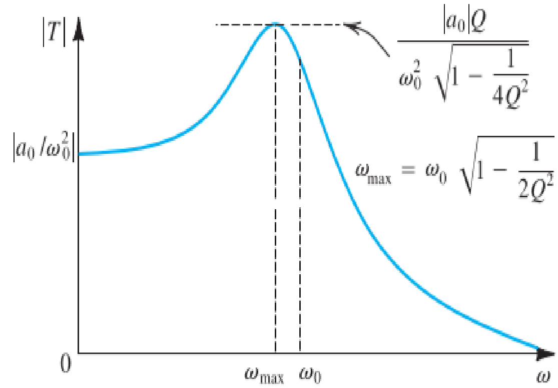
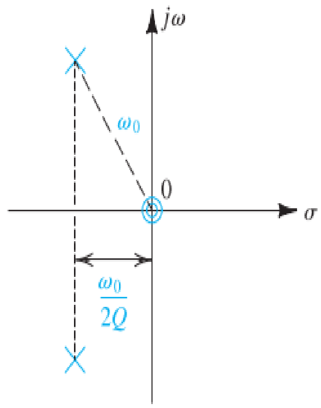
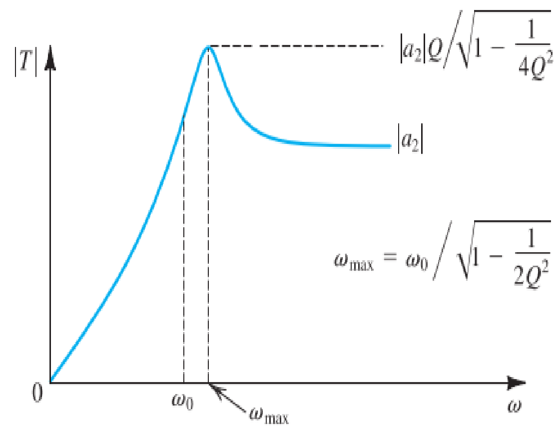
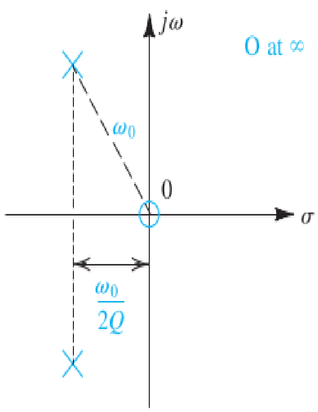
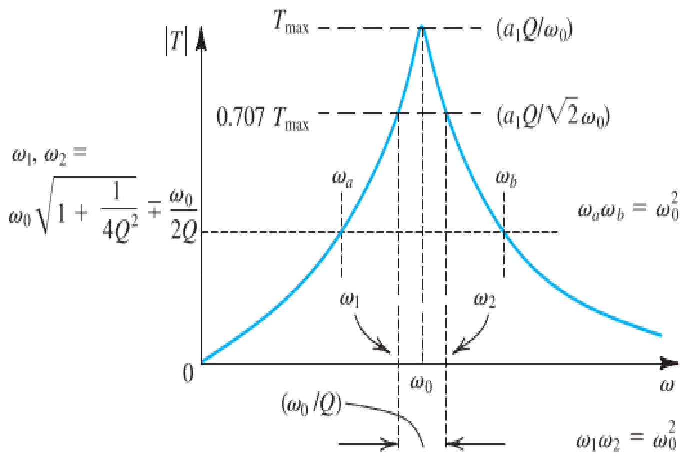
The general first-order transfer function is given by

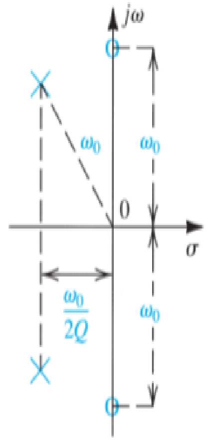
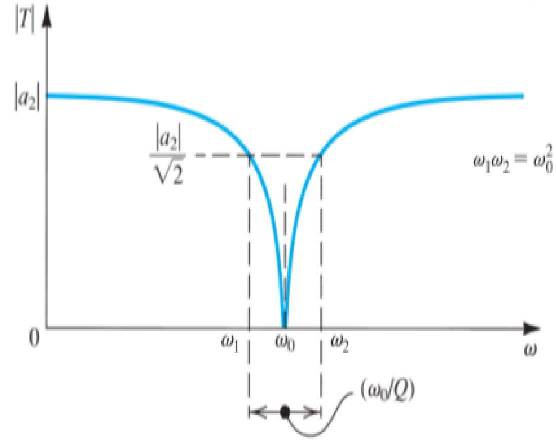
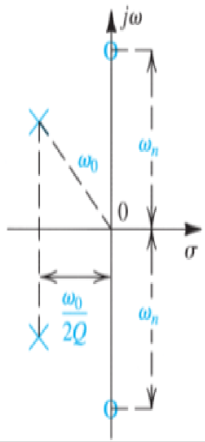
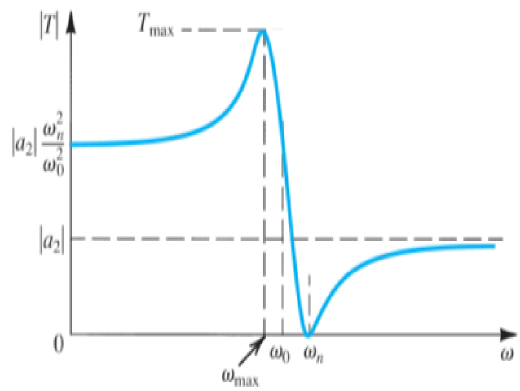
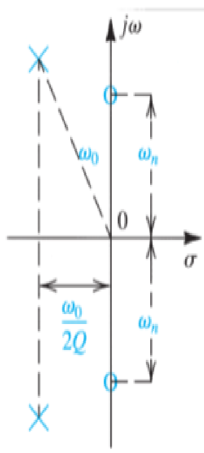
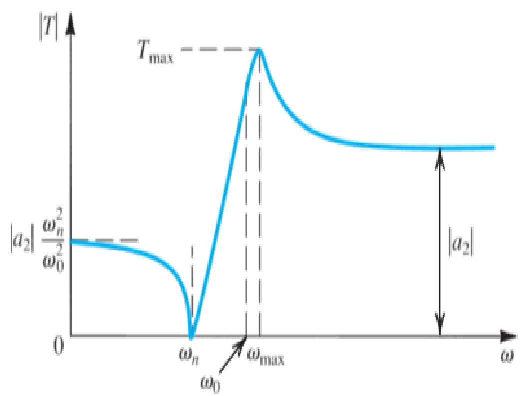
$$T(s) = \frac{a_1 s + a_0}{s + \omega_0}$$

