EX_4

EX 4.1

Consider a real periodic signal x(t), with fundamental frequency 2π, that is expressed in the complex exponential Fourier series as

$$x(t) = \sum_{k=-3}^{3} a_k e^{jk2\pi t}$$

where $a_0 = 1$, $a_1 = a_{-1} = 1/4$, $a_2 = a_{-2} = 1/2$, $a_3 = a_{-3} = 1/3$

$$x(t) = (1 + \frac{1}{4} (e^{j2\pi t} + e^{-j2\pi t})$$

$$+ \frac{1}{2} (e^{j4\pi t} + e^{-j4\pi t}) + \frac{1}{3} (e^{j6\pi t} + e^{j-6\pi t})$$

since $e^{j\theta} = \cos\theta + j\sin\theta$

$$\therefore e^{j\theta} + e^{-j\theta} = 2\cos\theta$$

$$\therefore x(t) = 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

EX 4.2

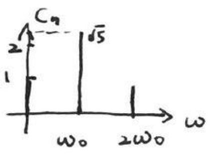
Consider the signal

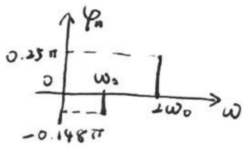
$$x(t) = 1 + \sin\omega_0 t + 2\cos\omega_0 t + \cos(2\omega_0 t + \frac{\pi}{4})$$

Plot the magnitude spectrum and phase spectrum of $x(t)$

$$x(t) = 1 + \sqrt{5}\cos(\omega_0 t - 0.148\pi) + \cos(2\omega_0 t + \frac{\pi}{4})$$

$$\begin{cases} C_0 = 1 \\ C_1 = \sqrt{5} \\ C_2 = 1 \end{cases} \qquad \begin{cases} \varphi_0 = 0 \\ \varphi_1 = -0.148\pi \\ \varphi_2 = 0.25\pi. \end{cases}$$

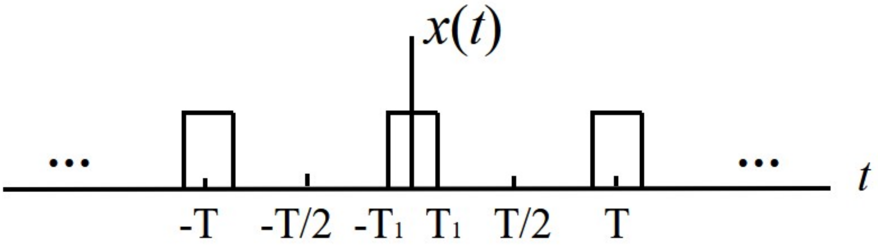




EX 4.3

The periodic square wave is defined over one period as:

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$



Represent it in Fourier series

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} dt = \frac{2T_1}{T}$$

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\,dt = -\frac{1}{jk\omega_0 T} e^{-jk\omega_0 t}\Big|_{-T_1}^{T_1}$$

notice: 上下限注意

$$= -\frac{1}{jk\omega_0 T}\left(e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}\right)$$

since $e^{j\theta} = \cos\theta + j\sin\theta$

$e^{j\theta} - e^{-j\theta} = 2j\sin\theta$

$$= -\frac{1}{jk\omega_0 T}\cdot -2j\sin(k\omega_0 T_1)$$

$$= \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}, \quad \text{since } \omega_0 = \frac{2\pi}{T}$$

$$= \frac{\sin(k\omega_0 T_1)}{k\pi} \quad (k \neq 0)$$

HW 3.1

Suppose we are given the following information about a signal x(t):

1. x(t) is real and odd
2. x(t) is periodic with period T = 2 and has Fourier coefficients a_k
3. $a_k = 0$ for $|k| > 1$
4. $\frac{1}{2}\int_0^2 |x(t)|^2 \mathrm{d}t = 1$

Specify two different signals that satisfy these conditions

1. since $x(t)$ is real and odd, a_k is purely imaginary and odd

$\Rightarrow$ $a_0 = 0$, $a_1 = -a_{-1}$

2. $T = 2$

3. for $|k| \geq 2$, $a_k = 0$

4. according to the Parseval's Relation

$$\frac{1}{2}\int_0^2 |x(t)|^2 dt = \frac{1}{T}\int_0^T |x(t)|^2 dt = \sum_{n=-\infty}^{\infty} |a_k|^2$$

$$= a_{-1}^2 + a_0^2 + a_1^2 = 1$$

$\therefore$ $a_1^2 = \frac{1}{2}$

case 1, $a_1 = \frac{\sqrt{2}}{2}j$, $a_{-1} = -\frac{\sqrt{2}}{2}j$

$w_0 = \frac{2\pi}{T} = \pi$

$$\therefore x_1(t) = \frac{\sqrt{2}}{2}j \cdot e^{j\pi t} - \frac{\sqrt{2}}{2}j e^{-j\pi t}$$

