Theory of Computation, Fall 2022
Assignment 5 (Due October 24 Monday 10:00 am)

Q1. Construct a PDA that accepts the following language.

$$\{w \in \{0,1\}^* : \text{no prefix of } w \text{ has more 0's than 1's}\}$$

Q2. Construct a PDA that accepts the following language.

$$\{w \in \{a,b\}^* : \#a\text{'s} = 2\#b\text{'s in } w\}$$

Q3. What is the language of the following CFG?

$$S \to 0S1 \mid A$$
$$A \to S$$

Q4. Convert the CFG in Q3 to a equivalent PDA. You should strictly follow the construction we used in the class.

Q5. Let A be a regular language. Prove that A is context-free. You may either use CFG or PDA to prove this, whichever you think is more convenient.

Q6. Let A and B be two regular languages. Prove that the following language is context-free.

$$A \diamond B = \{xy : x \in A, y \in B, \text{ and } |x| = |y|\}$$

More precisely, let $M_A = (K_A, \Sigma, \Delta_A, s_A, F_A)$ and $M_B = (K_B, \Sigma, \Delta_B, s_B, F_B)$ be FAs that accepts A and B, respectively. You should use them to construct a PDA that accepts $A \diamond B$. (Hint: $A \diamond B$ is quite similar to $A \circ B$. The only difference is that $A \diamond B$ requires $|x| = |y|$. Luckily, this can be done using the stack of PDA.)