

# CH\_7

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## 7.1 Entropy

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### Clausius Inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

This inequality is valid for **all cycles**

And this property is named as **entropy**, which designated S and defined as

$$dS = \left( \frac{\delta Q}{T} \right)_{int \ rev}$$

The entropy of a system during a process can be determined by integrating between the initial and the final states

$$\Delta S = S_2 - S_1 = \int_1^2 \left( \frac{\delta Q}{T} \right)_{int \ rev}$$

### Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left( \frac{\delta Q}{T} \right)_{int \ rev} = \int_1^2 \left( \frac{\delta Q}{T_0} \right)_{int \ rev} = \frac{1}{T_0} \int_1^2 (\delta Q)_{int \ rev}$$

$$\Delta S = \frac{Q}{T_0}$$

where  $T_0$  is the constant temperature of the system and  $Q$  is the heat transfer for the internally reversible process

## 7.2 The Increase of Entropy Principle

$$\oint \frac{\delta Q}{T} \leq 0$$

or

$$\int_1^2 \frac{\delta Q}{T} + \int_1^2 \left( \frac{\delta Q}{T} \right)_{int \ rev} \leq 0$$

while the second integral in the previous relation is recognized as the entropy change

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0$$

$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

which can also be arranged as in differential form as

$$dS \geq \frac{\delta Q}{T}$$

some entropy is generated or created during an irreversible process, which is due entirely to the presence of irreversibilities.

The entropy generated during a process is called **entropy generation** and is denoted by  $S_{gen}$

$$\Delta S_{sys} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + s_{gen}$$

for an isolated system, the heat transfer is zero and the former equation reduces to

$$\Delta S_{isolated} \geq 0$$

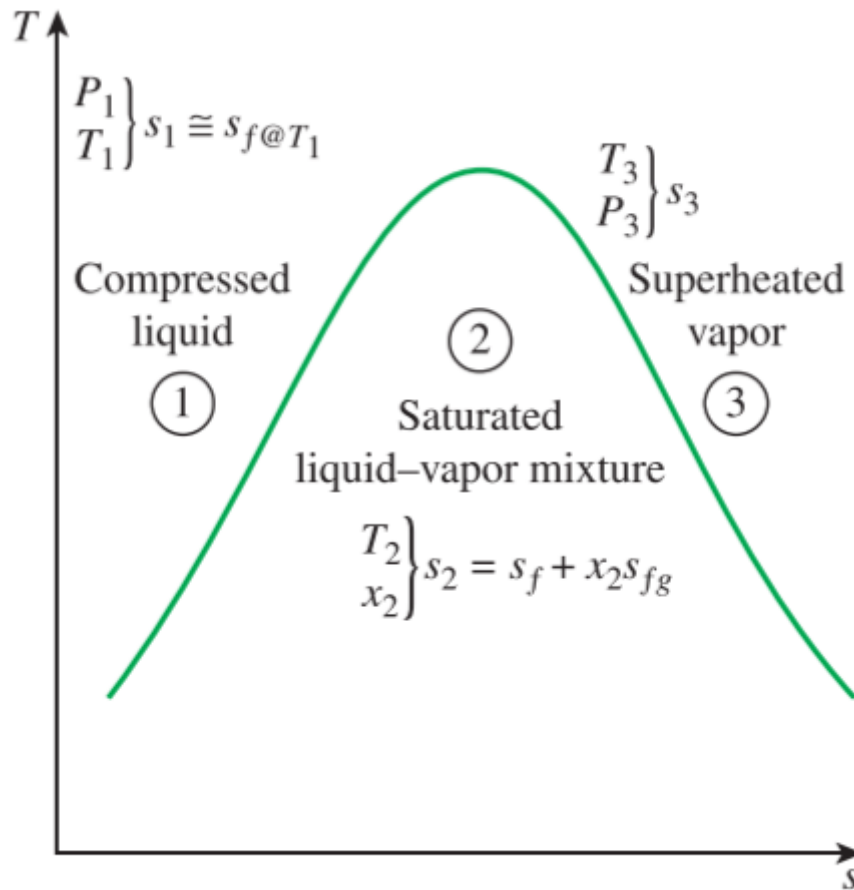
The equation can be expressed as **the entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant**. In other words, it *never* decreases. This is known as the **increase of entropy principle**

$$S_{gen} = \Delta S_{total} = \Delta S_{sys} + \Delta S_{surr} \geq 0$$

Since the entropy change of a system can be negative during a process, but entropy generation cannot. The increase of entropy can be summarized as follows:

$$S_{gen} \begin{cases} > 0 \text{ Irreversible Process} \\ = 0 \text{ Reversible Process} \\ < 0 \text{ Impossible Process} \end{cases}$$

