MKT382 Marketing Analytics II
Assignment 2
Due: February 26th, 11:59pm

Linear and Hierarchical Linear Models: Bayesian Estimation

In this exercise, we will practice Bayesian estimation for hierarchical linear models and regressions with random effects. We will use the same dataset "CreditCard_SOW_Data.csv" as in Assignment 1. The dataset has the following variables:

ConsumerID	ID's of the sampled consumers
History	How long (number of months) the customer has been using the card before the experiment
Income	The customer's annual income
WalletShare	The card's share of wallet in the consumer's total monthly spending
Promotion	Index of monthly promotion activity –higher index indicates more pomotions
Balance	The customer's unpaid balance at the beginning of the month

1). As in Assignment 1, convert consumer ID's to factors and create the following variable in the data frame: logSowRatio = log(WalletShare/(1-WalletShare)). Use the function MCMCregress() in the R package "MCMCpack" to estimate the linear regression

$$logSowRatio_{ij} = \beta_0 + \beta_1 \times History_i + \beta_2 \times Income_i + \beta_3 \times Balance_{ij} + \beta_4 \times Promotion_{ij} + \varepsilon_{ij}$$

Use the summary() function to find the results of the estimation. Copy and pastes the results here.

```
```{r}
l1=MCMCregress(logSowRatio~History+Income+Balance+Promotion, mcmc=6000, thin=6, data=dat)
summary(l1)
```
```

```
Iterations = 1001:6995
Thinning interval = 6
Number of chains = 1
Sample size per chain = 1000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

                  Mean        SD  Naive SE Time-series SE
(Intercept)  1.915e-01 1.699e-02 5.372e-04      5.372e-04
History      8.765e-03 2.233e-04 7.063e-06      7.063e-06
Income      -5.682e-07 1.508e-07 4.770e-09      5.041e-09
Balance     -4.960e-04 2.776e-06 8.780e-08      8.780e-08
Promotion    1.757e-01 9.001e-03 2.846e-04      2.998e-04
sigma2       4.332e-02 1.031e-03 3.259e-05      3.259e-05

2. Quantiles for each variable:

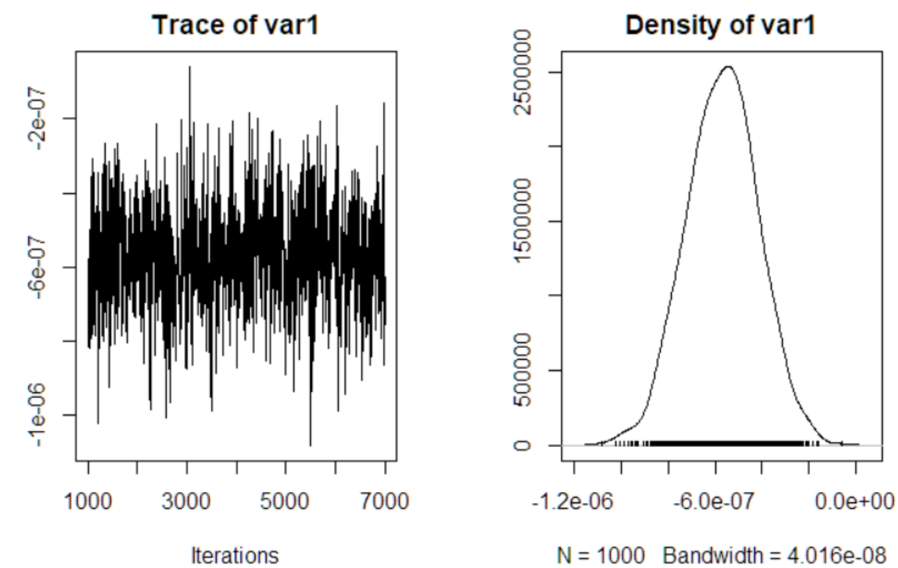
                  2.5%        25%        50%        75%      97.5%
(Intercept)  1.572e-01  1.812e-01  1.919e-01  2.030e-01  2.250e-01
History      8.293e-03  8.614e-03  8.769e-03  8.921e-03  9.189e-03
Income      -8.602e-07 -6.685e-07 -5.645e-07 -4.636e-07 -2.739e-07
Balance     -5.014e-04 -4.979e-04 -4.961e-04 -4.942e-04 -4.902e-04
Promotion    1.587e-01  1.697e-01  1.754e-01  1.819e-01  1.938e-01
sigma2       4.141e-02  4.256e-02  4.333e-02  4.400e-02  4.531e-02
```

From the Bayesian posterior intervals (use 2.5% and 97.5% quantiles of the simulated posterior distributions), are regression coefficients significant at the 5% level?

