Homework 1 (due 10/16)

Consider a band-pass signal $m(t) = \sum_{n=0}^{3} a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t)$ where

$\mathrm{n} = 0, f_0 = 16, a_0 = 1, b_0 = 1$

$\mathrm{n} = 1, f_1 = 18, a_1 = -1, b_1 = 1$

$\mathrm{n} = 2, f_2 = 22, a_2 = 1, b_2 = -1$

$\mathrm{n} = 3, f_3 = 24, a_3 = -1, b_3 = 1$

1. Plot the band pass signal $m(t)$ by using Matlab or any other software.

2. Is $m(t)$ a periodic signal? If yes, find its period.

3. What is the minimum sampling rate for $m(t)$?

4. Plot the spectrum (in frequency domain) $M'(f)$ of $m'(t)$, where $m'(t)$ is the baseband equivalent signal (complex envelope) of $m(t)$. You can draw the spectrum by hand.

5. Derive the formula for the base-band equivalent signal $m'(t)$.

6. Derive the formula for $m_I(t) = \mathrm{Re}[m'(t)]$ and $m_Q(t) = \mathrm{Im}[m'(t)]$. $m_I(t)$ and $m_Q(t)$ are in-phase component and quadrature component of $m(t)$, respectively.

7. Plot $m_I(t)$, $m_Q(t)$, and envelope $r(t) = \sqrt{m_I(t)^2 + m_Q(t)^2} = |x(t)|$ in the same figure by using Matlab or any other software.

8. Plot the band-pass signal $m(t)$ and envelope $r(t)$ in the same figure by using Matlab or any other software. What is the relationship between the two signals?

9. Let $a(t) = m_I(t) \cdot cos(2\pi f_c t)$, $b(t) = m_Q(t) \cdot sin(2\pi f_c t)$, where f_c=20 Hz is the central frequency. Plot

$$c(t) = a(t) - b(t)$$

by using Matlab or any other software. Compare $c(t)$ with $m(t)$. What is your conclusion?

2、

$f_0=16 \quad f_1=18 \quad f_2=22 \quad f_3=24$

$\frac{f_1}{f_0}=\frac{9}{8} \quad \frac{f_2}{f_0}=\frac{11}{8} \quad \frac{f_3}{f_0}=\frac{3}{2}$

⇒ 比例都是有理數，因此是週期性

$[f_0, f_1, f_2, f_3] = [2^4, 2\times3^2, 2\times11, 2^3\times3] = 2^4\times3^2\times11 = 1584\,(Hz)$

$T=\frac{1}{F}=\frac{1}{1584} \approx 6.31\times10^{-4}$ (second)

