Recommended Problems

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The following problems from Dummit and Foote review the basic ideas on modules and their homomorphisms.

Section 10.1

Problem 5.

If M is an R module and I is a left ideal of R, define

$$IM = \{\sum_{\text{finite}} a_i m_i : a_i \in I, m_i \in M\}$$

Prove that IM is a submodule of M.

Problem 8.

Given a ring R and an R-module M, let

$$\mathrm{Tor}(M) = \{m \in M : \exists r \in R, rm = 0\}$$

Prove that, if R is an integral domain, then $\mathrm{Tor}(M)$ is a submodule of M.

Show that if R is *not* an integral domain then $\mathrm{Tor}(M)$ need not be a submodule of M.

Problem 9.

If N is a submodule of an R-module M, defined the annihilator of N in R to be the set of $r \in R$ such that $rn = 0$ for all $n \in N$.

Show that the annihilator of N is a two-sided ideal of R.

Problem 19.

Let $V = \mathbb{R}^2$ and let T be the linear transformation given by projection onto the y-axis. Viewing V as an $\mathbb{R}[x]$ module with x acting as T, show that the only proper $\mathbb{R}[x]$ submodules of V are the x and y axes.

Section 10.2

Problem 8

Let $\phi : M \to N$ be a module homomorphism. Prove that $\phi(\mathrm{Tor}(M)) \subset \mathrm{Tor}(N)$.

Problem 10

If R is commutative with 1, prove that $\mathrm{Hom}_R(R, R)$ is isomorphic to R.

Let R be an integral domain ring and M be an R-module. The torsion submodule $\mathrm{Tor}(M)$ is the set of $m \in M$ such that $rm = 0$ for some $r \in R$.