Solutions of Assignment 2

> Theory of Computation, Fall 2022

Corrections

- **Q3.** A Kleene star (*) is missing:

$$L = \{w \in \{a,b,c\} : |w| \geq 1 \ldots\} \longrightarrow L = \{w \in \{a,b,c\}^* : |w| \geq 1 \ldots\} \tag{1}$$

- **Notes of Q2.** Please refer to Notes.

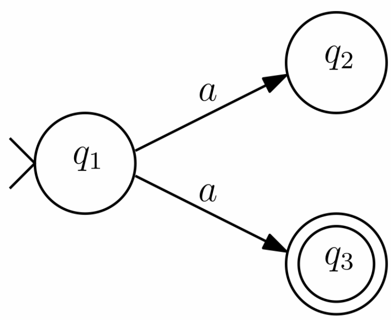
Keys

> For all problems, any other valid answer is also acceptable.

Q1

No.

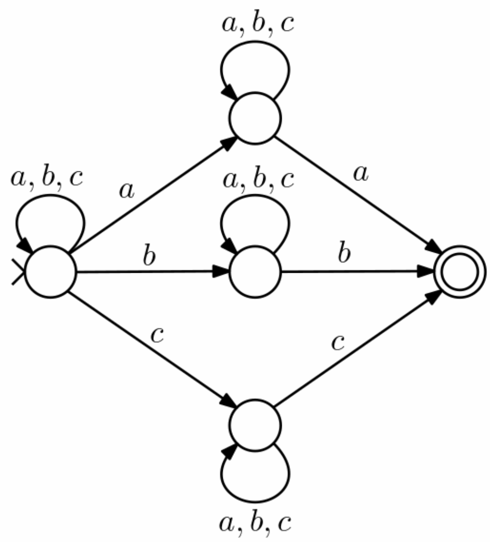
Let M denotes the following NFA over $\Sigma = \{a\}$, then $L(M) \cap L(M') = \{a\} \neq \emptyset$.



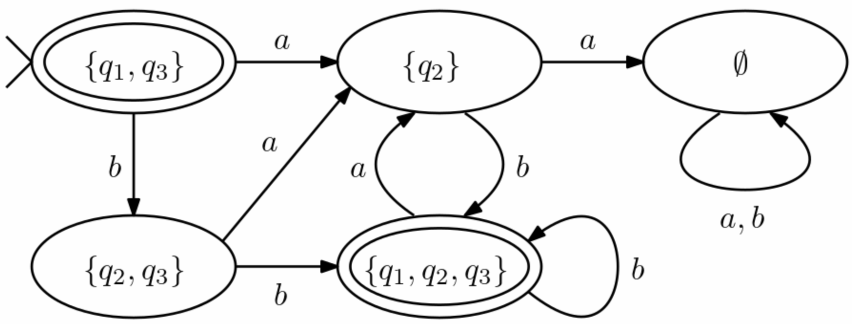
Q2

Construct a string $w = ba$. Then $w \in L(M')$ but $w \notin (L(M))^*$, hence $L(M') \neq (L(M))^*$.

Q3



Q4



Q5

(a) True.

(b) False.

$$\begin{gathered}\text{Suppose that } \Sigma = \{a\}, R = a \implies R\emptyset = \emptyset \\ L(R) = \{a\}, L(R\emptyset) = L(\emptyset) = \emptyset \implies L(R) \neq L\{R\emptyset\}\end{gathered} \tag{2}$$

(c) False.

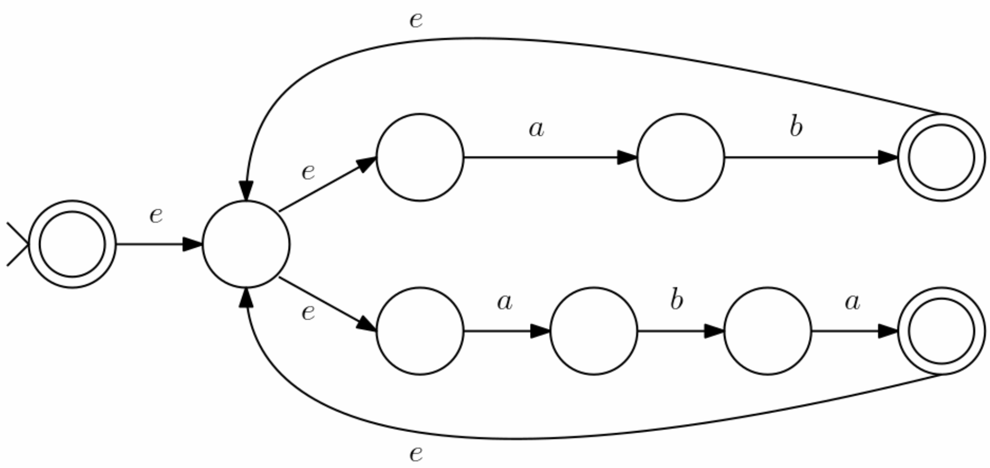
$$
\begin{aligned}
\emptyset^* &= e \\
\text{Suppose that } \Sigma = \{a\}, R = a &\implies R \cup \emptyset^* = a \cup e \\
L(R) = \{a\}, L(R \cup \emptyset^*) = \{a \cup e\} &\implies L(R) \neq L\{R \cup \emptyset^*\}
\end{aligned} \tag{3}
$$

(d) True.

Q6

$$a^*(a^*ba^*ba^*ba^*)^* \tag{4}$$

Q7



Grading

100 pts. in total

- Q1. 12 pts.
- Q2. 12 pts.
- Q3. 15 pts.
- Q4. 10 pts.
- Q5. 4 pts. for each judgement, and 5 pts. for each counterexample, totally 26 pts.
- Q6. 15 pts.
- Q7. 10 pts.

