CH_2

2.1 Hooke's Law

$$\sigma = E\varepsilon = \frac{E\delta}{L} = \frac{P}{A}$$

- E: the modulus of elasticity of Youngs' modulus, which has the stress units
- σ: stress
- ε: strain, which equals the ratio of the increasing length of the length

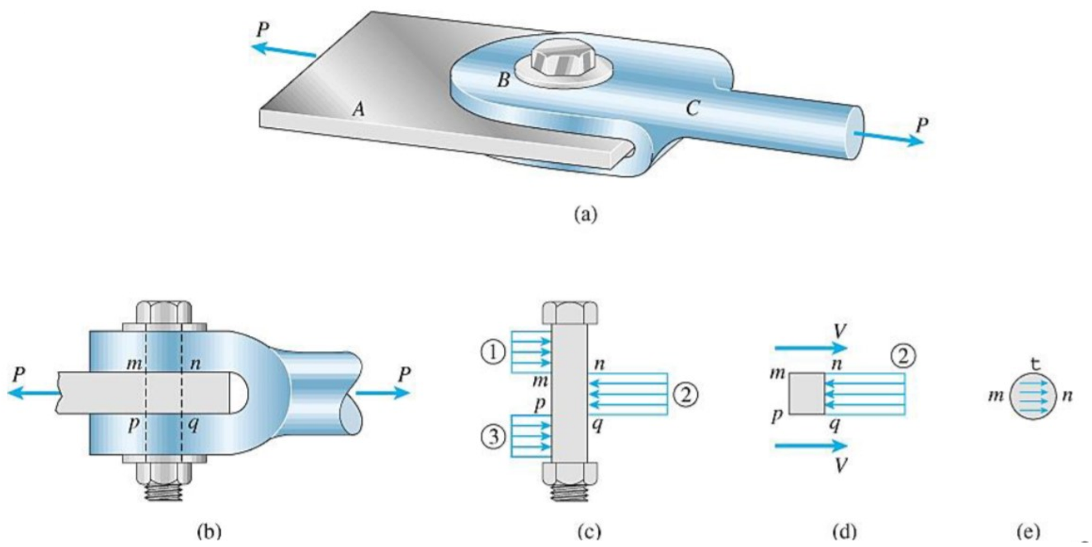
2.2 Poison Ratio

$$\nu = -\frac{\varepsilon'}{\varepsilon}$$

- ν: Poison Ratio
- ε: axial strain(轴向拉伸)
- ε': lateral strain(侧向拉伸)

2.3 Shear Stress

the stress component that act in the plane of the sectioned area



Average Bearing Stress

$$\sigma_b = \frac{F_b}{A_b}$$

- F_b: bearing force
- A_b: bearing area

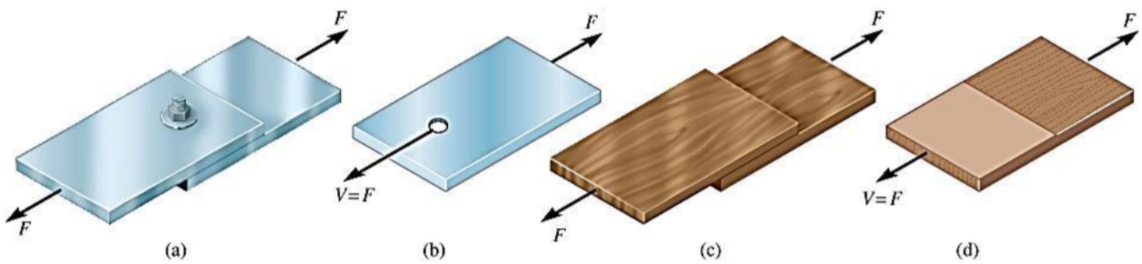
Average Shear Stress at Section

$$\tau_{aver} = \frac{V}{A}$$

- V: internal shear force at section determined from equations of equilibrium
- A: area of section

