# CH_2

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## 2.1 Hooke's Law

$$\sigma = E\varepsilon$$

- $E$: the modulus of elasticity of Youngs' modulus, which has the stress units

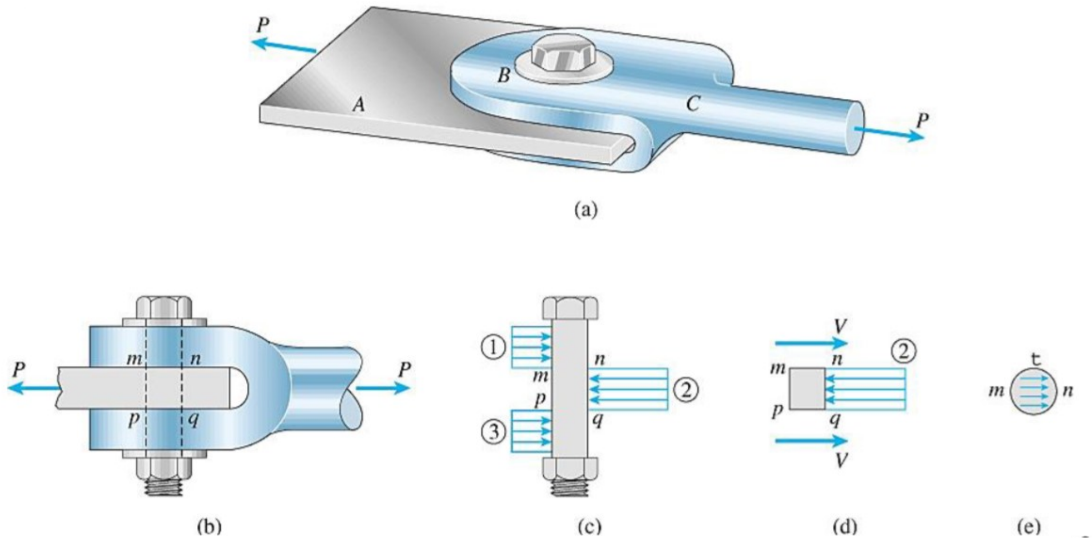
## 2.2 Poison Ratio

$$\nu = -\frac{\varepsilon'}{\varepsilon}$$

- $\nu$: Poison Ratio
- $\varepsilon$: axial strain
- $\varepsilon'$: lateral strain

## 2.3 Shear Stress

the stress component that act in the plane of the sectioned area




## Average Bearing Stress

$$\sigma_b = \frac{F_b}{A_b}$$

- $F_b$: bearing force
- $A_b$: bearing area

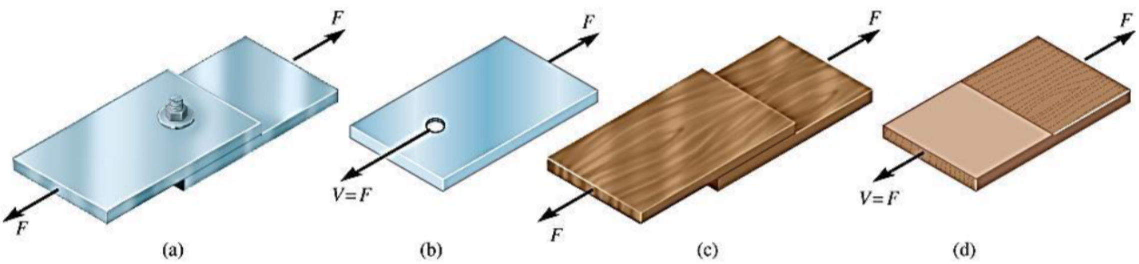
Average Shear Stress at Section

$$\tau_{aver} = \frac{V}{A}$$

- $V$: internal shear force at section determined from equations of equilibrium
- $A$: area of section

