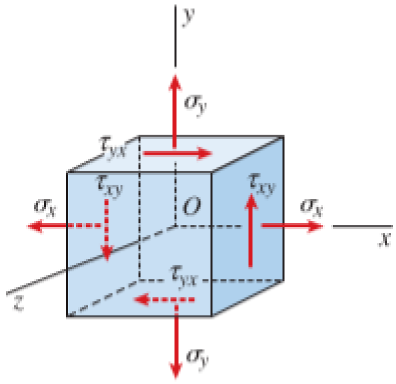
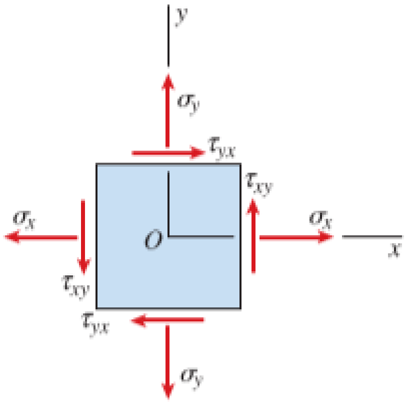
CH_7

7.1 Plane Stress

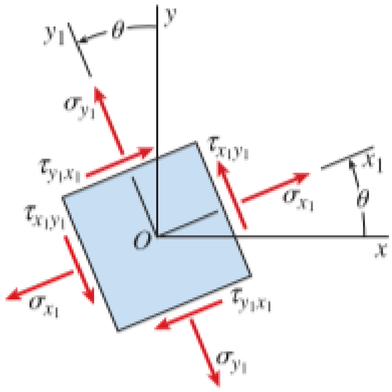
bars in tension and compression, shafts in torsion, and beams in bending are examples of a state of stress called **plane stress**



(a)



(b)

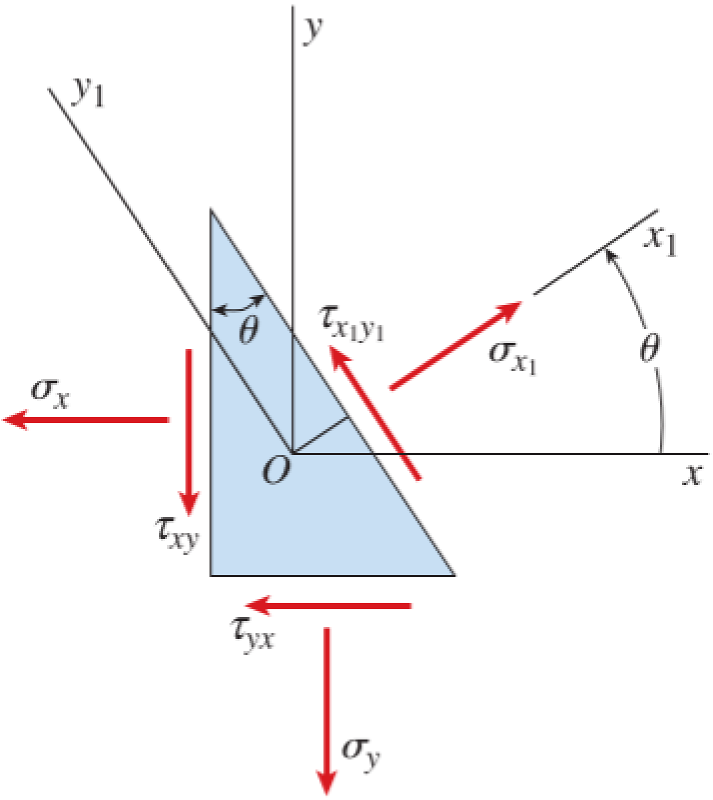


(c)

