CH_5

5.1 Pure Bending and Nonuniform Bending

Pure Bending

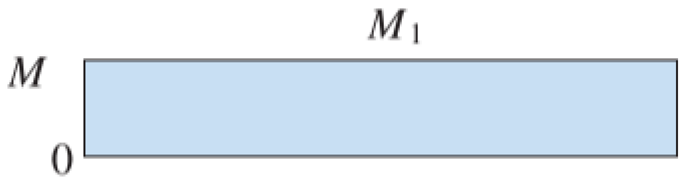
the flexure of a beam under a constant bending moment

Nonuniform Bending

the flexure in the presence of shear forces

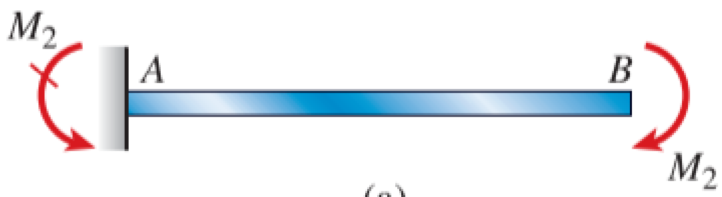
Simple beam in pure bending ($M = M_1$)

(a)

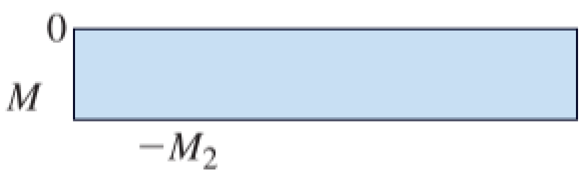


(b)

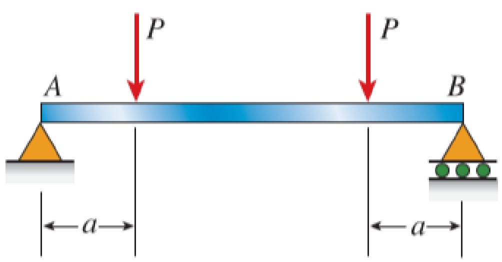
Cantilever beam in pure bending ($M = -M_2$)



(a)

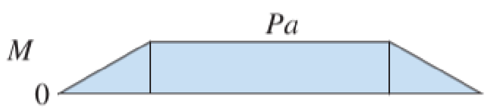


(b)



(a)

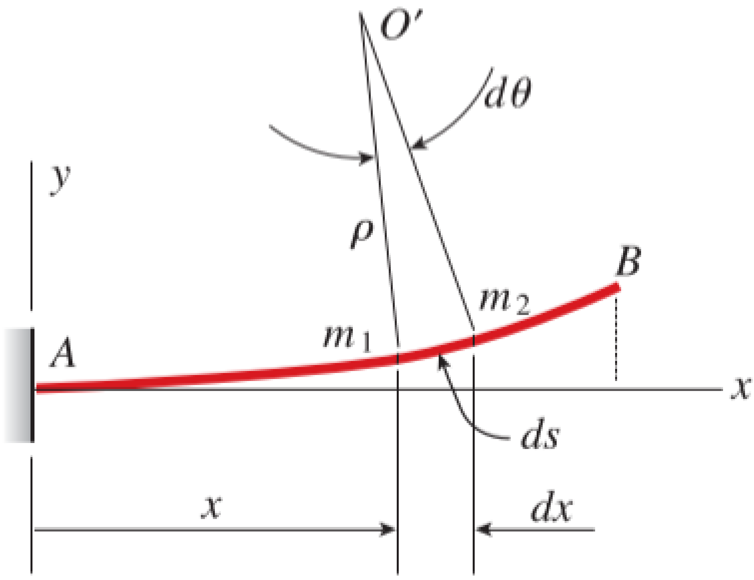
(b)



(c)

5.2 Curvature of a Beam

(a)



