

Theory of Computation, Fall 2022  
Assignment 9 (Due November 21 Monday 10:00am)

Q1. Let  $D$  be a DFA. Consider the following decision problem.

Given a string  $w$ , does  $D$  accept  $w$ ?

- (a) What is the language corresponding to the following problem?
- (b) Is this language recursive?
- (c) Prove that every regular language is recursive.

Q2. Let  $SB_{DFA} = \{ \langle D_1 \rangle \langle D_2 \rangle : D_1 \text{ and } D_2 \text{ are two DFAs with } L(D_1) \subseteq L(D_2) \}$ . Give a reduction from  $SB_{DFA}$  to  $E_{DFA}$ .

Q3. Let  $L = \{ w \in \{0,1\}^* : w \text{ contains an odd number of 1's} \}$ . Define

$$A_L = \{ \langle D \rangle : D \text{ is a DFA that accepts } L \}.$$

Give a reduction from  $A_L$  to  $E_{DFA}$ .

Q4. In class we have proved that  $E_{DFA}$  and  $EQ_{DFA}$  are recursive. What conclusion can you draw about  $SB_{DFA}$  and  $A_L$ ?

Q5. Let  $A$  and  $B$  be two languages. Suppose that we have a reduction  $f$  from  $A$  to  $B$ .

- (a) If  $B$  is recursively enumerable, what conclusion can you draw about  $A$ ? Prove your conclusion.
- (b) If  $A$  is NOT recursive, what conclusion can you draw about  $B$ ? Prove your conclusion.

Q6. Prove that every language is countable. You can use any theorem that we have proved in class.

Q7. Prove that there is an undecidable subset of  $\{1\}^*$ .