3、

根據 Nyquist Sampling Theorem

取樣頻率為最高頻的2倍

⇒ $f_s = 2\times24 = 48\,(Hz)$

4、

$m'(t)$ is complex envelope 基頻等效信號

$m(t)=\sum_{n=0}^{3} x_n(t)$

$x_n = I_n\cos(2\pi f_n t) - Q_n\sin(2\pi f_n t)$

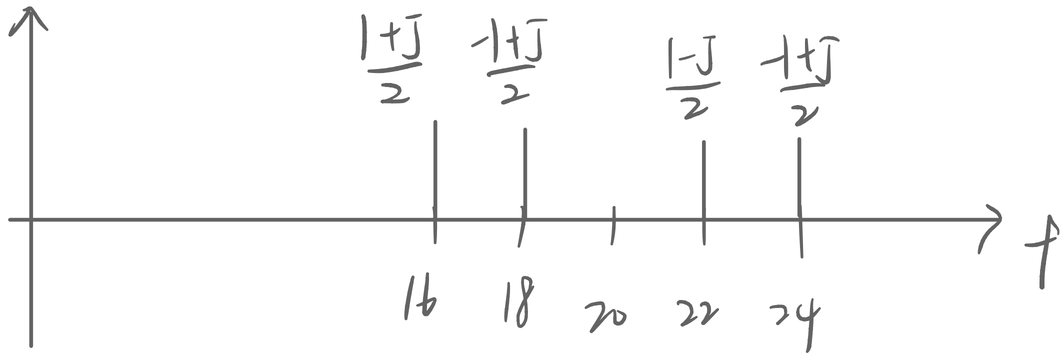
$n=0,\ f_0=16,\ I_0=1,\ Q_0=1$

$n=1,\ f_1=18,\ I_1=-1,\ Q_1=1$

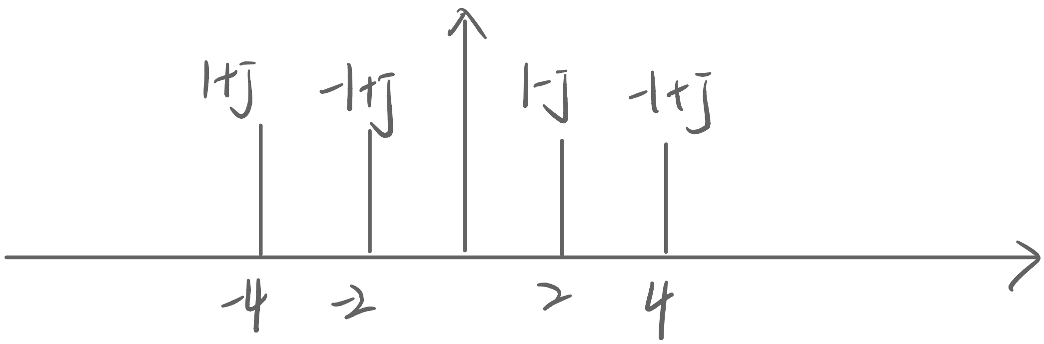
$n=2,\ f_2=22,\ I_2=1,\ Q_2=-1$

$n=3,\ f_3=24,\ I_3=-1,\ Q_3=1$

$M(f)$ （負頻省略不畫）



$M'(f)$



5.

$m'(t)$ is complex envelope 基頻等效信號

$$m'(t) = (1+j)e^{j2\pi(-4)t} + (-1+j)e^{j2\pi(-2)t} + (1-j)e^{j2\pi(2)t} + (-1+j)e^{j2\pi(4)t}$$

$$= \sum_{n=0}^{3} (a_n + jb_n) e^{j2\pi f_n t}$$

6.

$$m'(t) = m_I(t) + m_Q(t)$$

$$m'(t) = \sum_{n=0}^{3} (a_n + jb_n) e^{j2\pi f_n t} = \sum_{n=0}^{3} r_n(t) e^{j2\pi f_n t} \quad \rightarrow r_n = |a_n + jb_n|$$

$$m_I(t) = Re\{m'(t)\} = Re\left\{\sum_{n=0}^{3} (I_n + jQ_n)\left[\cos(2\pi f_n t) + j\sin(2\pi f_n t)\right]\right\}$$

$$= \sum_{n=0}^{3} I_n \cos(2\pi f_n t)$$

$$m_Q(t) = Im\{m(t)\} = Im\left\{\sum_{n=0}^{3} (I_n + jQ_n)\left[\cos(2\pi f_n t) + j\sin(2\pi f_n t)\right]\right\}$$

$$= -\sum_{n=0}^{3} Q_n \sin(2\pi f_n t)$$

$$m'(t) = m_I(t) + j m_Q(t)$$

$$= \sum_{n=0}^{3} I_n \cos(2\pi f_n t) - j Q_n \sin(2\pi f_n t)$$

$*$ $e^{j2\pi f_n t} = \cos(2\pi f_n t) + j\sin(2\pi f_n t)$

8.

$$m(t) = \sum_{n=0}^{3} a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t)$$

$$r(t) = \sqrt{m_I(t)^2 + m_Q(t)^2} = |x(t)|$$

$$m(t) = Re\{m'(t) e^{j2\pi f_n t}\} = \sum_{n=0}^{3} r(t) e^{j2\pi f_n t}$$

m(t)為實數信號，是信號原始形狀

r(t)是m(t)的瞬時振幅，是m(t)的包絡線

9.

c(t)與m(t)都是由$m_I(t)$、$m_Q(t)$所組成的

$$m(t) = Re\{m'(t) e^{j2\pi f_c t}\} = m_I \cos(2\pi f_c t) - m_Q \sin(2\pi f_c t)$$

$$c(t) = m_I(t) \cdot \cos(2\pi f_c t) - m_Q(t) \cdot \sin(2\pi f_c t) \qquad f_c = 20\,Hz$$

所以m(t)跟c(t)圖形一樣