CH_1

signals

communication system

a function conveys information about the behavior or attributes of some phenomenon

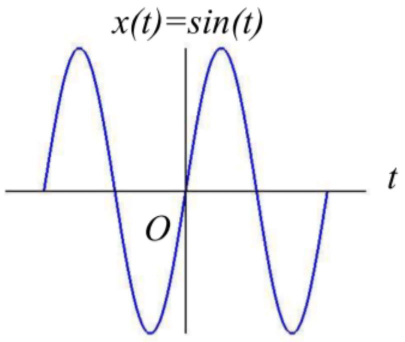
physical world

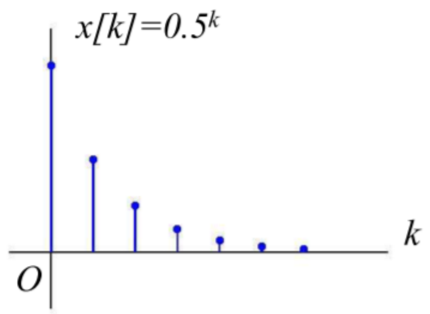
quantity exhibiting variation in time or variation in space

signals are mathematical functions

- Independent variable = time
- Dependent variable = voltage, flow rate, sound pressure, ...

signals can be represented by graph





classifications of signals

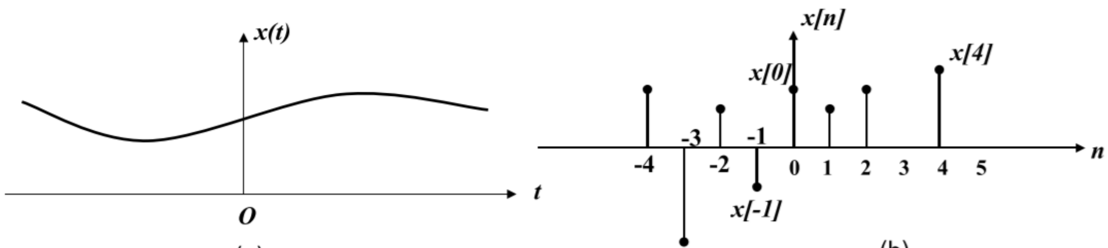
continuous-time and discrete-time signals

- continuous-time signals' independent variable is continuous

$$x(t) = e^t$$

- discrete-time signals are defined only at discrete times:

$$x[n] = 2^n$$



(a) (b)

- the independent variable is inherently discrete

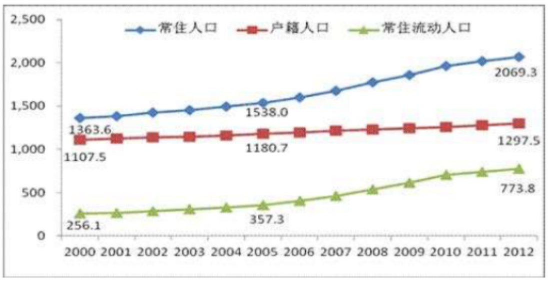
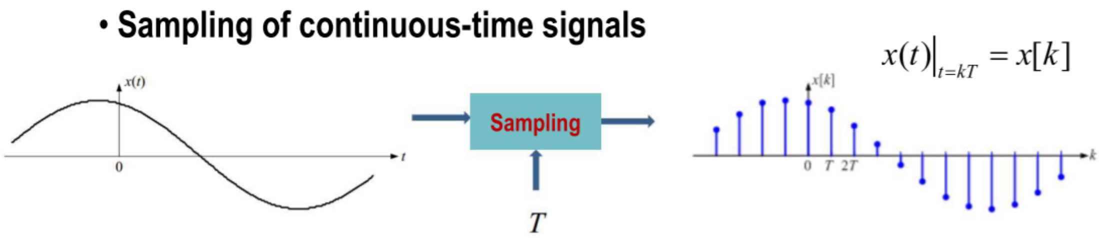


