This is the title

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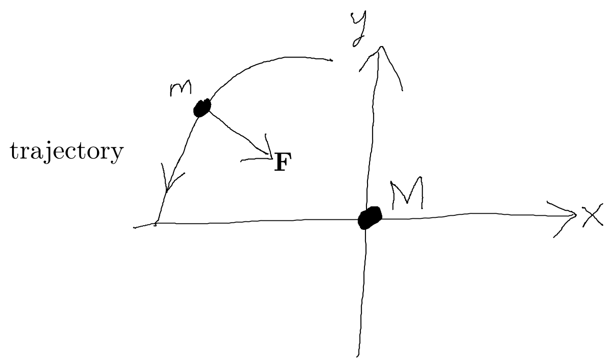
1 Introduction

Theorm 1.1 (Newton's 2nd law)**.** The force acting on a particle is related to

$$\mathbf{F} = m\mathbf{a} \tag{1}$$

The Kepler's problem in 2D is shown in the figure below. Where the force



Figuur 1: L1F1

follow the inverse square law

$$\left| \mathbf{F} = \frac{GMm}{x^2 + y^2} \right| \tag{2}$$

We switch to 2D polar coordinates (r, θ), where

$$x = r\cos\theta \tag{3}$$

$$y = r\sin\theta \tag{4}$$

The new basic vector is $\mathbf{e}_r, \mathbf{e}_\theta$, they are not fixed, thery are defined as

$$\mathbf{r} = r\mathbf{e}_r \tag{5}$$

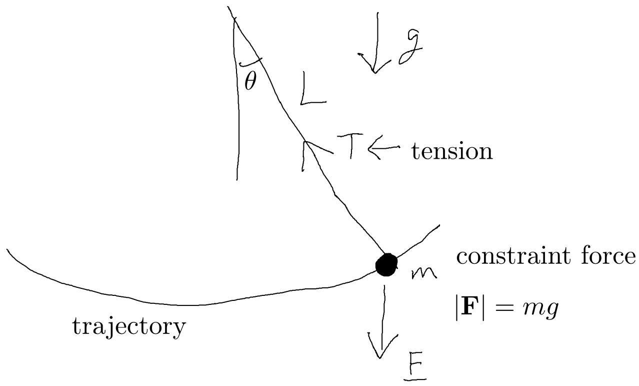
$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \tag{6}$$

$$\mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \tag{7}$$

The general coorginate system such as spherocal polar can be complicated.

1.1 Constrant force

Simple example is the simple pendulum The equation of motion is given by



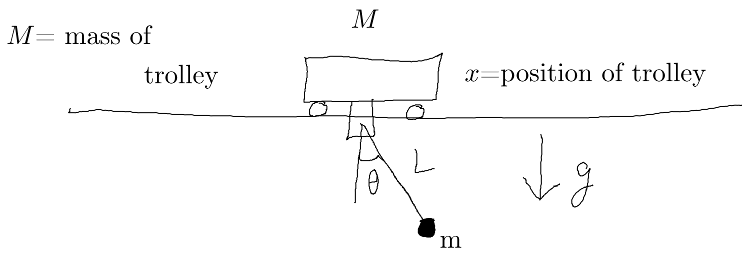
Figuur 2: L1F2

$$\ddot{\theta} = -\frac{g}{L}\sin\theta \tag{8}$$

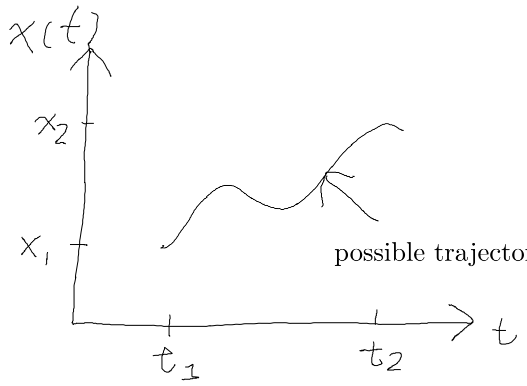
A more intricate problem is shown in figure 3

We cannot analyse constrant force, need a sidestep using a variational principle. 'Hamilton's principle' or 'principle of least action'.

Consider the trajectory of a particle (1d) mass m



Figuur 3: L1F3



Figuur 4: L1F4

$$x_1 = x(t_1) = \text{initial position} \tag{9}$$

$$x_2 = x(t_2) = \text{final position} \tag{10}$$

Consider the integral

$$S = \int_{t_1}^{t} L \text{ dt} \tag{11}$$

Where S is the action, L is the Lagrangian defined as

$$L = \frac{1}{2} m \dot{x}^2 - V(x) = T - V \tag{12}$$

Where $V(x)$ is the potential energy. It is defined through

$$F(x) = -\frac{dV}{dx} \tag{13}$$

Where the force F is conservative. One can compute the action S for any hyperthetical trajectory. The trajectory that minimizes S is the true trajectary of particle.

2 Calculus of Variation