Chapter 7

Introduction

We need to know the accelerations to calculate the dynamic forces from $F = ma$

The manual graphical method and the analytical solution for accelerations

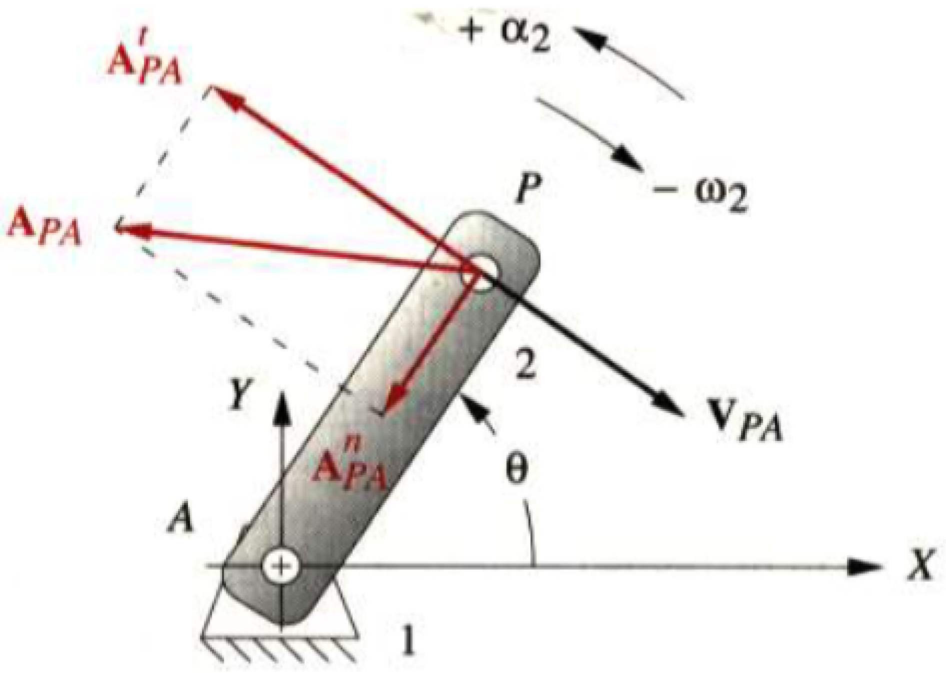
7.1 Definition of Acceleration

Definition

the rate of change of velocity with respect to time

$$\alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \qquad \boldsymbol{A} = \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t}$$

take the basic model for consideration



$$\boldsymbol{R}_{PA} = pe^{j\theta}$$

$$\boldsymbol{V}_{PA} = \omega jpe^{j\theta}$$

$$\boldsymbol{A}_{PA} = jp(\alpha e^{j\theta} + j\omega^2 e^{j\theta}) = -\omega^2 pe^{j\theta} + j\alpha pe^{j\theta}$$

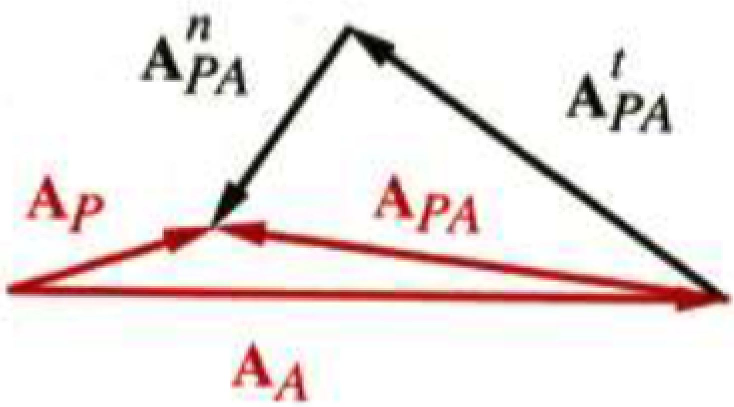
$$\boldsymbol{A}_{PA} = \boldsymbol{A}^n_{PA} + \boldsymbol{A}^t_{PA}$$

Acceleration Difference

Two points in the same body

$$\boldsymbol{A}_P = \boldsymbol{A}_A + \boldsymbol{A}_{PA}$$

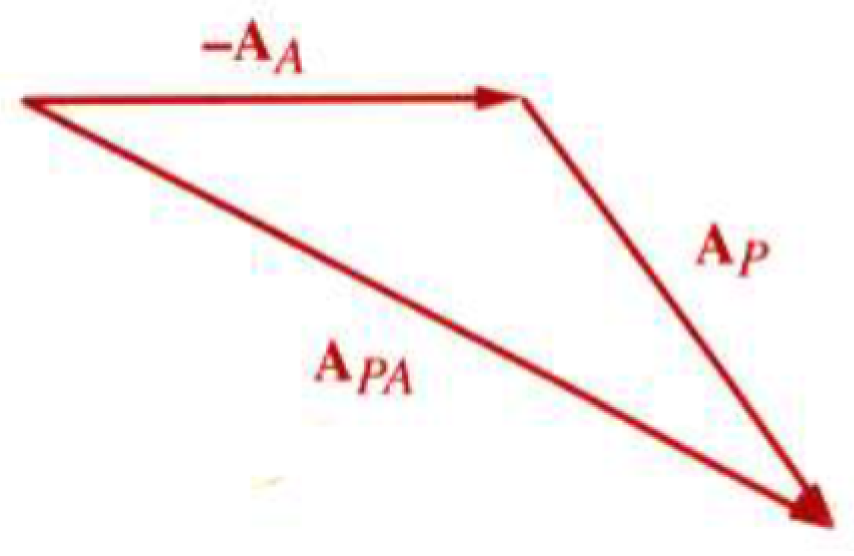
$$(\boldsymbol{A}^t_P + \boldsymbol{A}^n_P) = (\boldsymbol{A}^t_A + \boldsymbol{A}^n_A) + (\boldsymbol{A}^t_{PA} + \boldsymbol{A}^n_{PA})$$



