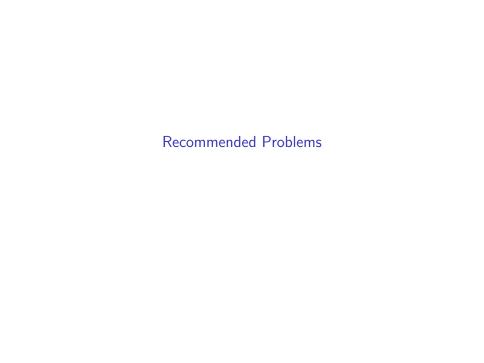
# Recommended Problems



Problem 1 (DF, Page 519, Number 7)

Prove that  $x^3 - nx + 2$  is irreducible unless n = -1, 3, 5.

### Problem 2 (DF Page 519, Number 2)

Let  $\theta$  be a root of  $x^3-2x-2$ . Write  $\frac{1+\theta}{1+\theta+\theta^2}$  in the form  $c_0+c_1\theta+c_2\theta^2$ .

## Problem 3 (DF, Page 529, Number 1)

Let F be a finite field of characteristic p. Prove that F has  $p^n$  elements for some positive integer n.

# Problem 4 (DF, Page 530, Number 4)

Find the degree over  $\mathbb Q$  of  $2+\sqrt{3}$  and  $1+\sqrt[3]{2}+\sqrt[3]{4}$ .

## Problem 5 (DF, Page 530, Number 7)

Prove that  $\mathbb{Q}(\sqrt{2}+\sqrt{3})=\mathbb{Q}(\sqrt{2},\sqrt{3})$ . Conclude that  $[\mathbb{Q}(\sqrt{2}+\sqrt{3}):\mathbb{Q}]=4$ . Find a minimal polynomial over  $\mathbb{Q}$  for  $\sqrt{2}+\sqrt{3}$ .

### Problem 6 (DF, Page 530, Number 15)

A field F is called *formally real* if -1 is not expressible as a sum of squares in F. Let F be a formally real field, let f(x) be an irreducible polynomial of odd degree, and let  $\alpha$  be a root of F. Prove that  $F(\alpha)$  is also formally real.

Hint: Choose a counterexample f(x) of minimal degree. Show that there is a g(x) of odd degree less than the degree of f(x), and polynomials  $p_1(x), \ldots, p_m(x)$  such that

$$-1 + f(x)g(x) = p_1(x)^2 + ... + p_m(x)^2.$$

Show that g(x) has a root  $\beta$  of odd degree over F and  $F(\beta)$  is not formally real, contradicting minimality of f(x).