$$= \frac{\sqrt{2}}{2}j\,(e^{j\pi t} - e^{-j\pi t})$$

$$= \frac{\sqrt{2}}{2}j \cdot 2j\sin\pi t = -\sqrt{2}\sin\pi t$$

case 2, $a_1 = -\frac{\sqrt{2}}{2}j$, $a_{-1} = \frac{\sqrt{2}}{2}j$

$$x_2(t) = -\frac{\sqrt{2}}{2}j e^{j\pi t} + \frac{\sqrt{2}}{2}j e^{-j\pi t}$$

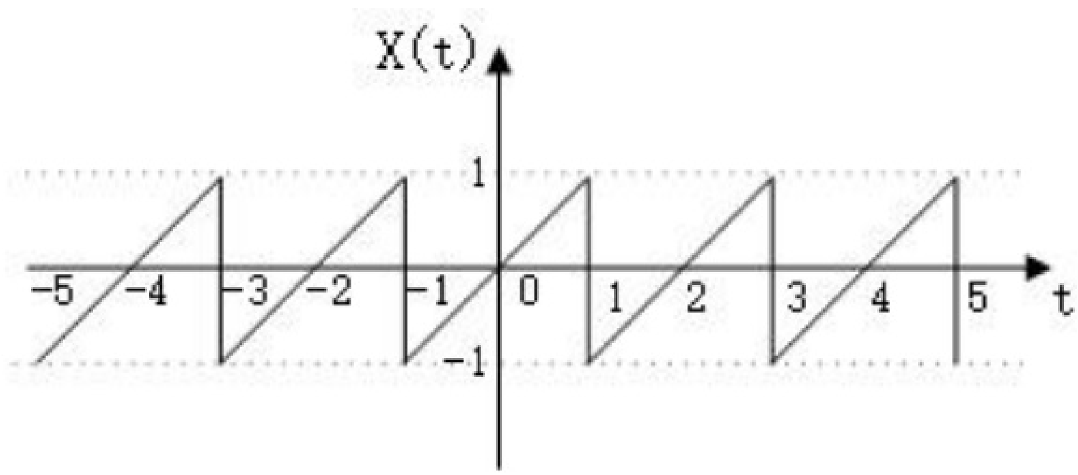
$$= -\frac{\sqrt{2}}{2}j\,(e^{j\pi t} - e^{-j\pi t})$$

$$= -\frac{\sqrt{2}}{2}j \cdot 2j\sin\pi t$$

$$= \sqrt{2}\sin\pi t.$$

HW 3.2

Determine the Fourier series representations for the following signals



$x(t) = t \quad (-1 < t < 1)$

since $x(t)$ is real and odd, same to a_k

$a_0 = 0$, $\omega = \frac{2\pi}{T} = \pi$.

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} t\, e^{-jk\pi t}\, dt$$

let $m = jk\pi$, $\int te^{-mt} = -\frac{(mt+1)e^{-mt}}{m^2} + C$

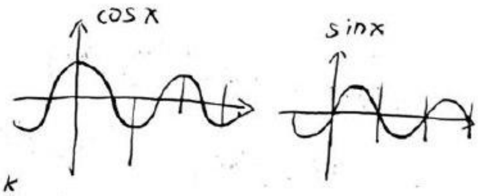
$$\therefore a_k = -\frac{1}{2} \left. \frac{(jk\pi t+1)e^{-jk\pi t}}{(jk\pi)^2} \right|_{-1}^{1}$$

$$= \frac{1}{2}\left[\frac{(jk\pi+1)e^{-jk\pi}}{k^2\pi^2} - \frac{(-jk\pi+1)e^{jk\pi}}{k^2\pi^2} \right].$$

$$= \frac{1}{2}\left[\frac{(jk\pi+1)\, e^{-jk\pi} + (jk\pi-1)e^{jk\pi}}{k^2\pi^2} \right]$$

$$= \frac{jk\pi\underbrace{(e^{jk\pi}+e^{-jk\pi})}_{2\cos(k\pi t)} - \underbrace{(e^{jk\pi}-e^{-jk\pi})}_{2j\sin(k\pi t)}}{2k^2\pi^2}$$

$$= \frac{j[k\pi\cos(k\pi t) - \sin(k\pi t)]}{k^2\pi^2}$$



$\sin(k\pi t) = 0$, $\cos(k\pi t) = (-1)^k$

$$\therefore a_k = \frac{(-1)^k j}{k\pi} \qquad \therefore x(t) = \sum_{n=-\infty}^{\infty} \frac{(-1)^k j}{k\pi} e^{j\pi k t}$$

> fix: a_k is purely imaginary and odd

HW 3.3

Suppose we are given the following information about a signal x(t):

1. x(t) is a real signal
2. x(t) is periodic with period T = 6 and has Fourier coefficients a_k
3. $a_k = 0$ for $k = 0$ and $k > 2$
4. $x(t) = -x(t-3)$
5. $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 \mathrm{d}t = \frac{1}{2}$

6. a_1 is a positive real number

Show that $x(t) = A\cos(Bt + C)$, and determine the values of the constants A, B and C

~~x(t)~~

$\omega = \frac{2\pi}{T} = \frac{\pi}{3}$

$\therefore x(t) = a_{-2} e^{-j\frac{2\pi}{3}t} + a_{-1} e^{-j\frac{\pi}{3}t} + a_1 e^{j\frac{\pi}{3}t} + a_2 e^{j\frac{2\pi}{3}t}.$

since ~~x(t) = x(t-3)~~

$x(t) = -x(t-3) = -x(t-\frac{T}{2})$

$\Rightarrow$ x(t) is half-wave symmetry, where ~~are~~

$a_{even} = 0.$

$x(t) = a_{-1} e^{-j\frac{\pi}{3}t} + a_1 e^{j\frac{\pi}{3}t}$

$|a_{-1}|^2 + |a_1|^2 = \frac{1}{2}$

$\because$ x(t) is real, $|a_1| = |a_{-1}|$,

and a_1 is real, $a_1 = a_{-1}^*$

$\Rightarrow |a_1| = |a_{-1}| = \frac{1}{2}$. $a_1 > 0$.

$\therefore a_1 = a_{-1} = \frac{1}{2}.$

$\therefore x(t) = \frac{1}{2} e^{j\frac{\pi}{3}t} + \frac{1}{2} e^{-j\frac{\pi}{3}t}$

$= \cos\frac{\pi}{3}t$

$A = 1,\ B = \frac{\pi}{3},\ C = 0.$