Relative Acceleration

Two points in different bodies

$$\boldsymbol{A}_{PA} = \boldsymbol{A}_P - \boldsymbol{A}_A$$



7.2 Graphical Acceleration Analysis

To solve any acceleration analysis problem graphically, we need only three equations

$$(\boldsymbol{A}_P^t + \boldsymbol{A}_P^n) = (\boldsymbol{A}_A^t + \boldsymbol{A}_A^n) + (\boldsymbol{A}_{PA}^t + \boldsymbol{A}_{PA}^n)$$
$$|\boldsymbol{A}^t| = A^t = r\alpha$$
$$|\boldsymbol{A}^n| = A^n = r\omega^2$$

Coriolis Acceleration

for the position vector $\boldsymbol{R}_P$

$$\boldsymbol{R}_P = pe^{j\theta_2}$$

we can simply get the velocity expression

$$\boldsymbol{V}_P = p\omega_2 e^{j\theta_2} + \dot{p}e^{j\theta_2}$$

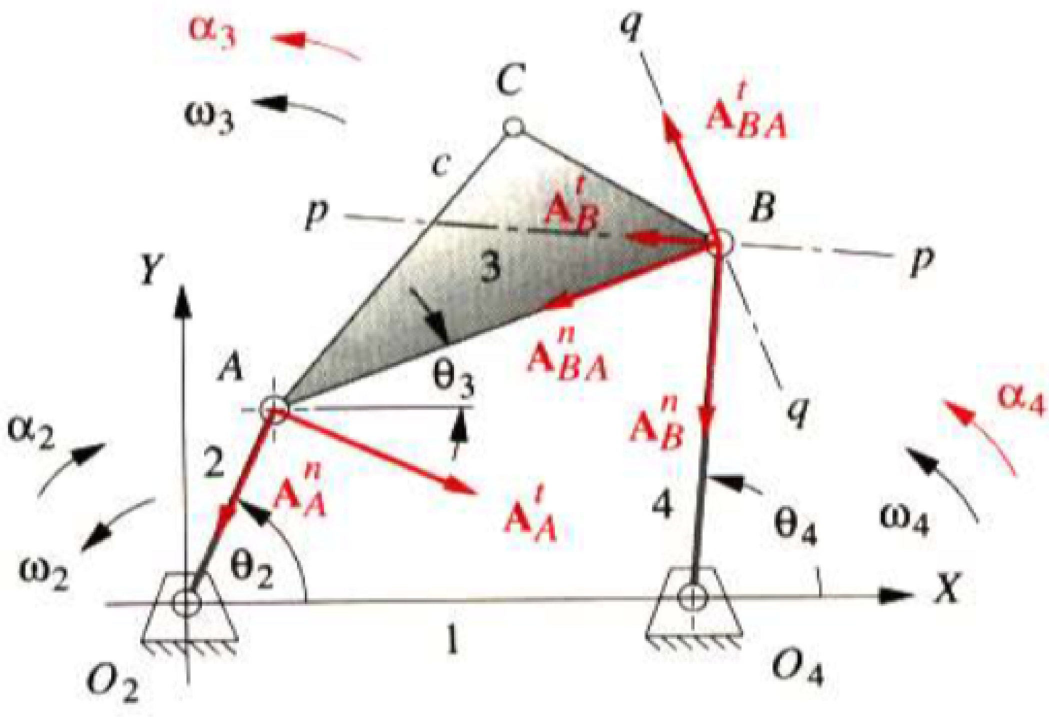
take a further step

$$\boldsymbol{A}_P = p\alpha_2 je^{j\theta_2} - p\omega_2^2 e^{j\theta_2} + 2\dot{p}\omega_2 je^{j\theta_2} + \ddot{p}e^{j\theta_2}$$

Example 1

Graphical Acceleration Analysis for One Position of a Fourbar Linkage

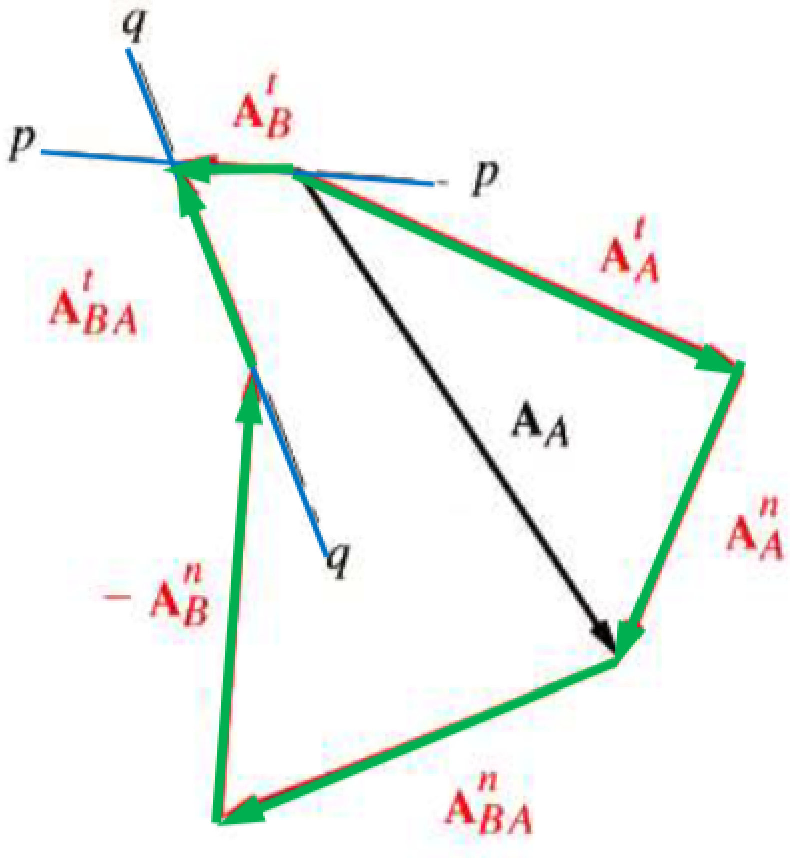
Given θ_2, θ_3, θ_4, ω_2, ω_3, ω_4, α_2, find α_3, α_4, $\boldsymbol{A}_A$, $\boldsymbol{A}_B$ and $\boldsymbol{A}_C$ by graphically method



