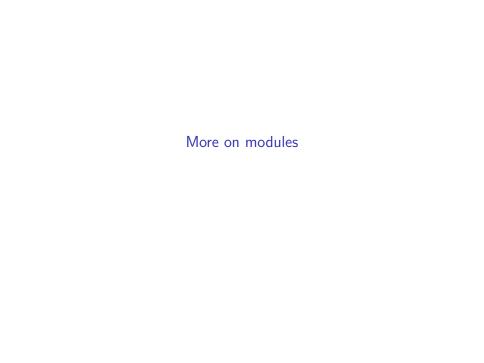
2. Modules (continued)



Sums of modules

Suppose that R is a ring and M is an R-module. Let N_1, \ldots, N_k be submodules of M. Then the sum $N_1 + \ldots + N_k$ is the collection

$$N_1 + \ldots + N_k = \{n_1 + \cdots + n_k : n_i \in N_i\}$$

It is a submodule of M and the smallest submodule containing all the N_i .

One can also consider infinite collections of submodules:

$$\sum_{i\in I} N_i = \{ \sum_{j\in J} n_j : n_j \in N_j, J \subset I \text{ finite } \}$$