Theory of Computation, Fall 2022 Assignment 10 (Due November 28 Monday 10:00am)

- Q1. Prove that every recursively enumerable language can be reduced to H.
- Q2. Let A and B be two languages. Prove that if $A \leq B$, then $\overline{A} \leq \overline{B}$.
- Q3. Let A be a recursively enumerable language. Prove that if $A \leq \overline{A}$, then A is recursive. (Hint: you may use the conclusion of Q2 in this homework and the conclusion of Q5(a) in homework 9)
- Q4. If $A \leq B$ and B is a regular language, does it imply that A is a regular language? Justify your answer.
- Q5. Let A be a language. If $H \leq A$, what can you say about \overline{A} ? (You may use the conclusion of Q2 and the fact that \overline{H} is not recursively enumerable.)
- Q6. Prove that the following language is not recursive.

 $\{\, \text{``}M_1\text{'''}M_2\text{'''}\text{'k''} \,:\, M_1 \text{ and } M_2 \text{ are two Turing machines with } |L(M_1)\cap L(M_2)| \geq k\}$

You can reduce from any non-recursive language we present in class.