1. 写出矢量关系式: $(\boldsymbol{A}_B^t + \boldsymbol{A}_B^n) = (\boldsymbol{A}_A^t + \boldsymbol{A}_A^n) + (\boldsymbol{A}_{AB}^t + \boldsymbol{A}_{AB}^n)$
2. 根据已知量完成 Magnitude-Direction 表， 确定已知量和未知量

Equation	$\boldsymbol{A}_B^n$	$\boldsymbol{A}_B^t$	$\boldsymbol{A}_A^n$	$\boldsymbol{A}_A^t$	$\boldsymbol{A}_{AB}^n$	$\boldsymbol{A}_{AB}^t$
Magnitude	1	0	1	1	1	0
Direction	1	1	1	1	1	1

3. 根据已知量和上表制图

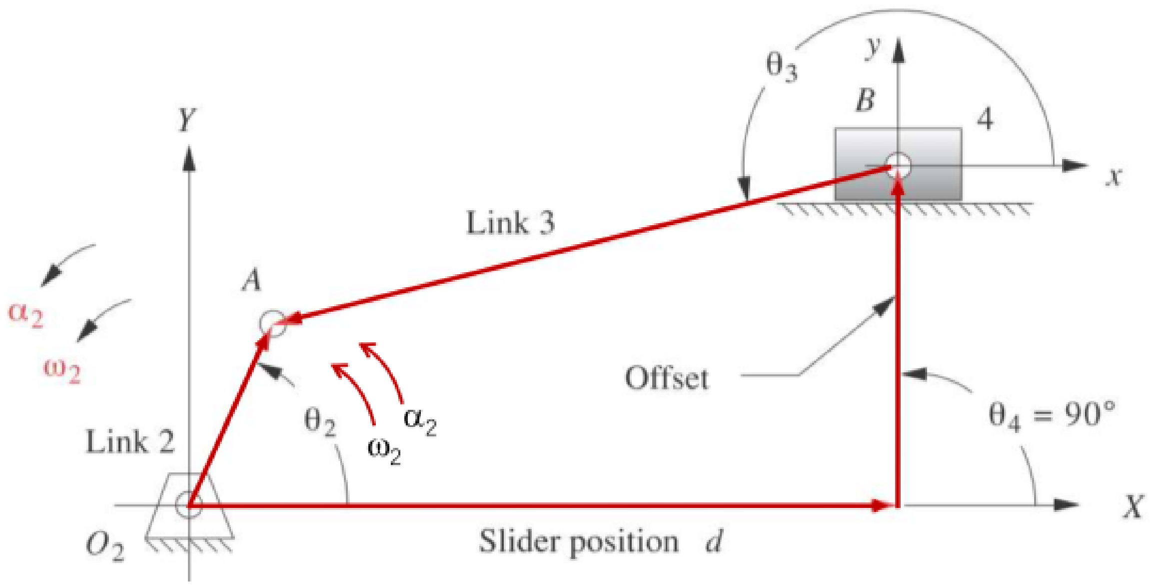


4. 利用图中长度的比例来求出未知量

Example 2

Graphically Acceleration Analysis for Fourbar Crank-Slider

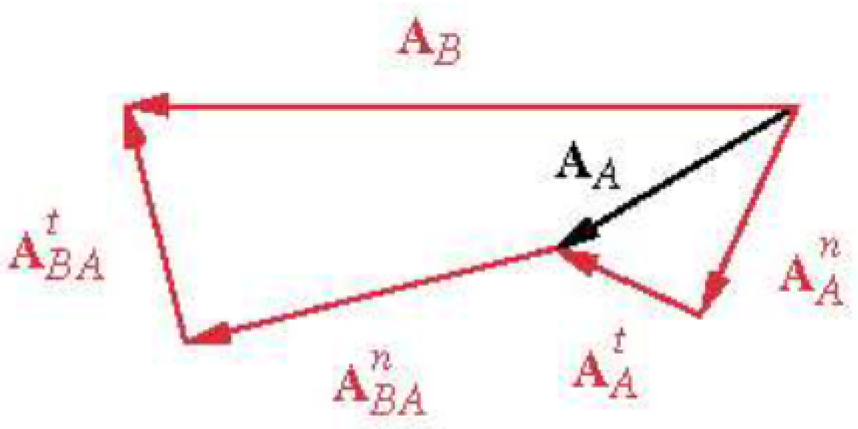
Given the value of θ_2, θ_3, θ_4, ω_2 **and** α_2



1. 列出加速度的矢量关系式: $\boldsymbol{A}_B = (\boldsymbol{A}_A^t + \boldsymbol{A}_A^n) + (\boldsymbol{A}_{AB}^t + \boldsymbol{A}_{AB}^n)$
2. 根据已知量完成 Magnitude-Direction 表

Equation	A_B	A^n_A	A^t_A	A^n_{AB}	A^t_{AB}
Magnitude	0	1	1	1	0
Magnitude	1	1	1	1	1