| Filter Type and $T(s)$ | $s$-Plane Singularities | Bode Plot for $\lvert T \rvert$ | Passive Realization | Op Amp–RC Realization |
|---|---|---|---|---|
| (a) Low pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$ |   |   |   $CR = \frac{1}{\omega_0}$ DC gain = 1 |   $CR_2 = \frac{1}{\omega_0}$ DC gain $= -\frac{R_2}{R_1}$ |
| (b) High pass (HP) $T(s) = \frac{a_1 s}{s + \omega_0}$ |   |   |   $CR = \frac{1}{\omega_0}$ High-frequency gain = 1 |   $CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$ |
| (c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$ |   |   |   $(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$ |   $C_2 R_2 = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$ |

| $T(s)$ | Singularities | $\lvert T \rvert$ and $\phi$ | Passive Realization | Op Amp–RC Realization |
|---|---|---|---|---|
| All pass (AP)<br>$T(s) = -a_1 \frac{s-\omega_0}{s+\omega_0}$<br>$a_1 > 0$ |  |  | <br>$CR = 1/\omega_0$<br>Flat gain $(a_1) = 0.5$ | <br>$CR = 1/\omega_0$<br>Flat gain $(a_1) = 1$<br>$\left\lvert \frac{V_o}{V_i} \right\rvert = 1$<br>$\phi(\omega) = -2 \tan^{-1}(\omega CR)$ |

# Second Order Transfer Functions

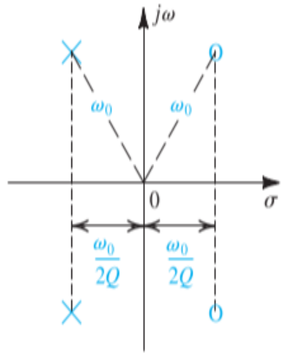
<table>
<tr><th>Filter Type and $T(s)$</th><th>$s$-Plane Singularities</th><th>$|T|$</th></tr>
<tr><td>(a) Low pass (LP)<br>$T(s) = \dfrac{a_0}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$<br>DC gain $= \dfrac{a_0}{\omega_0^2}$</td><td></td><td></td></tr>
<tr><td>(b) High pass (HP)<br>$T(s) = \dfrac{a_2 s^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$<br>High-frequency gain $= a_2$</td><td></td><td></td></tr>
<tr><td>(c) Bandpass (BP)<br>$T(s) = \dfrac{a_1 s}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$<br>Center-frequency gain $= \dfrac{a_1 Q}{\omega_0}$</td><td></td><td></td></tr>
</table>