Figure 1.8 Resident population of Beijing from 2000 to 2012

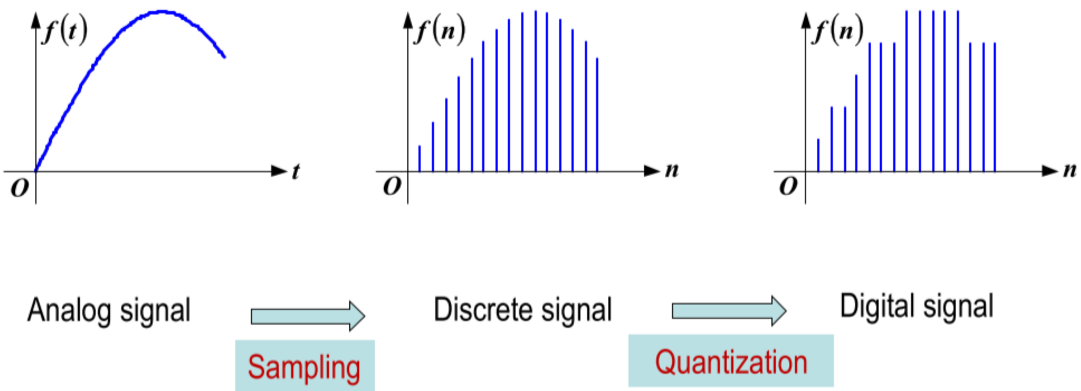
Figure 1.9 Shanghai Composite Index

- sampling of continuous-time signals



analog and digital signals

- analog signals: continuous in both time and amplitude
- digital signals: discrete in both time and amplitude



periodic and aperiodic signals

- for continuous-time

$$x(t) = x(t + T)$$

- for discrete-time

$$x[n] = x[n + N]$$

determinate and random signals

- **a determinate signal**: *x(t)*
- a random signal: cannot find a function to represent it

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E.g.: noise, speech
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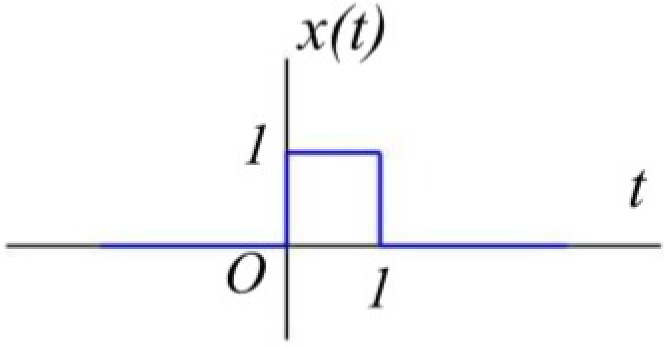
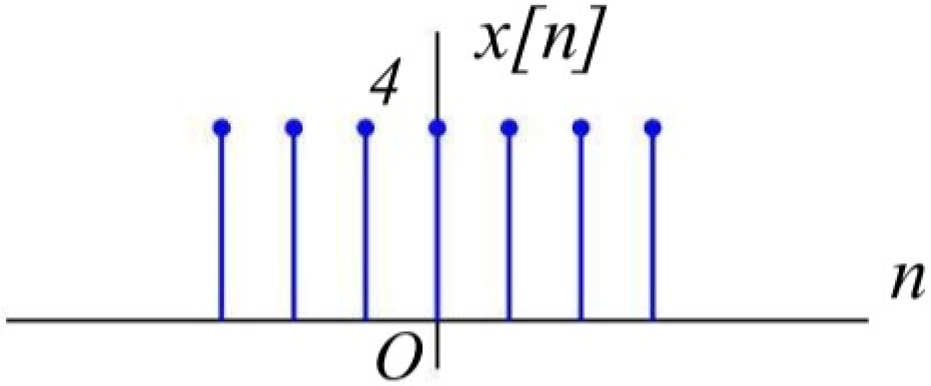
energy and power signals

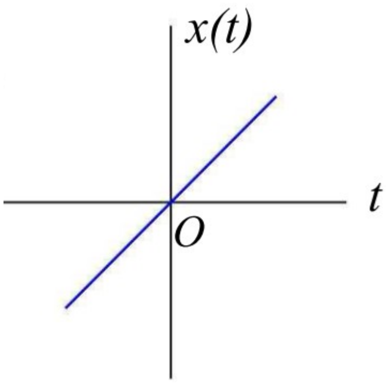
- energy signals $0 < E < \infty,\ P \to 0$



