

# 1. Modules

# Modules

- ▶ Modules are to rings as vector spaces are to fields.
- ▶ Modules are to rings as sets with group actions are to groups.

## Definition of (left) modules

**Definition:** Let  $R$  be a ring (for now, not necessarily commutative and not necessarily having a unit). A *left  $R$ -module* is an abelian group  $M$  together with a map  $R \times M \rightarrow M$  (written  $(r, m) \mapsto rm$ ) such that:

- ▶  $r(m_1 + m_2) = rm_1 + rm_2$
- ▶  $(r_1 + r_2)m = r_1m + r_2m$
- ▶  $r_1(r_2m) = (r_1r_2)m$

If  $R$  has a unit element  $1$ , we also require  $1m = m$  for all  $m \in M$ .

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