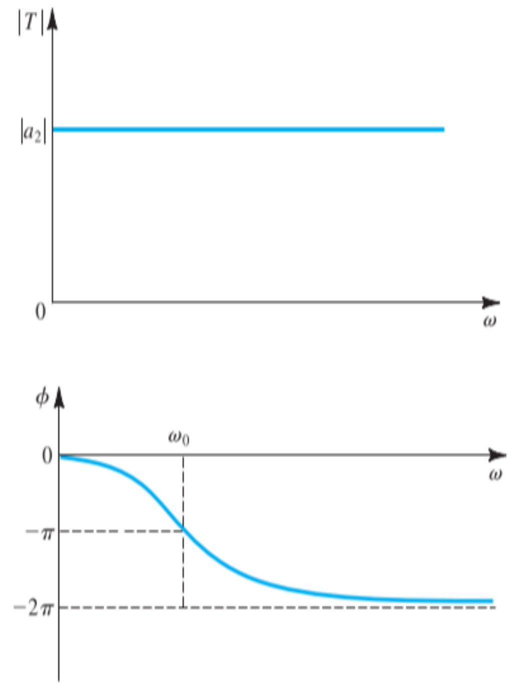
| Filter Type and $T(s)$ | $s$-Plane Singularities | $\lvert T \rvert$ |
|---|---|---|
| (d) Notch<br>$T(s) = a_2 \dfrac{s^2 + \omega_0^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$<br>DC gain =<br>High-frequency gain $= a_2$ |  |  |
| (e) Low-pass notch (LPN)<br>$T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$<br>$\omega_n \geq \omega_0$<br>DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$<br>High-frequency gain $= a_2$ |  | <br>$\omega_{\max} = \omega_0 \sqrt{\dfrac{\left(\dfrac{\omega_n^2}{\omega_0^2}\right)\left(1 - \dfrac{1}{2Q^2}\right) - 1}{\dfrac{\omega_n^2}{\omega_0^2} + \dfrac{1}{2Q^2} - 1}}$<br>$T_{\max} = \dfrac{\lvert a_2 \rvert \quad \lvert \omega_n^2 - \omega_{\max}^2 \rvert}{\sqrt{(\omega_0^2 - \omega_{\max}^2)^2 + \left(\dfrac{\omega_0}{Q}\right)^2 \omega_{\max}^2}}$ |
| (f) High-pass notch (HPN)<br>$T(s) = a_2 \dfrac{s^2 + \omega_n^2}{s^2 + s\dfrac{\omega_0}{Q} + \omega_0^2}$<br>$\omega_n \leq \omega_0$<br>DC gain $= a_2 \dfrac{\omega_n^2}{\omega_0^2}$<br>High-frequency gain $= a_2$ |  |  |

(g) All pass (AP)

$$T(s) = a_2 \frac{s^2 - s\frac{\omega_0}{Q} + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

Flat gain $= a_2$



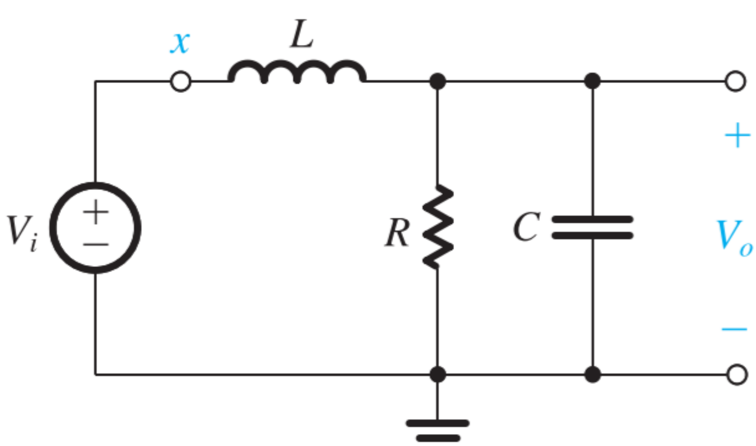




## Second-Order LCR Resonator

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 CR$$

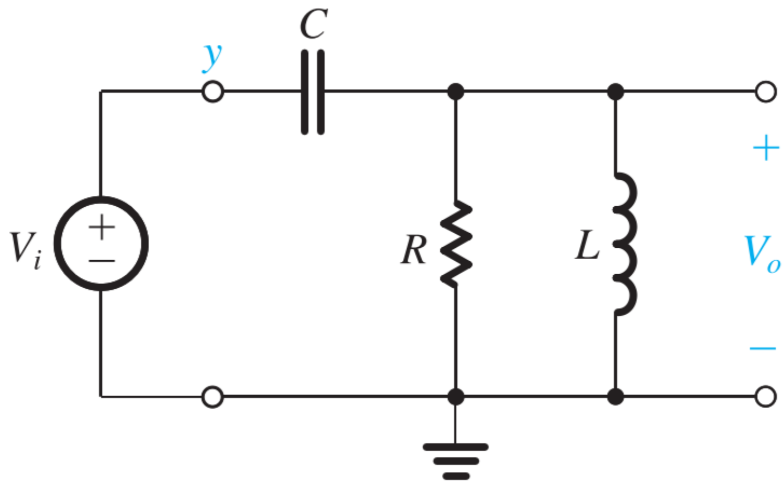
### Low-Pass Function




$$T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order low-pass filters have **2 zeros** at $\infty$

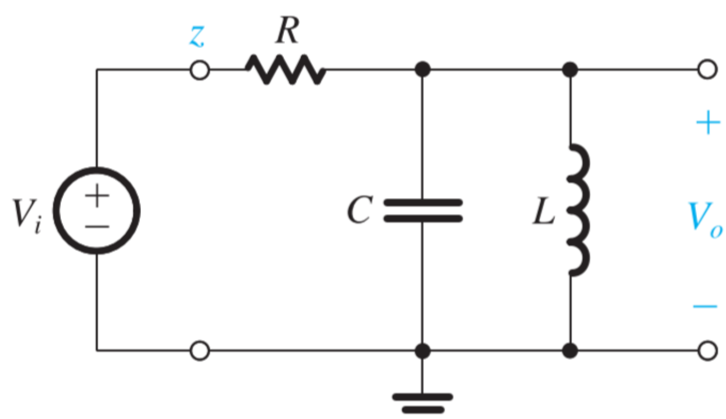
## High-Pass Function




$$T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{s^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order high-pass filters have **2 zeros** at $s = 0$

