Chapter 7

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Introduction

7.1 Definition of Acceleration

Definition

Acceleration Difference

Relative Acceleration

7.2 Graphical Acceleration Analysis

Coriolis Acceleration

Example 1

Example 2

Example 3

7.3 Analytical Solutions For Acceleration Analysis

Example 1

Example 2

Example 3

7.4 Unit Vector Method for Acceleration Analysis

Two Points on One Rigid Body

Two Coincident Points on Two Rigid Bodies

Introduction

We need to know the accelerations to calculate the dynamic forces from F=ma

The manual graphical method and the analytical solution for accelerations

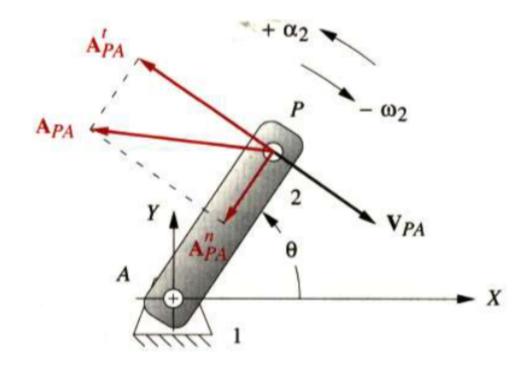
7.1 Definition of Acceleration

Definition

the rate of change of velocity with respect to time

$$lpha = rac{\mathrm{d}\omega}{\mathrm{d}t} \qquad m{A} = rac{\mathrm{d}m{V}}{\mathrm{d}t}$$

take the basic model for consideration

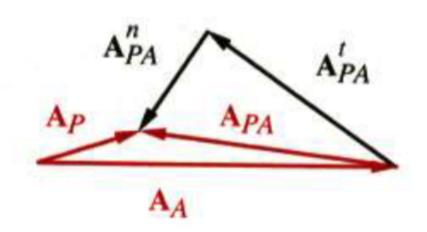


$$egin{align} m{R}_{PA} &= pe^{j heta} \ m{V}_{PA} &= \omega j p e^{j heta} \ m{A}_{PA} &= j p (lpha e^{j heta} + j \omega^2 e^{j heta}) = -\omega^2 p e^{j heta} + j lpha p e^{j heta} \ m{A}_{PA} &= m{A}_{PA}^n + m{A}_{PA}^t \end{aligned}$$

Acceleration Difference

Two points in the same body

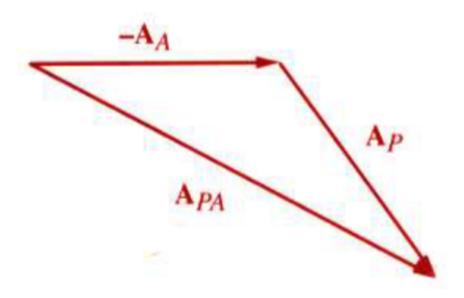
$$m{A}_P = m{A}_A + m{A}_{PA} \ m{(A}_P^t + m{A}_P^n) = m{(A}_A^t + m{A}_A^n) + m{(A}_{PA}^t + m{A}_{PA}^n)$$



Relative Acceleration

Two points in different bodies

$$oldsymbol{A}_{PA} = oldsymbol{A}_P - oldsymbol{A}_A$$



7.2 Graphical Acceleration Analysis

To solve any acceleration analysis problem graphically, we need only three equations

$$egin{split} (m{A}_P^t + m{A}_P^n) &= (m{A}_A^t + m{A}_A^n) + (m{A}_{PA}^t + m{A}_{PA}^n) \ &|m{A}^t| &= A^t = rlpha \ &|m{A}^n| &= r\omega^2 \end{split}$$