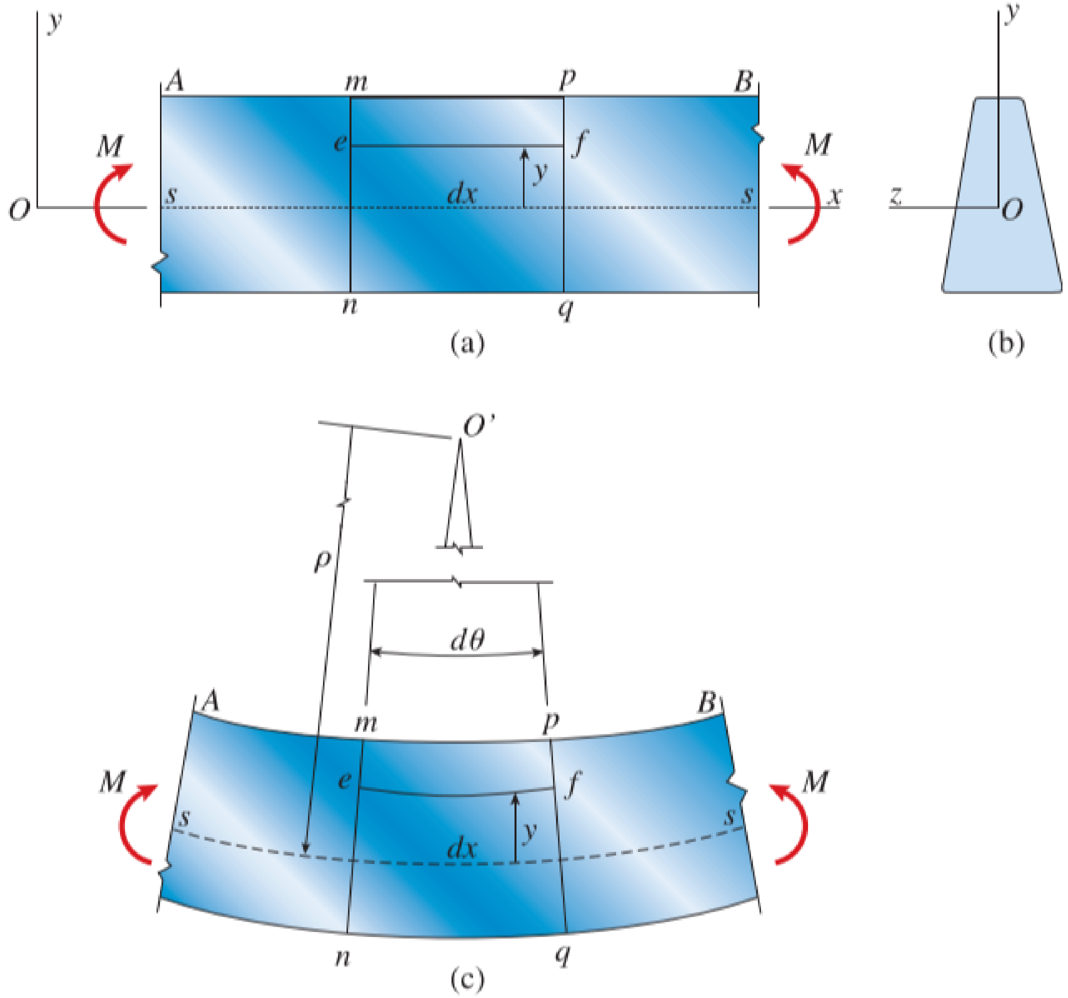
(b)

$$\rho d\theta = ds$$

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds}$$

$$= \frac{d\theta}{dx}$$

5.3 Longitudinal Strains in Beams



$$\widehat{ef} = (\rho - y)\mathrm{d}\theta \qquad \mathrm{d}x = \rho\mathrm{d}\theta$$

$$\varepsilon = \frac{\widehat{ef} - \mathrm{d}x}{\mathrm{d}x} = -\frac{y}{\rho}$$

5.4 Normal Stress in Beams

$$\sigma_x = E\varepsilon_x = \frac{Ey}{\rho} = -E\kappa y$$

Location of Neutral Axis

$$\int_A \sigma_x \mathrm{d}A = -\int_A E\kappa y \mathrm{d}A = 0$$
$$\int_A y\mathrm{d}A = 0$$