## Band-Pass Function

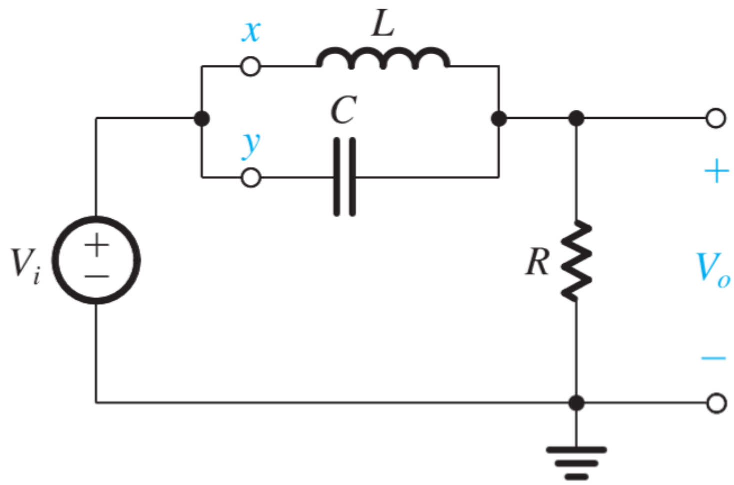



(d) BP

$$T(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{(\omega_0/Q)s}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

> 2nd-order band-pass filters have **1 zero** at $\infty$ and **1 zero** at $s = 0$

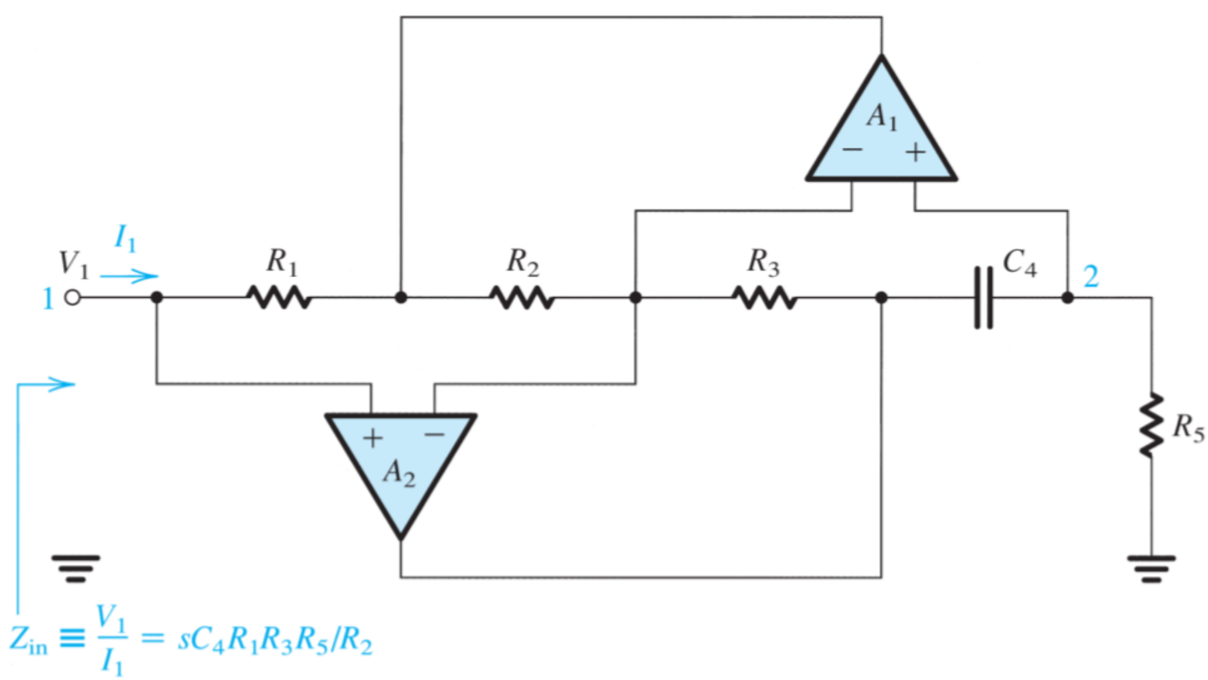
## Notch Function




$$T(s) = \frac{Z_2}{Z_1 + Z_2} = a_2 \frac{s^2 + \omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

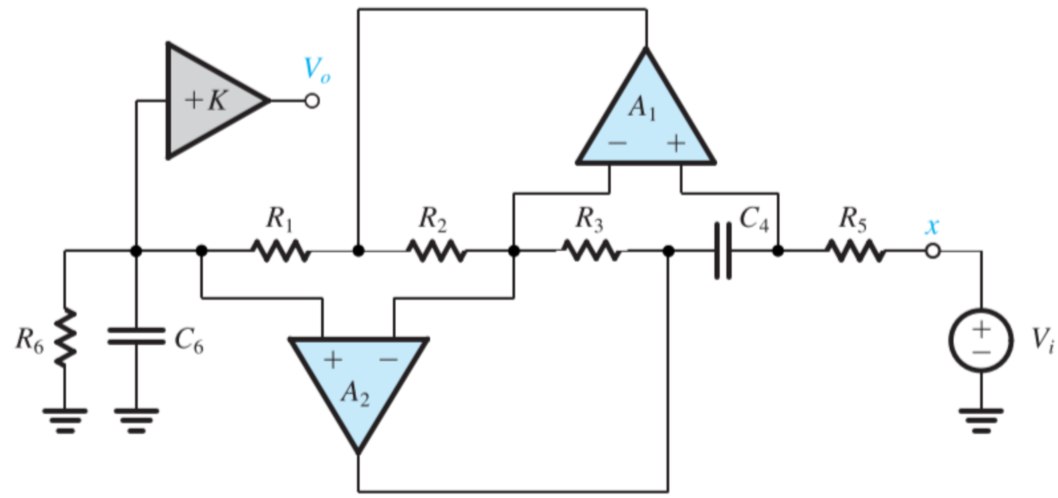
> 2nd-order notch(bandstop) filters have **2 zeros** at $j\omega$ axis

