

2. Modules (continued)

More on modules

Sums of modules

Suppose that R is a ring and M is an R -module. Let N_1, \dots, N_k be submodules of M . Then the sum $N_1 + \dots + N_k$ is the collection

$$N_1 + \dots + N_k = \{n_1 + \dots + n_k : n_i \in N_i\}$$

It is a submodule of M and the smallest submodule containing all the N_i .

One can also consider infinite collections of submodules:

$$\sum_{i \in I} N_i = \left\{ \sum_{j \in J} n_j : n_j \in N_j, J \subset I \text{ finite} \right\}$$