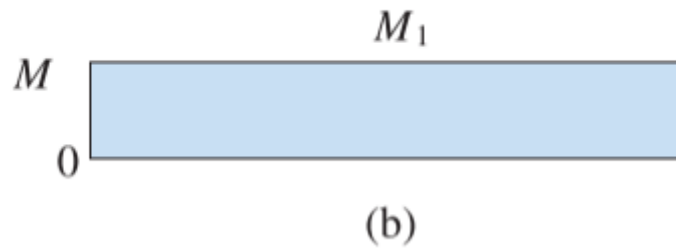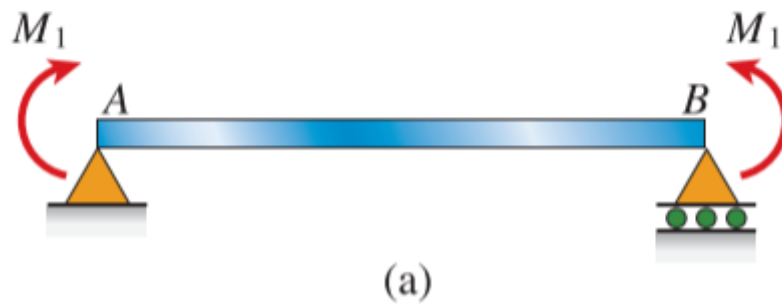# CH_5

5.1 Pure Bending and Nonuniform Bending

Pure Bending

the flexure of a beam under a constant bending moment

Nonuniform Bending

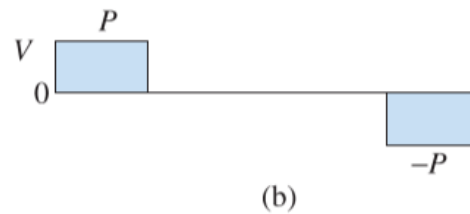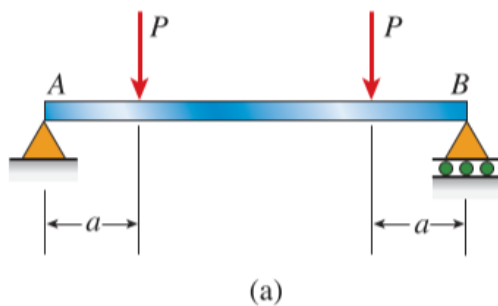the flexure in the presence of shear forces

Simple beam in pure bending $(M = M_1)$

(a)

(b)

Cantilever beam in pure bending $(M = -M_2)$

(a)

(b)

(a)

(b)

(c)
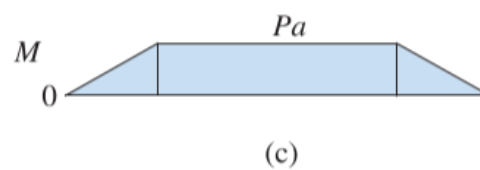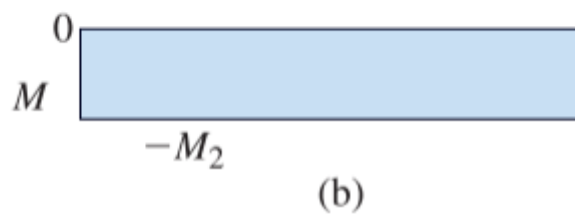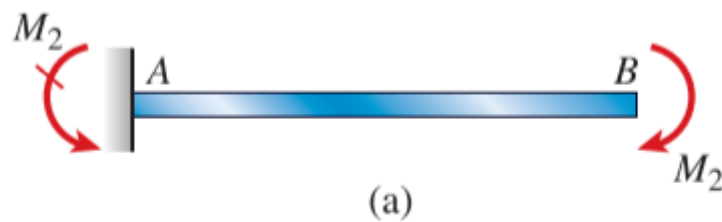
5.2 Curvature of a Beam

