Recommended Problems

Recommended Problems - Rational normal form

Problem 1 (DF, Problem 4, pg. 488)

Prove that two 3×3 matrices over a field F are similar if and only if they have the same characteristic and minimal polynomials. Give an explicit counterexample to this for 4×4 matrices.

Problem 2 (DF, Problem 12,m pg. 489)

Find all similarity classes of 3×3 matrices over $\mathbb{Z}/2\mathbb{Z}$ satisfying $A^6 = 1$. Do the same for 4×4 matrices satisfying $B^{20} = 1$.

Problem 3 (DF, Problem 17, p. 489)

Let $G = \mathrm{GL}_3(\mathbf{F}_2)$. This group has order $(7)(6)(4) = 168$. Find representatives for each of its conjugacy classes.

Problem 4 (DF Problems 22-25, pg 490)

These problems show that if A is an $n\times n$ matrix over F, then the matrix $xI - A$ is the "relation matrix" for the $F[x]$-module obtained from F^n with x acting as A. Consequently, to compute the rational normal form for A, one must use the "smith normal form" algorithm over $F[x]$ on this matrix to find the invariant factors. (Note: Actually the problem uses $xI - A^\intercal$ but this is just a question of conventions regarding rows vs. columns and doesn't really matter).

Recommended Problems - Jordan normal form

Computations

DF Problems 4-16 on pages 499-501 give a whole bunch of numerical examples; try some.

Problem 5 (DF, Problem 22, page 501)

Prove that an $n\times n$ matrix A over $\mathbf{C}$ satisfying $A^3 = A$ is diagonalizable. Is this true for any field F?

Problem 6 (DF, Problem 31, pag 502)

A matrix is nilpotent if $A^n = 0$ for some n. Prove that a nilpotent matrix is similar to a block diagonal matrix where each block has ones on the superdiagonal and zeros elsewhere.

Optional: Problems 40ff in Chapter 12.3 explore applications of the Jordan form to differential equations.