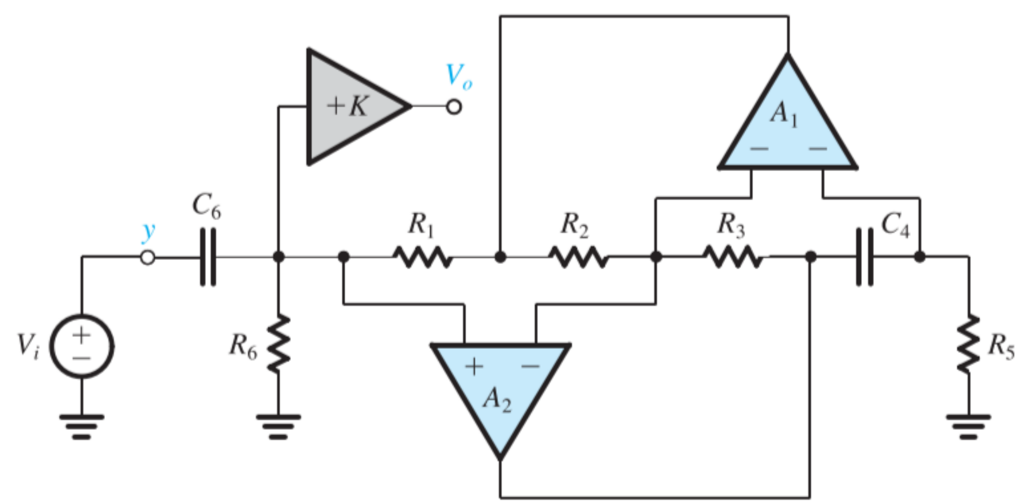
## Op Amp-RC Resonator

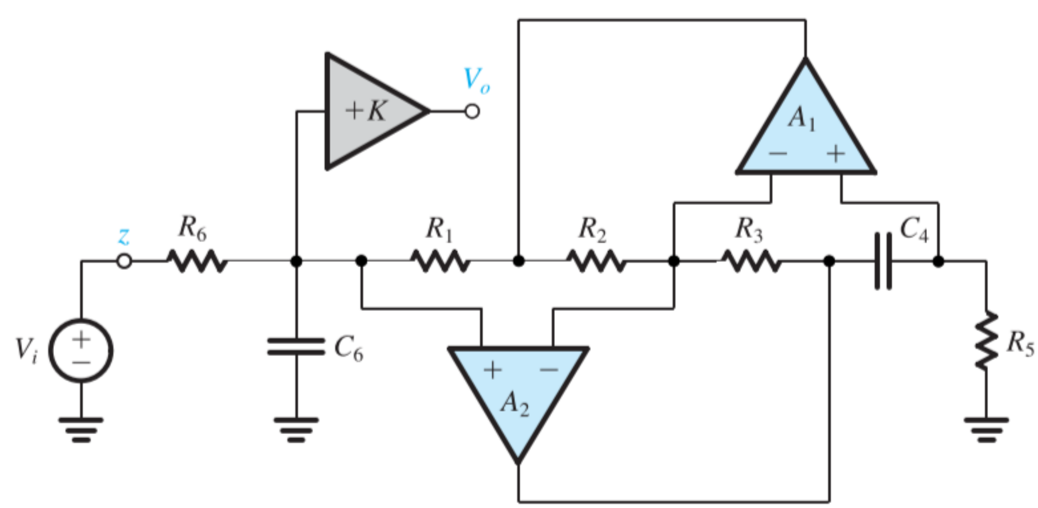


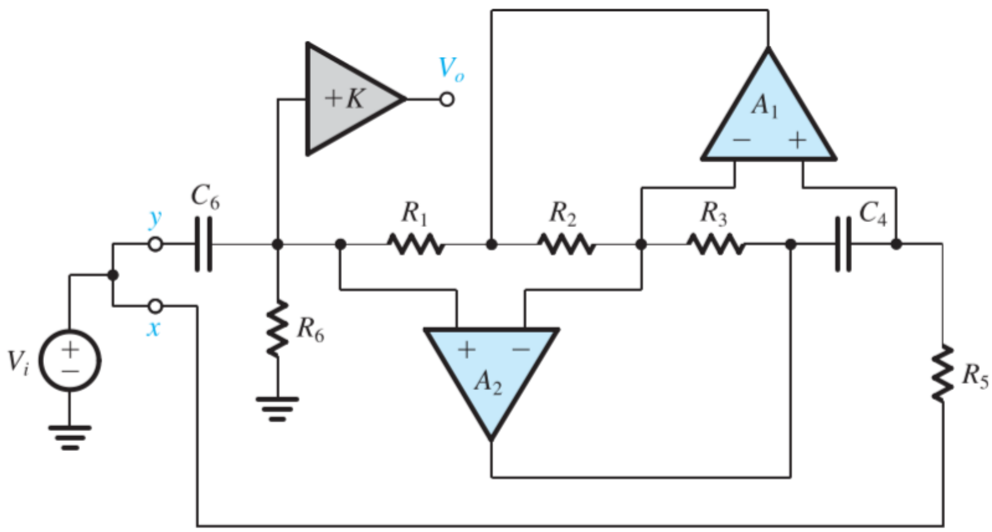

$$L = \frac{C_4 R_1 R_3 R_5}{R_2}$$

The design of the circuit is usually selecting $R_1 = R_2 = R_3 = R_5 = R$ and $C_4 = C$, which leads to $L = CR^2$