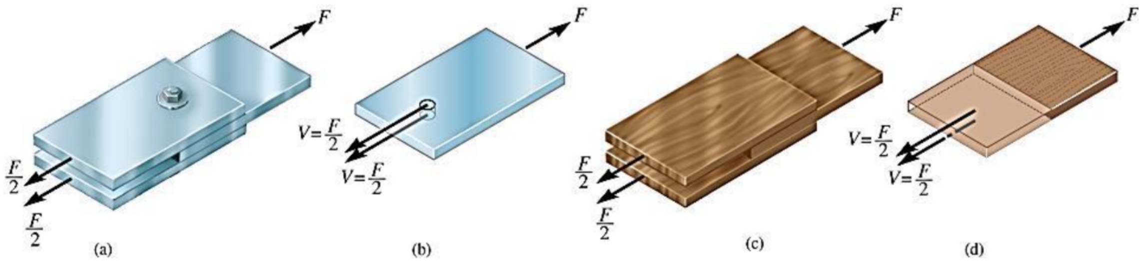
Single Shear

$$V = F$$



Double Shear

$$V = \frac{F}{2}$$



- Positive strain is when the angle between two positive faces is reduced
- Negative strain is when the angle between two positive faces is increased

Hooke's Law for Shear

$$\tau = G\gamma$$

$$G = \frac{E}{2(1+\nu)}$$

- G: shear modulus of elasticity

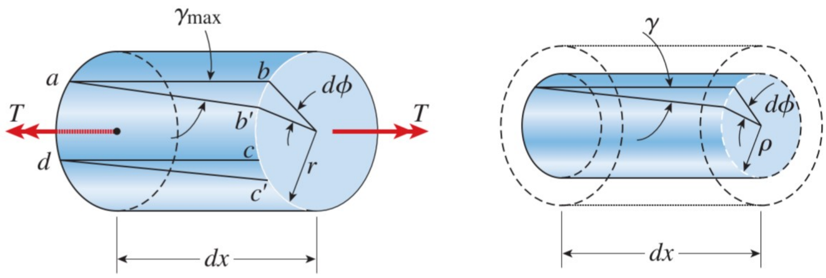
2.4 Allowable Stresses and Allowance Loads

When designing a structural member or mechanical element, the design interest is **strength**, that is *the capacity of the object to support or transmit loads*

- factor of safety (F.S.)

$$n = \frac{\text{Actual Strength}}{\text{Required Strength}}$$

- allowable strength



2.5 Design for Axial Loads and Direct Shear

- determine the area of section subjected to a normal force

$$A = \frac{P}{\sigma_{allow}}$$

- determine the area of section subjected to a shear force

$$A = \frac{V}{\tau_{allow}}$$

2.6 Changes in Lengths of An Axially Loaded Members

$$P = k\delta \qquad \delta = fP$$

$$k \text{ is the stiffness of the spring}$$

$$f \text{ is the flexibility of the spring}$$

- **stiffness**: is the force required to produce a unit elongation
- **flexibility**: is the elongation produced by a unit force

$$\begin{cases} \delta = L \cdot \varepsilon \\ \sigma = E \cdot \varepsilon \\ P = \sigma \cdot A \end{cases}$$

$$\Rightarrow \ \delta = \frac{L}{EA}P$$

Sign Convention

Sign	Forces	Displacement
Positive(+)	Tension	Elongation
Negative(-)	Compression	Contraction

Use Method of Sections

$$\begin{cases} \sigma = \frac{P(x)}{A(x)} = E\varepsilon \\ \varepsilon = \frac{d\delta}{dx} \end{cases}$$

$$\Rightarrow d\delta = \frac{P(x)}{A(x)E}dx$$

$$\delta = \int_0^L \frac{P(x)}{A(x)E}dx$$

- $P(x)$: internal normal force at the section, located a distance x from one end
- $A(x)$: x-sectional area of the bar, expressed as a function of x

Constant Load and x-Sectional Area

$$\delta = \frac{L}{EA}P$$

Bars with Intermediate Axial Loads

$$\delta = \sum \frac{L}{EA}P$$

2.7 Statically Indeterminate Structures

if the bar is **fixed at both ends**, then the unknown axial reactions occur, and the bar is statically indeterminate.

$$\delta_{A/B} = 0$$

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0$$

$$\Rightarrow F_A = P(\frac{L_C B}{L}) \qquad F_B = P(\frac{L_A C}{L})$$

2.8 Thermal Stress

expansion or contraction of material is linearly related to temperature increase or decrease that occurs

$$\varepsilon_T = \alpha \Delta T \qquad \delta_T = \alpha \Delta T L$$

α: liner coefficient of thermal expansion