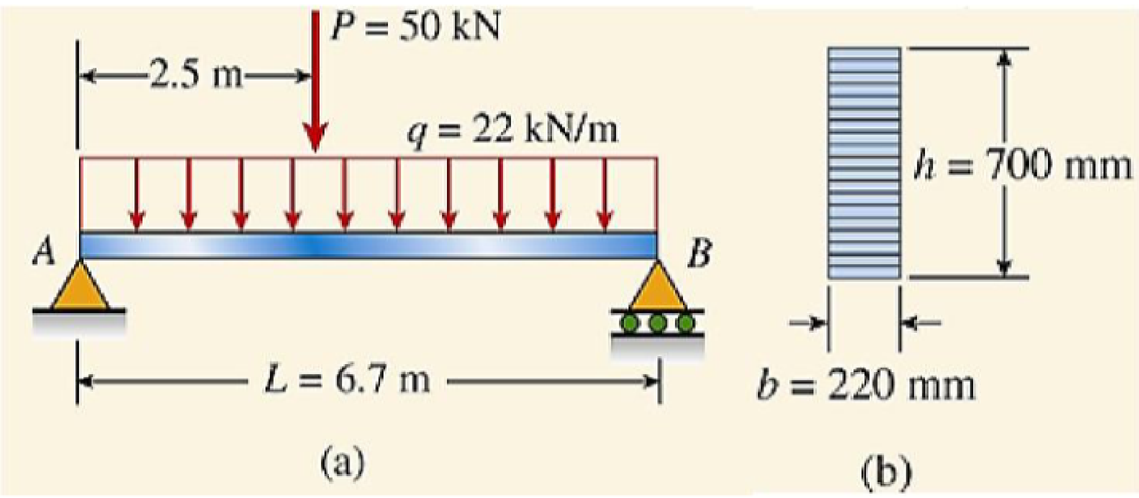
EX_5

EX 5.1

A simple beam AB of span length L=6.7m supports a uniform load of intensity q=22kN/m and a concentrated load P=50kN. The concentrated load acts at a point 2.5 m from the left-hand end of the beam. The beam is constructed of glued laminated wood and has a cross section of width b=220mm and height h=700mm.

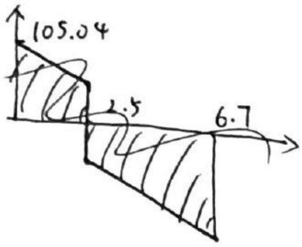
Determine the maximum tensile and compressive stresses in the beam due to loading.

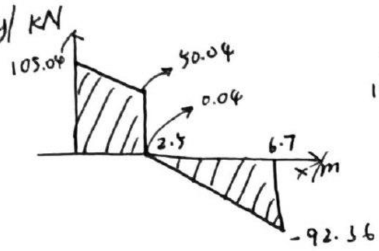


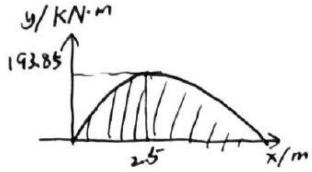
$$R_A + R_B = 50 + 22\times 6.7$$

$$R_B L = 50\times 2.5 + 22\times 6.7\times 6.7\times \frac{1}{2}$$

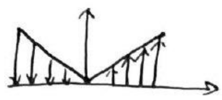
$$\Rightarrow \begin{cases} R_A = 105.04\ kN \\ R_B = 92.36\ kN \end{cases}$$







$$I = \frac{1}{12} b h^3 = 6.280\times 10^{-3}\ m^4$$



$\therefore\ \cancel{\sigma_1 = \sigma_2 = \frac{M\cdot\frac{h}{2}}{I} =}$

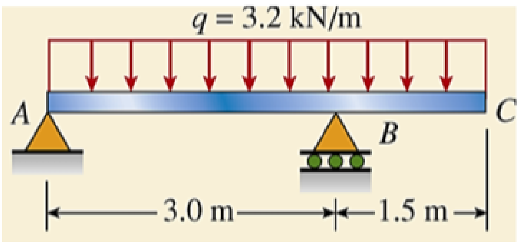
$$\sigma_1 = -\frac{M\left(\frac{h}{2}\right)}{I} = -10.80\ kPa$$