## 7.3 Entropy Change of Pure Substances



The value of entropy at a specified state is determined just like any other property. Similarly, in the saturated mixture region, it's determined from

$$s = s_f + x s_{fg}$$

The entropy change of a specified mass  $m$  during a process is simply

$$\Delta S = m \Delta s = m(s_2 - s_1)$$

## 7.4 Isentropic Processes

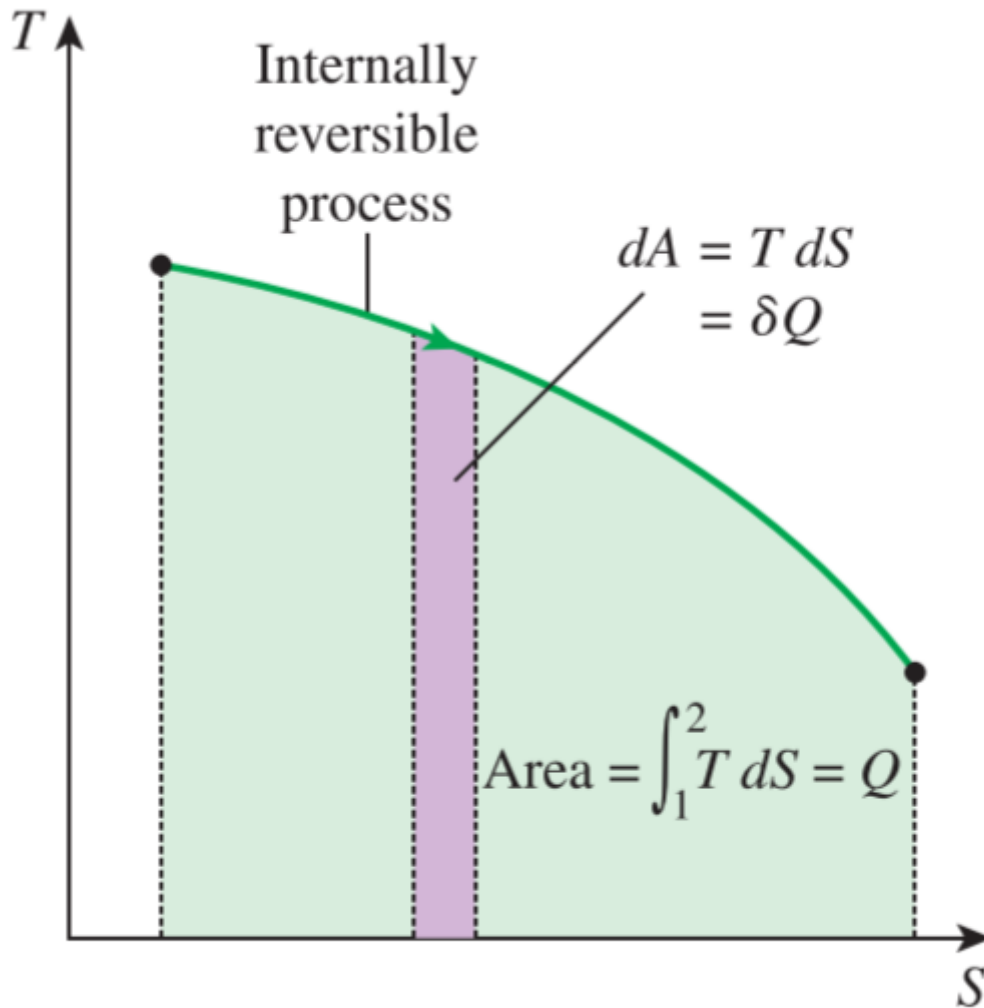
$$\Delta s = 0 \quad \text{or} \quad s_1 = s_2$$

It should be recognized that **a reversible adiabatic process** is necessarily **isentropic** ( $s_2 = s_1$ ), but an **isentropic process is not necessarily a reversible adiabatic process**.

