# **CH\_7**

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7.1 Plane Stress

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Special Cases of Plane Stress

**Uniaxial Stress** 

Pure Shear

**Biaxial Stress** 

7.2 Principal Stresses and Maximum Shear Stresses

**Principal Stresses** 

Principle Angles

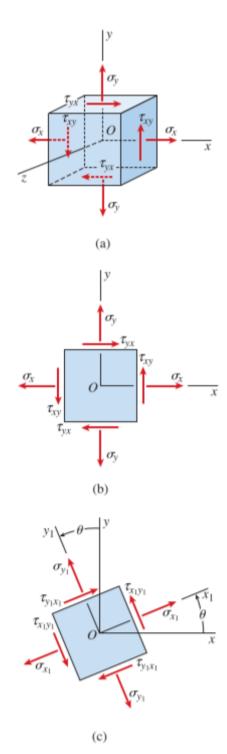
**Shear Stresses** 

**Special Cases** 

**Maximum Shear Stresses** 

### 7.1 Plane Stress

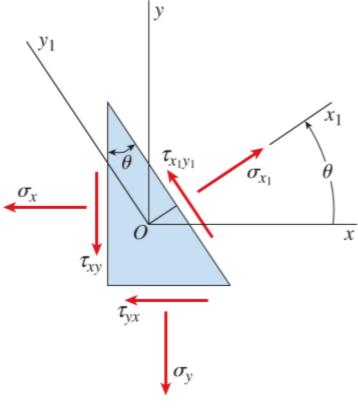
bars in tension and compression, shafts in torsion, and beams in bending are examples of a state of stress called **plane stress** 



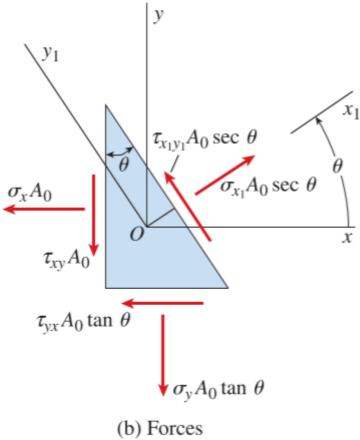
- **Normal Stress**: has a subscript that identifies the face on which the stress acts. For example:  $\sigma_x$  and  $\sigma_y$
- **Shear Stress**: has two subscripts: the first denotes the face on which the stress acts, and the second gives the direction on that face. For example:  $\tau_{xy}$  and  $\tau_{yx}$
- **Sign Convention for Shear Stresses**: a shear stress is positive when the directions associated with its subscripts are plus-plus or minus-minus; the stress is negative when the directions are plus-minus or minus-plus.

$$au_{xy} = au_{yx}$$

## **Transformation Equations for Plane Stress**



## (a) Stresses



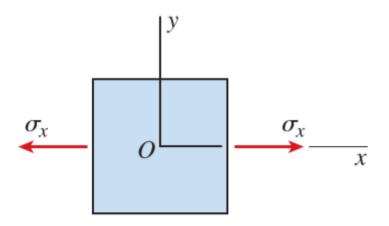
$$egin{aligned} \sigma_{x1} &= rac{\sigma_x + \sigma_y}{2} + rac{\sigma_x - \sigma_y}{2} \cos 2 heta + au_{xy} \sin 2 heta \ & \ \sigma_{y1} &= rac{\sigma_x + \sigma_y}{2} - rac{\sigma_x - \sigma_y}{2} \cos 2 heta - au_{xy} \sin 2 heta \ & \ au_{x_1y_1} &= -rac{\sigma_x - \sigma_y}{2} \sin 2 heta + au_{xy} \cos 2 heta \end{aligned}$$

## **Special Cases of Plane Stress**

#### **Uniaxial Stress**

$$\sigma_{x_1} = rac{\sigma_x}{2}(1+\cos 2 heta) \qquad au_{x_1y_1} = -rac{\sigma_x}{2}\sin 2 heta$$