Coriolis Acceleration

for the position vector $oldsymbol{R}_P$

$$oldsymbol{R}_P=pe^{j heta_2}$$

we can simply get the velocity expression

$$extbf{\emph{V}}_{P} = p \omega_{2} e^{j heta_{2}} + \dot{p} e^{j heta_{2}}$$

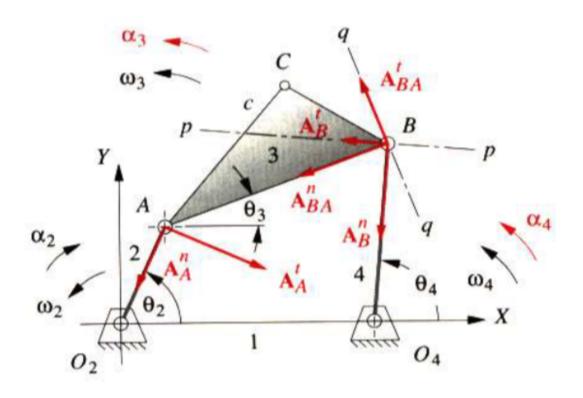
take a further step

$$m{A}_P = plpha_2 j e^{j heta_2} - p\omega_2^2 e^{j heta_2} + 2\dot{p}\omega_2 j e^{j heta_2} + \ddot{p}e^{j heta_2}$$

Example 1

Graphical Acceleration Analysis for One Position of a Fourbar Linkage

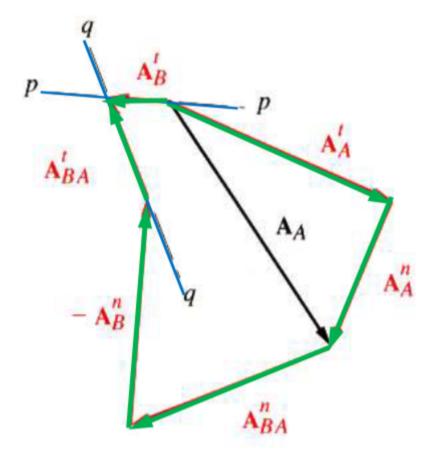
Given θ_2 , θ_3 , θ_4 , ω_2 , ω_3 , ω_4 , α_2 , find α_3 , α_4 , A_A , A_B and A_C by graphically method



- 1. 写出矢量关系式: $(\boldsymbol{A}_B^t + \boldsymbol{A}_B^n) = (\boldsymbol{A}_A^t + \boldsymbol{A}_A^n) + (\boldsymbol{A}_{AB}^t + \boldsymbol{A}_{AB}^n)$
- 2. 根据已知量完成 Magnitude-Direction 表 · 确定已知量和未知量

Equation	$oldsymbol{A}^n_B$	$oldsymbol{A}_B^t$	$oldsymbol{A}_A^n$	$oldsymbol{A}_A^t$	$oldsymbol{A}^n_{AB}$	$oldsymbol{A}^t_{AB}$
Magnitude	1	0	1	1	1	0
Direction	1	1	1	1	1	1

3. 根据已知量和上表制图

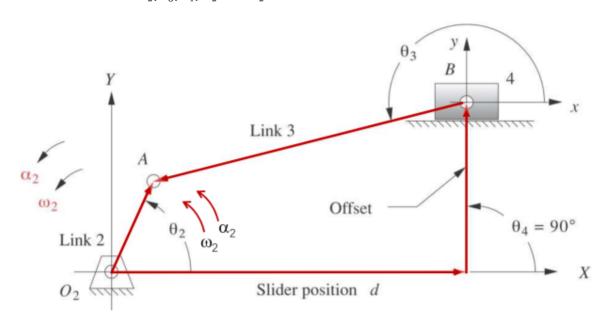


4. 利用图中长度的比例来求出未知量

Example 2

Graphically Acceleration Analysis for Fourbar Crank-Slider

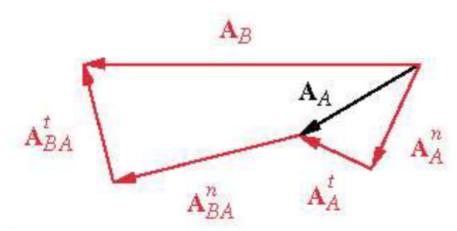
Given the value of θ_2 , θ_3 , θ_4 , ω_2 and α_2



