Homework 1 (due 10/16)

Consider a band-pass signal $m(t) = \sum_{n=0}^{3} a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t)$ where

$\mathrm{n} = 0, f_0 = 16, a_0 = 1, b_0 = 1$

$\mathrm{n} = 1, f_1 = 18, a_1 = -1, b_1 = 1$

$\mathrm{n} = 2, f_2 = 22, a_2 = 1, b_2 = -1$

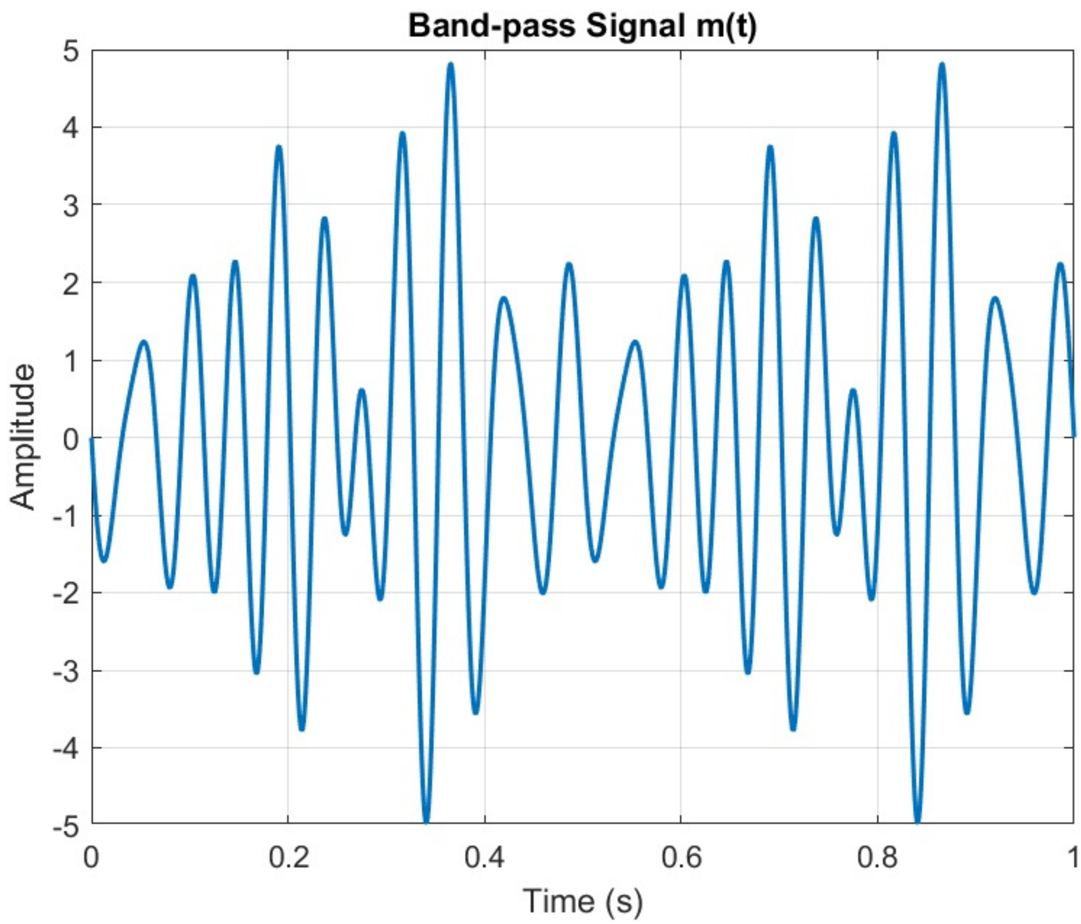
$\mathrm{n} = 3, f_3 = 24, a_3 = -1, b_3 = 1$