3. 根据上表制图

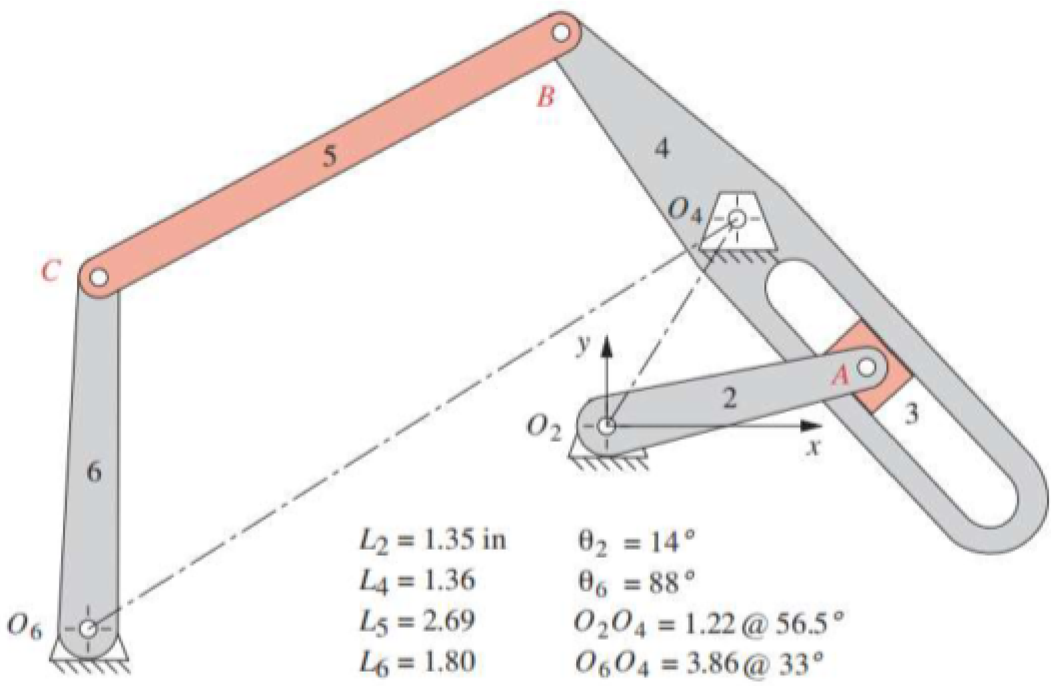


4. 根据比例尺量取图中长度

Example 3

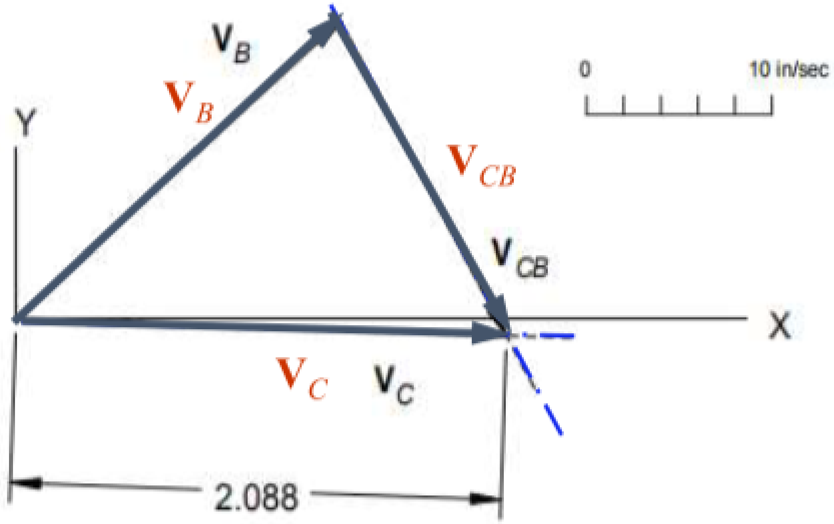
Graphically Acceleration Analysis for Slider

Given ω_2, θ_2, use a graphical method to calculate the accelerations of points A,B and C for the position shown



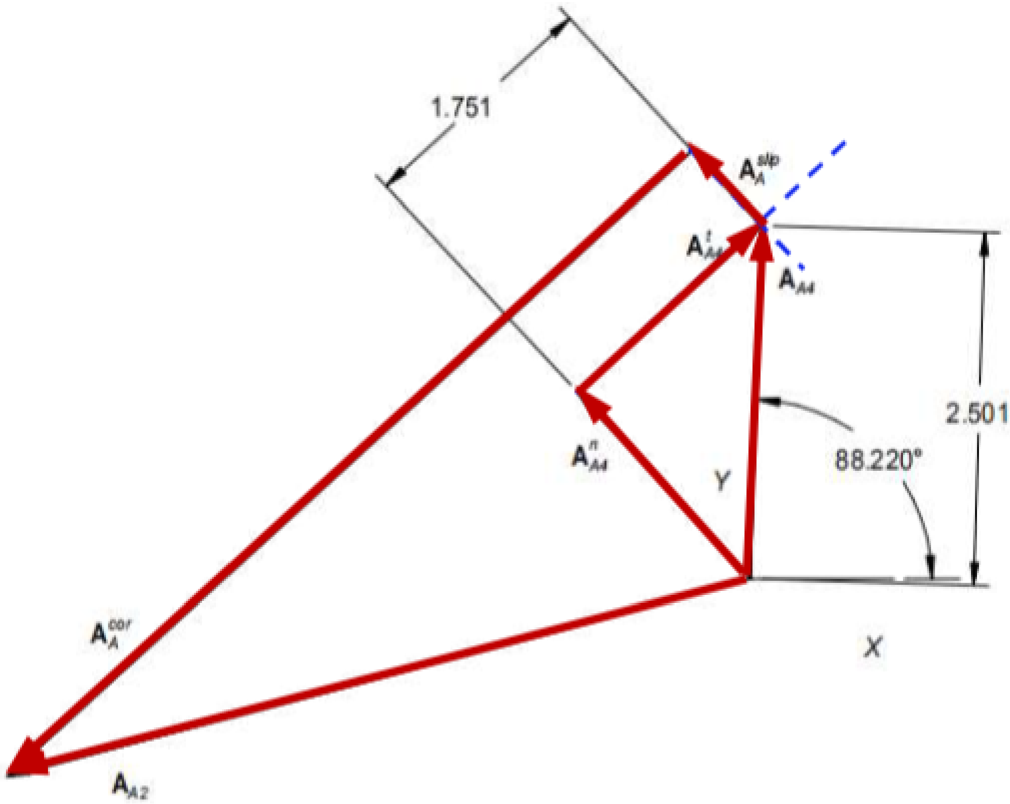
1. 沿杆 4 分解 A 点的速度，其垂直杆件 4 分量除以$|O_2|A$便为杆件 4 的角速度
2. 接下来则是求四杆机构 4、5、6、地面的各杆件速度
3. 列出矢量关系式$\vec{V_C} = \vec{V_B} + \vec{V_{BC}}$，列 Magnitude-Direction 表格，画图求得 C 点速度,顺带求出杆件 5、6 的角速度