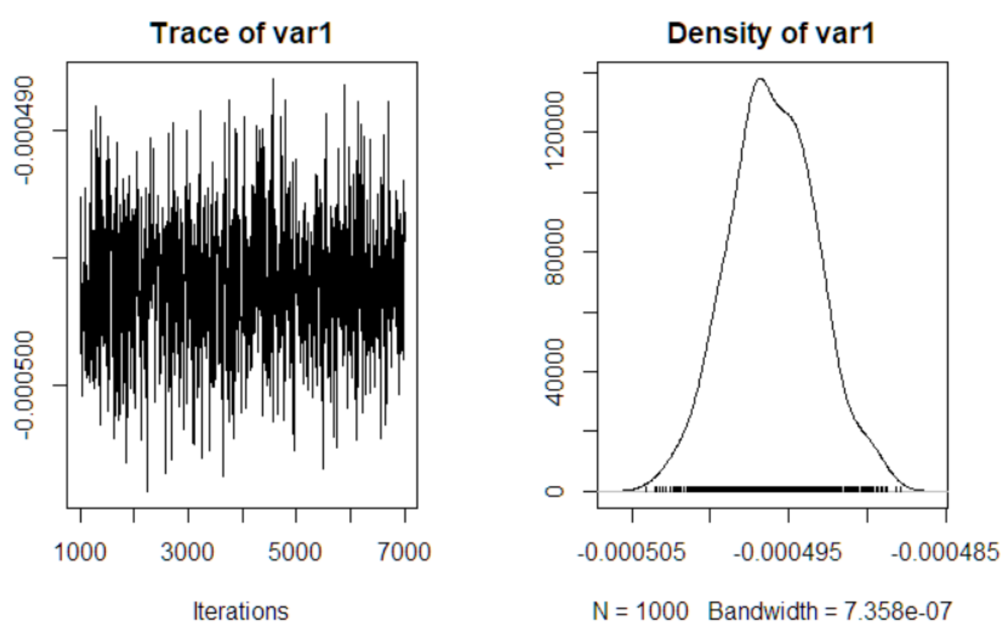
Yes. All the coefficients are significant at 5% level.

Use the plot() function to plot the posterior sampling chains and hist() to plot the posterior densities (histograms) for β_2 and β_3; copy and paste the results here.