- power signals $0 < P < \infty,\ E \to \infty$



- signals with neither finite E nor finite P $E \to \infty,\ P \to \infty$



Transformations of the Independent Variable of Signals

- time shift

$$x(t) \to x(t - t_0)$$

- time reversal *(left +, right -)*

$$x(t) \to x(-t)$$

- time scaling

$$x(t) \to x(k_0 t)$$

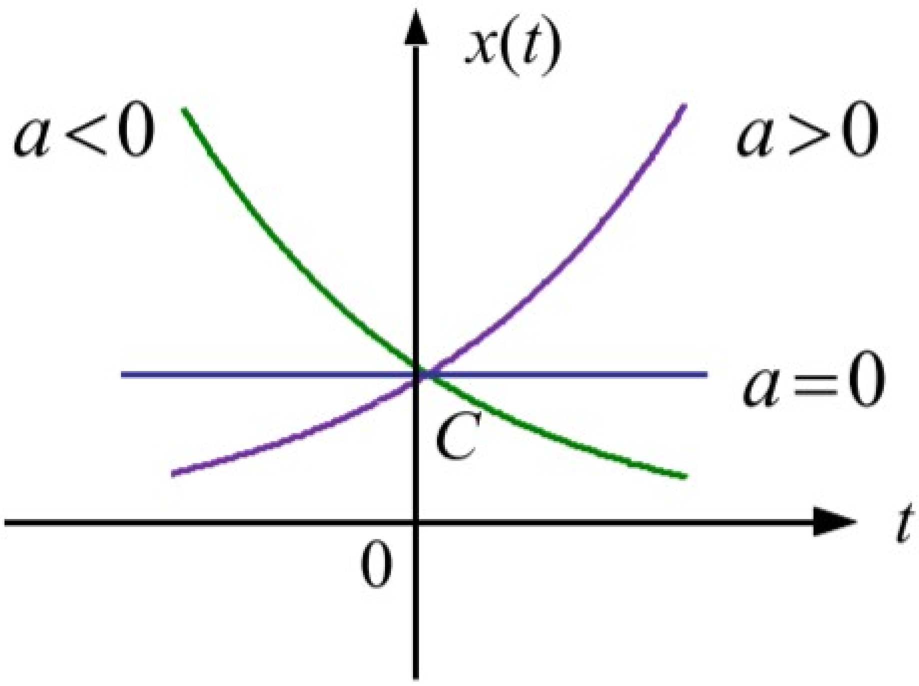
First Scaling, Second Shift

先尺缩，再平移

Some Useful Signal Modes

1. Real Exponential Signals

$$x(t) = Ce^{at}$$



2. Periodic Complex Exponential and Sinusoidal Signals

$$x(t) = Ce^{j\omega_0 t}$$

3. General Complex Exponential Signals

$$C = |C|e^{j\theta},\ a = r + j\omega_0$$

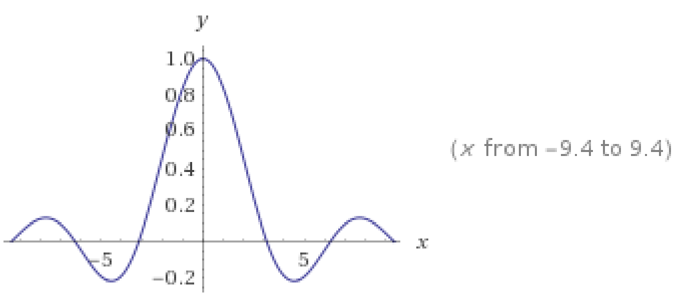
$$Ce^{at} = |C|e^{j\theta}e^{(r+j\omega_0)t} = |C|e^{rt}e^{j(\omega_0 t+\theta)}$$

$$Ce^{at} = |C|e^{rt}(\cos(\omega_0 t+\theta) + j\sin(\omega_0 t+\theta))$$

- When r=0, both parts are sinusoidal;
- When r>0, both parts are growing sinusoidal;
- When r<0, both parts are decaying sinusoidal (damped sinusoids).