Equation	$\vec{V_C}$	$\vec{V_B}$	$\vec{V_{BC}}$
Magnitude	0	1	0
Direction	1	1	1



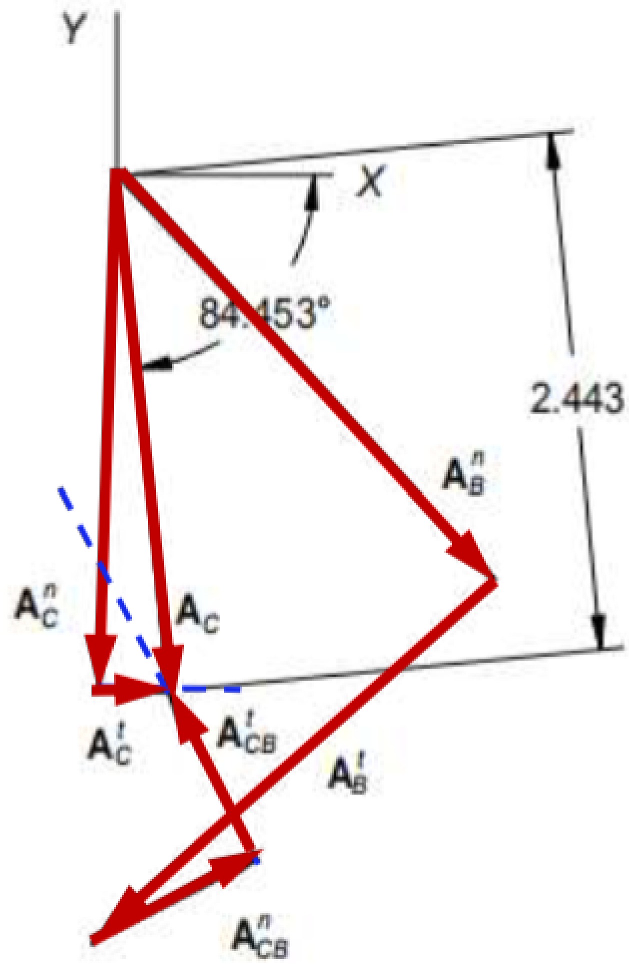
4. 写出 A 点的加速度矢量关系式:$A_2^n = A_4^n + A_4^t + A_4^{cor} + A_4^{slip}$, 根据已知量列出 Magnitude-Direction 表，并作图求得杆件 4 的角加速度

Equation	$\boldsymbol{A}_2^n$	$\boldsymbol{A}_4^n$	$\boldsymbol{A}_4^t$	$\boldsymbol{A}_4^{cor}$	$\boldsymbol{A}_4^{slip}$
Magnitude	1	1	0	1	0
Magnitude	1	1	1	1	1



5. 取杆件 4、5、6、地面组成的四杆机构，进行加速度分析，列出在 C 点的加速度矢量关系式: $(\boldsymbol{A}_C^t + \boldsymbol{A}_C^n) = (\boldsymbol{A}_B^t + \boldsymbol{A}_B^n) + (\boldsymbol{A}_{BC}^t + \boldsymbol{A}_{BC}^n)$, 列出 Magnitude-Direction 表，并作图求得杆件 6 的加速度