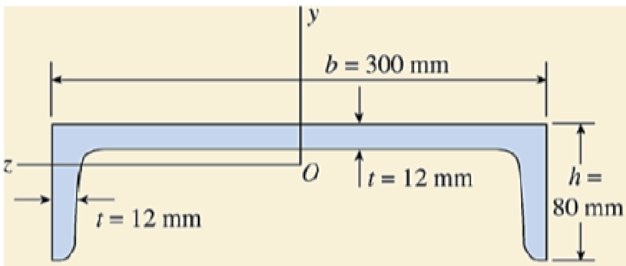
$$\sigma_2 = -\frac{M\left(-\frac{h}{2}\right)}{I} = 10.80\ kPa.$$

EX 5.2

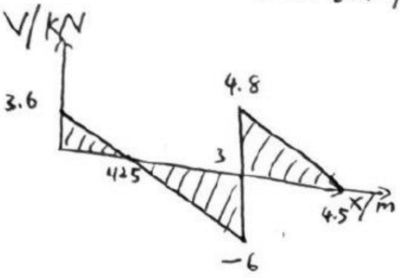
The beam ABC has simple supports at A and B and an overhang from B to C. The length of the span is 3 m and the length of the overhang is 1.5 m. A uniform load of intensity q=3.2 kN/m acts throughout the entire length of the beam. The beam has a cross section of channel shape.

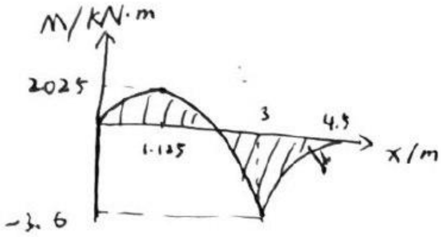
Determine the maximum tensile and compressive stresses in the beam due to loading.

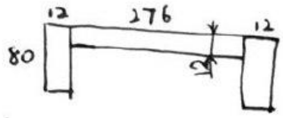




$$\left.\begin{aligned} R_A + R_B &= 3.2\times 4.5 \\ 3R_B &= 3.2\times 4.5\times \tfrac{1}{2}\times 4.5 \end{aligned}\right\} \Rightarrow \begin{cases} R_A = 3.6\ \text{kN} \\ R_B = 10.8\ \text{kN} \end{cases}$$







$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{2\times12\times80\times40 + 276\times12\times6}{2\times12\times80 + 276\times12} = 18.48\,mm$$

$$d_i = \bar{y}_i - \bar{y}$$

$$I = \sum (I_i + A_i d_i^2) = \frac{1}{12}\times276\times12^3 + 276\times12\times(\bar{y}-6)^2$$

$$+ 2\times\left(\frac{1}{12}\times12\times80^3 + 12\times80\times(\bar{y}-40)^2\right)$$