+K
$V_o$
$A_1$
$R_1$
$R_2$
$R_3$
$C_4$
$R_5$
$x$
$R_6$
$C_6$
$A_2$
$V_i$
(a) LP


+K
$V_o$
$A_1$
$C_6$
$y$
$R_1$
$R_2$
$R_3$
$C_4$
$V_i$
$R_6$
$A_2$
$R_5$
(b) HP

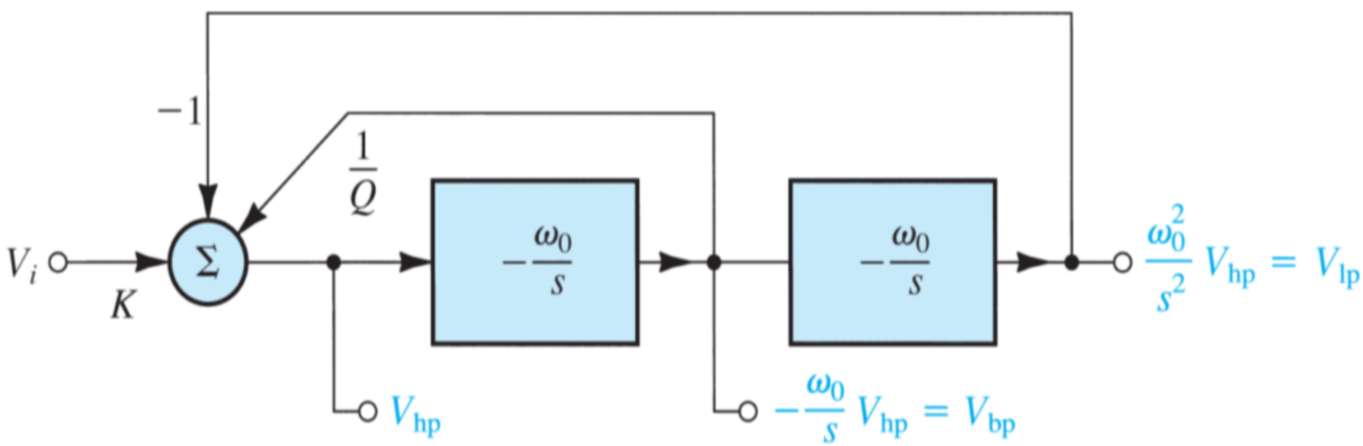

+K
$V_o$
$A_1$
$z$
$R_6$
$R_1$
$R_2$
$R_3$
$C_4$
$V_i$
$C_6$
$A_2$
$R_5$
(c) BP

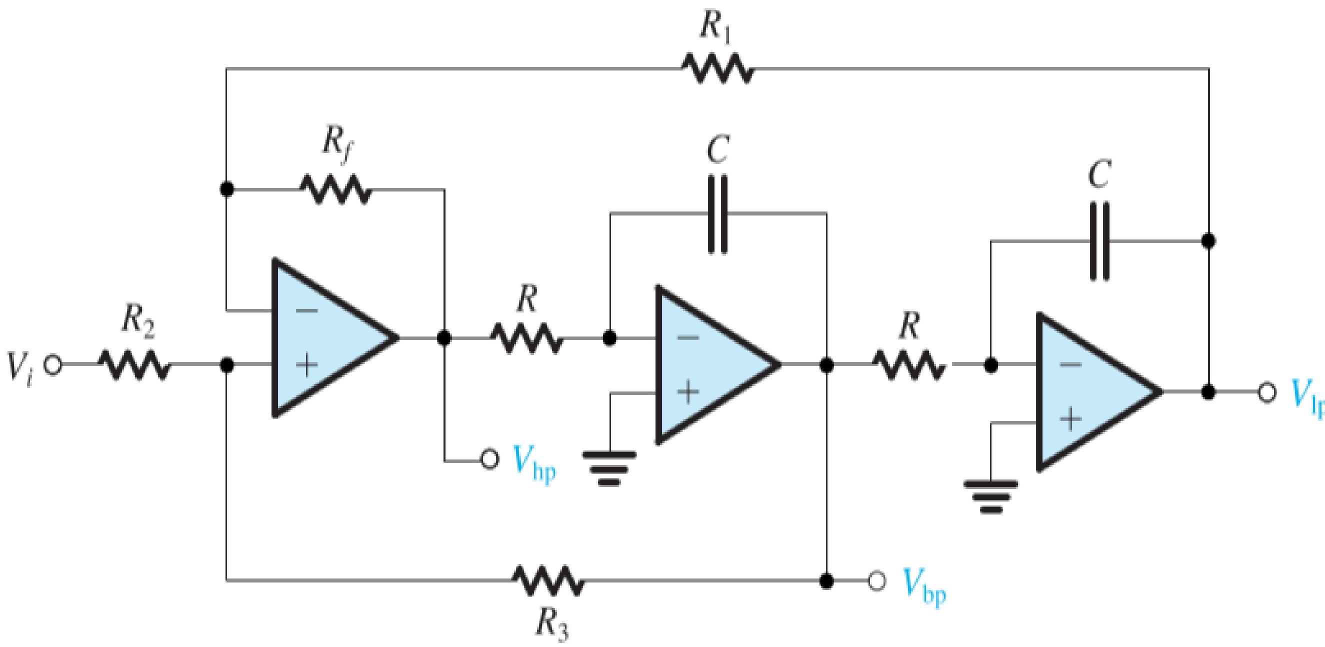



(d) Notch at $\omega_0$

# KHN Biquad

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