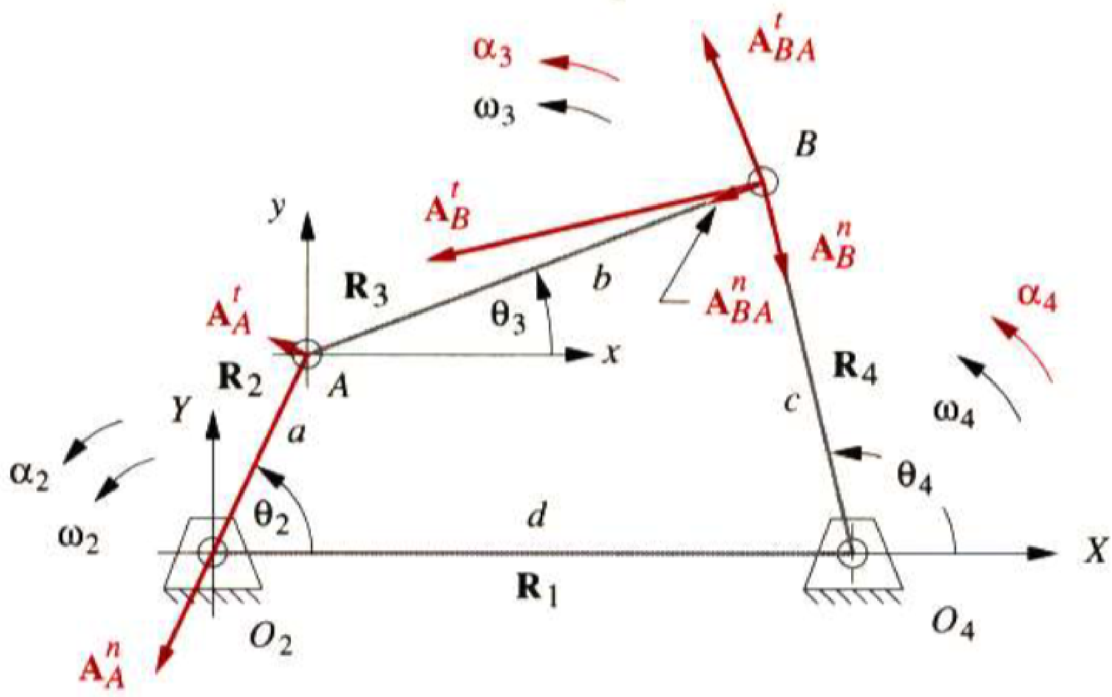
Equation	A_C^n	A_C^t	A_B^n	A_B^t	A_{BC}^n	A_{BC}^t
Magnitude	1	0	1	1	1	0
Direction	1	1	1	1	1	1



7.3 Analytical Solutions For Acceleration Analysis

Example 1

Find the acceleration of the four-bar linkage



$$R_2 + R_3 - R_4 - R_1 = 0$$

$$ae^{j\theta_2} + be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_4} = 0$$

$$a\omega_2 e^{j\theta_2} + b\omega_3 e^{j\theta_3} - c\omega_4 e^{j\theta_4} = 0$$

$$(-a\omega_2^2 e^{j\theta_2} + a\alpha_2 je^{j\theta_2}) + (-b\omega_3^2 e^{j\theta_3} + b\alpha_3 je^{j\theta_3}) - (-c\omega_4^2 e^{j\theta_4} + c\alpha_4 je^{j\theta_4}) = 0$$

$$\boldsymbol{A}_A + \boldsymbol{A}_{BA} - \boldsymbol{A}_B = 0$$

separate the real parts and imaginary parts, get alpha accelerations

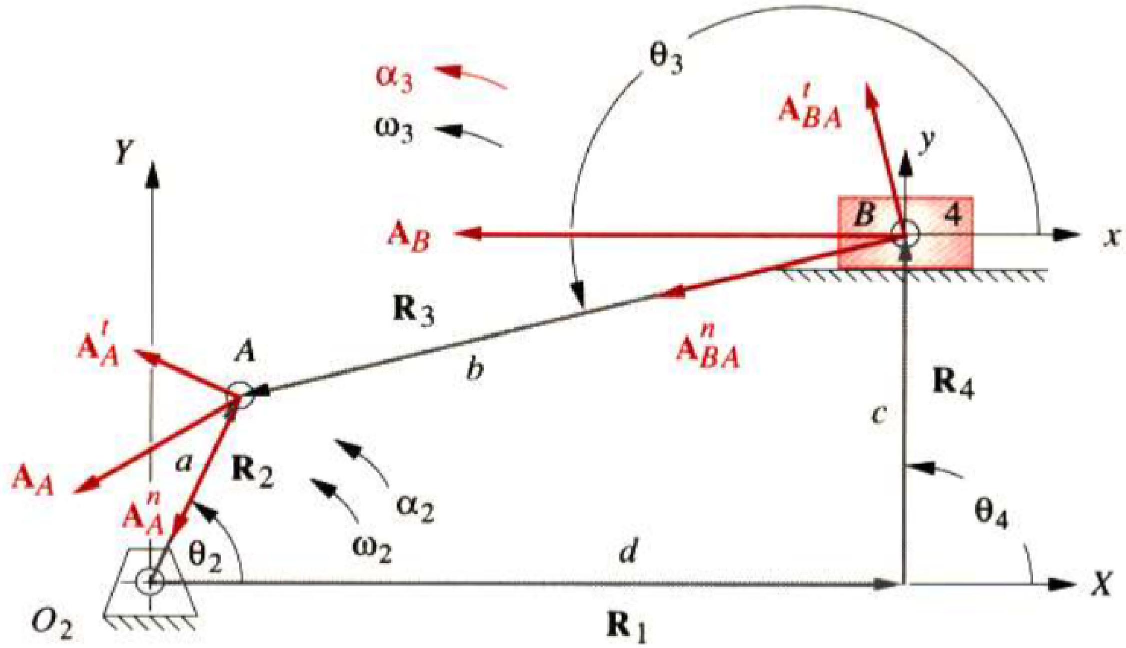
$$\alpha_3 = \frac{CD - AF}{AE - BD}$$
$$\alpha_3 = \frac{CE - BF}{AE - BD}$$

where $A = c\sin\theta_4$, $B = b\sin\theta_3$, $C = a\alpha_2 \sin\theta_2 + a\omega_2^2 \cos\theta_2 + b\omega_3^2 \cos\theta_3 - c\omega_4^2 \cos\theta_4$

$D = c\cos\theta_4$, $E = b\cos\theta_3$ and $F = a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 - b\omega_3^2 \sin\theta_3 + c\omega_4^2 \sin\theta_4$

Example 2

Find the acceleration of the fourbar slider-crank



$$R_3 - R_3 - R_4 - R_1 = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$$ja\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{d} = 0$$