$$= 2.469\times10^{-6}\ m^4$$

$$\sigma_c = \frac{M\bar{y}}{I} \qquad \sigma_t = -\frac{M(\bar{y}-h)}{I}$$

for positive moment, $\cancel{\sigma_c < 0,\ \sigma_t > 0}$

$$\sigma_c = -\frac{M\bar{y}}{I} = -15.16\ MPa$$

$$\sigma_t = -\frac{M(\bar{y}-80)}{I} = 50.46\ MPa$$

$$\sigma_c = -\frac{M(\bar{y}-h)}{I}, \quad \sigma_t = -\frac{M\bar{y}}{I}$$

for negative moment $\cancel{\sigma_c > 0,\ \sigma_t < 0}$

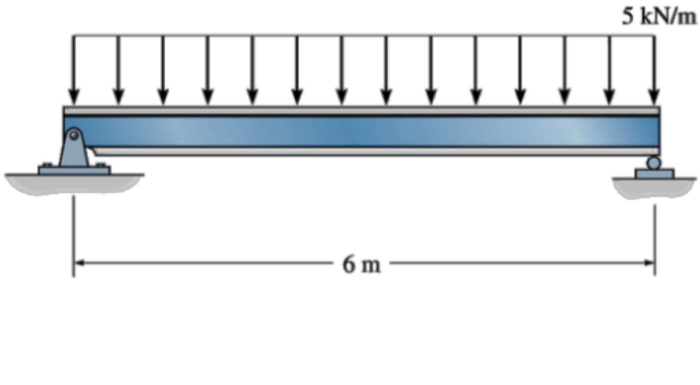
$$\sigma_c = -\frac{M(\bar{y}-h)}{I} = -89.70\ MPa$$

$$\sigma_t = -\frac{M\bar{y}}{I} = 26.95\ MPa$$

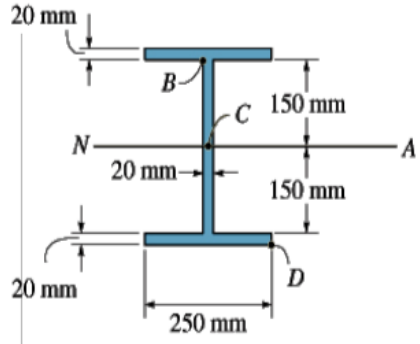
EX 5.3

The simply supported beam in the following figure (a) has the cross-sectional area shown in figure (b).

Determine the absolute maximum bending stress in the beam and draw the stress distribution over the cross section at this location.



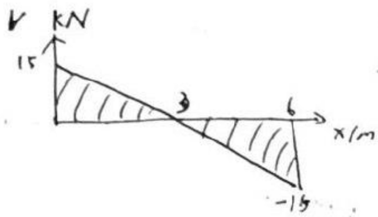
(a)

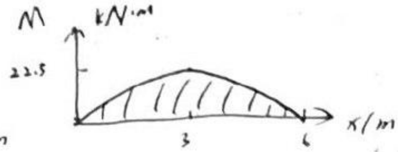


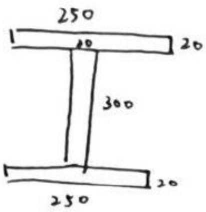
(b)

$2R = 5 \times 6$

$R = 15\,N$







due to the symmetry, $\bar{y} = \frac{300+20+20}{2} = 170\,mm$

$$I = \sum (I_i + A_i d_i^2)$$

$$= \frac{1}{12} \times 250 \times 20^3 + 250 \times 20 \times (170-10)^2$$

$$+ \frac{1}{12} \times 20 \times 300^3 + 20 \times 300 \times (170-170)^2$$

$$+ \frac{1}{12} \times 250 \times 20^3 + 20 \times 250 \times (330-170)^2$$

$$= \cancel{3.18 \times 10^{-4}\,m^4}\ 3.013 \times 10^{-4}\,m^4$$

for positive bending moment $\sigma_c = -\frac{M\bar{y}}{I}$, $\sigma_t = -\frac{M(\bar{y}-h)}{I}$