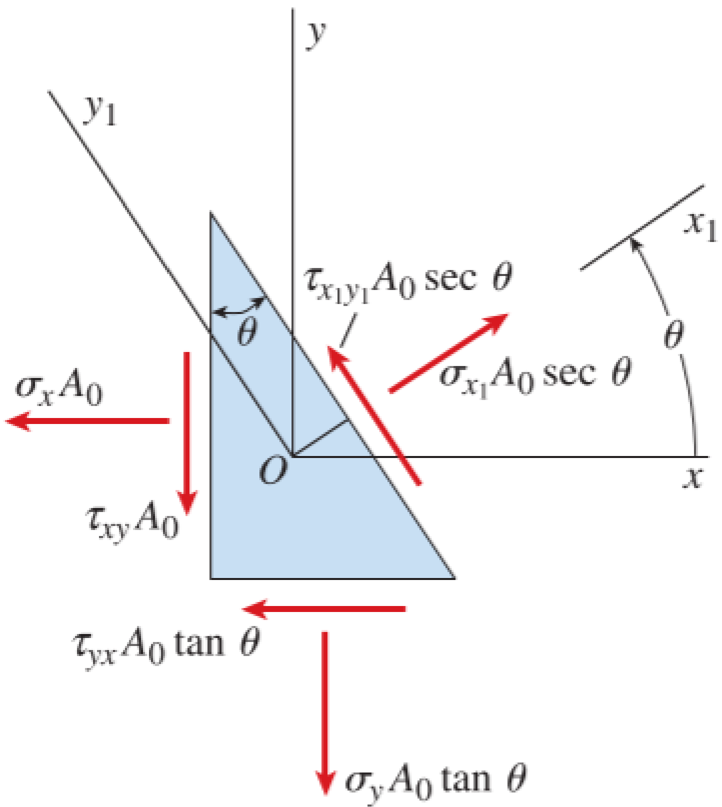
- **Normal Stress**: has a subscript that identifies the face on which the stress acts. For example: σ_x and σ_y
- **Shear Stress**: has two subscripts: the first denotes the face on which the stress acts, and the second gives the direction on that face. For example: τ_{xy} and τ_{yx}
- **Sign Convention for Shear Stresses**: a shear stress is positive when the directions associated with its subscripts are plus-plus or minus-minus; the stress is negative when the directions are plus-minus or minus-plus.

$$\tau_{xy} = \tau_{yx}$$

Transformation Equations for Plane Stress



(a) Stresses



(b) Forces

$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

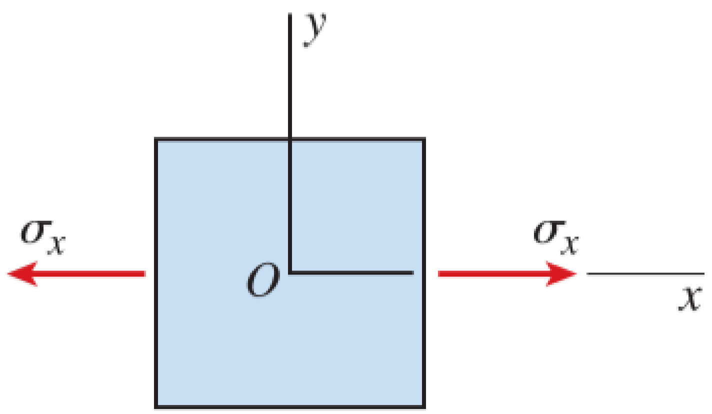
$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Special Cases of Plane Stress

Uniaxial Stress

$$\sigma_{x_1} = \frac{\sigma_x}{2}(1 + \cos 2\theta) \qquad \tau_{x_1y_1} = -\frac{\sigma_x}{2}\sin 2\theta$$