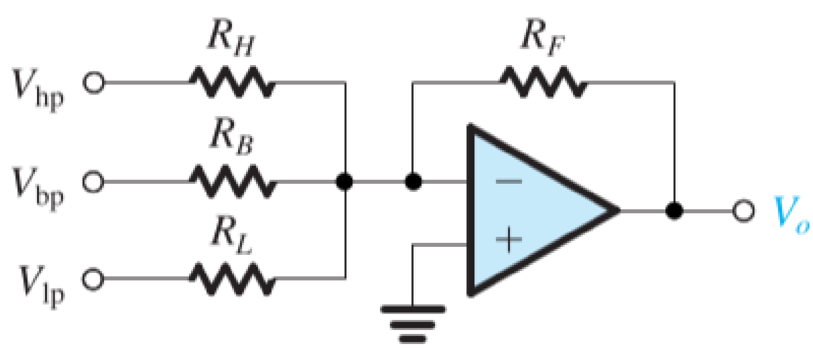





$$V_{hp} = V_i \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) + V_{bp}\frac{R_2}{R_2 + R3}\left(1 + \frac{R_f}{R_1}\right) - V_{lp}\frac{R_f}{R_1}$$

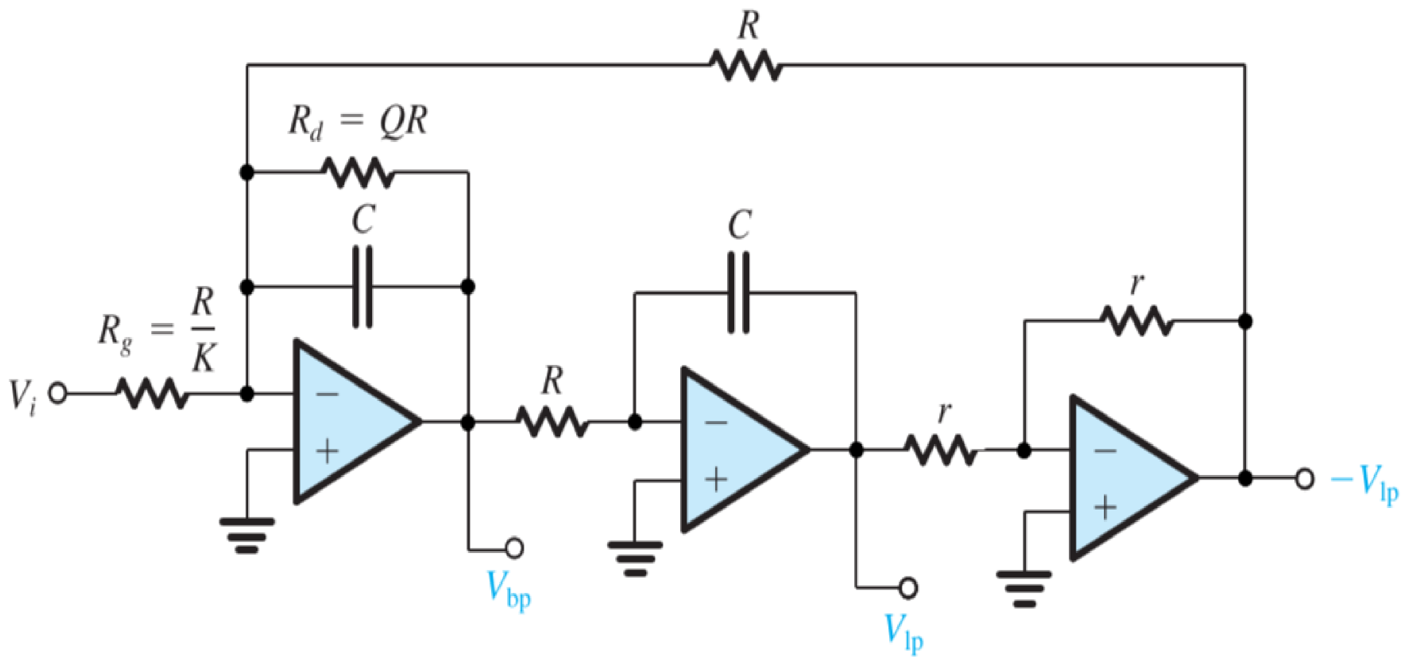
$$= V_i \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) + \left(-\frac{\omega_0}{s}V_{hp}\right)\frac{R_2}{R_2 + R3}\left(1 + \frac{R_f}{R_1}\right) - \left(\frac{\omega_0^2}{s^2}V_{hp}\right)\frac{R_f}{R_1}$$