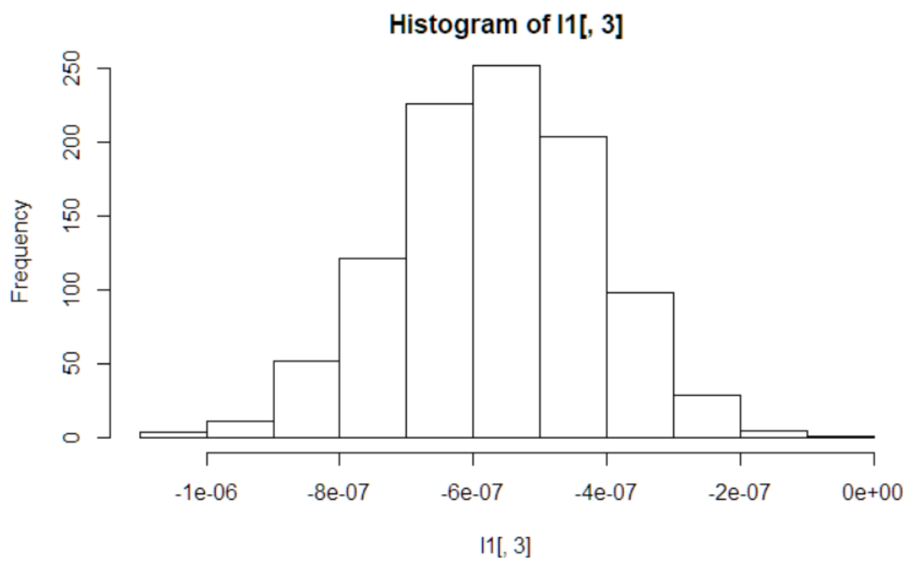
```{r}
plot(l1[,3],type="l")
```



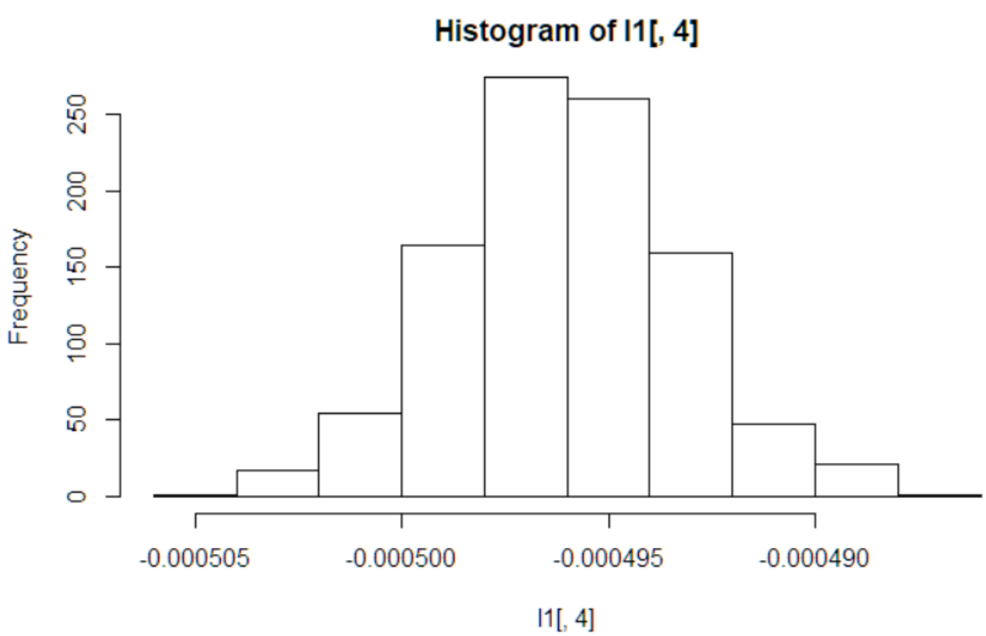

```{r}
plot(l1[,4],type="l")
```



```{r}
hist(l1[,3])
```




```{r}
hist(l1[,4])
```



2). For the hierarchical linear model below,

$$logSowRatio_{ij} = \beta_{0i} + \beta_1 \times Balance_{ij} + \beta_{2i} \times Promotion_{ij} + \varepsilon_{ij}$$

$$\beta_{0i} = \mu_0 + \mu_1 \times History_i + \zeta_i$$

$$\beta_{2i} = \gamma_0 + \gamma_1 \times History_i + \gamma_2 \times Income_i + \xi_i$$

use the function MCMChregress( ) in the R package "MCMCpack" for its Bayesian estimation.

Please copy and paste the Bayesian estimation results of the fixed effects (same fixed effects as in (3)) in the model using summary("*yourBayesianModelName*"$mcmc[,1:6]). From the Bayesian posterior intervals, are the fixed effects significant at the 5% level?

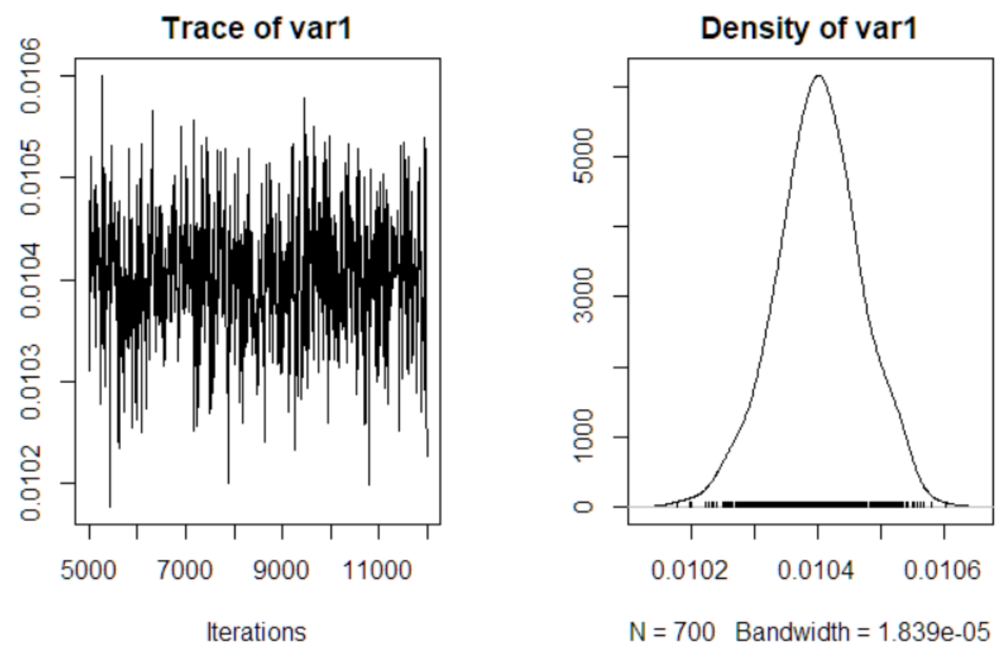
Yes, the fixed effects are significant at the 5% level.

```{r}
summary(rp1$mcmc[,1:6])
```

