

Recommended Problems

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Problem 1

Find the splitting fields of the following polynomials over \mathbb{Q} :

a. $x^4 - 2$

Hints:

- ▶ irreducible by Eisenstein
- ▶ $\mathbb{Q}(\sqrt[4]{2})$ is real of degree 4.
- ▶ If $\alpha = \sqrt[4]{2}$, then the four roots are $\pm\alpha, \pm i\alpha$.

b. $x^4 + 2$

Hints:

- ▶ irreducible by Eisenstein
- ▶ If $\alpha = \sqrt[4]{-2}$, then the four roots are $\pm\alpha, \pm i\alpha$.
- ▶ The square root of i is $e^{\pi/4} = \sqrt{2}/2 + i\sqrt{2}/2$, so in fact $\mathbb{Q}(\sqrt[4]{2})$ is in $\mathbb{Q}(i, \alpha)$.

c. $x^4 + x^2 + 1$

Hints:

- ▶ If α is a root of this polynomial, then α^2 is a root of $x^2 + x + 1 = 0$ and the roots of this polynomial are

Problem 2 (DF, Problem 6, page 545)