## 7.5 Property Diagrams Involving Entropy

$$\delta Q_{int rev} = T ds$$

$$Q_{int rev} = \int_1^2 T ds$$



## 7.6 What is Entropy

### Boltzmann Relation

$$S = k \ln W$$

### Gibbs's Formulation

$$S = -k \sum p_i \log p_i \quad p_i = \frac{1}{W} \ll 1$$

### Third Law of Thermodynamics

the entropy of a pure crystalline substance at absolute zero temperature is zero

## 7.7 The $T ds$ Relations

$$\delta Q_{int rev} - \delta W_{int rev,out} = dU$$

And

$$\delta Q_{int rev} = T dS$$

$$\delta W_{int rev,out} = P dV$$

Therefore,

$$Tds = du + Pdv$$

$$ds = \frac{1}{T}(du + Pdv)$$

the second equation is obtained by h

$$dh = du + PdV + vdP = Tds + vdP$$

$$ds = \frac{1}{T}(dh - vdP)$$

## 7.8 Entropy Change of Liquids and Solids

since liquids and solids can be approximated as **incompressible** substances

$$ds = \frac{du}{T} = \frac{cdT}{T}$$

$$s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \approx c_{avg} \ln \frac{T_2}{T_1}$$

when the process is isentropic, the entropy change equals to 0

## 7.9 The Entropy Change of Ideal Gases

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1} = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

## Variable Specific Heats

choose absolute zero as the reference temperature and define a function  $s^o$  as

$$s^o = \int_0^T c_p(T) \frac{dT}{T}$$

$$s_2 - s_1 = s_2^o - s_1^o - R \ln \frac{P_2}{P_1}$$

## Constant Specific Heat

during the **isentropic progress**, the change of the entropy equals to 0, therefore,

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \left( \frac{v_2}{v_1} \right)$$

$$\ln \frac{T_2}{T_1} = \ln \left( \frac{v_1}{v_2} \right)^{R/c_v}$$

$$\left( \frac{T_2}{T_1} \right) = \left( \frac{v_1}{v_2} \right)^{R/c_v} = \left( \frac{v_1}{v_2} \right)^{k-1}$$

where  $R = c_p - c_v$ ,  $k = c_p/c_v$  and  $R/c_v = k - 1$

similarly, the another equation comes to

$$\left( \frac{T_2}{T_1} \right) = \left( \frac{P_2}{P_1} \right)^{(k-1)/k}$$

combing the two equation

$$\left(\frac{P_2}{P_1}\right) = \left(\frac{v_1}{v_2}\right)^k$$

## Relative Pressure and Relative Specific Volume

similarly, from the equation we can get

$$\frac{P_2}{P_1} = \exp \frac{s_2^o - s_1^o}{R} = \frac{\exp s_2^o/R}{\exp s_1^o/R}$$

define  $\exp s^o/R$  as the relative pressure  $P_r$

$$\left(\frac{P_2}{P_1}\right)_{s=const} = \frac{P_{r2}}{P_{r1}} \quad \left(\frac{v_2}{v_1}\right)_{s=const} = \frac{v_{r2}}{v_{r1}}$$

## 7.10 Reversible Steady-Flow Work

the energy transform for a steady-flow device undergoing an internally reversible process can be expresses in differential form as