- 1. 列出加速度的矢量关系式: $oldsymbol{A}_B = (oldsymbol{A}_A^t + oldsymbol{A}_A^n) + (oldsymbol{A}_{AB}^t + oldsymbol{A}_{AB}^n)$
- 2. 根据已知量完成 Magnitude-Direction 表

Equation	$oldsymbol{A}_B$	$oldsymbol{A}_A^n$	$oldsymbol{A}_A^t$	$oldsymbol{A}^n_{AB}$	$oldsymbol{A}^t_{AB}$
Magnitude	0	1	1	1	0
Magnitude	1	1	1	1	1

3. 根据上表制图

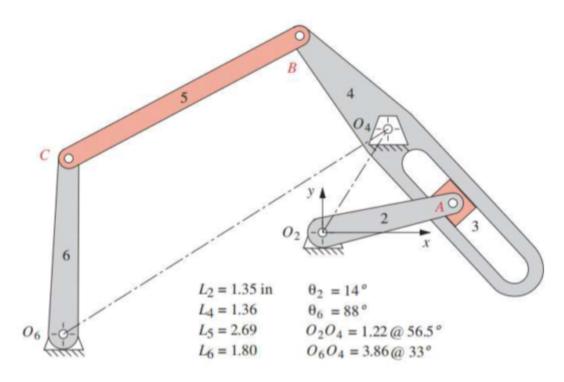


4. 根据比例尺量取图中长度

Example 3

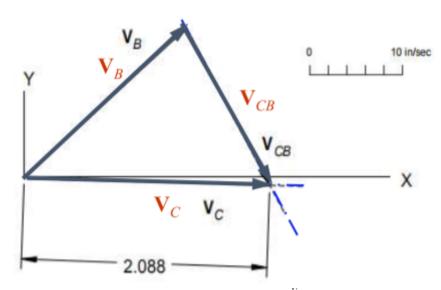
Graphically Acceleration Analysis for Slider

Given ω_2 , θ_2 , use a graphical method to calculate the accelerations of points A,B and C for the position shown



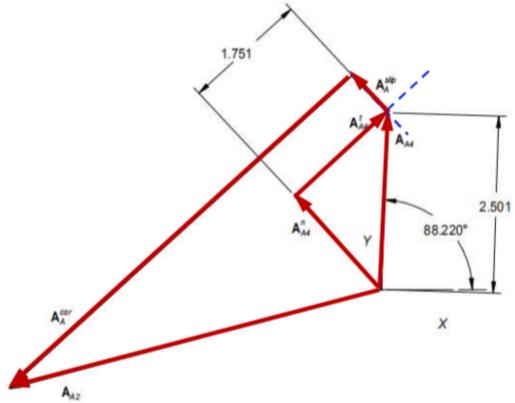
- 1. 沿杆 4 分解 A 点的速度,其垂直杆件 4 分量除以 $|O_2|$ A 便为杆件 4 的角速度
- 2. 接下来则是求四杆机构 4、5、6、地面的各杆件速度
- 3. 列出矢量关系式 $\overrightarrow{V_C}=\overrightarrow{V_B}+\overrightarrow{V_{BC}}$,列 Magnitude-Direction 表格,画图求得 C 点速度,顺带求出 杆件 5、6 的角速度

Equation	$\overrightarrow{V_C}$	$\overrightarrow{V_B}$	$\overrightarrow{V_{BC}}$
Magnitude	0	1	0
Direction	1	1	1



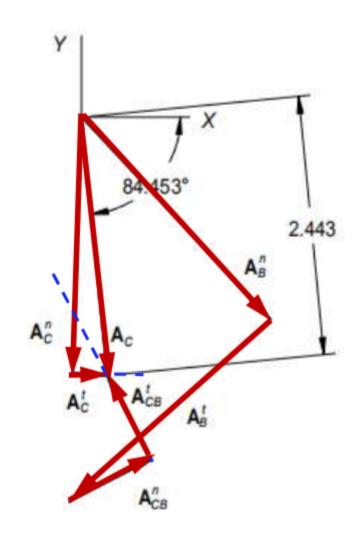
4. 写出 A 点的加速度矢量关系式: $A_2^n=A_4^n+A_4^t+A_4^{cor}+A_4^{slip}$, 根据已知量列出 Magnitude-Direction 表,并作图求得杆件 4 的角加速度

