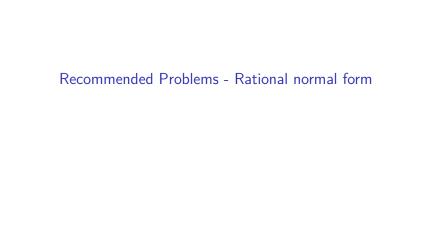
# Recommended Problems



#### Problem 1 (DF, Problem 4, pg. 488)

Prove that two  $3\times 3$  matrices over a field F are similar if and only if they have the same characteristic and minimal polynomials. Give an explicit counterexample to this for  $4\times 4$  matrices.

# Problem 2 (DF, Problem 12,m pg. 489)

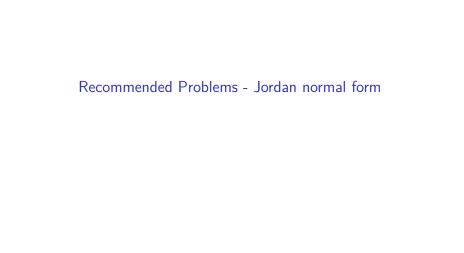
Find all similarity classes of  $3 \times 3$  matrices over  $\mathbb{Z}/2\mathbb{Z}$  satisfying  $A^6 = 1$ . Do the same for  $4 \times 4$  matrices satisfying  $B^{20} = 1$ .

### Problem 3 (DF, Problem 17, p. 489)

Let  $G = GL_3(\mathbf{F}_2)$ . This group has order (7)(6)(4) = 168. Find representatives for each of its conjugacy classes.

### Problem 4 (DF Problems 22-25, pg 490)

These problems show that if A is an  $n \times n$  matrix over F, then the matrix xI - A is the "relation matrix" for the F[x]-module obtained from  $F^n$  with x acting as A. Consequently, to compute the rational normal form for A, one must use the "smith normal form" algorithm over F[x] on this matrix to find the invariant factors. (Note: Actually the problem uses  $xI - A^T$  but this is just a question of conventions regarding rows vs. columns and doesn't really matter).



#### Computations

DF Problems 4-16 on pages 499-501 give a whole bunch of numerical examples; try some.

# Problem 5 (DF, Problem 22, page 501)

Prove that an  $n \times n$  matrix A over  $\mathbf{C}$  satisfying  $A^3 = A$  is diagonalizable. Is this true for any field F?

### Problem 6 (DF, Problem 31, pag 502)

A matrix is nilpotent if  $A^n = 0$  for some n. Prove that a nilpotent matrix is similar to a block diagonal matrix where each