$$\delta q_{rev} - \delta w_{rev} = dh + dke + dpe$$

since  $\delta q_{rev} = dh - v dP$

$$-\delta w_{rev} = v dP + dke + dpe$$

therefore,

$$w_{rev} = - \int_1^2 v dP - \Delta ke - \Delta pe$$

when the changes in energy are negligible

$$w_{rev} = - \int_1^2 v dP$$

to avoid the negative sign, define the work input to steady flow system as

$$w_{rev,in} = \int_1^2 v dP + \Delta ke + \Delta pe$$

for steady flow system that involves no work interactions

$$w_{rev} = v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

which becomes the famous **Bernoulli Equation**

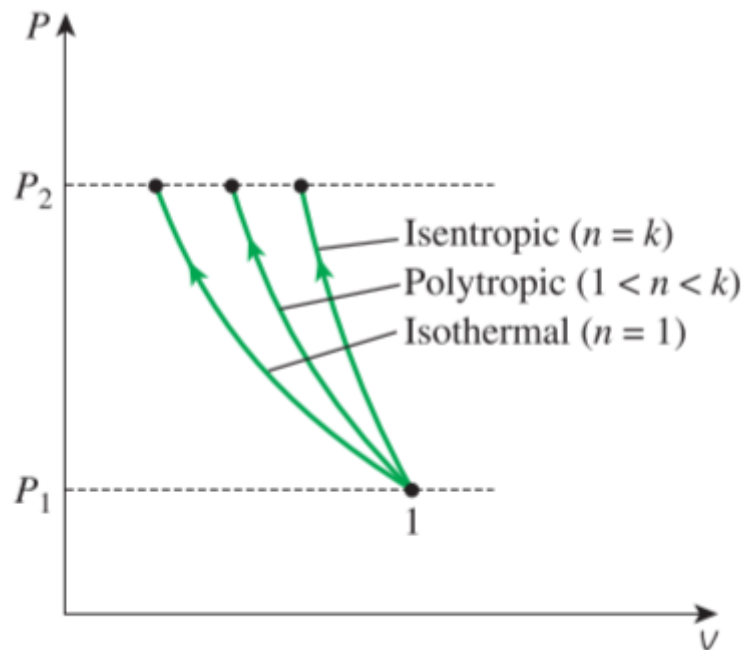
## 7.11 Minimizing the Compressor Work

from the former equation, we can claim that when the changes in kinetic and potential energies are negligible, the compressor work is given by

$$w_{rev,in} = \int_1^2 v dP$$

- minimizing the irreversibilities such as friction, turbulence and nonquasi-equilibrium compression
- keep the specific volume of the gas as small as possible during the compression process

the second minimizing could be done by **maintaining the temperature of the gas as low as possible during compression** since the specific volume of a gas is proportional to temperature



- Isentropic ( $Pv^k = \text{constant}$ )

$$w_{comp,in} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

- Polytropic ( $Pv^n = \text{constant}$ )

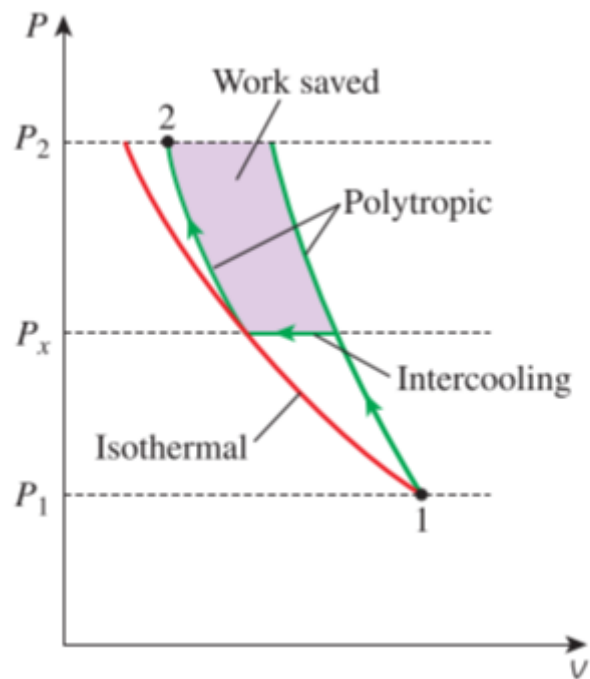
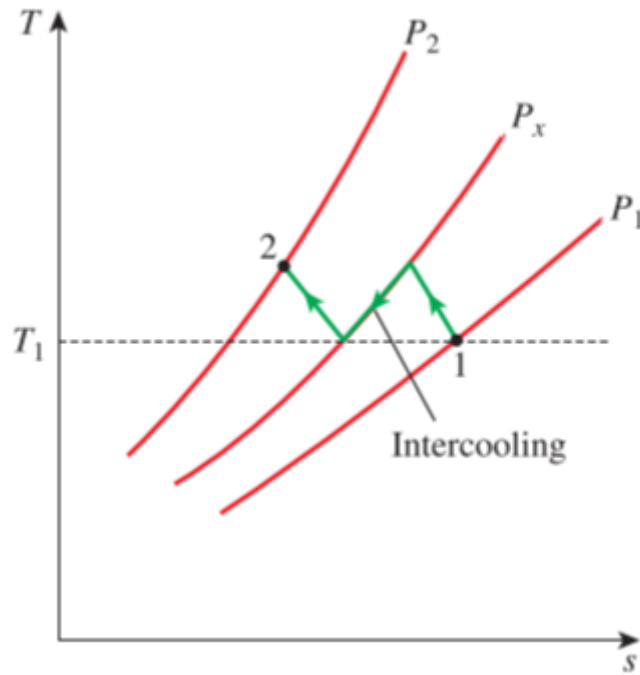
$$w_{comp,in} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[ \left( \frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

