

# CH\_3

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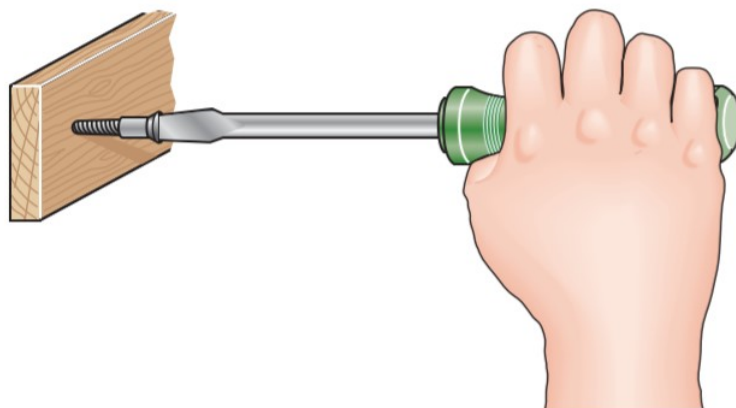
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## 3.1 Introduction

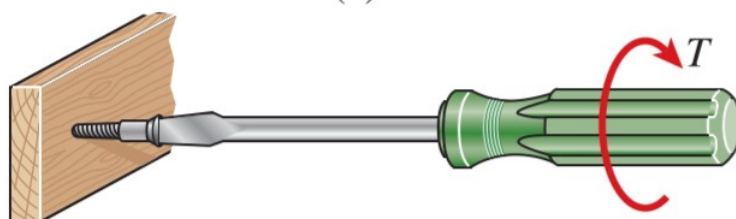
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### Torsion

the twisting of a straight bar when it is loaded by moment (or torque) that tends to produce rotation about the longitudinal axis of the bar



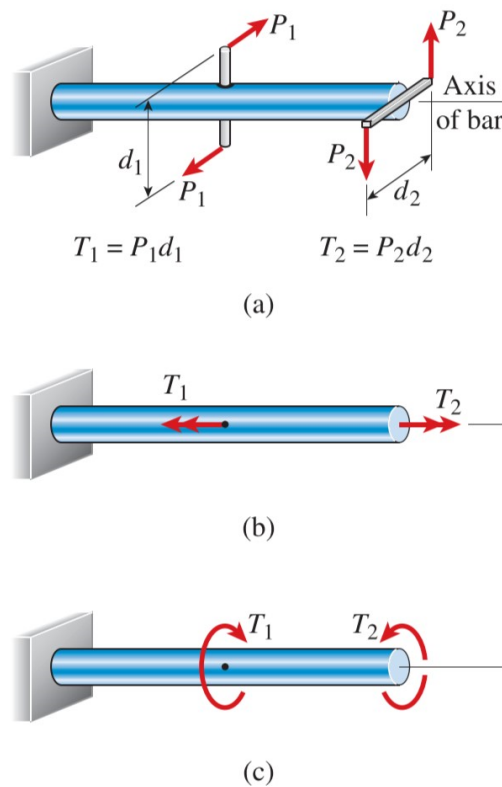
(a)



(b)

### Torque

a moment that twists/deforms a member about its longitudinal axis



## Shaft

cylindrical members that are subjected to torques and transmit power through rotation