(a)
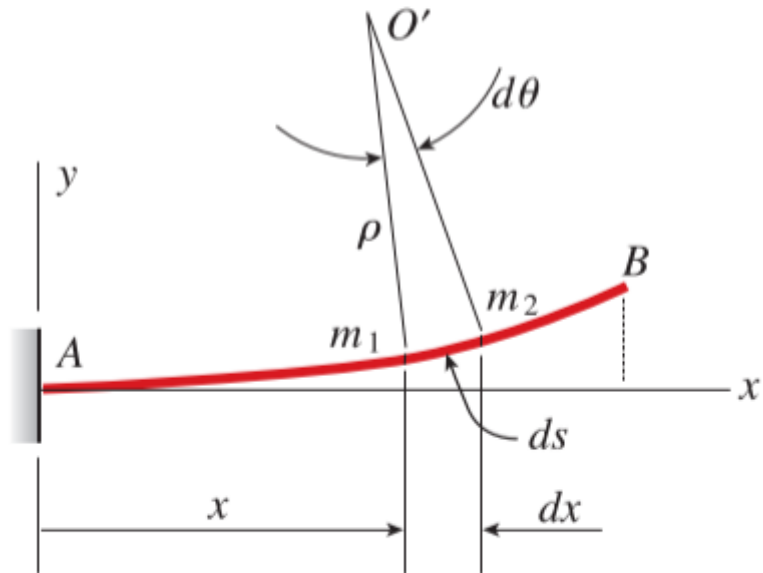
(b)

$$\rho \mathrm{d}\theta = \mathrm{d}s$$

$$\kappa = \frac{1}{\rho} = \frac{\mathrm{d}\theta}{\mathrm{d}s}$$

$$= \frac{\mathrm{d}\theta}{\mathrm{d}x}$$

5.3 Longitudinal Strains in Beams

(a)

(b)

(c)
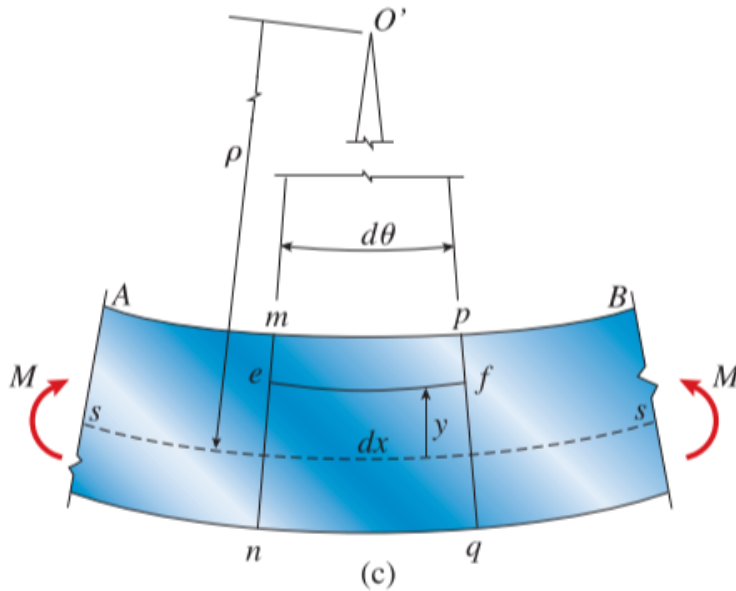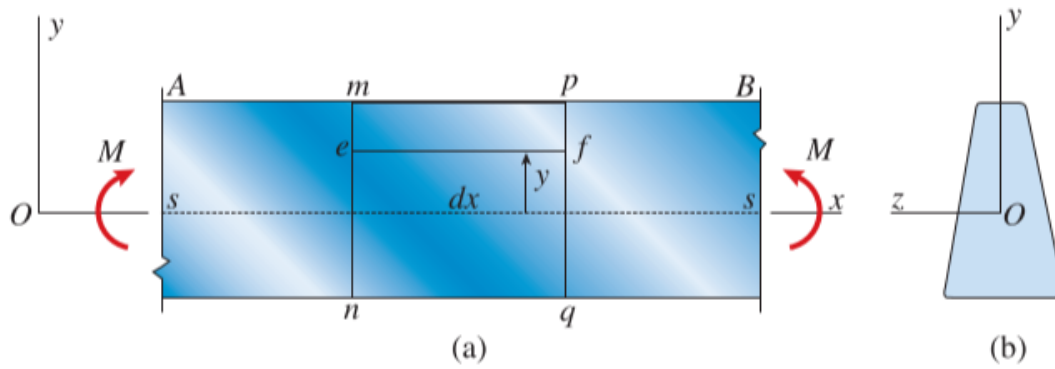
$$\widehat{ef} = (\rho - y)\mathrm{d}\theta \qquad \mathrm{d}x = \rho\mathrm{d}\theta$$

