CH_2

Intro Linear Time-invariant system

- linear
- time-invariant

2.1 Linear Constant-Coefficient Differential Equation

Linear constant-coefficient differential equation is the mathematical repression of a continuous-time LTI system.

Classical Solution of Differential Equations

$$\begin{aligned} & C_0\frac{d^n r(t)}{dt^n}+C_1\frac{d^{n-1}r(t)}{dt^{n-1}}+\cdots+C_{n-1}\frac{dr(t)}{dt}+C_n r(t) \\ & =E_0\frac{d^m e(t)}{dt^m}+E_1\frac{d^{m-1}e(t)}{dt^{m-1}}+\cdots+E_{m-1}\frac{de(t)}{dt}+E_m e(t) \end{aligned} \tag{1}$$

$$C_0\frac{d^n r(t)}{dt^n}+C_1\frac{d^{n-1}r(t)}{dt^{n-1}}+\cdots+C_{n-1}\frac{dr(t)}{dt}+C_n r(t)=0 \tag{2}$$

- Homogeneous solution (natural response)
  1. **Characteristic equation**
     $$C_0\alpha^n+C_1\alpha^{n-1}+\cdots+C_{n-1}\alpha+C_n=0 \tag{3}$$
  2. **Characteristic roots** $\alpha_1,\alpha_2,\cdots,\alpha_n$ satisfy tje characteristic equation
  3. **Homogeneous solution**
     $r_h(t)=\sum_{i=1}^n A_i e^{\alpha_i t}$
     If α_1 is the k-th repeated roots, then there will be k number of α_1:$\sum_{i=1}^k A_i t^{k-i}e^{\alpha_1 t}$
- Particular solution (forced response)
  1. Table look-up

Exciting $e(t)$	**Response** $r(t)$
t^p	$B_1t^p + B_2t^{p-1} + L + B_pt + B_{p+1}$
$e^{\alpha t}$	$Be^{\alpha t}$
$\sin(\omega t)/\cos(\omega t)$	$B_1 \sin(\omega t) + B_2 \cos(\omega t)$
$t^pe^{\alpha t} \sin(\omega t)/$ $t^pe^{\alpha t} \cos(\omega t)$	$(B_1t^p + B_2t^{p-1} + L + B_pt + B_{p+1})e^{\alpha t} \sin(\omega t) + (D_1t^p + D_2t^{p-1} + L + D_pt + D_{p+1})e^{\alpha t} \cos(\omega t)$

- Auxiliary conditions (initial conditions)
    1. Determine the coefficients

$$r(t_0), \frac{dr(t_0)}{dt}, \frac{d^2r(t_0)}{dt^2}, L, \frac{d^{n-1}r(t_0)}{dt^{n-1}}$$

2.2 Partial Initial Conditions

The Law of Switching

$$v_c(0_-) = v_c(0_+),\ i_L(0_-) = i_L(0_+)$$

Impulse Function Matching Method

$$\delta(t) \quad \delta'(t) \quad \delta''(t) \quad \cdots$$

The Laplace Transform

$$X(s) = \int_0^\infty x(t)e^{-st}\mathrm{d}t$$