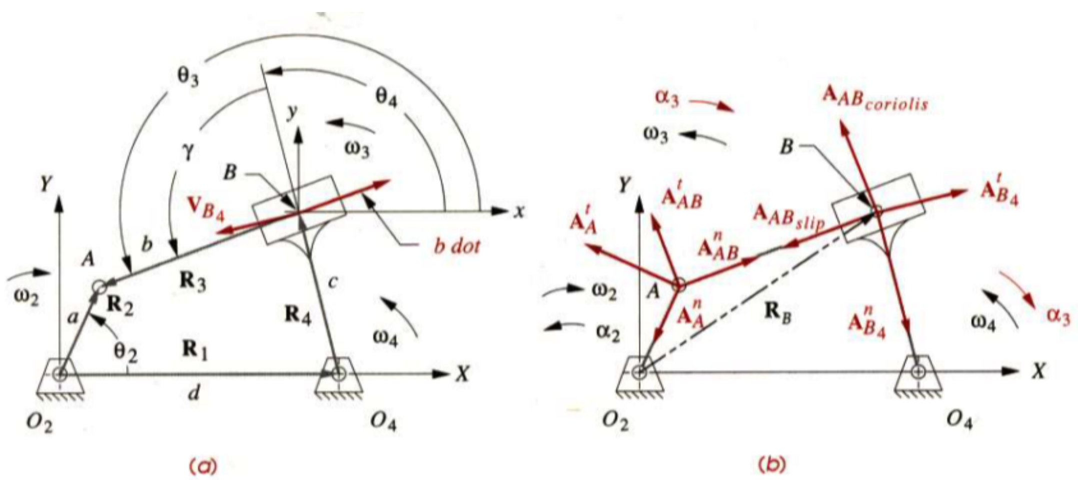
$$(ja\alpha_2 e^{j\theta_2} - a\omega_2^2 e^{j\theta_2}) - (b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3}) - \ddot{d} = 0$$

separate the real and imaginary parts, we can get

$$\alpha_3 = \frac{a\alpha_2 \cos\theta_2 - a\omega_2^2 \sin\theta_2 + b\omega_3^2 \sin\theta_3}{b\cos\theta_3}$$
$$\ddot{d} = -a\alpha_2 \sin\theta_2 - a\omega_2^2 \cos\theta_2 + b\alpha_3 \sin\theta_3 + b\omega_3^2 \cos\theta_3$$

Example 3

Find the acceleration of the inverted crank slider



$$R_2 - R_3 - R_4 - R_1 = 0$$

$$ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} = 0$$

$$j\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{b}e^{j\theta_3} - jc\omega_4 e^{j\theta_4} = 0$$

$$(a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}) - (b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3} + 2\dot{b}\omega_3 je^{j\theta_3} + \ddot{b}e^{j\theta_3}) - (c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}) = 0$$

and we know that $\omega_3 = \omega_4$ and $\alpha_3 = \alpha_4$

separate the imaginary and real parts, we can get the solution

$$\alpha_4 = \frac{a[\alpha_2\cos(\theta_3-\theta_2)+\omega_2^2\sin(\theta_3-\theta_2)]+c\omega_4^2\sin(\theta_4-\theta_3)-2\dot{b}\omega_3}{b+c\cos(\theta_3-\theta_4)}$$

$$\ddot{b} = \frac{a\omega_2^2[b\cos(\theta_3-\theta_2)+c\cos(\theta_4-\theta_2)]+a\alpha_2[b\sin(\theta_2-\theta_3)-c\sin(\theta_4-\theta_2)]+2\dot{b}c\omega_4\sin(\theta_4-\theta_3)-\omega_4^2[b^2=c^2=2bc\cos(\theta_4-\theta_3)]}{b+c\cos(\theta_3-\theta_4)}$$

7.4 Unit Vector Method for Acceleration Analysis

Acceleration of point P

$$\boldsymbol{A}_P = \boldsymbol{A}_O + \boldsymbol{A} + 2\omega\times\boldsymbol{V} + \alpha\times\boldsymbol{R} + \omega\times(\omega\times\boldsymbol{R})$$

$\boldsymbol{A}_P$: the absolute acceleration of point P

$\boldsymbol{A}_O$: the absolute acceleration of the origin of moving axis system

$\boldsymbol{A}$: the relative acceleration of point P with respect to the moving axis system

$\boldsymbol{V}$ and $\boldsymbol{R}$: the velocity and position vectors of point P with respect to the moving axis system