- Isothermal ( $Pv = \text{constant}$ )

$$w_{comp,in} = RT \ln \frac{P_2}{P_1}$$

## Multistage Compression with Intercooling

the technique where gas is compressed in stages and cooled between each stage by passing it through a heat exchanger called an intercooler



$$w_{comp,in} = w_{compI,in} + w_{compII,in}$$

$$= \frac{nRT_1}{n-1} \left[ \left( \frac{P_x}{P_1} \right)^{(n-1)/n} - 1 \right] + \frac{nRT_1}{n-1} \left[ \left( \frac{P_2}{P_x} \right)^{(n-1)/n} - 1 \right]$$

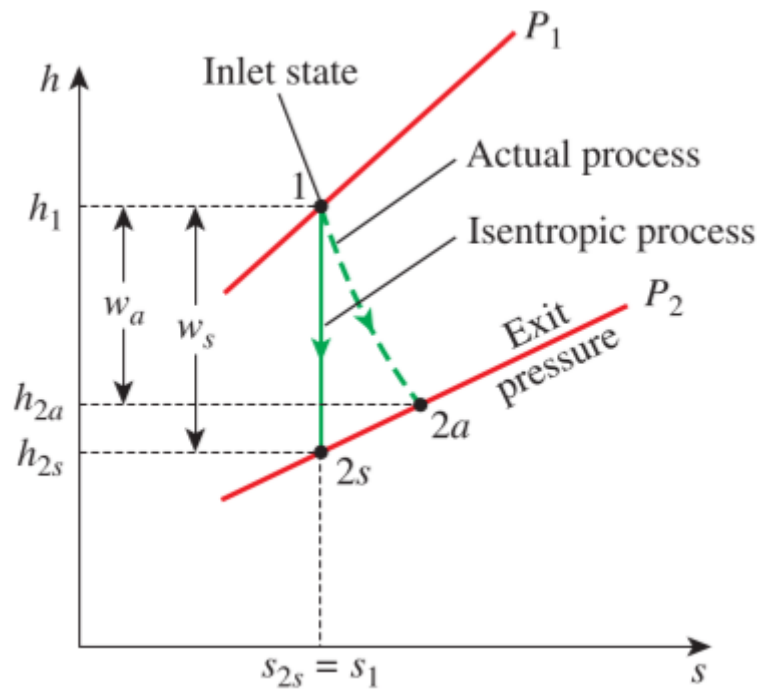
$$P_x = (P_1 P_2)^{1/2} \quad \frac{P_x}{P_1} = \frac{P_2}{P_x}$$

## 7.12 Isentropic Efficiencies of Steady-Flow Devices

### Isentropic Efficiency of Turbines

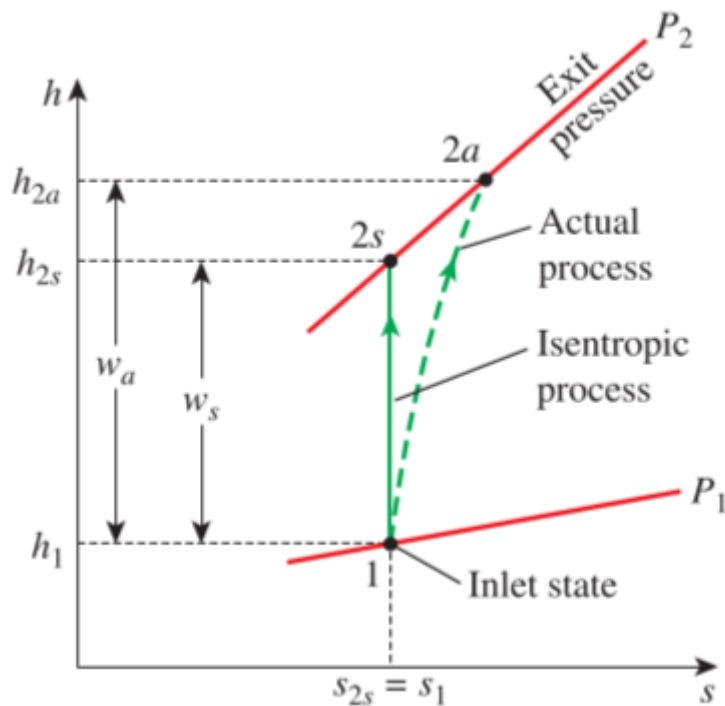
$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$





## Isentropic Efficiency of Compressors and Pumps

$$\eta_C = \frac{\text{Isentropic turbine work}}{\text{Actual turbine work}} = \frac{w_s}{w_a} \approx \frac{h_{2s} - h_1}{h_{2a} - h_1}$$



## Isentropic Efficiency of Nozzles

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2} \approx \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$