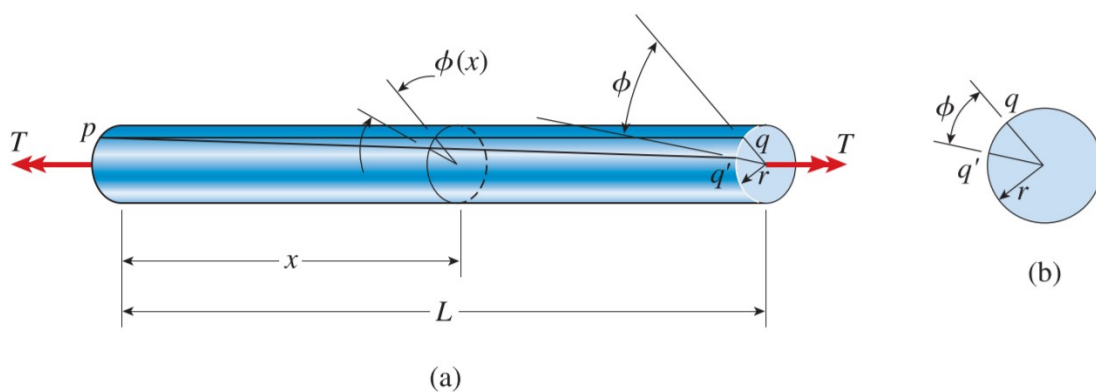
## 3.2 Torsional Deformation of a Circular Bar

### Pure Torsion

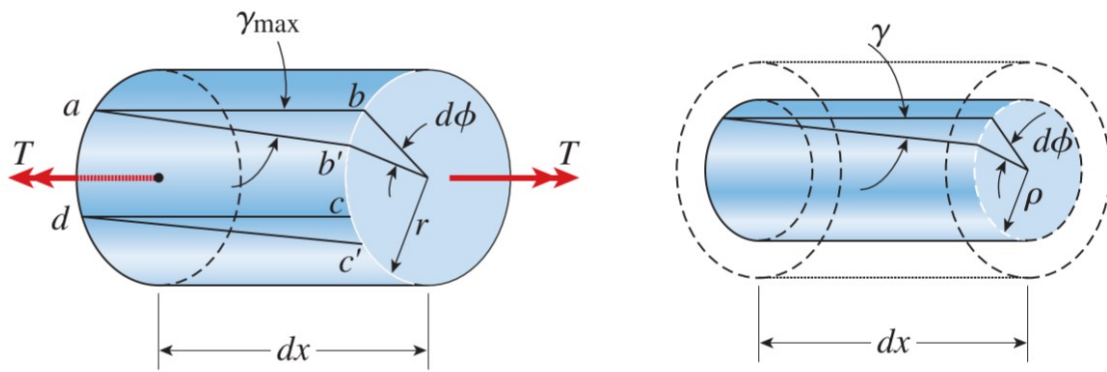
the bar is in pure torsion when every cross section of the bar is identical and subjected to the same internal torque  $T$

### Angle of Twist

under the action of the torque  $T$ , the right-hand end will rotate (with respect to the left-hand end) through a small angle  $\phi$