4. Sampling Signals

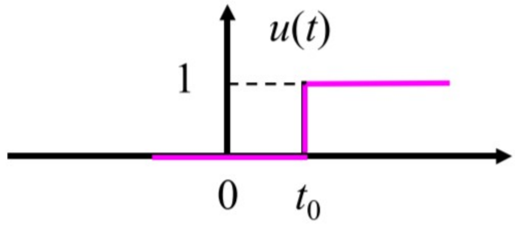
$$Sa(t) = \frac{\sin t}{t}$$



The Unit Impulse and Unit Step Functions

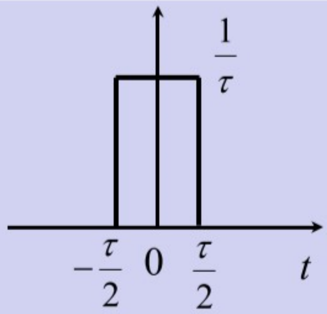
- Unit Step Function

$$u(t-t_0)=\begin{cases}0, & t<t_0\\ 1, & t>t_0\end{cases}$$

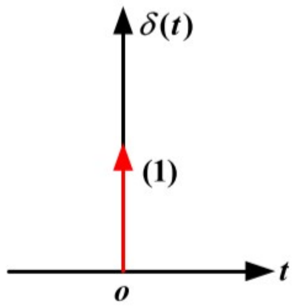


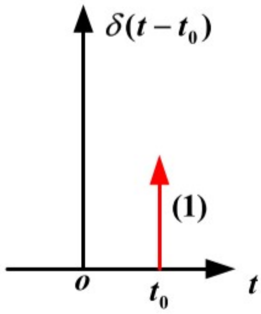
- Unit Impulse Function

$$\begin{cases}\int_{-\infty}^{+\infty}\delta(t)\,\mathrm{d}t=1\\ \delta(t)=0 \quad (t\neq 0)\end{cases} \qquad \int_{-\infty}^{+\infty}\delta(t)\,\mathrm{d}t=\int_{0_-}^{0_+}\delta(t)\,\mathrm{d}t$$



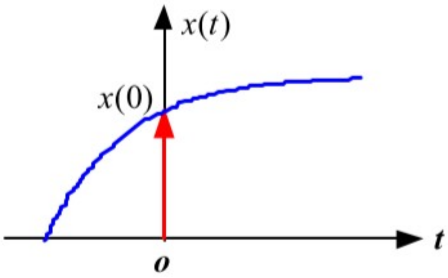
As $\tau\rightarrow 0$, the pulse height $\rightarrow\infty$.





$$x(t)=\frac{1}{\tau}\left[u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)\right]$$

1. **Sampling Property**



$$x(t)\delta(t) = x(0)\delta(t)$$

$$\int_{-\infty}^{+\infty} x(t)\delta(t)dt = x(0)$$

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

2. Even Function

$$\delta(t) = \delta(-t)$$

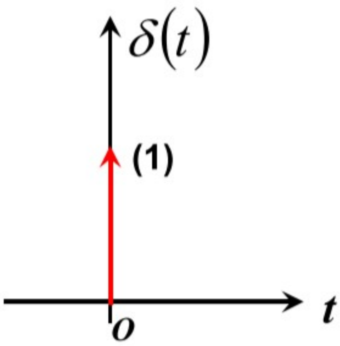
3. Time Scaling

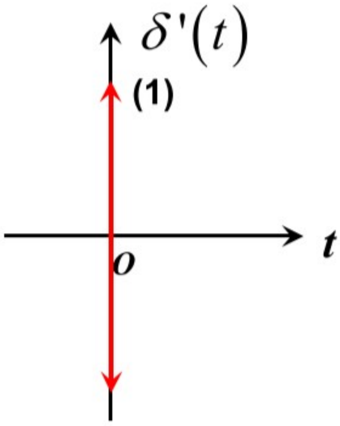
$$\delta(kt) = \frac{1}{|k|}\delta(t)$$

4. **Relationship to Unit Step Function**

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau \quad \delta(t) = \frac{\mathrm{d}u(t)}{\mathrm{d}t}$$