- $CR = 1/\omega_0$
- $R_f = R_1$
- $R_3/R_2 = 2Q - 1$
- $K = 2 - (1/Q)$



$$
\begin{aligned}
V_o &= -(\frac{R_F}{R_H}V_{hp} + \frac{R_F}{R_B}V_{bp} + \frac{R_F}{R_L}V_{lp}) \\
&= -V_i(\frac{R_F}{R_H}T_{hp} + \frac{R_F}{R_B}T_{bp} + \frac{R_F}{R_L}T_{lp}) \\
\frac{V_o}{V_i} &= -K\frac{(R_F/R_H)s^2 - s(R_F/R_B)\omega_0 + (R_F/R_L)\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2}
\end{aligned}
$$

## Tow–Thomas Biquad




$$
\frac{V_o}{V_i} = -\frac{s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{1}{QCR} + \frac{1}{C^2R^2}}
$$

- $r = \text{arbitrary}$
- $C = \text{arbitrary}$
- $R = 1/\omega_0$

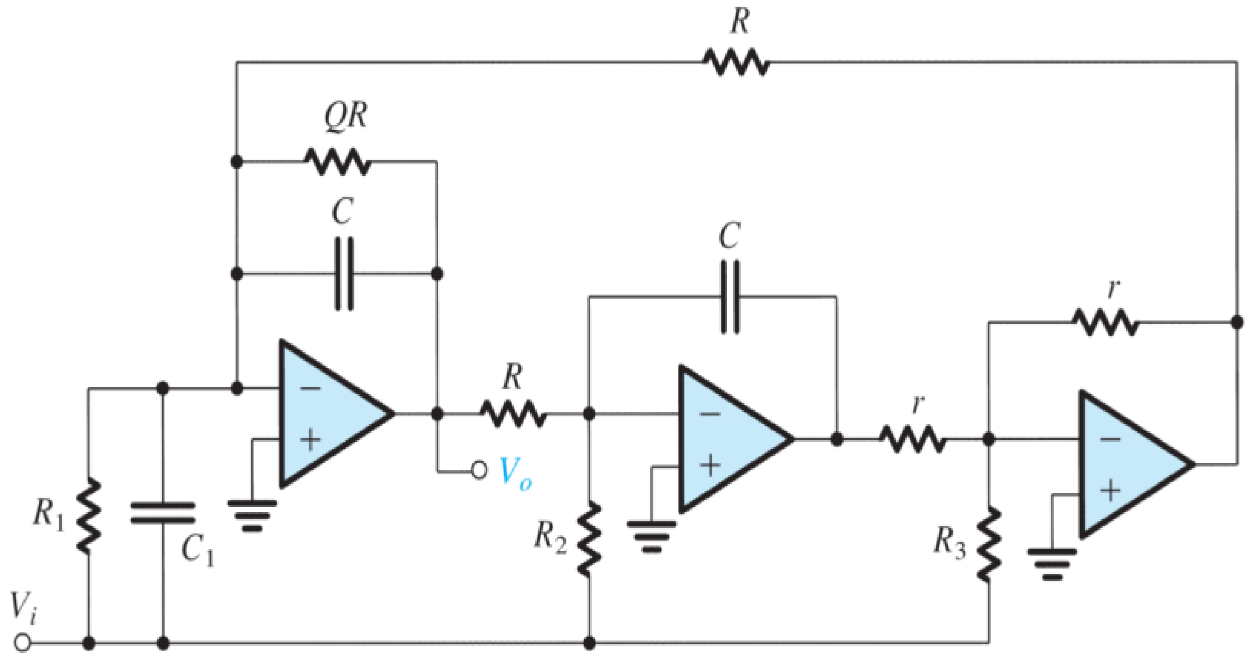



**Figure 17.26** The Tow–Thomas biquad with feedforward. The transfer function of Eq. (17.68) is realized by feeding the input signal through appropriate components to the inputs of the three op amps. This circuit can realize all special second-order functions. The design equations are given in Table 17.2.

**Table 17.2** Design Data for the Circuit in Fig. 17.26

| | |
|---|---|
| All cases | $C =$ arbitrary, $R = 1/\omega_0 C$, $r =$ arbitrary |
| LP | $C_1 = 0$, $R_1 = \infty$, $R_2 = R/\text{dc gain}$, $R_3 = \infty$ |
| Positive BP | $C_1 = 0$, $R_1 = \infty$, $R_2 = \infty$, $R_3 = Qr/\text{center-frequency gain}$ |
| Negative BP | $C_1 = 0$, $R_1 = QR/\text{center-frequency gain}$, $R_2 = \infty$, $R_3 = \infty$ |
| HP | $C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = \infty$, $R_3 = \infty$ |
| Notch (all types) | $C_1 = C \times$ high-frequency gain, $R_1 = \infty$, $R_2 = R(\omega_0/\omega_n)^2/\text{high-frequency gain}$, $R_3 = \infty$ |
| AP | $C_1 = C \times$ flat gain, $R_1 = \infty$, $R_2 = R/\text{gain}$, $R_3 = Qr/\text{gain}$ |