Notes

Q1

To construct a valid counterexample, just construct 2 states that a single state (or equivalent states) moves to after reading the same symbol, one of which is a final state while the other one is not.

In fact, $L(M) \cap L(M') = \emptyset$ stands when the operation happens on DFAs.

Q2

A correction:

$$L(M') = (bb^*a)^* \longrightarrow L(M') = (bb^* \cup bb^*a)^* \tag{5}$$

The following description is now correct.

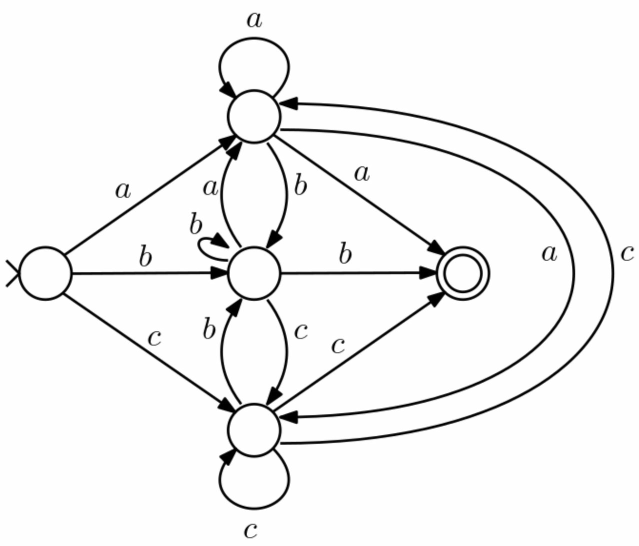
$L(M') = (bb^* \cup bb^*a)^*$, which means it accepts the empty string and all strings in $\{a, b\}^*$ with initial b without consecutive a's.

However, $L(M) = b(ab \cup b)^*$, $(L(M))^* = (b(ab \cup b)^*)^*$. That means strings ending with a are rejected by $(L(M))^*$.

To construct an equivalent NFA of $(L(M))^*$, a new initial state (also set as a final state) should be added. The way setting the original initial state a final state is invalid, since there is some state transitioning to it.

Q3

Another valid solution (A little more complex):



Q4

No comment.

Q5

No comment, although you guys perform badly on this question.

Q6

Notice the case of strings full with a without b.

Another valid solutions (You may exchange expressions on both sides of $\cup$):

$$a^* \cup (a^*ba^*ba^*ba^*)^* \tag{6}$$

$$a^*(ba^*ba^*ba^*)^* \tag{7}$$

$$(a^*ba^*ba^*b)^*a^* \tag{8}$$

These are invalid:

$$a^* \cup (ba^*ba^*ba^*)^* \tag{9}$$

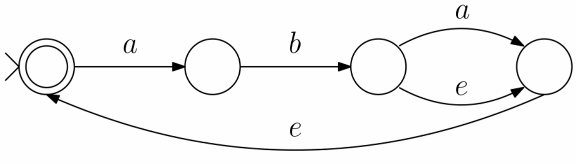
$$(a^*ba^*ba^*b)^* \cup a^* \tag{10}$$

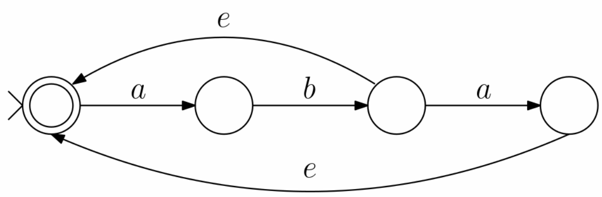
Try $abababa$ if you do not believe it.

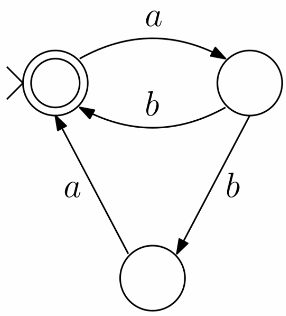
Q7

In this question, a construction without a new initial state but setting the original initial state as a final state is also valid. However, this way fails at most cases --- you may refer to Q2.

Since the way to construct the NFA is not required, these 3 constructions below are also valid. There may also be some other valid answer.

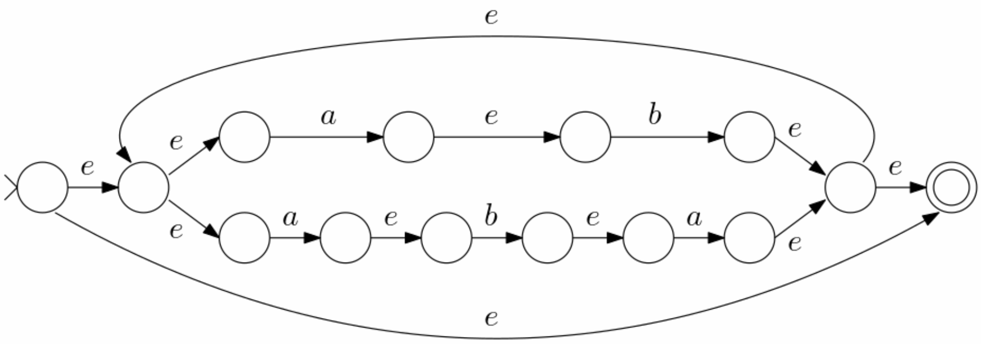






However, some ones constructing in these ways fail to compose correctly...

Another valid solution (You will have to learn this in the course Compiler Principles and Technology):



Some unnecessary e transitions are allowed to be omitted.