CH_3

Introduction

- Linear Constant-Coefficient Difference Equation Representations
- Block Diagram Representations
- Advantages of Discrete-Time System

3.1 Discrete-Time Signal/Sequence

Representation

- Function Representation
- Graphical Representation

Classifications

- **One-sided Sequence**: $n \geq 0$
- **Two-sided Sequence**: $-\infty \leq n \leq \infty$
- **Limited Sequence**: $n_1 \leq n \leq n_2$

Operations

- Addition: $z[n] = x[n] + y[n]$
- Multiplication: $z[n] = x[n] \cdot y[n]$
- Scalar Multiplication: $z[n] = \alpha x[n]$
- Time Shift: $\begin{cases} z[n] = x[n-m] & \text{Right Shift} \\ z[n] = x[n+m] & \text{Left Shift} \end{cases}$
- Time Reversal: $z[n] = x[-n]$
- Difference: $\begin{cases} \Delta x[n] & = x[n+1] - x[n] \qquad \text{Forward} \\ \nabla x[n] & = x[n] - x[n-1] \qquad \text{Backward} \end{cases}$

- Accumulation: $z[n] = \sum_{k=\infty}^{\infty} x[k]$
- Time Scaling (Decimation and Interpolation): $x[n] \to x[an] \qquad x[n] \to x[\frac{n}{a}]$
- Energy: $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$

Some Useful Modes

- Unit Impulse (Unit Sample)

$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

$$\delta[n-t] = \begin{cases} 0 & n \neq t \\ 1 & n = t \end{cases}$$

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\delta[n-m]$$

- Unit Step

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$u[n] = \sum_{k=0}^{\infty} \delta[n-k] = \sum_{k=-\infty}^{n} \delta[k]$$

$$\delta[n] = u[n] - u[n-1]$$

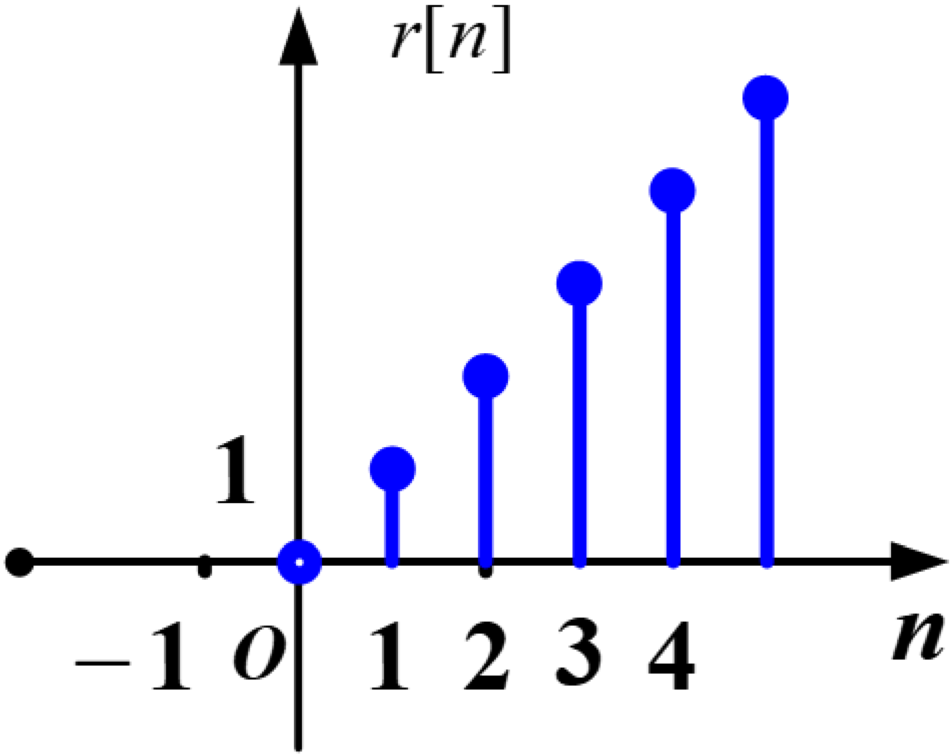
- Rectangular Sequence

$$R_N[n] = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$R_N[n] = u[n] - u[n-N] = \sum_{m=0}^{N-1} \delta[n-m]$$

