Recommended Problems

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Problem 1 (DF, Page 519, Number 7)

Prove that $x^3 - nx + 2$ is irreducible unless $n = -1, 3, 5$.

Problem 2 (DF Page 519, Number 2)

Let θ be a root of $x^3 - 2x - 2$. Write $\frac{1+\theta}{1+\theta+\theta^2}$ in the form $c_0 + c_1\theta + c_2\theta^2$.

Problem 3 (DF, Page 529, Number 1)

Let F be a finite field of characteristic p. Prove that F has p^n elements for some positive integer n.

Problem 4 (DF, Page 530, Number 4)

Find the degree over $\mathbb{Q}$ of $2+\sqrt{3}$ and $1+\sqrt[3]{2}+\sqrt[3]{4}$.

Problem 5 (DF, Page 530, Number 7)

Prove that $\mathbb{Q}(\sqrt{2}+\sqrt{3}) = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Conclude that $[\mathbb{Q}(\sqrt{2}+\sqrt{3}) : \mathbb{Q}] = 4$. Find a minimal polynomial over $\mathbb{Q}$ for $\sqrt{2}+\sqrt{3}$.

Problem 6 (DF, Page 530, Number 15)

A field F is called *formally real* if -1 is not expressible as a sum of squares in F. Let F be a formally real field, let $f(x)$ be an irreducible polynomial of odd degree, and let α be a root of F. Prove that $F(\alpha)$ is also formally real.

Hint: Choose a counterexample $f(x)$ of minimal degree. Show that there is a $g(x)$ of odd degree less than the degree of $f(x)$, and polynomials $p_1(x), \ldots, p_m(x)$ such that

$$-1 + f(x)g(x) = p_1(x)^2 + ... + p_m(x)^2.$$

Show that $g(x)$ has a root β of odd degree over F and $F(\beta)$ is not formally real, contradicting minimality of $f(x)$.