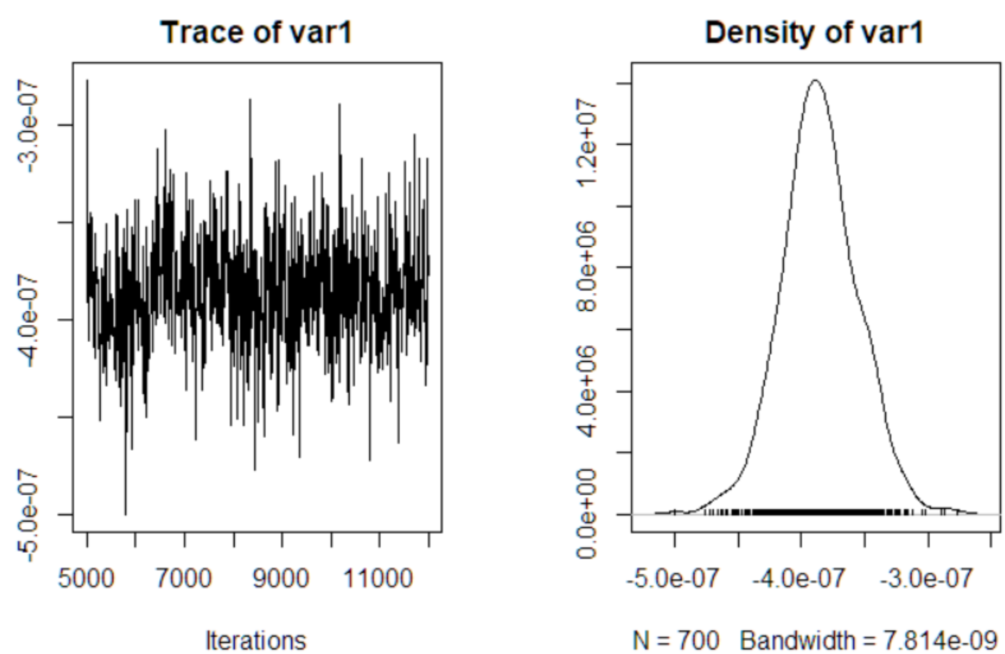
```
Iterations = 5001:11991
Thinning interval = 10
Number of chains = 1
Sample size per chain = 700

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

                             Mean        SD  Naive SE Time-series SE
beta.(Intercept)        9.659e-02 2.425e-03 9.164e-05      9.164e-05
beta.History            1.040e-02 6.699e-05 2.532e-06      2.532e-06
beta.Balance           -5.008e-04 1.473e-07 5.569e-09      5.569e-09
beta.Promotion          2.940e-01 3.083e-03 1.165e-04      1.380e-04
beta.History:Promotion -2.574e-03 4.256e-05 1.609e-06      1.818e-06
beta.Promotion:Income  -3.855e-07 3.006e-08 1.136e-09      1.136e-09

2. Quantiles for each variable:

                             2.5%        25%        50%        75%      97.5%
beta.(Intercept)        9.198e-02  9.500e-02  9.646e-02  9.811e-02  1.017e-01
beta.History            1.026e-02  1.036e-02  1.040e-02  1.045e-02  1.053e-02
beta.Balance           -5.011e-04 -5.009e-04 -5.008e-04 -5.007e-04 -5.005e-04
beta.Promotion          2.873e-01  2.921e-01  2.940e-01  2.961e-01  2.998e-01
beta.History:Promotion -2.654e-03 -2.604e-03 -2.574e-03 -2.545e-03 -2.492e-03
beta.Promotion:Income  -4.433e-07 -4.039e-07 -3.864e-07 -3.673e-07 -3.244e-07
```

Use the plot() and hist() function to plot the posterior sampling chains and posterior densities for μ_1 and γ_2; copy and paste the results here.

