TEST5

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#Questions

Consider again the application in lecture 5.5, where we have analyzed response to a direct mailing using the following logit specification

$$Pr[resp_i = 1] = \frac{exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

for i = 1, . . . , 925. The maximum likelihood estimates of the parameters are given by

Variable	Coefficient	Std. Error	t-value	*p*-value
Intercept	-2.488	0.890	-2.796	0.005
Male	0.954	0.158	6.029	0.000
Active	0.914	0.185	4.945	0.000
Age	0.070	0.036	1.964	0.050
$(\text{Age}/10)^2$	-0.069	0.034	-2.015	0.044

(a)

The marginal effect of activity status is defined as

$$\frac{\partial Pr[resp_i = 1]}{\partial active_i} = Pr[resp_i = 1]Pr[resp_i = 0]\beta_2.$$

We could use this result to construct an activity status elasticity

$$\frac{\partial Pr[resp_i = 1]}{\partial active_i} = \frac{active_i}{Pr[resp_i = 1]} = Pr[resp_i = 0]active_i\beta_2.$$

Use this results to compute the elasticity effect of active status for a 50 years old active male customer. Do the same for an 50 years old inactive male customer.

answer (a)

$$Pr[resp_i = 1] = \frac{exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}{1 + exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$Pr[resp_i = 0] = 1 - Pr[resp_i = 1]$, so,

$$Pr[resp_i = 0] = \frac{1}{1 + exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

In active,

$Pr[resp_i = 0, male_i = 1, active_i = 1, age_i = 50], \beta_2 = 0.914$

$elasticity = Pr[resp_i = 0] * active_i * \beta_2$

$Pr[resp_i = 0] = \frac{1}{1+exp(\beta_0+\beta_1 male_i+\beta_2 active_i+\beta_3 age_i+\beta_4(age_i/10)^2)} = \frac{1}{1+exp(-2.488+0.954*1+0.914*1+0.070*50+(-0.069*(\frac{50}{10})^2)}$

$elasticity = \frac{1}{1+exp(-2.488+0.954*1+0.914*1+0.070*50+(-0.069*(\frac{50}{10})^2)} * 1 * 0.914$

$$= \frac{1}{1 + exp(1.155)} * 1 * 0.914 = \frac{1}{1 + 3.1740} * 1 * 0.914 = 0.219$$

In inactive,

$Pr[resp_i = 0, male_i = 1, active_i = 0, age_i = 50], \beta_2 = 0.914$

$$elasticity = Pr[resp_i = 0] * active_i * \beta_2 = 0$$

$(active_i = 0)$

(b)

The activity status variable is only a dummy variable and hence it can take only two values. It is therefore better to define the elasticity as

$$\frac{Pr[resp_i = 1|active_i = 1] - Pr[resp_i = 1|active_i = 0]}{Pr[resp_i = 1|active_i = 0]}$$

Show that you can simplify the expression for the elasticity as

$$(exp(\beta_2) - 1)Pr[resp_i = 0|active_i = 1].$$

answer (b)

$$exp(active_i\beta_2)exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2 = 2i * exp(\beta_2 active_i)$$

$$= \frac{Pr[resp_i = 1|active_i = 1] - Pr[resp_i = 1|active_i = 0]}{Pr[resp_i = 1|active_i = 0]}$$

$$= \frac{\frac{exp(\beta_2)2i}{1+exp(\beta_1)2i} - \frac{2i}{1+2i}}{\frac{2i}{1+2i}} | \frac{1+2i}{2i}$$

$$= \frac{exp(\beta_2)(1 + 2i)}{1 + exp(\beta_1)2i} - 1$$

$$= \frac{exp(\beta_2) - 1 + 2i(exp(\beta_2) - exp(\beta_1))}{1 + exp(\beta_1)2i}$$

$$\approx \frac{exp(\beta_2) - 1}{1 + exp(\beta_1)2i}$$

$$= (exp(\beta_2) - 1)Pr[resp_i = 0|active_i = 1]$$

(c)

Use the formula in (b) to compute the activity elasticity of 50 years old male active customer.

answer (c)

$$elasticity = (exp(\beta_2) - 1)Pr[resp_i = 0|active_i = 1]$$

so,

$$elasticity = (exp(\beta_2) - 1)Pr[resp_i = 0, male_i = 1, active_i = 1, age_i = 50], \beta_2 = 0.914$$

$$= (exp(\beta_2) - 1)\frac{1}{1 + exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 age_i + \beta_4 (age_i/10)^2)}$$

$$= (exp(0.914) - 1)\frac{1}{1 + exp(-2.488 + 0.954 * 1 + 0.914 * 1 + 0.070 * 50 + (-0.069 * (\frac{50}{10})^2)}$$

$$= 1.494 * 0.24$$

$$= 0.36$$