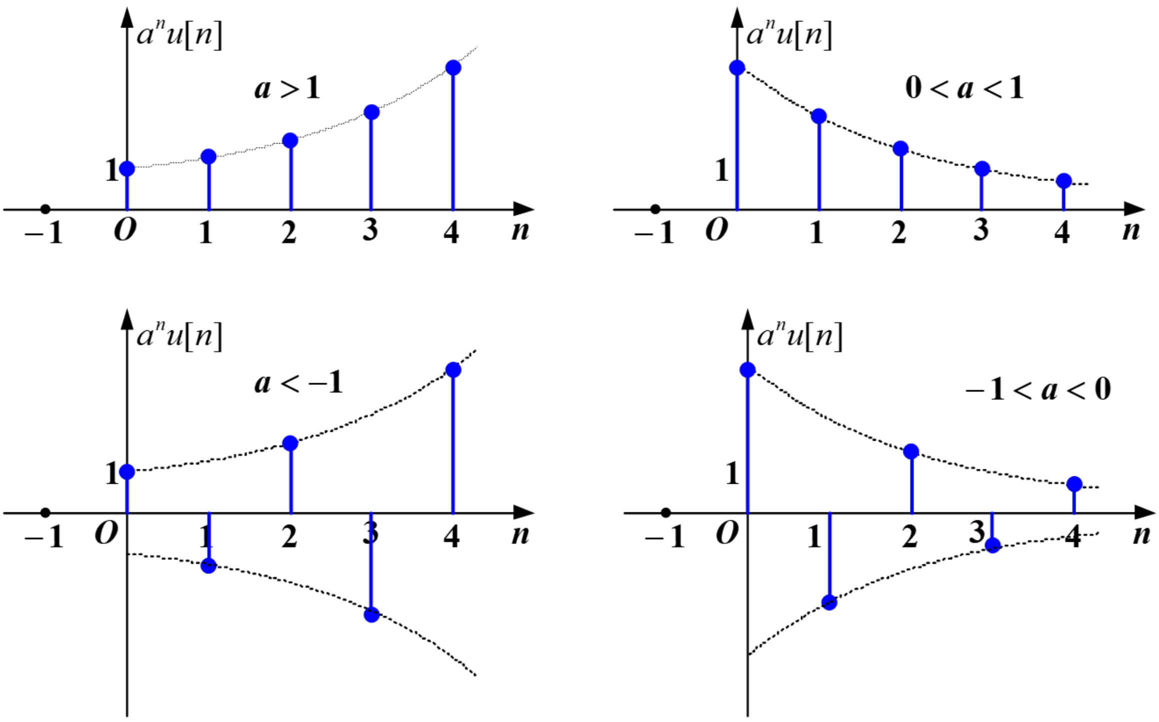
- Ramp Sequence

$$r[n] = nu[n]$$



- Real Exponential Sequence

$$x[n] = a^n u[n]$$



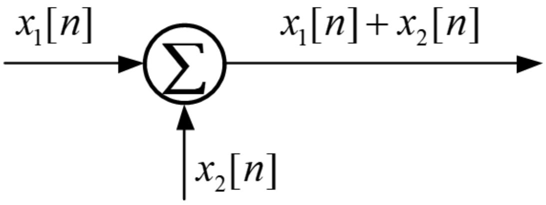
- Sinusoidal Sequence

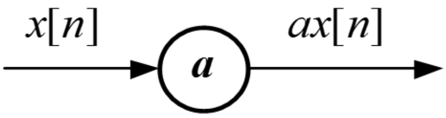
$$x[n] = \sin n\omega_0$$

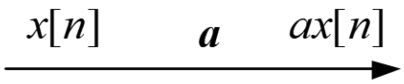
- Complex Exponential Sequence

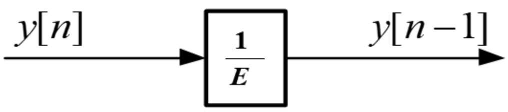
$$x[n] = e^{j\omega_0 n} = \cos\omega_0 n + j\sin\omega_0 n$$

