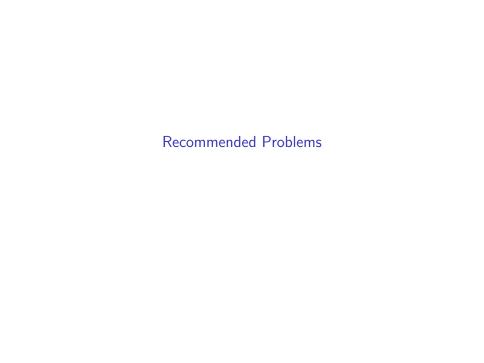
## Recommended Problems



Problem 5.

If M is an R module and I is a left ideal of R, define

$$IM = \{ \sum_{\text{finite}} a_i m_i : a_i \in I, m_i \in M \}$$

Prove that IM is a submodule of M.

Problem 8.

Given a ring R and an R-module M, let

$$Tor(M) = \{ m \in M : \exists r \in R, rm = 0 \}$$

Prove that, if R is an integral domain, then Tor(M) is a submodule of M.

Show that if R is *not* an integral domain then Tor(M) need not be a submodule of M.

Problem 9.

If N is a submodule of an R-module M, defined the annihilator of N in R to be the set of  $r \in R$  such that rn = 0 for all  $n \in N$ .

Show that the annihilator of N is a two-sided ideal of R.

### Problem 19.

Let  $V=\mathbb{R}^2$  and let T be the linear transformation given by projection onto the y-axis. Viewing V as an  $\mathbb{R}[x]$  module with x acting as T, show that the only proper  $\mathbb{R}[x]$  submodules of V are the x and y axes.

# Section 10.2

### Problem 8

Let  $\phi: M \to N$  be a module homomorphism. Prove that  $\phi(\text{Tor}(M)) \subset \text{Tor}(N)$ .

### Problem 10

If R is commutative with 1, prove that  $\operatorname{Hom}_R(R,R)$  is isomorphic to R.