

1. Karnaugh maps

- Method of simplifying Boolean expressions visually
- Hamming distance – minimum number of bits needed to change a binary number to another number
 - 101 to 110 has a Hamming distance of two
- Karnaugh maps have a Hamming distance of one between all adjacent cells
 - Why the order on a 2-variable row/column goes 00, 01, 11, 10, see below
- Map wraps around top, bottom, sides
- Place a 1 wherever the function is true, 0 elsewhere
- Examples:

		AB			
		00	01	11	10
C	0	0	d	0	1
	1	0	1	0	1

Three variable Karnaugh map

		AB			
		00	01	11	10
CD	00	0	4	12	8
	01	1	5	13	9
	11	3	7	15	11
	10	2	6	14	10

Four variable Karnaugh map

2. Terminology

- Literal – each variable in a product term, either uncomplemented or complemented
 - Example: A in $\overline{A}BC$
- Don't cares – combinations of inputs that will never occur (represented by a **d** or **D**)
 - Thus, the output at that point can be either 1 or 0
 - Binary to decimal converters – if they use 4 bits to represent a decimal digit, we'll never see 1010, 1011, 1100, 1101, and 1111 and thus those are don't cares
 - Don't cares are useful for simplifying function further
- Implicant – product term for which the function is 1 (e.g., 11 for AND)
- Prime implicant – the largest possible implicant
 - Essential prime implicant – prime implicant that contains a 1 that no other prime implicant has
 - Don't cares can be included in these
- Cover – set of implicants that cover all the 1's in the map
- Cost of a circuit – number of gates + the total number of inputs to the gates

3. Minimization

- Generate all prime implicants
 - Draw rectangles around entries that include 1s and not 0s
 - Size of rectangles must be powers of 2 (remember, 1 is a power of 2 as well!)
 - Make sure rectangles are as large as possible
 - Remember that you can wrap around sides
- Eliminate prime implicants that overlap until you find the essential implicants
 - Other considerations: may want to minimize cost

4. Examples

a. $f_1 = m_0 + m_1 + m_4 + m_5 + m_7 = \Sigma(0, 1, 4, 5, 7) = \overline{B} + AC$

		AB			
		00	01	11	10
C	0	1	0	0	1
	1	1	0	1	1

b. $f_2 = \Sigma(6, 8, 9, 10, 11, 12, 13, 14) = A\overline{C} + A\overline{B} + BC\overline{D}$

		AB			
		00	01	11	10
C D	00	0	0	1	1
	01	0	0	1	1
	11	0	0	0	1
	10	0	1	1	1

c. Further examples with don't cares and wrapping

i. $f_2 = m0 + D2 + D5 + D7 + m8 + m10 = \overline{B}\overline{D}$

		AB			
		00	01	11	10
C D	00	1	0	0	1
	01	0	d	0	0
	11	0	d	0	0
	10	d	0	0	1

d. Whether or not don't cares are included depends on your desired use case

- i. Example: whenever we see an illegal input, raise a flag
- ii. Wouldn't want to include don't cares in this case