1. Plot the band pass signal $m(t)$ by using Matlab or any other software.
2. Is $m(t)$ a periodic signal? If yes, find its period.
3. What is the minimum sampling rate for $m(t)$?
4. Plot the spectrum (in frequency domain) $M'(f)$ of $m'(t)$, where $m'(t)$ is the baseband equivalent signal (complex envelope) of $m(t)$. You can draw the spectrum by hand.
5. Derive the formula for the base-band equivalent signal $m'(t)$.
6. Derive the formula for $m_I(t) = \mathrm{Re}[m'(t)]$ and $m_Q(t) = \mathrm{Im}[m'(t)]$. $m_I(t)$ and $m_Q(t)$ are in-phase component and quadrature component of $m(t)$, respectively.
7. Plot $m_I(t)$, $m_Q(t)$, and envelope $r(t) = \sqrt{m_I(t)^2 + m_Q(t)^2} = |x(t)|$ in the same figure by using Matlab or any other software.
8. Plot the band-pass signal $m(t)$ and envelope $r(t)$ in the same figure by using Matlab or any other software. What is the relationship between the two signals?
9. Let $a(t) = m_I(t) \cdot cos(2\pi f_c t)$, $b(t) = m_Q(t) \cdot sin(2\pi f_c t)$, where f_c=20 Hz is the central frequency. Plot

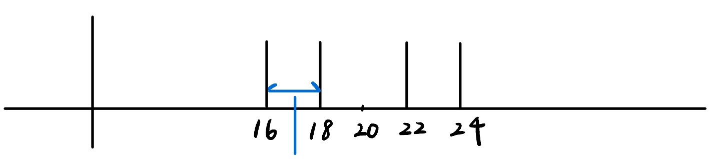
$$c(t) = a(t) - b(t)$$

by using Matlab or any other software. Compare $c(t)$ with $m(t)$. What is your conclusion?

1、



2、 $f_0 = 16$, $f_1 = 18$, $f_2 = 22$, $f_3 = 24$



基本頻 f

$\rightarrow T = \frac{1}{f} = \frac{1}{2} = 0.5$ #

(對照 matlab 所畫的 m(t)，符合 T = 0.5)

3、 根據 Nyquist Sampling Theorem，取樣頻率為最高頻的2倍

$\Rightarrow f_s = 2 \times 24 = 48$ (sample / second) #

4、 m'(t) is complex envelope 基頻等效信號

$m(t) = \sum_{n=0}^{3} x_n(t)$

$x_n = I_n \cos(2\pi f_n t) - Q_n \sin(2\pi f_n t)$

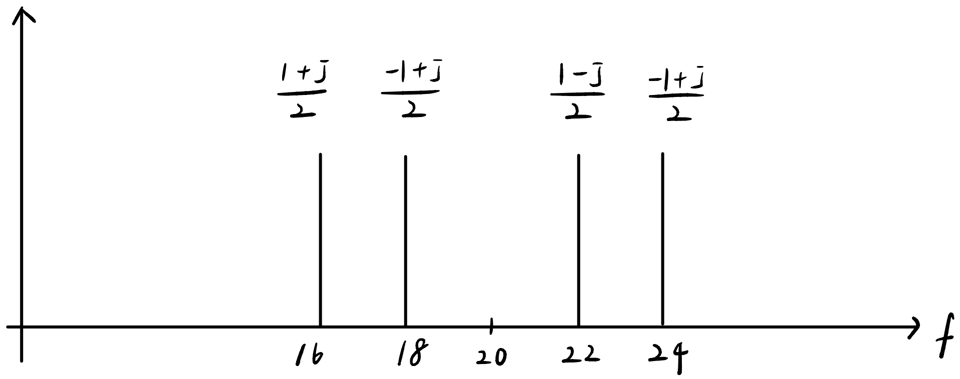
$n = 0$, $f_0 = 16$, $I_0 = 1$, $Q_0 = 1$