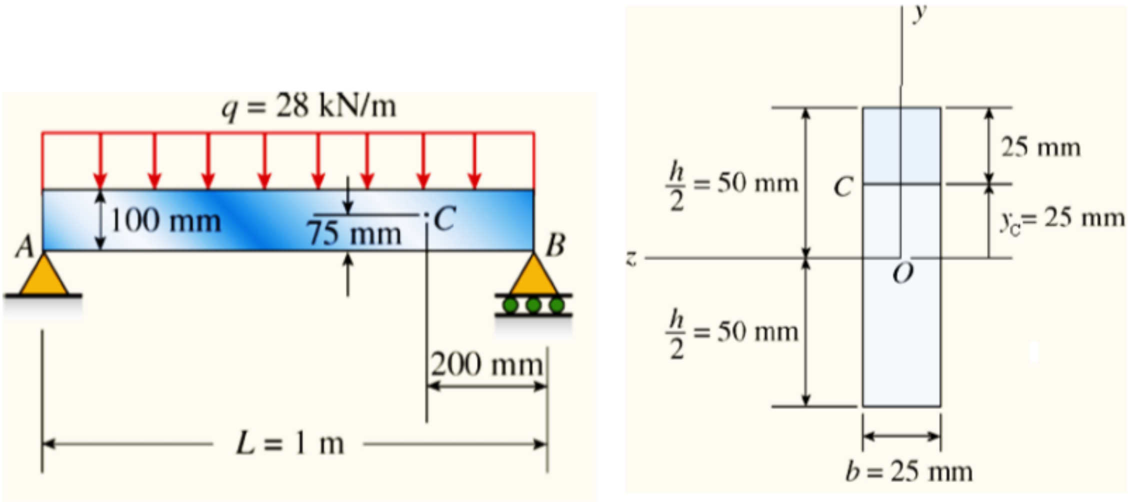
$$\sigma_c = -\frac{22.5 \times 170}{3.013 \times 10^{-4}} = \cancel{-12.03\,MPa}\ -12.69\,MPa$$

$$\sigma_t = -\frac{M(\bar{y}-h)}{I} = \cancel{12.03\,MPa}\ 12.69\,MPa$$

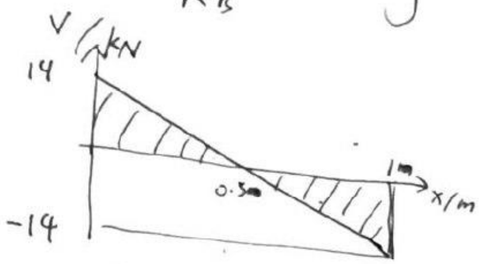
EX 5.3 (torsion)

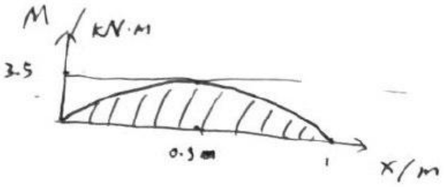
A metal beam with span L = 1 m is simply supported at points A and B. The uniform load on the beam is q=28 kN/m. The cross section of the beam is rectangular with width b=25 mm and height h=100 mm.

Determine the normal stress σ_C and shear stress τ_C at point C, which is located 25 mm below the top of the beam and 200 mm from the righthand support. Show these stresses on a sketch of a stress element at point C.



$$\left.\begin{aligned} R_A + R_B &= 28 \times 1 \\ R_A &= R_B \end{aligned}\right\} \Rightarrow R_A = R_B = 14\ kN$$