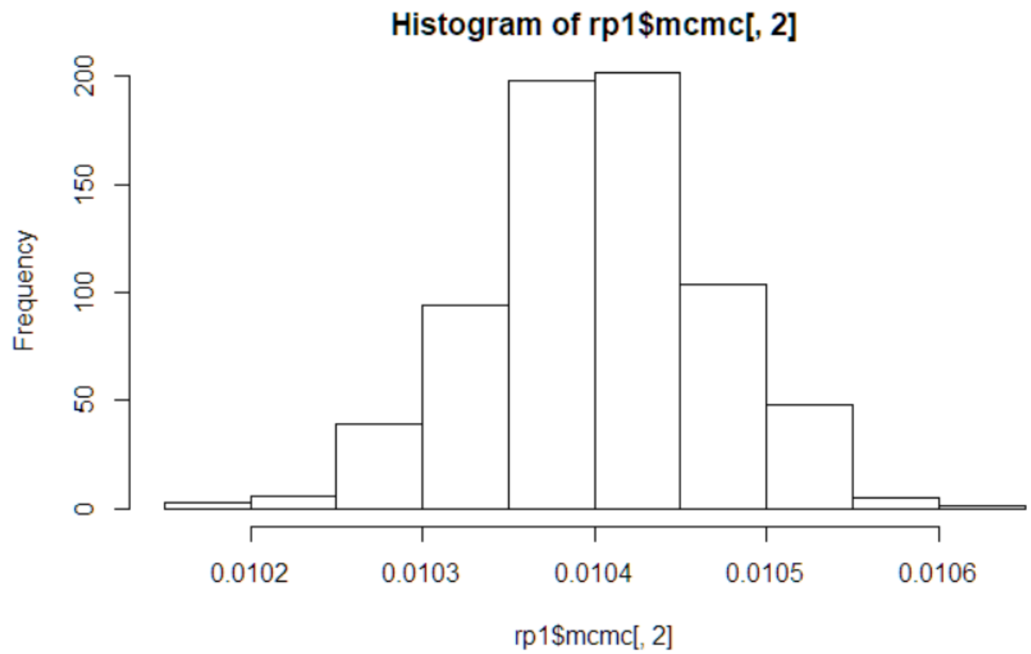
```{r}
plot(rp1$mcmc[,2],type="l")
```



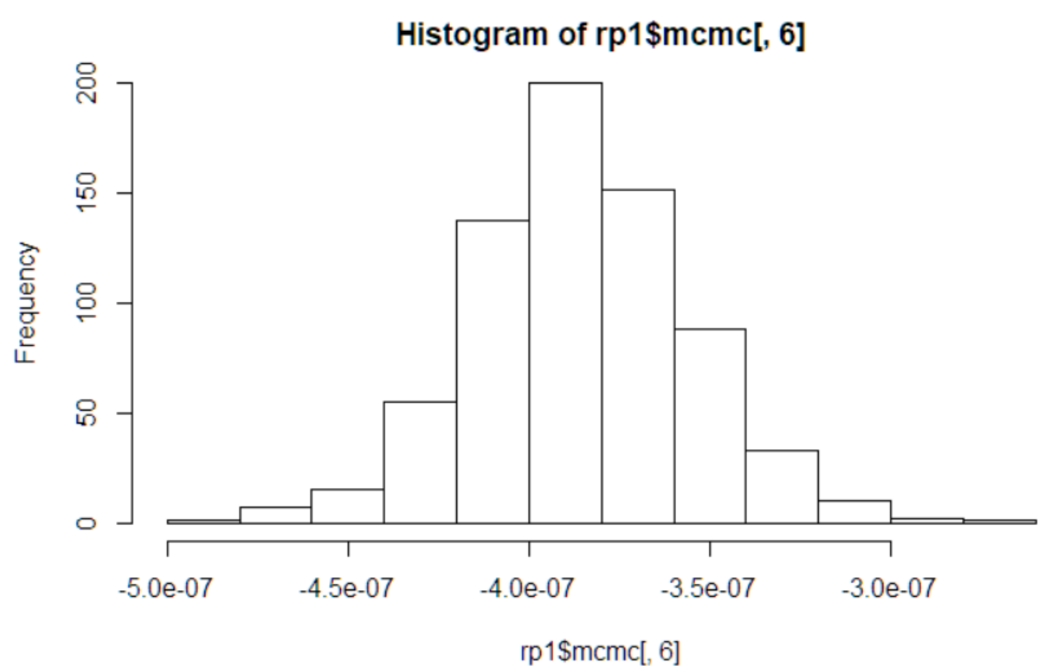

```{r}
plot(rp1$mcmc[,6],type="l")
```



```{r}
hist(rp1$mcmc[,2])
```




```{r}
hist(rp1$mcmc[,6])
```



Binary Data Regression Models for Bank Customer Attrition

This exercise is similar to the bank customer acquisition problem that we discussed in our class. Imagine that you are hired as a consultant. For the analysis, the management has given you access to 2505 customers, among whom 449 (about 18%) have closed their accounts within one year. As a consultant, you would like to know what demographic and behavioral variables contribute to higher attrition/churn rates among these customers.