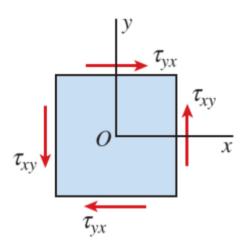
### Element in uniaxial stress



#### **Pure Shear**

$$\sigma_{x_1} = au_{xy} \sin 2 heta \qquad au_{x_1y_1} = au_{xy} \cos 2 heta$$

## Element in pure shear



#### **Biaxial Stress**

$$\sigma_{x_1} = rac{\sigma_x + \sigma_y}{2} + rac{\sigma_x - \sigma_y}{2} \cos 2 heta$$
 $au_{x_1y_1} = -rac{\sigma_x - \sigma_y}{2} \sin 2 heta$ 

## 7.2 Principal Stresses and Maximum Shear Stresses

#### **Principal Stresses**

$$egin{aligned} rac{\mathrm{d}\sigma_{x_1}}{\mathrm{d} heta} &= -(\sigma_x - \sigma_y)\sin2 heta + 2 au_{xy}\cos2 heta = 0 \ an2 heta_p &= rac{2 au_{xy}}{\sigma_x - \sigma_y} \end{aligned}$$

Define 
$$R=\sqrt{\left(rac{\sigma_x-\sigma_y}{2}
ight)^2+ au_{xy}^2}$$
 , then  $\cos 2 heta_p=rac{\sigma_x-\sigma_y}{2R}$   $\sin 2 heta_p=rac{ au_{xy}}{R}$ 

then  $\sigma_1$  is denoted as:

$$egin{aligned} \sigma_1 &= rac{\sigma_x + \sigma_y}{2} + rac{\sigma_x - \sigma_y}{2} \cos 2 heta_p + au_{xy} \sin 2 heta_p \ &= rac{\sigma_x + \sigma_y}{2} + rac{\sigma_x - \sigma_y}{2} rac{\sigma_x - \sigma_y}{2R} + au_{xy} rac{ au_{xy}}{R} \ &= rac{\sigma_x + \sigma_y}{2} + \sqrt{\left(rac{\sigma_x - \sigma_y}{2}
ight)^2 + au_{xy}^2} \end{aligned}$$

and  $\sigma_1+\sigma_2=\sigma_x+\sigma_y$ , then  $\sigma_2$  is denoted as

$$egin{aligned} \sigma_2 &= \sigma_x + \sigma_y - \sigma_1 \ &= rac{\sigma_x + \sigma_y}{2} - \sqrt{\left(rac{\sigma_x - \sigma_y}{2}
ight)^2 + au_{xy}^2} \end{aligned}$$

where  $\sigma_1$  and  $\sigma_2$  are the principle stresses

### **Principle Angles**

$$\cos 2 heta_{p1} = rac{\sigma_x - \sigma_y}{2R} \qquad \sin 2 heta_{p1} = rac{ au_{xy}}{R}$$

the value is unique for  $\theta_{p1}$ , and the angle for  $\theta_{p2}$  is perpendicular to the previous one, which is 90 degree larger or less than  $\theta_{p1}$ 

#### **Shear Stresses**

shear stresses are zero on the principle planes

### **Special Cases**

- ullet uniaxial stress (biaxial stress):  $an 2 heta_p = 0$
- ullet pure shear:  $an 2 heta_p = \infty$ , stresses come to  $\sigma_1 = au_{xy}$  and  $\sigma_2 = - au_{xy}$

#### **Maximum Shear Stresses**

similar to the principle stresses

$$rac{\mathrm{d} au_{xy}}{\mathrm{d} heta} = -(\sigma_x - \sigma_y)\cos 2 heta + 2 au_{xy}\sin 2 heta = 0 
onumber \ an 2 heta_s = -rac{\sigma_x - \sigma_y}{2 au_{xy}}$$

since 
$$an 2 heta_p = rac{2 au_{xy}}{\sigma_x - \sigma_y}$$
, then  $\cos(2 heta_s - 2 heta_p) = 0$ 

$$egin{align} heta_s &= heta_p \pm 45^\circ \ &\sigma_{s_1} &= heta_{p_1} - 45^\circ \ & au_{max} &= \sqrt{\left(rac{\sigma_x - \sigma_y}{2}
ight)^2 + au_{xy}^2} = rac{\sigma_1 - \sigma_2}{2} \ &\sigma_{aver} &= rac{\sigma_x + \sigma_y}{2} \ \end{gathered}$$