Equation	$oldsymbol{A}_2^n$	$oldsymbol{A}_4^n$	$oldsymbol{A}_4^t$	$m{A}_4^{cor}$	$m{A}_4^{slip}$
Magnitude	1	1	0	1	0
Magnitude	1	1	1	1	1



5. 取杆件 4、5、6、地面组成的四杆机构,进行加速度分析,列出在 C 点的加速度矢量关系式: $(\boldsymbol{A}_C^t + \boldsymbol{A}_C^n) = (\boldsymbol{A}_B^t + \boldsymbol{A}_B^n) + (\boldsymbol{A}_{BC}^t + \boldsymbol{A}_{BC}^n),$ 列出 Magnitude-Direction 表,并作图求得杆件 6 的加速度

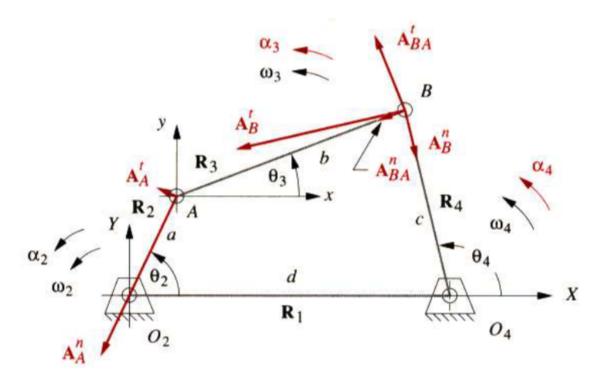
Equation	$oldsymbol{A}^n_C$	$oldsymbol{A}_C^t$	$oldsymbol{A}^n_B$	$oldsymbol{A}_B^t$	$oldsymbol{A}^n_{BC}$	$oldsymbol{A}_{BC}^t$
Magnitude	1	0	1	1	1	0
Direction	1	1	1	1	1	1



7.3 Analytical Solutions For Acceleration Analysis

Example 1

Find the acceleration of the four-bar linkage



$$egin{align*} R_2 + R_3 - R_4 - R_1 &= 0 \ ae^{j heta_2} + be^{j heta_3} - ce^{j heta_4} - de^{j heta_4} &= 0 \ a\omega_2 e^{j heta_2} + b\omega_3 e^{j heta_3} - c\omega_4 e^{j heta_4} &= 0 \ (-a\omega_2^2 e^{j heta_2} + alpha_2 je^{j heta_2}) + (-b\omega_3^2 e^{j heta_3} + blpha_3 je^{j heta_3}) - (-c\omega_4^2 e^{j heta_4} + clpha_4 je^{j heta_4}) &= 0 \ egin{align*} oldsymbol{A}_A + oldsymbol{A}_{BA} - oldsymbol{A}_B &= 0 \ \end{pmatrix}$$

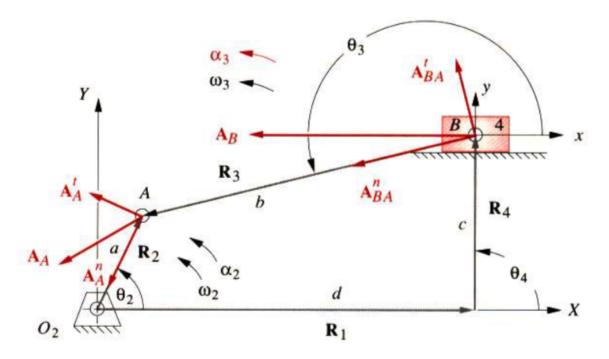
separate the real parts and imaginary parts, get alpha accelerations

