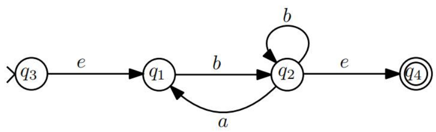
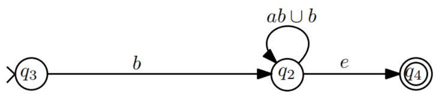
Theory of Computation, Fall 2022
Assignment 3 Solutions

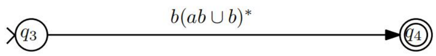
Q1. (15 pts)
The regular expression is $b(ab \cup b)^*$



(a) Step 1



(b) Step 2



(c) Step 3

Q2. (35 pts, 5 pts for each)

(a) False. The non-regular language $\{0^i1^i : i \geq 0\}$ is a subset of the regular language $(0\cup1)^*$.

(b) True. It can be proved by recursively applying the theorem that the union of two regular languages is regular.

(c) True. A language that contains a finite number of strings can be seen as a union of a finite number of languages that contains only one string. By (b), such a language must be regular.

(d) False. For every $i \geq 0$, define $L_i = \{a^ib^i\}$. Clearly every L_i is regular. But their union $\cup_{i=0}^{\infty} L_i = \{a^ib^i : i \geq 0\}$ is not regular.

(e) False. Consider any union of infinite number of regular languages $\cup_{i=0}^{\infty} L_i$. By De Morgan's Laws, we have $\cup_{i=0}^{\infty} L_i = \overline{\cap_{i=0}^{\infty} \overline{L_i}}$. We already know that the complement of a regular language is regular. If statement (e) is true, then by above formula, we can conclude that $\cup_{i=0}^{\infty} L_i$ is regular. But in (d), we know that $\cup_{i=0}^{\infty} L_i$ is not necessarily regular. Contradiction.

(f) False. Let $A = (0 \cup 1)^*$, $B = \{0^i1^i : i \geq 0\}$, $A \cup B = A$. In this case, A is regular, and $A \cup B$ is regular, but B is non-regular.

(g) True. Because when $A \cap B = \emptyset$, $B = (A \cup B) \cap \bar{A}$. According to the closure property of regular language, the intersection of regular language is regular. In this case, both $A \cup B$ and A are regular, so B is regular.

Q3. (15 pts)
True. Following the proof of pumping theorem, we know that w can be written as $w = xyz$ such that $xy^iz \in L$ for any $i \geq 0$. This implies that L contains an infinite number of string.

Q4. (20 pts)

Let $L = \{ww : w \in a, b^*\}$. Suppose, for the sake of contradiction, that this language is regular. Let p be the pumping length given by the pumping theorem. Consider the string $w = a^p b a^p b \in L$. By pumping theorem, w can be written as $w = xyz$ such that
(i) $xy^i z \in L$ for any $i \geq 0$
(ii) $|y| \geq 0$, and
(iii)$|xy| \leq p$
(ii) and (iii) imply that $y = a^k$ for some $k \geq 0$. Consider the string $xy^0 z = a^{p-k} b a^p b$. Clearly, this string does not belong to L. This contradicts with (i). Therefore, L is non-regular.

Q5. (15 pts)

Assume $L_1 = \{0^m 1^n : m \neq n\}$ is regular. And it is easy to prove that $L = \{0^* 1^*\}$ is regular.

According to the closure property of regular language, the difference of two regular languages is regular. So $L - L_1$ is regular. However, $L - L_1 = \{0^n 1^n : n \geq 0\}$ is not regular. Contradiction. So $L_1 = \{0^m 1^n : m \neq n\}$ is non-regular.
*Prove the difference of two regular languages is regular: since $A - B = A \cap \bar{B}$, we already know the intersection and the complement of regular languages are regular, so the difference of two regular languages is regular.