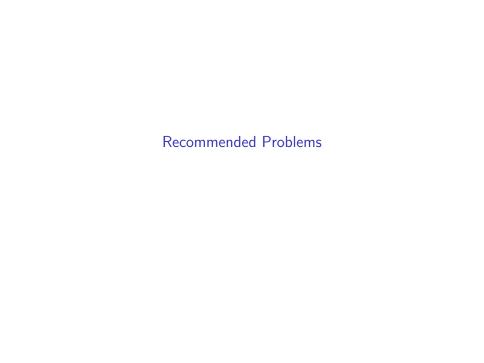
Recommended Problems



Problem 5.

If M is an R module and I is a left ideal of R, define

$$IM = \{ \sum_{\text{finite}} a_i m_i : a_i \in I, m_i \in M \}$$

Prove that IM is a submodule of M.

Problem 8.

Given a ring R and an R-module M, let

$$\mathrm{Tor}(M)=\{m\in M: \exists r\in R, rm=0, r\neq 0\}$$

Prove that, if R is an integral domain, then Tor(M) is a submodule of M.

Show that if R is *not* an integral domain then Tor(M) need not be a submodule of M.

Problem 9.

If N is a submodule of an R-module M, defined the annihilator of N in R to be the set of $r \in R$ such that rn = 0 for all $n \in N$.

Show that the annihilator of N is a two-sided ideal of R.

Problem 19.

Let $V=\mathbb{R}^2$ and let T be the linear transformation given by projection onto the y-axis. Viewing V as an $\mathbb{R}[x]$ module with x acting as T, show that the only proper $\mathbb{R}[x]$ submodules of V are the x and y axes.

Section 10.2

Problem 8

Let $\phi: M \to N$ be a module homomorphism. Prove that $\phi(\text{Tor}(M)) \subset \text{Tor}(N)$.

Problem 10

If R is commutative with 1, prove that $\operatorname{Hom}_R(R,R)$ is isomorphic to R.