

CH_2

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2.1 Hooke's Law

$$\sigma = E\epsilon$$

- E : the modulus of elasticity of Youngs' modulus, which has the stress units

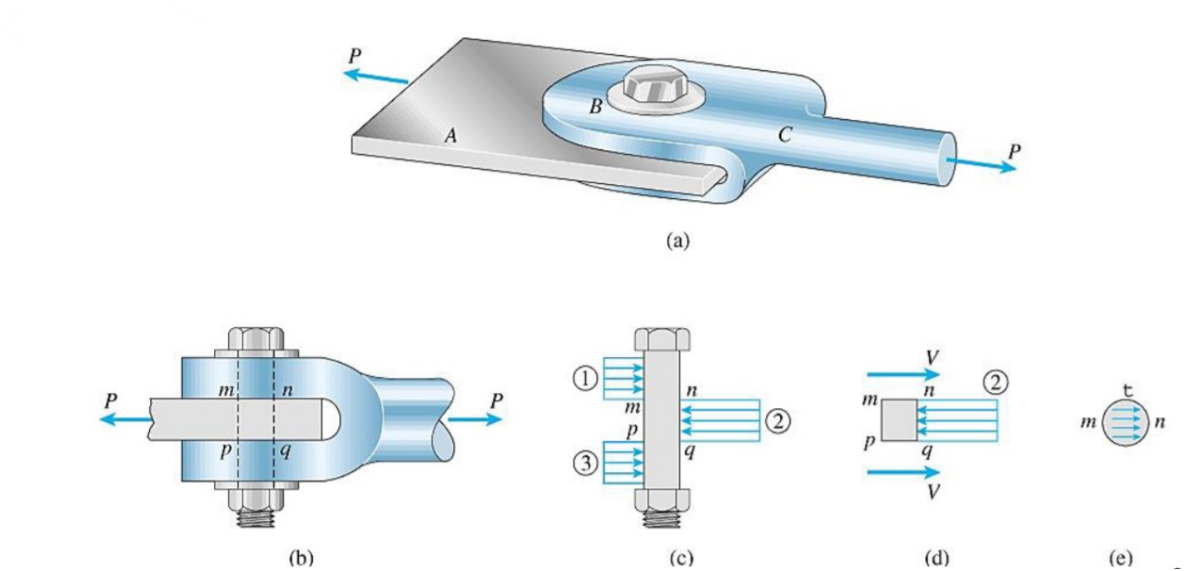
2.2 Poison Ratio

$$\nu = -\frac{\epsilon'}{\epsilon}$$

- ν : Poison Ratio
- ϵ : axial strain
- ϵ' : lateral strain

2.3 Shear Stress

the stress component that act in the plane of the sectioned area



2.4 Average Bearing Stress

$$\sigma_b = \frac{F_b}{A_b}$$

- F_b : bearing force
- A_b : bearing area

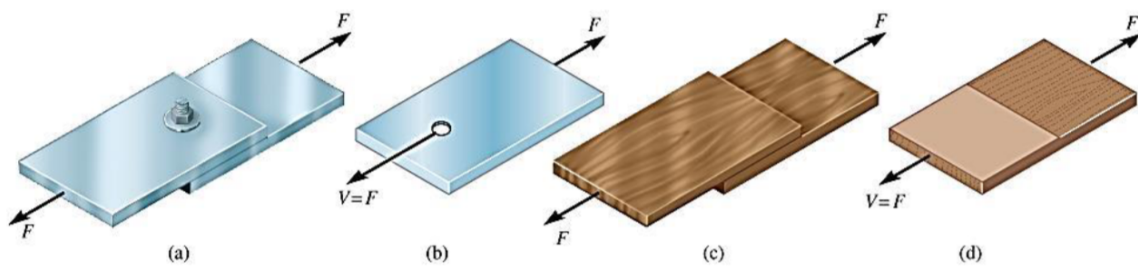
Average Shear Stress at Section

$$\tau_{aver} = \frac{V}{A}$$

- V : internal shear force at section determined from equations of equilibrium
- A : area of section