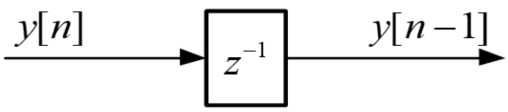
3.2 Discrete-Time System Block Diagram Representations











3.3 Linear Constant-Coefficient Difference Equation

General Nth-order Linear Constant-coefficient Difference Solution

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

Solution of Difference Equations

- Recursive Method
- Homogeneous solution + Particular Solution
  - Natural Response:
    - Eigenvalue from equation
    - Homogeneous solution

$$\sum_{k=0}^{N} a_k y[n-k] = 0$$

$$y[n] - \alpha y[n-1] = 0$$

$$\alpha = \frac{y[n]}{y[n-1]}$$

$$\Rightarrow \qquad y[n] = C\alpha^n$$

$$
\begin{aligned}
a_0 C\alpha^n + a_1 C\alpha^{n-1} + \cdots + a_{N-1} C\alpha^{n-(N-1)} + a_N C\alpha^{n-N} &= 0 \\
a_0 \alpha^n + a_1 \alpha^{n-1} + \cdots + a_{N-1} \alpha^{n-(N-1)} + a_N \alpha^{n-N} &= 0 && \text{Eigen Equation} \\
\alpha_1, \alpha_2, \cdots, \alpha_N & && \text{Eigenvalue} \\
C_1\alpha_1^n + C_2\alpha_2^n + \cdots + C_N\alpha_N^n & && \text{Homogeneous Solution}
\end{aligned}
$$

- Forced Response: Table look-up

Input: $x[n]$	**Output:** $y[n]$
e^{an}	Ae^{an}
$e^{j\omega n}$	$Ae^{j\omega n}$
$\cos(\omega n)$	$A\cos(\omega n + \theta)$
$\sin(\omega n)$	$A\sin(\omega n + \theta)$
n^k	$A_k n^k + A_{k-1} n^{k-1} + \cdots + A_1 n + A_0$
A	C
r^n	Cr^n

- Zero-input Solution + Zero-state Response
  - homogeneous solution
  - total solution
- Z-Transform Method

3.4 The Convolution Sum

the Representation of Discrete-time Signals in Terms of Impulses

$$
\begin{aligned}
x[n] = & \cdots + x[-1]\delta[n-1] + x[0]\delta[n] + x[1]\delta[n-1] + \\
& \cdots + x[m]\delta[n-m] + \cdots \\
= & \sum_{m=\infty}^{\infty} x[m]\delta[n-m]
\end{aligned}
$$

for convolution sum $\delta[n] \to h[n]$

Time-Invariant $\delta[n-m] \to h[n-m]$

Homogeneity $x[m]\delta[n-m] \to x[m]h[n-m]$

Additivity $y[n] = \sum_{m=\infty}^{\infty} x[m]h[n-m] = x[n] * h[n]$

Properties of Convolution Sum

Commutative Property

$x[n] * h[n] = h[n] * x[n]$

Distributive Property

$x[n] * [h_1[n] + h_2[n]] = x[n] * h_1[n] + x[n] * h_2[n]$

Associative Property

$x[n] * h_1[n] * h_2[n] = x[n] * (h_1[n] * h_2[n])$

Calculation of Convolution Sum

- Analytic Method
- Graphic Method
- Properties Method
- Upright Formula Method

3.5 Discrete-Time System Properties

- **Stability**
  $\sum |h[n]| = P < \infty$
- **Causality**

$$h[n] = 0 \quad \text{for} \quad n < 0$$

$$h[n] = h[n]u[n]$$