$n = 1$, $f_1 = 18$, $I_1 = -1$, $Q_1 = 1$

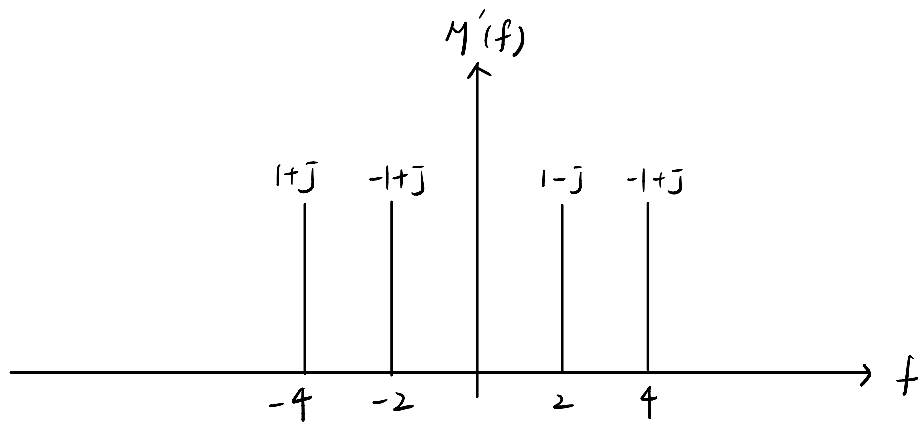
$n = 2$, $f_2 = 22$, $I_2 = 1$, $Q_2 = -1$

$n = 3$, $f_3 = 24$, $I_3 = -1$, $Q_3 = 1$

M(f) (負頻省略不畫)



◀ M(f)



◀ M'(f)

5、

$m'(t)$ is complex envelope 基頻等效信號

$$m'(t) = (1+j)e^{j2\pi(-4)t} + (-1+j)e^{j2\pi(-2)t} + (1-j)e^{j2\pi(2)t} + (-1+j)e^{j2\pi(4)t} \quad \#$$

$$= \sum_{n=0}^{3}(a_n + j b_n)e^{j2\pi f_n t}$$

6、

$$m'(t) = m_I(t) + m_Q(t)$$

$$* \; e^{j2\pi f_n t} = \cos(2\pi f_n t) - j\sin(2\pi f_n t)$$

$$m'(t) = \sum_{n=0}^{3}(a_n + j b_n)e^{j2\pi f_n t}$$

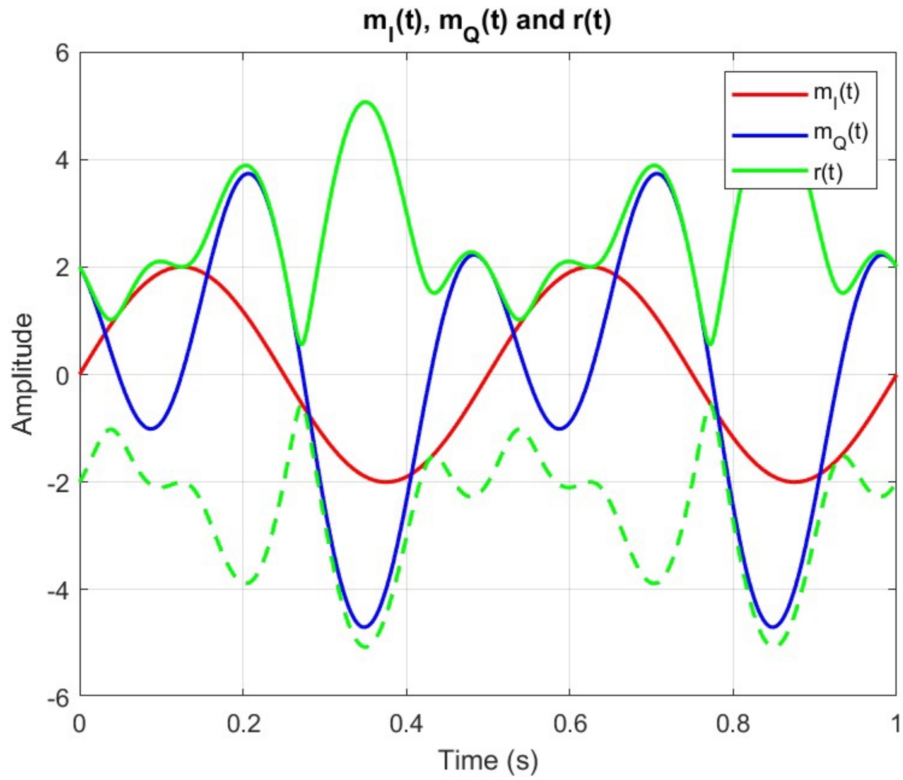
$$m'(t) = \sum_{n=0}^{3}(a_n + j b_n)(\cos(2\pi f_n t) + j\sin(2\pi f_n t))$$