The data file is "Bank_Retention_Data.csv" on Canvas. It has the following variables:

Age	The customer's age
Income	The customer's income
HomeVal	The customer's home value
TractID	A label/ID of the census tract of the customer's residence
Tenure	How long this person has been a customer of the bank
DirectDeposit	Indicator dummy=1 if the customer uses direct deposit and 0 otherwise
LoanInd	Loan indicator dummy = 1 if the customer has ever taken loans from her bank and 0 if not
Dist	Distance from customer's home to the nearest bank branch
MktShare	Bank's market share in the customer's market
Churn	Indicator dummy = 1 if the customer has closed her/his accounts (s/he has churned) with the bank and 0 if not

3). Read the data into R. Convert TractID into a factor variable.

Estimate the following binary data regression model using the R function glm( ).

$$Churn_i \sim \beta_0 + \beta_1 \times Age_i + \beta_2 \times Income_i + \beta_3 \times HomeVal_i + \beta_4 \times Tenure_i$$
$$+ \beta_5 \times DirectDeposit_i + \beta_6 \times LoanInd_i + \beta_7 \times Dist_i + \beta_8 \times MktShare_i$$

Use both of the logit (for logistic regression) and probit (for probit regression) link functions of the binomial family and paste results here.

```{r}
glm1=glm(Churn~Age+Income+HomeVal+Tenure+DirectDeposit+Loan+Dist+MktShare,data=bank,family=binomial(link="logit"))
summary(glm1)
```

```
Call:
glm(formula = Churn ~ Age + Income + HomeVal + Tenure + DirectDeposit +
    Loan + Dist + MktShare, family = binomial(link = "logit"),
    data = bank)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.2054  -0.6823  -0.5328  -0.3401   2.6266

Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)   -0.606224   0.296596  -2.044 0.040960 *
Age           -0.016103   0.004150  -3.881 0.000104 ***
Income         0.107067   0.015985   6.698 2.11e-11 ***
HomeVal       -0.026059   0.005477  -4.758 1.95e-06 ***
Tenure        -0.029709   0.006549  -4.536 5.73e-06 ***
DirectDeposit -0.465836   0.110617  -4.211 2.54e-05 ***
Loan           0.099376   0.124380   0.799 0.424310
Dist           0.267618   0.061958   4.319 1.57e-05 ***
MktShare      -0.082440   0.325551  -0.253 0.800089
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2355.9  on 2504  degrees of freedom
Residual deviance: 2189.4  on 2496  degrees of freedom
AIC: 2207.4

Number of Fisher Scoring iterations: 5
```

```{r}
glm2=glm(Churn~Age+Income+HomeVal+Tenure+DirectDeposit+Loan+Dist+MktShare,data=bank,family=binomial(link="probit"))
summary(glm2)
```

```
Call:
glm(formula = Churn ~ Age + Income + HomeVal + Tenure + DirectDeposit +
    Loan + Dist + MktShare, family = binomial(link = "probit"),
    data = bank)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.1714  -0.6886  -0.5374  -0.3252   2.7140

Coefficients:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)   -0.397967   0.168825  -2.357   0.0184 *
Age           -0.009050   0.002314  -3.910 9.22e-05 ***
Income         0.059194   0.008871   6.673 2.51e-11 ***
HomeVal       -0.014360   0.002922  -4.914 8.90e-07 ***
Tenure        -0.016430   0.003550  -4.628 3.69e-06 ***
DirectDeposit -0.263070   0.062851  -4.186 2.84e-05 ***
Loan           0.057756   0.070224   0.822   0.4108
Dist           0.154712   0.036313   4.261 2.04e-05 ***
MktShare      -0.045443   0.184547  -0.246   0.8055
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2355.9  on 2504  degrees of freedom
Residual deviance: 2188.6  on 2496  degrees of freedom
AIC: 2206.6

Number of Fisher Scoring iterations: 6
```

How do you interpret β_1, β_2, β_3, β_4, β_5, β_6, β_7, β_8? Are they statistically significant in the logistic and probit models? Please also calculate the AIC and BIC of the logistic and probit models using the R functions AIC( ) and BIC( ). Which model (logistic or probit) fits the data better based on AIC and BIC?

In both logistic and probit models, β_6 and β_8 are not statistically significant, but β_1, β_2, β_3, β_4, β_5 and β_7 are.

β_1 means when Age increases, the probability of churn will decrease.