- Impulse Doublet Signal





$$\delta'(t) = \frac{d\delta(t)}{dt}$$

$$\delta'(t) = \lim_{\tau \to 0} x'(t)$$

1. **Sampling Property**

$$x(t) = \delta'(t-t_0) = x(t_0)\delta'(t-t_0) - x'(t_0)\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} x(t_0)\delta'(t-t_0)\mathrm{d}t = -x'(t_0)$$

2. Scaling

$$\delta'(kt) = \frac{1}{k|k|}\delta'(t),\ k \neq 0$$

3. Odd Function

$$\delta'(t) = -\delta'(-t)$$

Signal Decompositions and Components of a Signal

Even and Odd Components

$$x(t) = \underbrace{\frac{1}{2}\left[x(t)+x(-t)\right]}_{\text{Even，偶分量}} + \underbrace{\frac{1}{2}\left[x(t)-x(-t)\right]}_{\text{Odd，奇分量}}$$

$$x_e(t) = x_e(-t) \qquad x_o(t) = -x_o(-t)$$

Real and Imaginary Components

$$x(t) = x_r(t) + jx_i(t)$$

$$x^*(t) = x_r(t) - jx_i(t) \qquad \text{Complex conjugate}$$

$$x_r(t) = \frac{1}{2}\left[x(t)+x^*(t)\right] \qquad x_i(t) = \frac{1}{2j}\left[x(t)-x^*(t)\right]$$

Systems

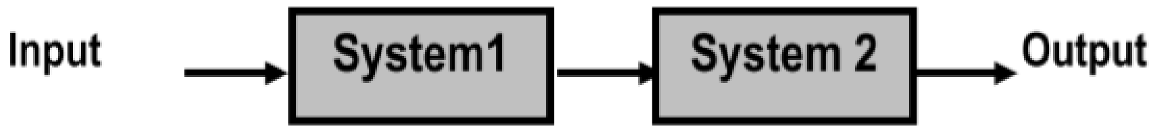
a process in which **input signals** are transformed by the system or cause the system to respond in some way, resulting in other signals as **outputs**

Descriptions

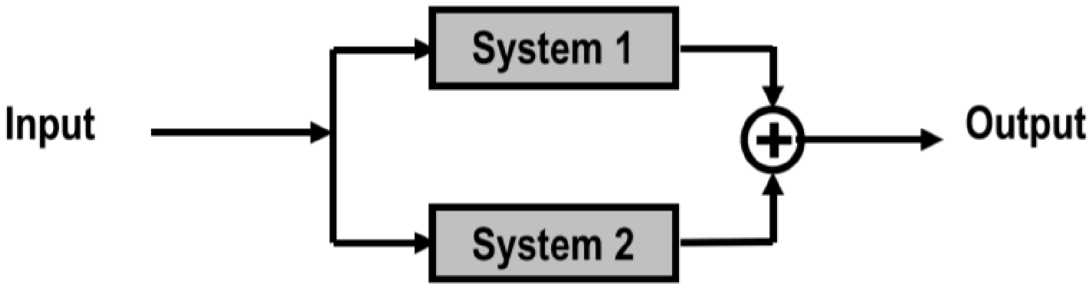
- input and output description
  N-order linear differential equation
- state-space description
  N first-order differential equations