$$I = \frac{1}{12} b h^3 = 2.08 \times 10^{-6}\ m^4$$

$$V(x) = 14 - 28x \quad (kN) \qquad V(0.8) = -8.4\ kN$$

$$M(x) = 14x(1-x) \quad (kN \cdot m). \qquad M(0.8) = 2.24\ kN \cdot m$$

$$y_c = \frac{h}{2} = 50mm$$

$$\therefore \quad \sigma_c = -\frac{M(0.8)(y_c - 25)}{I} = -26.92\ MPa$$

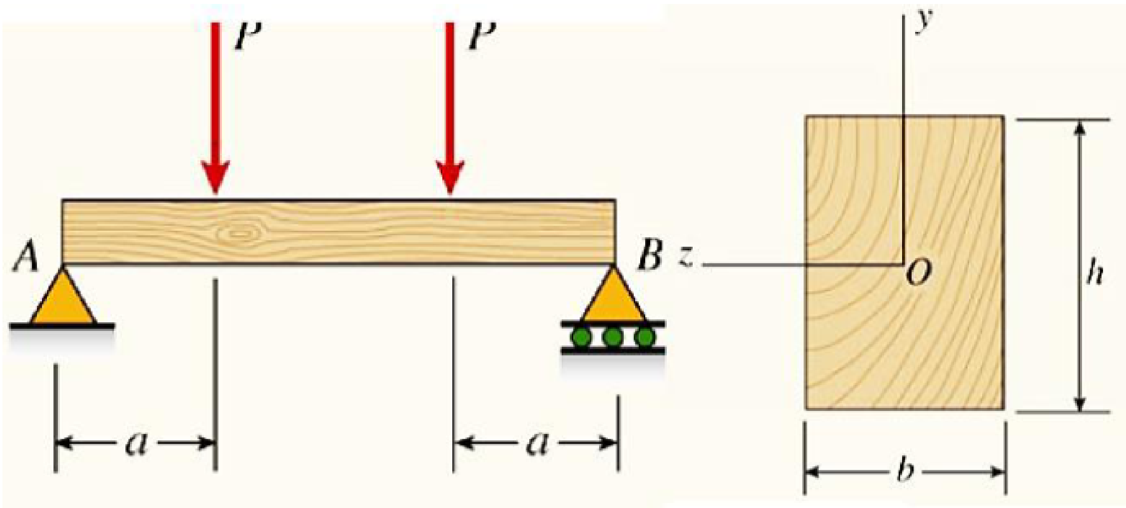
$$Q = \frac{1}{2} b \left(\frac{h^2}{4} - (y_c - 25)^2\right) = 2.34 \times 10^{-5}\ m^3$$

$$\tau = -\frac{VQ}{Ib} = 3.78\ MPa.$$

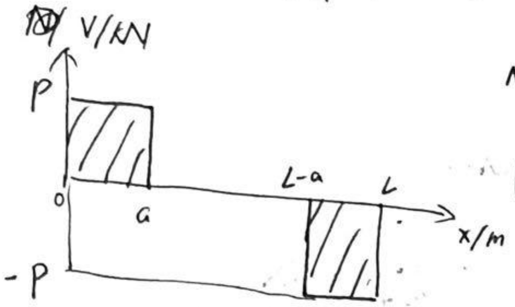
EX 5.4

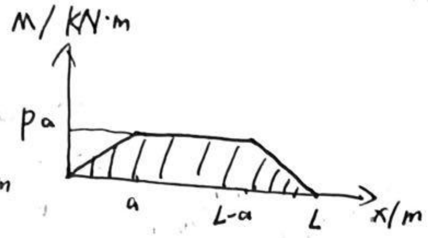
A wood beam AB supporting two concentrated loads P has a rectangular cross section of width b=100 mm and height h=150 mm. The distance from each end of the beam to the nearest load is a=0.5m.

Determine the maximum permissible value P_{max} of the loads if the allowable stress in bending is $\sigma_{allow} = 11MPa$ and the allowable stress in horizontal shear is $\tau_{allow} = 1.2MPa$.



$R_A = R_B = P$





$M_{max} = Pa$, $V_{max} = P$, $I = \frac{1}{12} bh^3 = \cancel{2.8125 \times 10^{-5} m^4}$

$\cancel{Q} \; \bar{y} = \frac{h}{2}$ $\quad \cancel{Q = \frac{1}{2} b \left(\frac{h^2}{4} - \frac{h^2}{4}\right) = 0}$.

$$Q_{max} = \frac{1}{2} b \left(\frac{h^2}{4} - 0\right) = \frac{1}{8} bh^2$$

$$\sigma_{max} = \frac{M_{max} \cdot \frac{h}{2}}{I} = \frac{Pa\,h}{\frac{1}{6} bh^3} = \frac{6Pa}{bh^2}$$

$$\tau_{max} = \frac{VQ}{Ib} = \frac{P \cdot \frac{1}{8} bh^2}{\frac{1}{12} bh^3 \cdot b} = \frac{3P}{2bh}$$

$$\Rightarrow \begin{cases} P_{max,1} = \frac{\sigma_{max} \cdot bh^2}{6a} = 8.25\,kN \\ P_{max,2} = \frac{\tau_{max} \cdot 2bh}{3} = 12\,kN \end{cases} \Rightarrow P_{max} = \min\{P_{m1}, P_{m2}\} = 8.25\,kN.$$