### Shear Strain at the Outer Surface



$$\gamma_{max} = \frac{bb'}{ab}$$

$$\Rightarrow \gamma_{max} = \frac{r d\phi}{dx}$$

denote the symbol  $\theta$  as the **rate of twist**

$$\theta = \frac{d\phi}{dx}$$

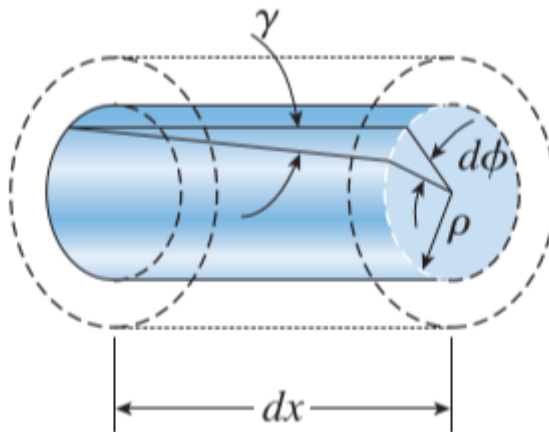
$$\Rightarrow \gamma_{max} = r\theta$$

**for pure torsion only**, we can obtain that

$$\gamma_{max} = \frac{r\phi}{L}$$

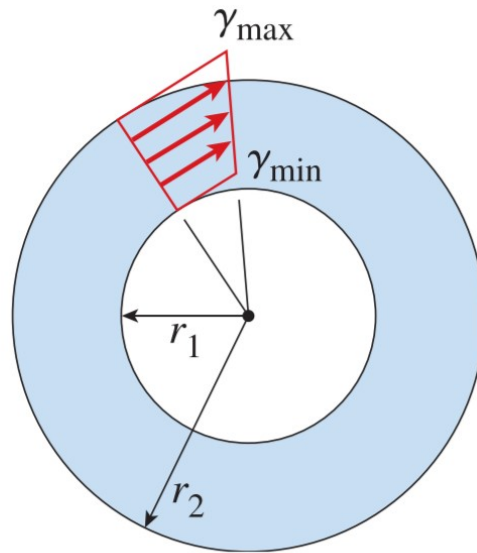
## Shear Strain within the bar

$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{max}$$



## Circular Tube

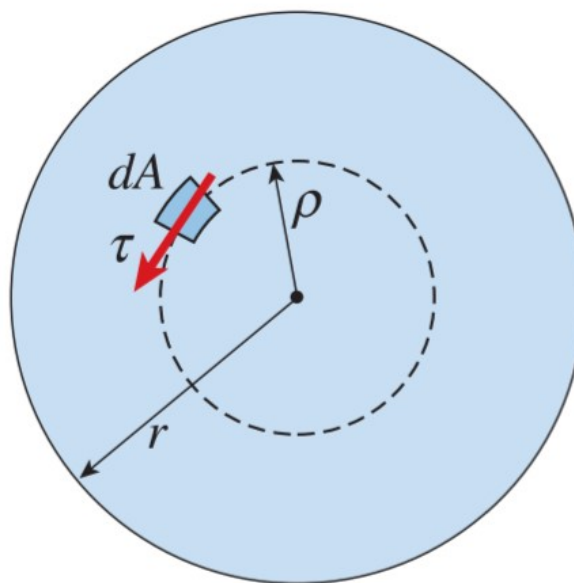
$$\gamma_{max} = \frac{r_2\phi}{L} \quad \gamma_{min} = \frac{r_1}{r_2}\gamma_{max} = \frac{r_1\phi}{L}$$



### 3.3 Circular Bars of Linearly Elastic Materials

the shear stresses acting on a cross-sectional plane are accompanied by shear stresses of the same magnitude acting on longitudinal planes.

#### the Torsion Formula



$$dM = \tau \rho dA = \frac{\tau_{max}}{r} \rho^2 dA$$

the resultant moment is the summation over the entire cross-sectional area

$$T = \int_A dM = \frac{\tau_{max}}{r} \int_A \rho^2 dA$$

in which

$$I_P = \int_A \rho^2 dA$$

is the **polar moment of inertia** of the circular cross section

The shear stress at distance  $\rho$  from the center of the bar is

$$\tau_{max} = \frac{Tr}{I_P} \quad \tau = \frac{T\rho}{I_P} \quad (\text{torsion formula})$$

## Solid Shaft

$$dA = 2\pi\rho d\rho$$

$$I_P = \frac{\pi}{32} d^4$$

## Tubular Shaft

$$I_P = \frac{\pi}{32} (d_0^4 - d_i^4)$$

## Angle of Twist

according to Hooke's Law in shear

$$\tau = G\gamma = G\rho\theta = \frac{\rho}{r}\tau_{max}$$

using the torsion formula

$$G\rho\theta = \frac{\rho}{r} \frac{Tr}{I_P}$$

$$\Rightarrow \theta = \sum \frac{TL}{GI_P} = \frac{d\phi}{dx}$$

for a bar in **pure torsion**, the total angle of the twist  $\phi$ , equal to the rate of twist time the length of the bar

$$\phi = \frac{TL}{GI_P} = \int_0^L \frac{T(x)dx}{GI_P(x)}$$

## Torsional Stiffness and Torsional Flexibility

$$k_T = \frac{GI_P}{L} \quad f_T = \frac{L}{GI_P}$$