# 1. Modules

## Modules

- Modules are to rings as vector spaces are to fields.
- Modules are to rings as sets with group actions are to groups.

## Definition of (left) modules

**Definition:** Let $R$ be a ring (for now, not necessarily commutative and not necessarily having a unit). A *left $R$-module* is an abelian group $M$ together with a map $R \times M \to M$ (written $(r, m) \mapsto rm$) such that:

- $r(m_1 + m_2) = rm_1 + rm_2$
- $(r_1 + r_2)m = r_1m + r_2m$
- $r_1(r_2m) = (r_1r_2)m$

If $R$ has a unit element 1, we also require $1m = m$ for all $m \in M$.

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