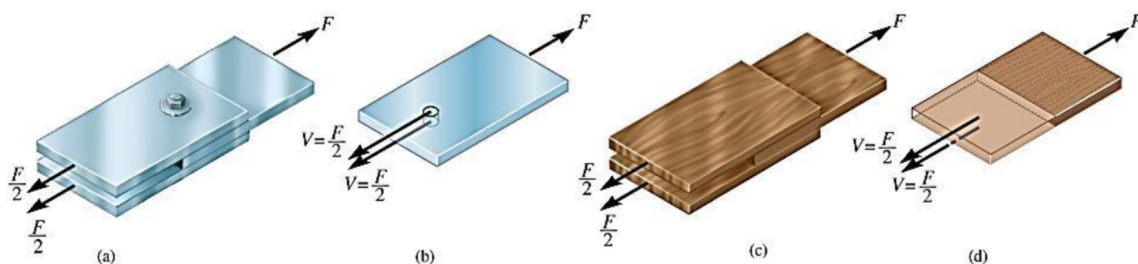
Single Shear

$$V = F$$



Double Shear

$$V = \frac{F}{2}$$



- Positive strain is when the angle between two positive faces is reduced
- Negative strain is when the angle between two positive faces is increased

Hooke's Law for Shear

$$\tau = G\gamma$$

$$G = \frac{E}{2(1 + \nu)}$$

- G : shear modulus of elasticity

2.5 Allowable Stresses and Allowance Loads

When designing a structural member or mechanical element, the design interest is **strength**, that is *the capacity of the object to support or transmit loads*

- factor of safety (F.S.)

$$n = \frac{\text{Actual Strength}}{\text{Required Strength}}$$

- allowable strength

$$\sigma_{\text{allow}} = \frac{\sigma_Y}{n_1}$$

$$\tau_{\text{allow}} = \frac{\tau_Y}{n_2}$$

- σ_Y and τ_Y are yield stresses
- n_1 and n_2 are the corresponding factors of safety.

2.6 Design for Axial Loads and Direct Shear

- determine the area of section subjected to a normal force

$$A = \frac{P}{\sigma_{\text{allow}}}$$

- determine the area of section subjected to a shear force

$$A = \frac{V}{\tau_{\text{allow}}}$$

2.7 Changes in Lengths of An Axially Loaded Members

$$P = k\delta \quad \delta = fP$$

k is the stiffness of the spring

f is the flexibility of the spring

- **stiffness**: is the force required to produce a unit elongation
- **flexibility**: is the elongation produced by a unit force

$$\begin{cases} \delta = L \cdot \varepsilon \\ \sigma = E \cdot \varepsilon \\ P = \sigma \cdot A \end{cases}$$

$$\Rightarrow \delta = \frac{L}{EA} P$$

Sign Convention

Sign	Forces	Displacement
Positive(+)	Tension	Elongation
Negative(-)	Compression	Contraction

Use Method of Sections

$$\begin{cases} \sigma = \frac{P(x)}{A(x)} = E\varepsilon \\ \varepsilon = \frac{d\delta}{dx} \end{cases}$$

$$\Rightarrow d\delta = \frac{P(x)}{A(x)E} dx$$

$$\delta = \int_0^L \frac{P(x)}{A(x)E} dx$$

- $P(x)$: internal normal force at the section, located a distance x from one end
- $A(x)$: x-sectional area of the bar, expressed as a function of x

Constant Load and x-Sectional Area

$$\delta = \frac{L}{EA} P$$

Bars with Intermediate Axial Loads

$$\delta = \sum \frac{L}{EA} P$$

2.8 Statically Indeterminate Structures

if the bar is **fixed at both ends**, then the unknown axial reactions occur, and the bar is statically indeterminate.

$$\delta_{A/B} = 0$$

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$$

$$\Rightarrow F_A = P\left(\frac{L_{CB}}{L}\right) \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

2.9 Thermal Stress

expansion or contraction of material is linearly related to temperature increase or decrease that occurs

$$\varepsilon_T = \alpha \Delta T \quad \delta_T = \alpha \Delta T L$$

α : liner coefficient of thermal expansion