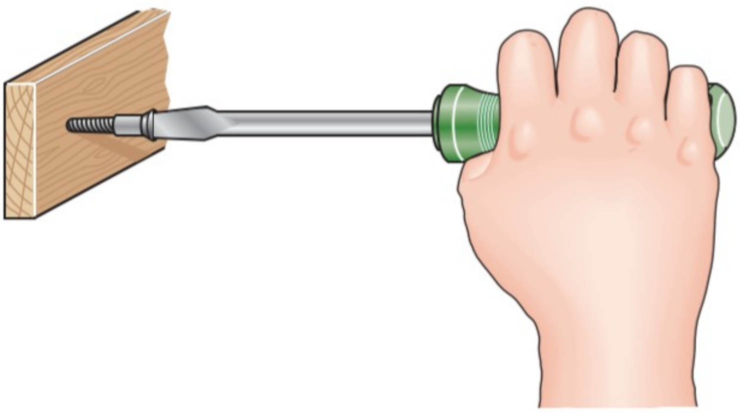
CH_3

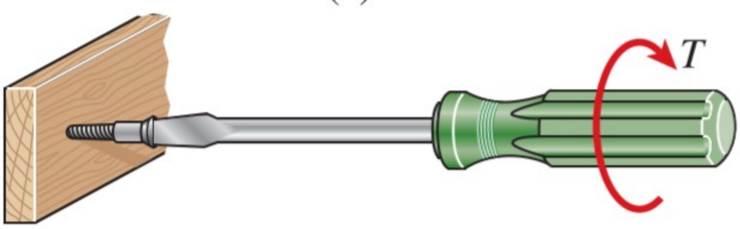
3.1 Introduction

Torsion

the twisting of a straight bar when it is loaded by moment (or torque) that tends to produce rotation about the longitudinal axis of the bar



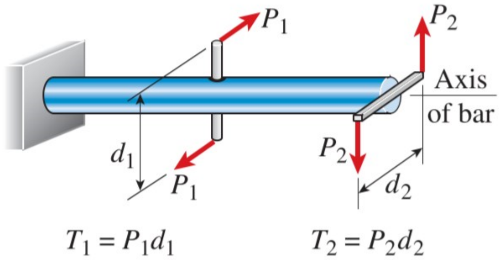
(a)



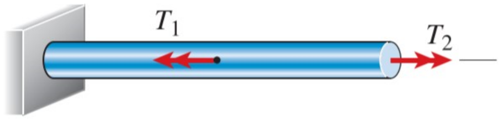
(b)

Torque

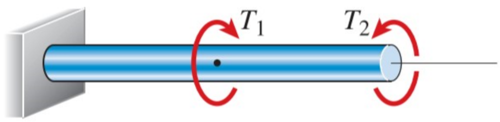
a moment that twists/deforms a member about its longitudinal axis



(a)



(b)



(c)

Shaft

cylindrical members that are subjected to torques and transmit power through rotation

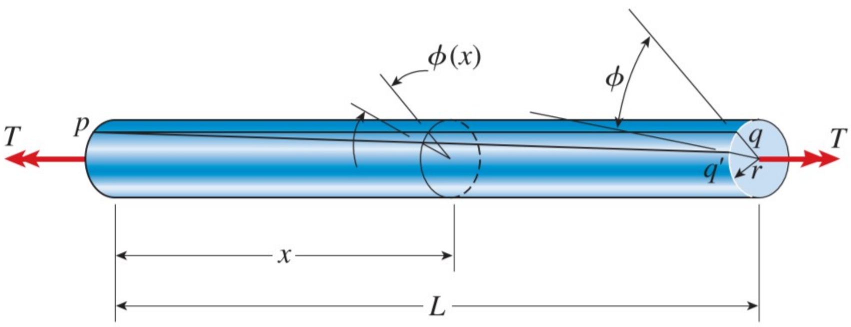
3.2 Torsional Deformation of a Circular Bar

Pure Torsion

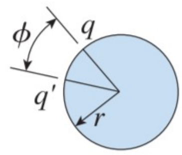
the bar is in pure torsion when every cross section of the bar is identical and subjected to the same internal torque *T*

Angle of Twist

under the action of the torque *T*, the right-hand end will rotate (with respect to the left-handed) through a small angle ϕ

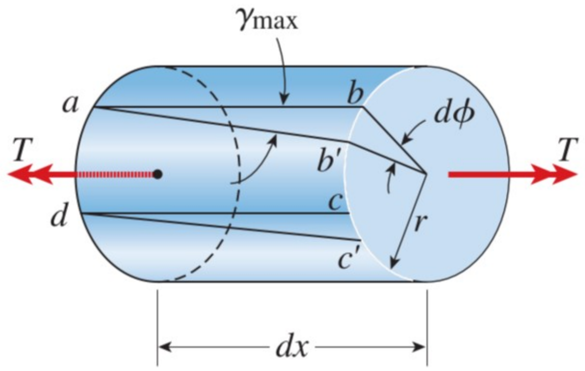


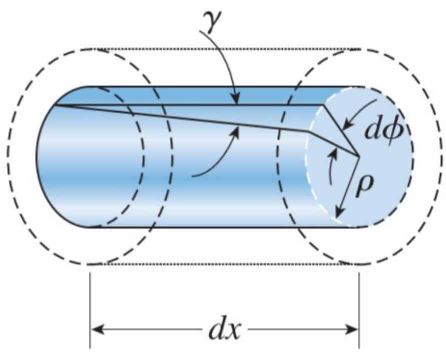
(a)



(b)