β_2 means when Income increases, the probability of churn will also increase.

β_3 means when HomeVal increases, the probability of churn will decrease.

β_4 means when Tenure increases, the probability of churn will decrease.
β_5 means when DirectDeposit increases, the probability of churn will decrease.
β_7 means when Dist increases, the probability of churn will also increase.

```{r}
AIC(glm1)
```

```
[1] 2207.358
```

```{r}
AIC(glm2)
```

```
[1] 2206.626
```

```{r}
BIC(glm1)
```

```
[1] 2259.793
```

```{r}
BIC(glm2)
```

```
[1] 2259.06
```

According to AIC and BIC, the probit model is slightly better than the logistic model.

4). Next we will use a random effect grouped by TractID in the logistic regression.
Use the function glmer( ) in the "lme4" package in R to fit

$$Churn_i \sim \beta_{0p} + \beta_1 \times Age_i + \beta_2 \times Income_i + \beta_3 \times HomeVal_i + \beta_4 \times Tenure_i + \beta_5 \times DirectDeposit_i + \beta_6 \times LoanInd_i + \beta_7 \times Dist_i + \beta_8 \times MktShare_i$$

where β_{0p} is the random effect for the p-th census tract (TractID). Paste results here.

Check the fixed effect estimates of β_1, β_2, β_3, β_4, β_5, β_6 , β_7, β_8 again. Are they still statistically significant? Please also calculate the AIC and BIC of this model using the R functions AIC( ) and BIC( ). Based on the AIC and BIC, compare the model fit of this model to the models in (3).

It's the same: β_6 and β_8 are not statistically significant, but β_1, β_2, β_3, β_4, β_5 and β_7 are.
Based on AIC and BIC, this model does not fit the dataset as well as the models in (3)

```{r}
AIC(glmer1)
```

```
[1] 2208.686
```

```{r}
BIC(glmer1)
```

```
[1] 2266.947
```

```{r}
summary(glmer1)
```

```
Generalized linear mixed model fit by maximum likelihood (Laplace Approximation) ['glmerMod']
 Family: binomial  ( logit )
Formula: Churn ~ Age + Income + HomeVal + Tenure + DirectDeposit + Loan +
    Dist + MktShare + (1 | TractID)
   Data: bank
Control: glmerControl(optimizer = "bobyqa", optCtrl = list(maxfun = 1e+05))

     AIC      BIC   logLik deviance df.resid
  2208.7   2266.9  -1094.3   2188.7     2495

Scaled residuals:
    Min      1Q  Median      3Q     Max
-1.0913 -0.5118 -0.3894 -0.2447  5.3463

Random effects:
 Groups  Name        Variance Std.Dev.
 TractID (Intercept) 0.01988  0.141
Number of obs: 2505, groups:  TractID, 26

Fixed effects:
               Estimate Std. Error z value Pr(>|z|)
(Intercept)   -0.561878   0.305951  -1.836   0.0663 .
Age           -0.016503   0.004178  -3.950 7.81e-05 ***
Income         0.106973   0.016078   6.653 2.87e-11 ***
HomeVal       -0.026715   0.005692  -4.693 2.69e-06 ***
Tenure        -0.029232   0.006564  -4.453 8.46e-06 ***
DirectDeposit -0.461198   0.111002  -4.155 3.25e-05 ***
Loan           0.099832   0.124633   0.801   0.4231
Dist           0.266895   0.063377   4.211 2.54e-05 ***
MktShare       0.006009   0.373151   0.016   0.9872
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

5). For the model in (4), use the MCMCpack function MCMChlogit() to estimate the same parameters with Bayesian estimation. Because the model only has a random intercept, specify random=~1 and r=2, R=1 in the MCMChlogit() function. Please also set burnin=10000, mcmc=20000 and thin=20.

```{r}
a=MCMChlogit(Churn~Age+Income+HomeVal+Tenure+DirectDeposit+Loan+Dist+MktShare,random=~1,group="TractID", data=bank,r=2,R=1,
burin=10000, mcmc=20000, thin=20)
```

```
Running the Gibbs sampler. It may be long, keep cool :)