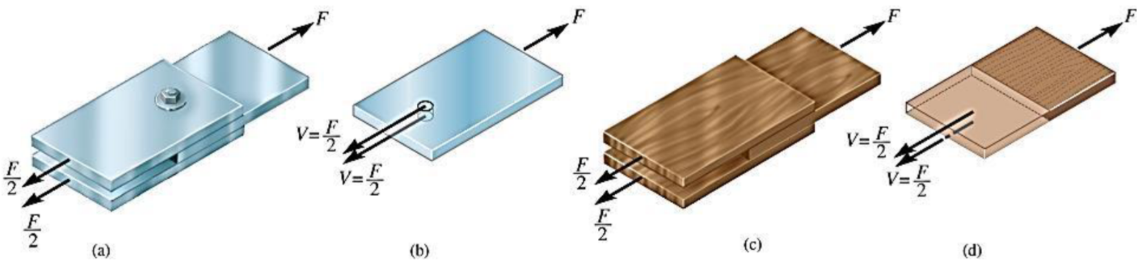
### Single Shear

$$V = F$$




### Double Shear

$$V = \frac{F}{2}$$



- Positive strain is when the angle between two positive faces is reduced
- Negative strain is when the angle between two positive faces is increased

### Hooke's Law for Shear

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)}$$

- $G$: shear modulus of elasticity

## 2.4 Allowable Stresses and Allowance Loads

When designing a structural member or mechanical element, the design interest is **strength**, that is *the capacity of the object to support or transmit loads*

- factor of safety (F.S.)

$$n = \frac{\text{Actual Strength}}{\text{Required Strength}}$$

- allowable strength

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{n_1} \qquad \tau_{\text{allow}} = \frac{\tau_Y}{n_2}$$

- $\sigma_Y$ and $\tau_Y$ are yield stresses
- $n_1$ and $n_2$ are the corresponding factors of safety.

## 2.5 Design for Axial Loads and Direct Shear

- determine the area of section subjected to a normal force

$$A = \frac{P}{\sigma_{allow}}$$

- determine the area of section subjected to a shear force

$$A = \frac{V}{\tau_{allow}}$$

## 2.6 Changes in Lengths of An Axially Loaded Members

$$P = k\delta \qquad \delta = fP$$

$k$ is the stiffness of the spring

$f$ is the flexibility of the spring

- **stiffness**: is the force required to produce a unit elongation
- **flexibility**: is the elongation produced by a unit force

$$
\begin{cases}
\delta = L \cdot \varepsilon \\
\sigma = E \cdot \varepsilon \\
P = \sigma \cdot A
\end{cases}
$$

$$
\Rightarrow \ \delta = \frac{L}{EA} P
$$

## Sign Convention

| Sign | Forces | Displacement |
| --- | --- | --- |
| Positive(+) | Tension | Elongation |
| Negative(-) | Compression | COntraction |

## Use Method of Sections

$$
\begin{cases}
\sigma = \frac{P(x)}{A(x)} = E\varepsilon \\
\varepsilon = \frac{d\delta}{dx}
\end{cases}
$$

$$
\Rightarrow d\delta = \frac{P(x)}{A(x)E} dx
$$

$$
\delta = \int_0^L \frac{P(x)}{A(x)E} dx
$$

- $P(x)$: internal normal force at the section, located a distance x from one end
- $A(x)$: x-sectional area of the bar, expressed as a function of x

#### Constant Load and x-Sectional Area

$$
\delta = \frac{L}{EA} P
$$

#### Bars with Intermediate Axial Loads

$$
\delta = \sum \frac{L}{EA} P
$$

# 2.7 Statically Indeterminate Structures

if the bar is **fixed at both ends**, then the unknown axial reactions occur, and the bar is statically indeterminate.

$$
\delta_{A/B} = 0
$$

$$
\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0
$$

$$
\Rightarrow F_A = P(\frac{L_C B}{L}) \qquad F_B = P(\frac{L_A C}{L})
$$

# 2.8 Thermal Stress

expansion or contraction of material is linearly related to temperature increase or decrease that occurs

$$\varepsilon_T = \alpha \Delta T \qquad \delta_T = \alpha \Delta T L$$

$\alpha$: liner coefficient of thermal expansion