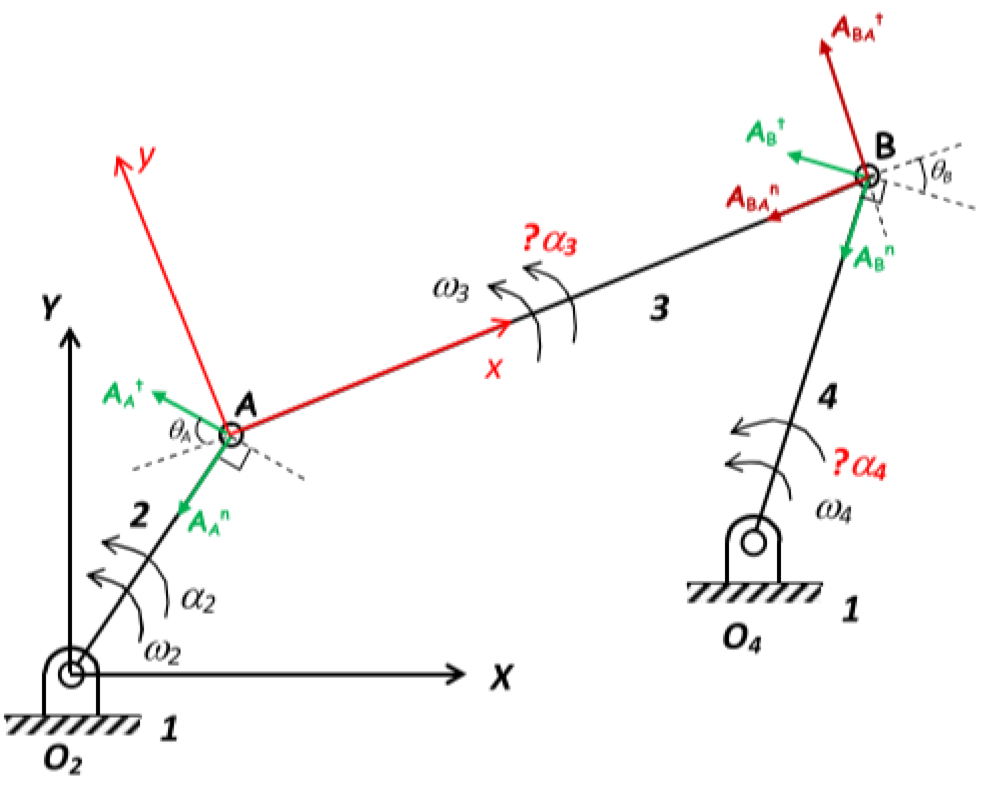
ω and α: the angular velocity and acceleration of the moving axis system

Two Points on One Rigid Body

$\boldsymbol{V}=0$ and $\boldsymbol{A}=0$

$$A_P^n + A_P^t = A_O^n + A_O^t + A_{PO}^n + A_{PO}^t$$

Basic Four Bar Mechanism



$$A_B^n + A_B^t = A_A^n + A_A^t + A_{BA}^n + A_{BA}^t$$

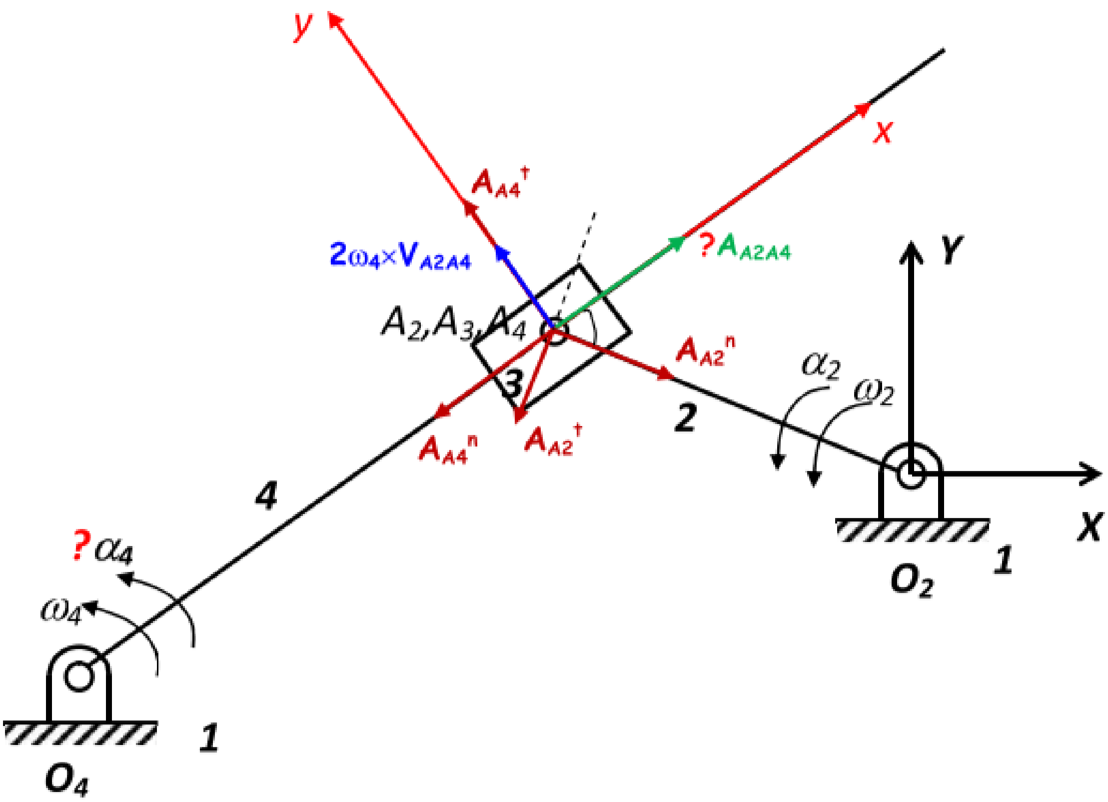
Equation	A_B^n	A_B^t	A_A^n	A_A^t	A_{BA}^n	A_{BA}^t
Magnitude	1	0	1	1	1	0
Direction	$B \rightarrow O_4$	$\perp O_4B$	$A \rightarrow O_2$	$\perp O_2A$	$-i$	$(+?-)j$

Two Coincident Points on Two Rigid Bodies

$R = 0$

$$A_P^n + A_P^t = A_O^n + A_O^t + A + A_{Coriolis}$$

Sliding Connection



$$A_{A_2}^n + A_{A_2}^t = A_{A_4}^n + A_{A_4}^t + A_{slip} + 2\omega_4 \times V_{slip}$$

Equation	$A_{A_2}^n$	$A_{A_2}^t$	$A_{A_4}^n$	$A_{A_4}^t$	A_{slip}	$\omega_4 \times V_{slip}$
Magnitude	1	1	1	0	0	1
Direction	$A \rightarrow O_2$	$\perp O_2A$	$A \rightarrow O_4$	$\perp O_4A$	$(+?-)i$	j