$$lpha_3 = rac{CD - AF}{AE - BD}$$
 $lpha_3 = rac{CE - BF}{AE - BD}$

where $A=c\sin\theta_4$, $B=b\sin\theta_3$, $C=a\alpha_2\sin\theta_2+a\omega_2^2\cos\theta_2+b\omega_3^2\cos\theta_3-c\omega_4^2\cos\theta_4$ $D=c\cos\theta_4$, $E=b\cos\theta_3$ and $F=a\alpha_2\cos\theta_2-a\omega_2^2\sin\theta_2-b\omega_3^2\sin\theta_3+c\omega_4^2\sin\theta_4$

Example 2

Find the acceleration of the fourbar slider-crank



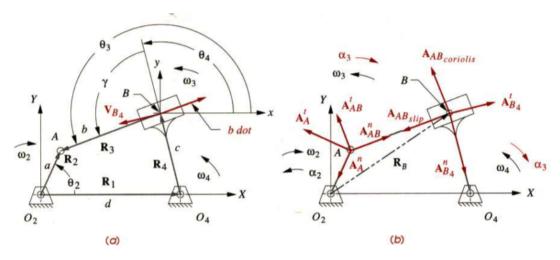
$$egin{align*} R_3 - R_3 - R_4 - R_1 &= 0 \ &ae^{j heta_2} - be^{j heta_3} - ce^{j heta_4} - de^{j heta_1} &= 0 \ &ja\omega_2 e^{j heta_2} - jb\omega_3 e^{j heta_3} - \dot{d} &= 0 \ &(jalpha_2 e^{j heta_2} - a\omega_2^2 e^{j heta_2}) - (blpha_3 je^{j heta_3} - b\omega_3^2 e^{j heta_3}) - \ddot{d} &= 0 \ \end{gathered}$$

separate the real and imaginary parts, we can get

$$\alpha_3 = \frac{a\alpha_2\cos\theta_2 - a\omega_2^2\sin\theta_2 + b\omega_3^2\sin\theta_3}{b\cos\theta_3}$$
$$\ddot{d} = -a\alpha_2\sin\theta_2 - a\omega_2^2\cos\theta_2 + b\alpha_3\sin\theta_3 + b\omega_3^2\cos\theta_3$$

Example 3

Find the acceleration of the inverted crank slider



$$\begin{split} R_2 - R_3 - R_4 - R_1 &= 0 \\ ae^{j\theta_2} - be^{j\theta_3} - ce^{j\theta_4} - de^{j\theta_1} &= 0 \\ j\omega_2 e^{j\theta_2} - jb\omega_3 e^{j\theta_3} - \dot{b}e^{j\theta_3} - jc\omega_4 e^{j\theta_4} &= 0 \\ (a\alpha_2 je^{j\theta_2} - a\omega_2^2 e^{j\theta_2}) - (b\alpha_3 je^{j\theta_3} - b\omega_3^2 e^{j\theta_3} + 2\dot{b}\omega_3 je^{j\theta_3} + \ddot{b}e^{j\theta_3}) - (c\alpha_4 je^{j\theta_4} - c\omega_4^2 e^{j\theta_4}) &= 0 \end{split}$$

and we know that $\omega_3=\omega_4$ and $lpha_3=lpha_4$

separate the imaginary and real parts, we can get the solution

$$\alpha_4 = \frac{a[\alpha_2\cos(\theta_3 - \theta_2) + \omega_2^2\sin(\theta_3 - \theta_2)] + c\omega_4^2\sin(\theta_4 - \theta_3) - 2\dot{b}\omega_3}{b + c\cos(\theta_3 - \theta_4)} \\ \ddot{b} = \frac{a\omega_2^2[b\cos(\theta_3 - \theta_2) + c\cos(\theta_4 - \theta_2)] + a\alpha_2[b\sin(\theta_2 - \theta_3) - c\sin(\theta_4 - \theta_2)] + 2\dot{b}c\omega_4\sin(\theta_4 - \theta_3) - \omega_4^2[b^2 = c^2 = 2bc\cos(\theta_4 - \theta_3)]}{b + c\cos(\theta_3 - \theta_4)}$$

