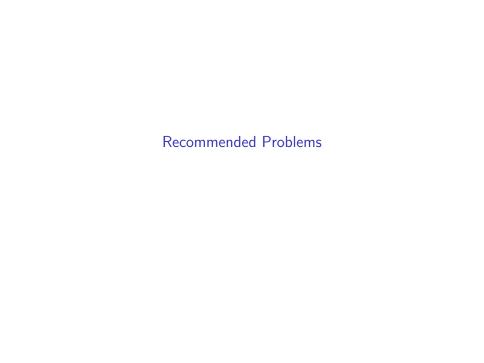
Recommended Problems



Section 10.1

Problem 5.

If M is an R module and I is a left ideal of R, define

$$IM = \{ \sum_{\text{finite}} a_i m_i : a_i \in I, m_i \in M \}$$

Prove that IM is a submodule of M.

Problem 8.

Given a ring R and an R-module M, let

$$Tor(M) = \{ m \in M : \exists r \in R, rm = 0 \}$$

Prove that, if R is an integral domain, then Tor(M) is a submodule of M.

Show that if R is *not* an integral domain then Tor(M) need not be a submodule of M.

Problem 9.

If N is a submodule of an R-module M, defined the annihilator of N in R to be the set of $r \in R$ such that rn = 0 for all $n \in N$.

Section 10.2

Problem 8

Let $\phi:M\to N$ be a module homomorphism. Prove that $\phi(\operatorname{Tor}(M))\subset\operatorname{Tor}(N)$.

Problem 10

If R is commutative with 1, prove that $\operatorname{Hom}_R(R,R)$ is isomorphic to R.