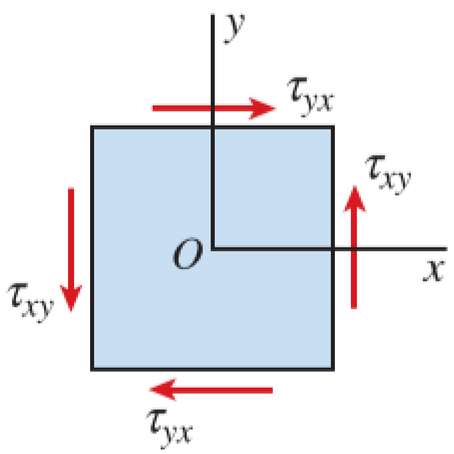
Element in uniaxial stress



Pure Shear

$$\sigma_{x_1} = \tau_{xy}\sin 2\theta \qquad \tau_{x_1y_1} = \tau_{xy}\cos 2\theta$$

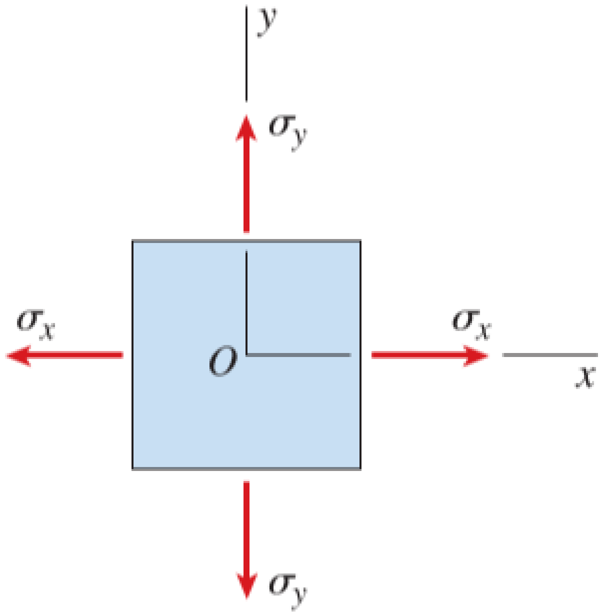
Element in pure shear



Biaxial Stress

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$\tau_{x_1 y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$



7.2 Principal Stresses and Maximum Shear Stresses

Principal Stresses

$$\frac{\mathrm{d}\sigma_{x_1}}{\mathrm{d}\theta} = -(\sigma_x - \sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Define $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$, then $\cos 2\theta_p = \frac{\sigma_x - \sigma_y}{2R} \quad \sin 2\theta_p = \frac{\tau_{xy}}{R}$

then σ_1 is denoted as:

$$\begin{aligned}\sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta_p + \tau_{xy}\sin 2\theta_p \\ &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\frac{\sigma_x - \sigma_y}{2R} + \tau_{xy}\frac{\tau_{xy}}{R} \\ &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\end{aligned}$$

and $\sigma_1 + \sigma_2 = \sigma_x + \sigma_y$, then σ_2 is denoted as

$$\begin{aligned}\sigma_2 &= \sigma_x + \sigma_y - \sigma_1 \\ &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}\end{aligned}$$

where σ_1 and σ_2 are the principle stresses

Principle Angles

$$\cos 2\theta_{p1} = \frac{\sigma_x - \sigma_y}{2R} \qquad \sin 2\theta_{p1} = \frac{\tau_{xy}}{R}$$

the value is unique for θ_{p1}, and the angle for θ_{p2} is perpendicular to the previous one, which is 90 degree larger or less than θ_{p1}

Shear Stresses

shear stresses are zero on the principle planes

Special Cases

- uniaxial stress (biaxial stress): $\tan 2\theta_p = 0$
- pure shear: $\tan 2\theta_p = \infty$, stresses come to $\sigma_1 = \tau_{xy}$ and $\sigma_2 = -\tau_{xy}$

Maximum Shear Stresses

similar to the principle stresses

$$\frac{\mathrm{d}\tau_{xy}}{\mathrm{d}\theta} = -(\sigma_x - \sigma_y)\cos 2\theta + 2\tau_{xy}\sin 2\theta = 0$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

since $\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$, then $\cos(2\theta_s - 2\theta_p) = 0$

$$\theta_s = \theta_p \pm 45^\circ$$

$$\sigma_{s_1} = \theta_{p_1} - 45^\circ$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2}$$