Interconnections

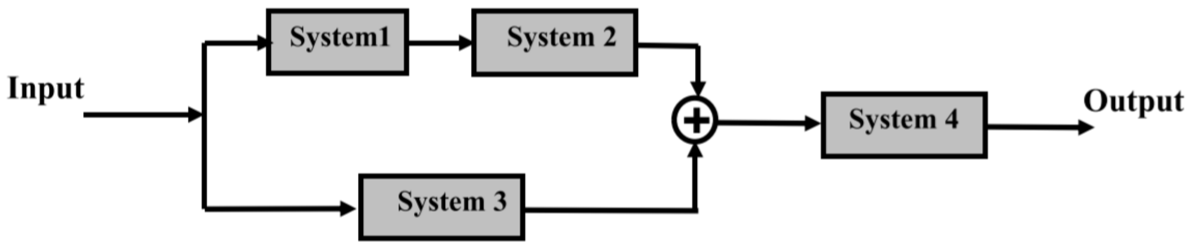
- series interconnection



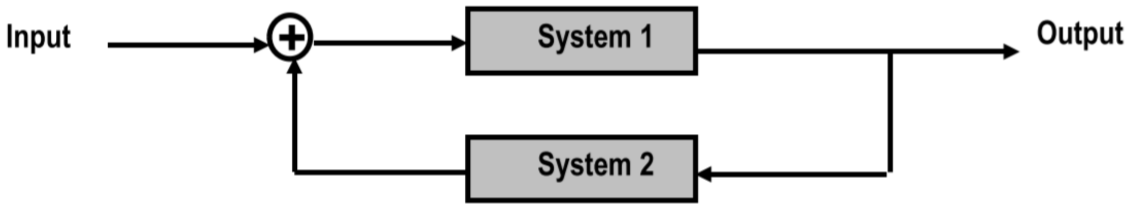
- parallel interconnection



- feedback connection



- complicated connection which combine the former two interconnections



Basic System Properties

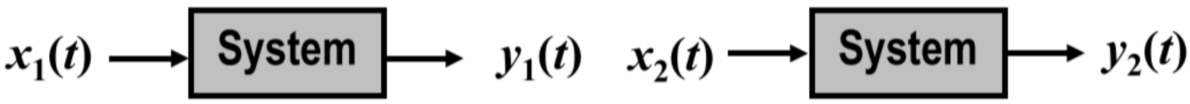
Linearity

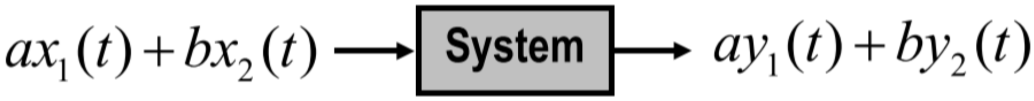
- additivity

$$\begin{matrix} x_1(t) \to y_1(t) \\ x_2(t) \to y_2(t) \end{matrix} \quad \Rightarrow x_1(t) + x_2(t) \to y_1(t) + y_2(t)$$

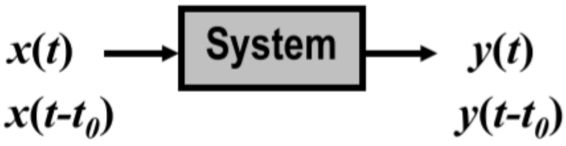
- homogeneity

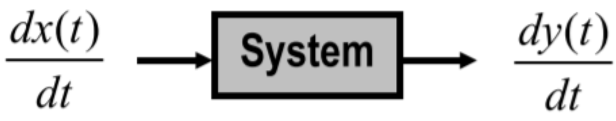
$$x(t) \rightarrow y(t) \quad \Rightarrow kx(t) \rightarrow ky(t)$$

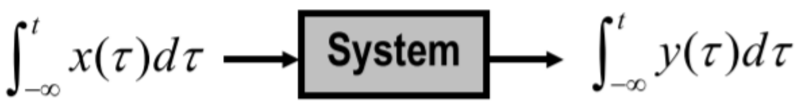




Time-Invariant







With Memory

- memoryless

  its output for each value of the independent variable at a given time is dependent **only on the input at that same time**
- system with memory

  it **retains or stores information** about input values at time other than the current time
  E.g.: Accumulator, Delay;

Causality

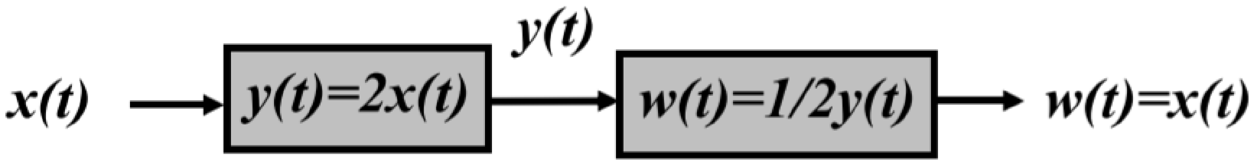
- casual system

  output depends **only on the input at present time** and in the past

  All memoryless systems are casual systems
- non-casual system
  E.g.: Data Smoothing

Invertibility

distinct input lead to distinct output



Stability

if the input to a stable system is bounded, then the output must also be bounded