Recommended Problems

Recommended Problems

Problem 1

Find the splitting fields of the following polynomials over $\mathbb{Q}$:

a. $x^4 - 2$

Hints:

- irreducible by Eisenstein
- $\mathbb{Q}(\sqrt[4]{2})$ is real of degree 4.
- If $\alpha = \sqrt[4]{2}$, then the four roots are $\pm\alpha$, $\pm i\alpha$.

b. $x^4 + 2$

Hints:

- irreducible by Eisenstein
- If $\alpha = \sqrt[4]{-2}$, then the four roots are $\pm\alpha$, $\pm i\alpha$.
- The square root of i is $e^{\pi/4} = \sqrt{2}/2 + i\sqrt{2}/2$, so in fact $\mathbb{Q}(\sqrt[4]{2})$ is in $\mathbb{Q}(i, \alpha)$.

c. $x^4 + x^2 + 1$

Hints:

- If α is a root of this polynomial, then α^2 is a root of $x^2 + x + 1 = 0$ and the roots of this polynomial are

$$\frac{-1 \pm \sqrt{-3}}{2} = e^{\pm 2\pi i/3}.$$

d. $x^6 - 4$

Hints:

- this polynomial is $(x^3 - 2)(x^3 + 2)$.

Problem 2 (DF, Problem 6, page 545)

Prove that, if K_1/F and K_2/F are splitting fields, then so are the composite K_1K_2 and the intersection $K_1 \cap K_2$.