CH_9

9.1 Introduction

Deflection Curve

definition: the axis is deformed to into a curve while loading by a lateral forces

9.2 Differential Equations of The Deflection Curve

Beams with Small Angles of Rotation

$$\kappa = \frac{1}{\rho} = \frac{\mathrm{d}\theta}{\mathrm{d}x}$$

$$\theta \approx \tan\theta = \frac{\mathrm{d}v}{\mathrm{d}x}$$

$$\kappa = \frac{1}{\rho} = \frac{\mathrm{d}\theta}{\mathrm{d}x} = \frac{\mathrm{d}^2 v}{\mathrm{d}x^2}$$

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

$$\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} = \frac{M}{EI}$$

Nonprismatic Beams

In the case of a nonprismatic beam, the flexural rigidity EI is variable, and therefore

$$EI_x \frac{\mathrm{d}^2 v}{\mathrm{d}x^2} = M$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(EI_x \frac{\mathrm{d}^2 v}{\mathrm{d}x^2}) = \frac{\mathrm{d}M}{\mathrm{d}x} = V$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}(EI_x \frac{\mathrm{d}^2 v}{\mathrm{d}x^2}) = \frac{\mathrm{d}V}{\mathrm{d}x} = -q$$

Prismatic Beams

In the case of a prismatic beam, the flextural rigidity is constant, and therefore

$$EI\frac{\mathrm{d}^2v}{\mathrm{d}x^2} = M \qquad EI\frac{\mathrm{d}^3v}{\mathrm{d}x^3} = V \qquad EI\frac{\mathrm{d}^2v}{\mathrm{d}x^2} = -q$$

$$EIv'' = M \qquad EIv''' = V \qquad EIv'''' = -q$$

Exact Expression for Curvature

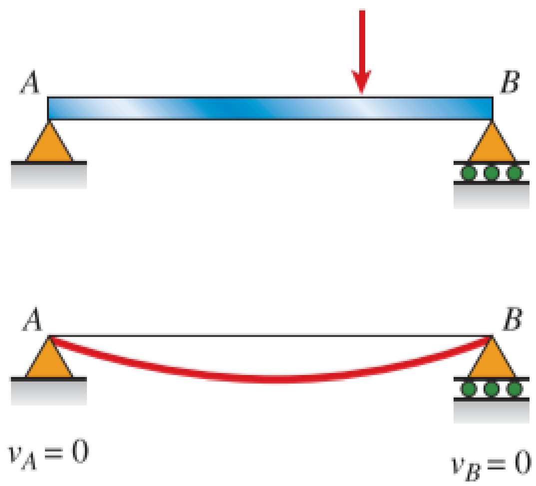
Like the curvature in calculus, the exact expression can be written as

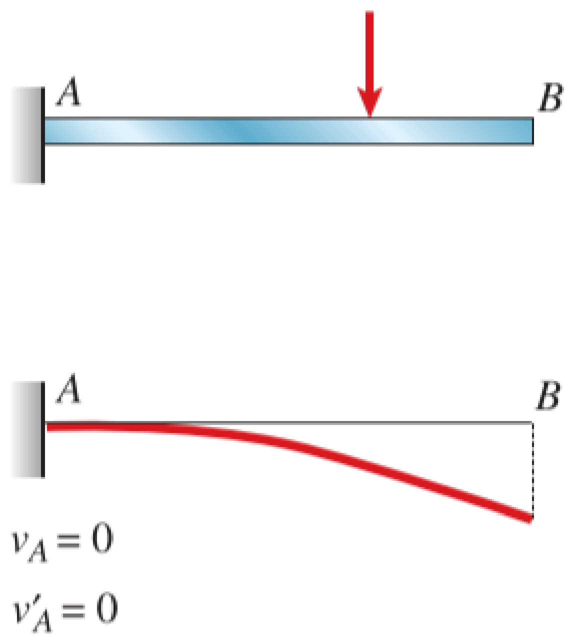
$$\kappa = \frac{1}{\rho} = \frac{v''}{[1+(v')^2]^{3/2}}$$

9.3 Deflections by Integration of the Bending-Moment Equation

Boundary Conditions

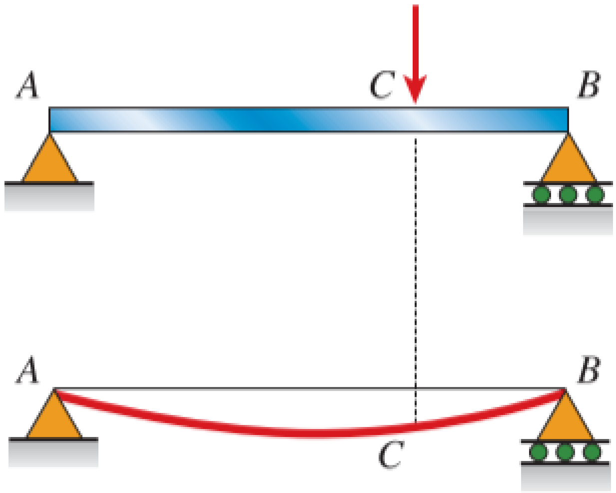
the deflections and slope at the supports of a beam





Continuity Conditions

occur at points where the regions of integration meet



At point C: $(v)_{AC} = (v)_{CB}$

$(v')_{AC} = (v')_{CB}$

Symmetry Conditions

the beam is loaded by uniformly distributed