

Recommended Problems

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The following problems from Dummit and Foote review the basic ideas on modules and their homomorphisms.

Problem 5.

If M is an R module and I is a left ideal of R , define

$$IM = \left\{ \sum_{\text{finite}} a_i m_i : a_i \in I, m_i \in M \right\}$$

Prove that IM is a submodule of M .

Problem 8.

Given a ring R and an R -module M , let

$$\text{Tor}(M) = \{m \in M : \exists r \in R, rm = 0, r \neq 0\}$$

Prove that, if R is an integral domain, then $\text{Tor}(M)$ is a submodule of M .

Show that if R is *not* an integral domain then $\text{Tor}(M)$ need not be a submodule of M .

Problem 9.

If N is a submodule of an R -module M , defined the annihilator of N in R to be the set of $r \in R$ such that $rn = 0$ for all $n \in N$.

Show that the annihilator of N is a two-sided ideal of R .

Problem 19.

Let $V = \mathbb{R}^2$ and let T be the linear transformation given by projection onto the y -axis. Viewing V as an $\mathbb{R}[x]$ module with x acting as T , show that the only proper $\mathbb{R}[x]$ submodules of V are the x and y axes.

Section 10.2

Problem 8

Let $\phi : M \rightarrow N$ be a module homomorphism. Prove that $\phi(\text{Tor}(M)) \subset \text{Tor}(N)$.

Problem 10

If R is commutative with 1, prove that $\text{Hom}_R(R, R)$ is isomorphic to R .