Homework 1 (due 10/16)

Consider a band-pass signal $m(t) = \sum_{n=0}^{3} a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t)$ where
n $= 0, f_0 = 16, a_0 = 1, b_0 = 1$
n $= 1, f_1 = 18, a_1 = -1, b_1 = 1$
n $= 2, f_2 = 22, a_2 = 1, b_2 = -1$
n $= 3, f_3 = 24, a_3 = -1, b_3 = 1$

1. Plot the band pass signal $m(t)$ by using Matlab or any other software.
2. Is $m(t)$ a periodic signal? If yes, find its period.
3. What is the minimum sampling rate for $m(t)$?
4. Plot the spectrum (in frequency domain) $M'(f)$ of $m'(t)$, where $m'(t)$ is the baseband equivalent signal (complex envelope) of $m(t)$. You can draw the spectrum by hand.
5. Derive the formula for the base-band equivalent signal $m'(t)$.
6. Derive the formula for $m_I(t) = \mathrm{Re}[m'(t)]$ and $m_Q(t) = \mathrm{Im}[m'(t)]$. $m_I(t)$ and $m_Q(t)$ are in-phase component and quadrature component of $m(t)$, respectively.
7. Plot $m_I(t)$, $m_Q(t)$, and envelope $r(t) = \sqrt{m_I(t)^2 + m_Q(t)^2} = |x(t)|$ in the same figure by using Matlab or any other software.
8. Plot the band-pass signal $m(t)$ and envelope $r(t)$ in the same figure by using Matlab or any other software. What is the relationship between the two signals?
9. Let $a(t) = m_I(t) \cdot cos(2\pi f_c t)$, $b(t) = m_Q(t) \cdot sin(2\pi f_c t)$, where f_c=20 Hz is the central frequency. Plot

$$c(t) = a(t) - b(t)$$

by using Matlab or any other software. Compare $c(t)$ with $m(t)$. What is your conclusion?