Shear Strain at the Outer Surface





$$\gamma_{max} = \frac{bb'}{ab}$$

$$\Rightarrow \quad \gamma_{max} = \frac{r\mathrm{d}\phi}{\mathrm{d}x}$$

denote the symbol θ as the **rate of twist**

$$\theta = \frac{\mathrm{d}\phi}{\mathrm{d}x}$$

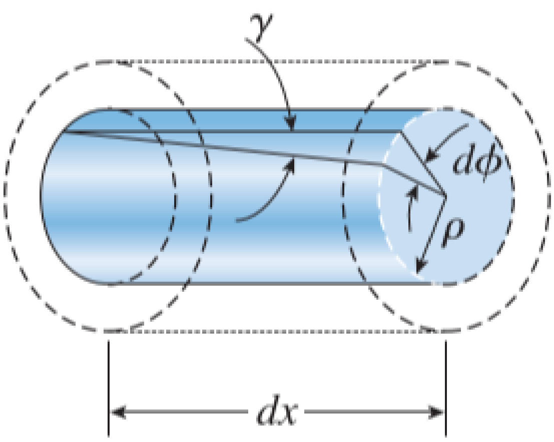
$$\Rightarrow \quad \gamma_{max} = r\theta$$

for pure torsion only, we can obtain that

$$\gamma_{max} = \frac{r\phi}{L}$$

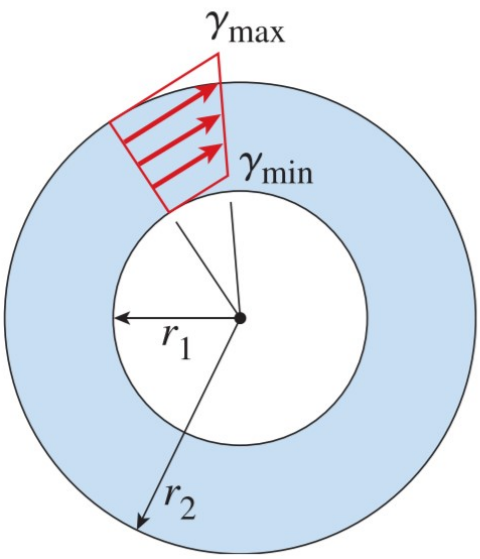
Shear Strain within the bar

$$\gamma = \rho\theta = \frac{\rho}{r}\gamma_{max}$$



Circular Tube

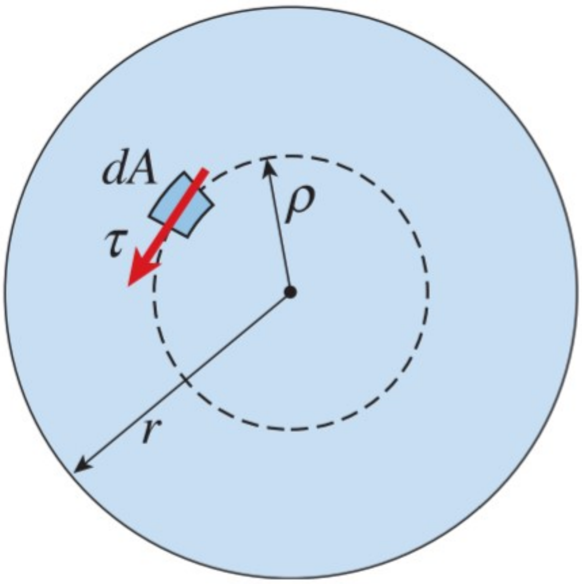
$$\gamma_{max} = \frac{r_2\phi}{L} \qquad \gamma_{min} = \frac{r_1}{r_2}\gamma_{max} = \frac{r_2\phi}{L}$$



3.3 Circular Bars of Linearly Elastic Materials

the shear stresses acting on a cross-sectional plane are accompanied by shear stresses of the same magnitude acting on longitudinal planes.

the Torsion Formula



$$\mathrm{d}M = \tau\rho\mathrm{d}A = \frac{\tau_{max}}{r}\rho^2\mathrm{d}A$$

the resultant moment is the summation over the entire cross-sectional area

$$T = \int_A \mathrm{d}M = \frac{\tau_{max}}{r}\int_A \rho^2\mathrm{d}A$$

in which

$$I_P = \int_A \rho^2\mathrm{d}A$$

is the **polar moment of inertia** of the circular cross section

The shear stress at distance ρ from the center of the bar is

$$\tau_{max} = \frac{Tr}{I_P} \qquad \tau = \frac{T\rho}{I_P} \tag{torsion formula}$$

Solid Shaft

$$\mathrm{d}A = 2\pi\rho\mathrm{d}\rho$$

$$I_P = \frac{\pi}{32}d^4$$

Tubular Shaft

$$I_P = \frac{\pi}{32}(d_0^4 - d_i^4)$$

Angle of Twist

according to Hooke's Law in shear

$$\tau = G\gamma = G\rho\theta = \frac{\rho}{r}\tau_{max}$$

using the torsion formula

$$G\rho\theta = \frac{\rho}{r}\frac{Tr}{I_P}$$
$$\Rightarrow \theta = \sum \frac{T}{GI_P} = \frac{\mathrm{d}\phi}{\mathrm{d}x}$$

for a bar in **pure torsion**, the total angle of the twist ϕ, equal to the rate of twist time the length of the bar

$$\phi = \frac{TL}{GI_P} = \int_0^L \frac{T(x)\mathrm{d}x}{GI_P(x)}$$

Torsional Stiffness and Torsional Flexibility

$$k_T = \frac{GI_P}{L} \qquad f_T = \frac{L}{GI_P}$$