**********:10.0%, mean accept. rate=0.374
**********:20.0%, mean accept. rate=0.416
**********:30.0%, mean accept. rate=0.459
**********:40.0%, mean accept. rate=0.445
**********:50.0%, mean accept. rate=0.565
**********:60.0%, mean accept. rate=0.485
**********:70.0%, mean accept. rate=0.544
**********:80.0%, mean accept. rate=0.536
**********:90.0%, mean accept. rate=0.473
**********:100.0%, mean accept. rate=0.545
```

Please copy and paste the Bayesian estimation results of the fixed effects (same fixed effects as in (4)) in the model using summary("*yourBayesianModelName*"$mcmc[,1:9]). From the Bayesian posterior intervals, are the fixed effects significant at the 5% level?

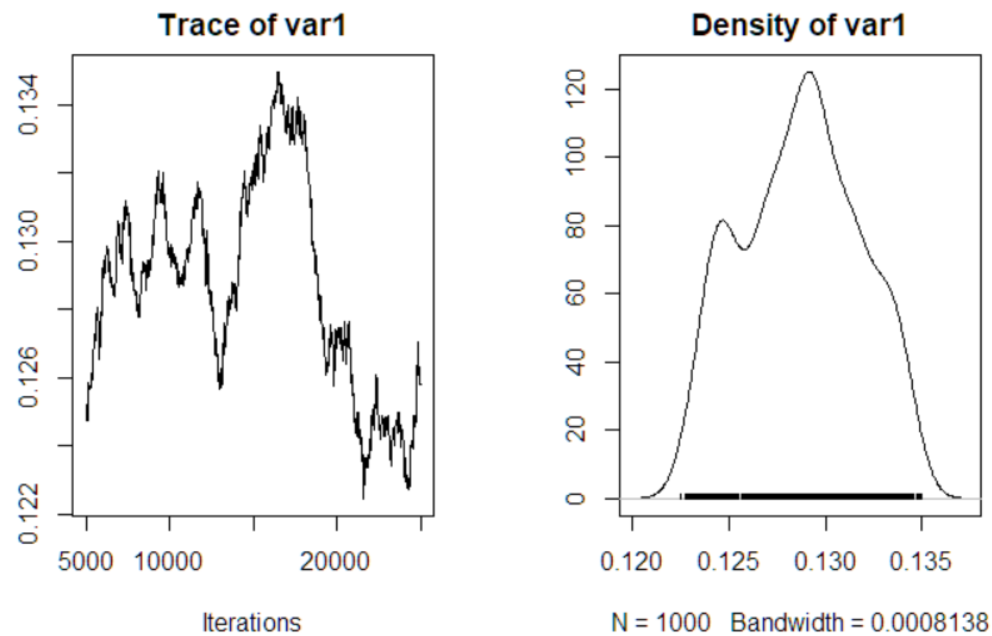
Yes, the fixed effects are significant at the 5% level

```{r}
summary(a$mcmc[,1:9])
```

```
Iterations = 5001:24981
Thinning interval = 20
Number of chains = 1
Sample size per chain = 1000

1. Empirical mean and standard deviation for each variable,
   plus standard error of the mean:

                        Mean        SD  Naive SE Time-series SE
beta.(Intercept)    -0.45298 0.1126221 3.561e-03      0.0650863
beta.Age            -0.01592 0.0007637 2.415e-05      0.0003775
beta.Income          0.12871 0.0030566 9.666e-05      0.0018888
beta.HomeVal        -0.03239 0.0004787 1.514e-05      0.0002394
beta.Tenure         -0.04029 0.0008128 2.570e-05      0.0002731
beta.DirectDeposit -0.57058 0.0285845 9.039e-04      0.0156295
beta.Loan            0.22673 0.0285611 9.032e-04      0.0136874
beta.Dist            0.26031 0.0212924 6.733e-04      0.0138720
beta.MktShare       -0.19183 0.1132437 3.581e-03      0.0603667

2. Quantiles for each variable:

                        2.5%      25%      50%      75%    97.5%
beta.(Intercept)    -0.61111 -0.56889 -0.46363 -0.35185 -0.24824
beta.Age            -0.01728 -0.01649 -0.01606 -0.01521 -0.01451
beta.Income          0.12343  0.12636  0.12886  0.13110  0.13415
beta.HomeVal        -0.03347 -0.03271 -0.03239 -0.03200 -0.03149
beta.Tenure         -0.04162 -0.04101 -0.04017 -0.03963 -0.03902
beta.DirectDeposit -0.60220 -0.59035 -0.58099 -0.56050 -0.50647
beta.Loan            0.18521  0.20597  0.22662  0.24088  0.30619
beta.Dist            0.23376  0.24633  0.25331  0.26551  0.30882
beta.MktShare       -0.35181 -0.30624 -0.19467 -0.08703 -0.01197
```

Use the plot() function to plot the posterior sampling chains and posterior densities for β_2 and β_5; copy and paste the results here.

```{r}
plot(a$mcmc[,3])
```




```{r}
plot(a$mcmc[,6])
```