$$\epsilon = \frac{\widehat{ef} - \mathrm{d}x}{\mathrm{d}x} = -\frac{y}{\rho}$$

5.4 Normal Stress in Beams

$$\sigma_x = E\epsilon_x = \frac{Ey}{\rho} = -E\kappa y$$

Location of Neutral Axis

$$\int_A \sigma_x \mathrm{d}A = -\int_A E\kappa y \mathrm{d}A = 0$$

$$\int_A y\mathrm{d}A = 0$$

Moment Curvature Relationship

$$\mathrm{d}M = -\sigma_x y \mathrm{d}A$$

$$M = -\int_A \sigma_x y \mathrm{d}A$$

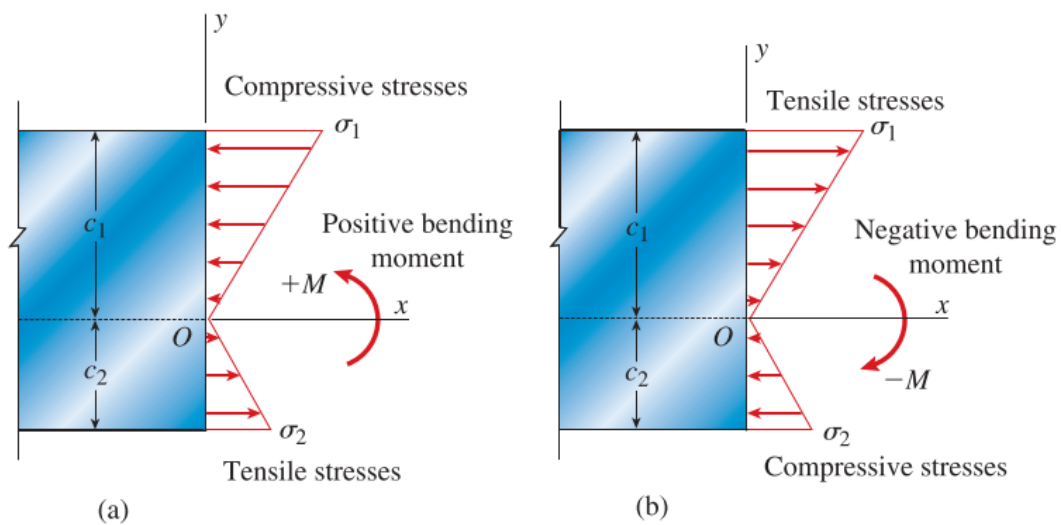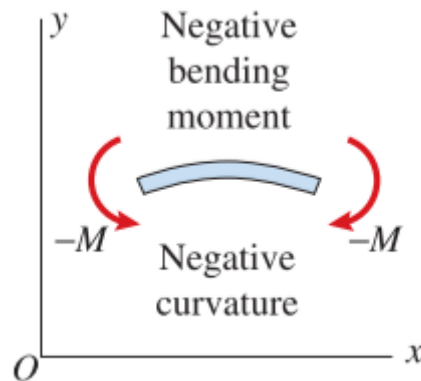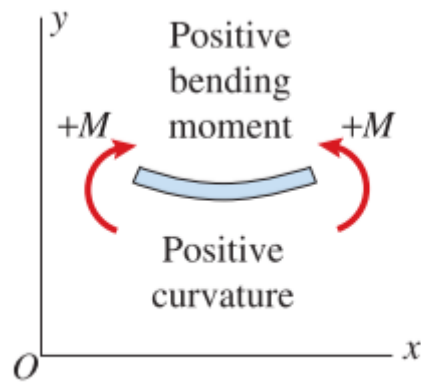$$= \int_A \kappa E y^2 \mathrm{d}A = \kappa E \int_A y^2 \mathrm{d}A$$

$$= \kappa E I$$

$$\kappa = \frac{1}{\rho} = \frac{M}{EI}$$

Flexure Formula

bending stresses (flexure stresses)

$$\sigma_x = -\frac{My}{I}$$

(a)   (b)

Maximum Stresses at a Cross Section

the *maximum tensile* and *compressive bending stresses* acting at any given cross section occur at points **located farthest from the neutral axis**

$$\sigma_{max} = -\frac{Mc}{I} = -\frac{M}{S}$$

$$S = \frac{I}{c}$$

where S is known as the **section moduli** of the cross-sectional area

Doubly Symmetric Shapes

Rectangular Cross Section

$$I = \frac{bh^3}{12} \qquad S = \frac{bh^3}{6}$$

Circular Cross Section

$$I = \frac{\pi d^4}{64} \qquad S = \frac{\pi d^3}{32}$$