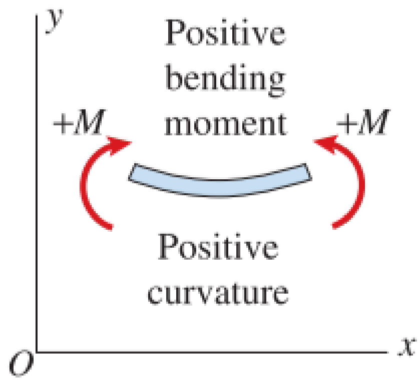
Moment Curvature Relationship

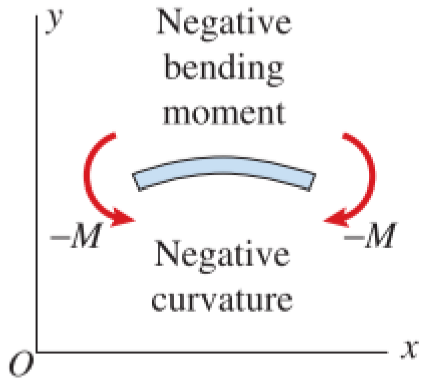
$$
\begin{aligned}
\mathrm{d}M &= -\sigma_x y \mathrm{d}A \\
M &= -\int_A \sigma_x y \mathrm{d}A \\
&= \int_A \kappa E y^2 \mathrm{d}A = \kappa E \int_A y^2 \mathrm{d}A \\
&= \kappa E I \\
\kappa &= \frac{1}{\rho} = \frac{M}{EI}
\end{aligned}
$$

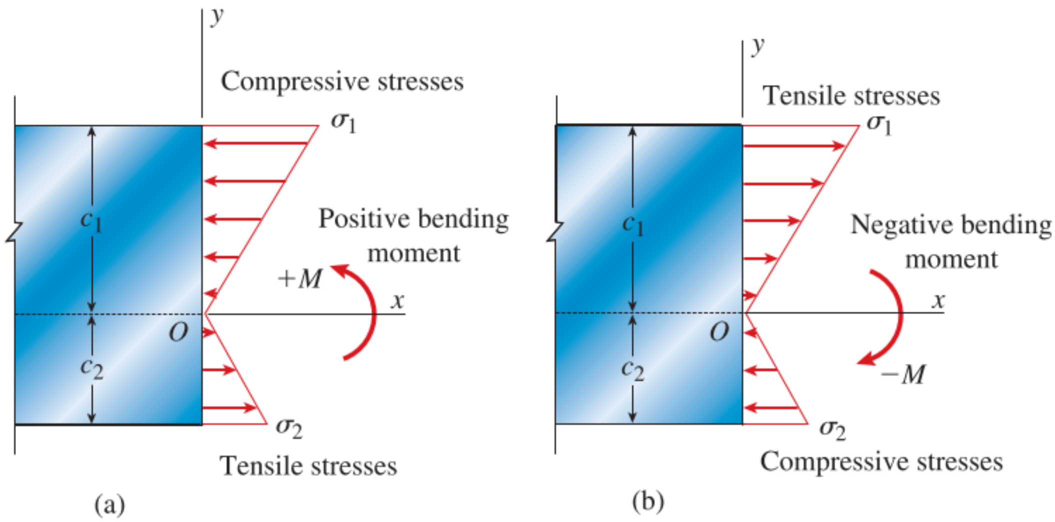
Flexure Formula

bending stresses (flexure stresses)

$$
\sigma_x = -\frac{My}{I}
$$







> For positive moment: $\sigma_c = -\frac{M\bar{y}}{I}$ $\sigma_t = -\frac{M(\bar{y}-h)}{I}$
> For negative moment: $\sigma_c = -\frac{M(\bar{y}-h)}{I}$ $\sigma_t = -\frac{M\bar{y}}{I}$

Maximum Stresses at a Cross Section

the *maximum tensile* and *compressive bending stresses* acting at any given cross section occur at points **located farthest from the neutral axis**

$$\sigma_{max} = -\frac{Mc}{I} = -\frac{M}{S}$$

$$S = \frac{I}{c}$$

where S is known as the **section moduli** of the cross-sectional area

Doubly Symmetric Shapes

Rectangular Cross Section

$$I = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$

Circular Cross Section

$$I = \frac{\pi d^4}{64} \qquad S = \frac{\pi d^3}{32}$$

5.8 Shear Stresses in Rectangular Beams

Shear Formula

$$\tau = \frac{VQ}{I_z b}$$

- V: internal resultant shear force
- I: moment of inertia of entire x-sectional area computed about the neutral axis
- b: width of the member's x-sectional area, measured at the pt where τ is to be determined
- **Q: $\frac{1}{2}b(\frac{h^2}{4} - y^2)$, where y is measured from neutral axis**

$$\tau = \frac{6V}{bh^3}(\frac{h^2}{4} - y^2)$$