$$= \sum_{n=0}^{3}[a_n\cos(2\pi f_n t) - b_n\sin(2\pi f_n t)] + j[a_n\sin(2\pi f_n t) + b_n\cos(2\pi f_n t)] \quad \#$$

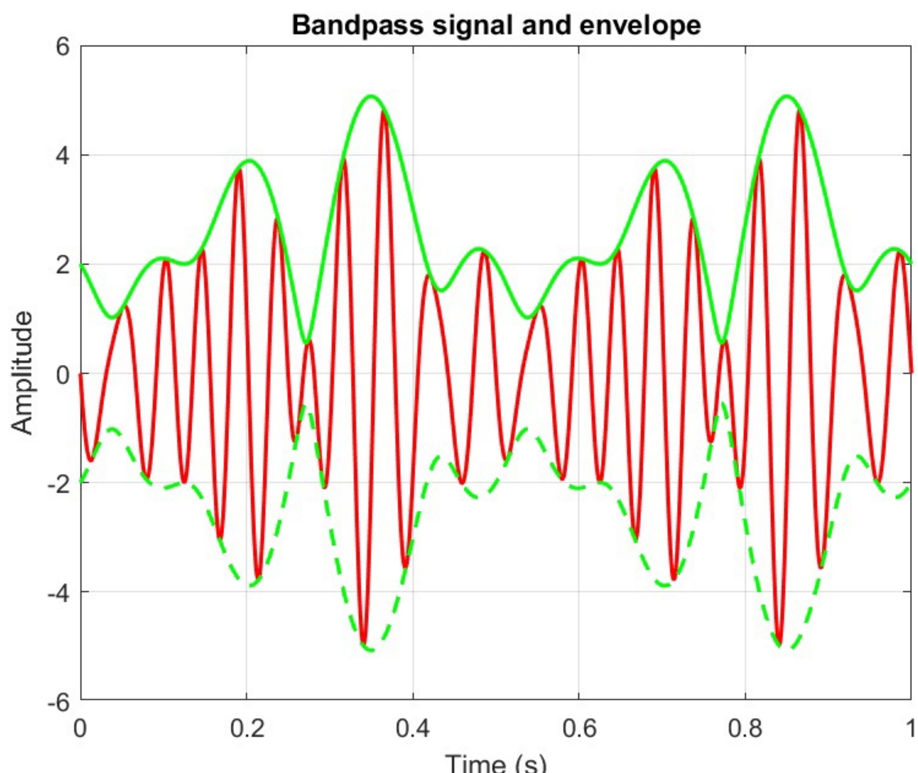
$$m_I(t) = Re\{m'(t)\} = \sum_{n=0}^{3} a_n\cos(2\pi f_n t) - b_n\sin(2\pi f_n t)$$

$$m_Q(t) = Im\{m't\} = \sum_{n=0}^{3} a_n\sin(2\pi f_n t) + b_n\cos(2\pi f_n t)$$

