# Theory of Computation, Fall 2022
## Assignment 6 (Due October 31 Monday 10:00 am)

Q1. Show that if $L$ is context-free, so is

$$L^R = \{w^R : w \in L\}.$$

Q2. Let $a, b, c$ be three distinct symbols. Use pumping theorem to show that $A = \{wcw : w \in \{a, b\}^*\}$ is not context-free.

Q3. Let $A$ be a context-free language. Let $B$ be a regular language. Prove that $A \cap B$ is context-free. You may assume that $A$ and $B$ are defined over the same alphabet $\Sigma$. (Hint: let $P_A$ be a PDA accepting $A$. Let $M_B$ be an NFA accepting $B$. Construct a PDA $P_\cap$ that conceptually runs $P_A$ and $M_B$ in parallel.)

Q4. Let $A = \{w \in \{a, b, c\}^* : w \text{ has same number of } a\text{'s}, b\text{'s, and } c\text{'s}\}$.

   (a) Prove that $A$ is not context-free. (Hint: It is not necessary to use pumping theorem. You may try the conclusion of Q3.)

   (b) Show that $\overline{A}$ is context-free. (Hint: it suffices to show that $\overline{A}$ is a union of several context-free languages.)

   (c) What conclusion can you draw from (a) and (b)?

Q5. Suppose that $L$ is context-free and $R$ is regular. Is $L - R$ necessarily context-free? What about $R - L$? Justify your answer.