7.4 Unit Vector Method for Acceleration Analysis

Acceleration of point P

$$oldsymbol{A}_P = oldsymbol{A}_O + oldsymbol{A} + 2\omega imes oldsymbol{V} + lpha imes oldsymbol{R} + \omega imes (\omega imes oldsymbol{R})$$

 $oldsymbol{A}_P$: the absolute acceleration of point P

 $oldsymbol{A}_O$: the absolute acceleration of the origin of moving axis system

A: the relative acceleration of point P with respect to the moving axis system

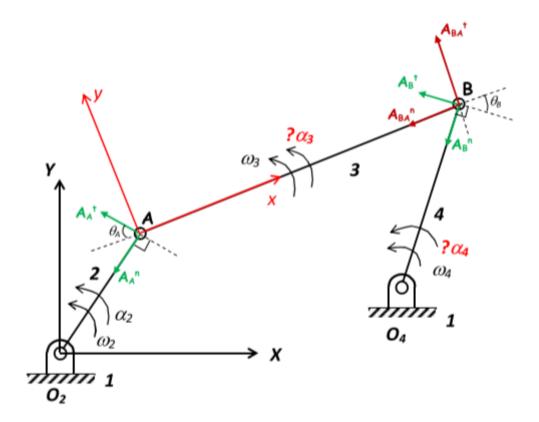
 $m{V}$ and $m{R}$: the velocity and position vectors of point P with respect to the moving axis system ω and α : the angular velocity and acceleration of the moving axis system

Two Points on One Rigid Body

 $oldsymbol{V}=0$ and $oldsymbol{A}=0$

$$A_P^n + A_P^t = A_O^n + A_O^t + A_{PO}^n + A_{PO}^t$$

Basic Four Bar Mechanism



$$A_B^n + A_B^t = A_A^n + A_A^t + A_{BA}^n + A_{BA}^t \label{eq:abs}$$

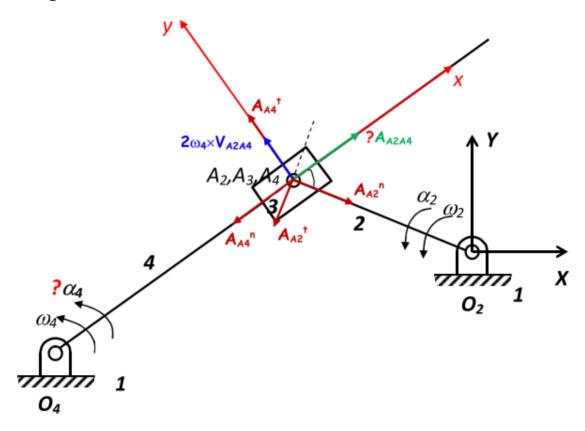
Equation	A_B^n	A_B^t	A_A^n	A_A^t	A^n_{BA}	A^t_{BA}
Magnitude	1	0	1	1	1	0
Direction	$B o O_4$	$\perp O_4 B$	$A o O_2$	$\perp O_2 A$	-i	(+?-)j

Two Coincident Points on Two Rigid Bodies

R = 0

$$A_P^n + A_P^t = A_O^n + A_O^t + A + A_{Coriolis}$$

Sliding Connection



$$A^{n}_{A_{2}}+A^{t}_{A_{2}}=A^{n}_{A_{4}}+A^{t}_{A_{4}}+A_{slip}+2\omega_{4} imes V_{slip}$$

Equation	$A^n_{A_2}$	$A^t_{A_2}$	$A^n_{A_4}$	$A^t_{A_4}$	A_{slip}	$\omega_4 imes V_{slip}$
Magnitude	1	1	1	0	0	1
Direction	$A o O_2$	$\perp O_2 A$	$A o O_4$	$\perp O_4 A$	(+?-)i	j