7、



8、



$$m(t) = \sum_{n=0}^{3} a_n \cos(2\pi f_n t) - b_n \sin(2\pi f_n t)$$

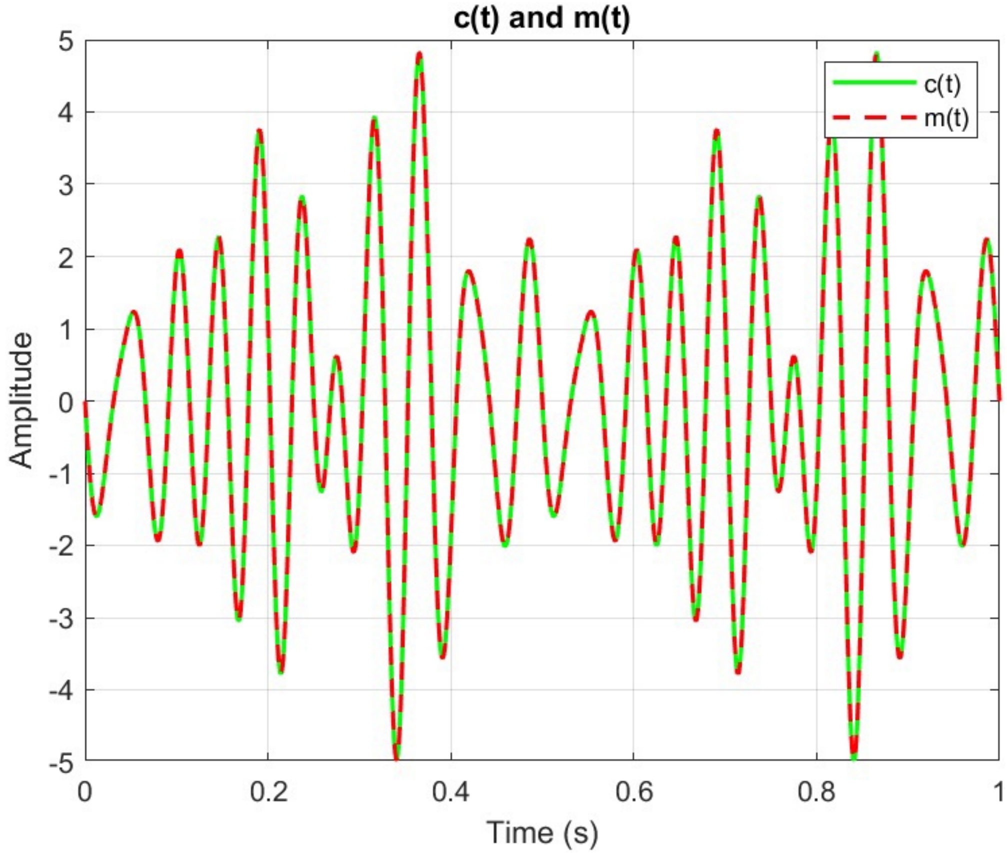
$$r(t) = \sqrt{m_I(t)^2 + m_Q(t)^2} = |x(t)|$$

$$m(t) = \mathrm{Re}\{m'(t)\, e^{j2\pi f_n t}\} = \sum_{n=0}^{3} r(t)\, e^{j2\pi f_n t}$$

$m(t)$ 為實數信號，是信號原始形狀

$r(t)$ 是 $m(t)$ 的瞬時振幅，是 $m(t)$ 的包絡線

9、



$c(t)$ 與 $m(t)$ 都是由 $m_I(t)$、$m_Q(t)$ 所組成的

$m(t) = Re\{m'(t)e^{j2\pi f_c t}\} = m_I\cos(2\pi f_c t) - m_Q\sin(2\pi f_c t)$

$c(t) = m_I(t)\cdot\cos(2\pi f_c t) - m_Q(t)\cdot\sin(2\pi f_c t)$ $f_c = 20\ Hz$

$\therefore$ $m(t)$ 跟 $c(t)$ 圖形一樣