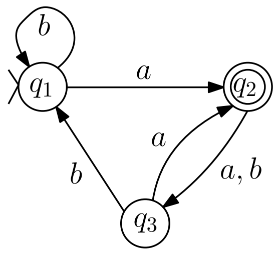
Theory of Computation, Fall 2022
Assignment 1 (Due September 19 Monday 10:00 am)

The finite automata we learned in the first class is deterministic, so they are also called deterministic finite automata (DFA).

Q1. Consider the following DFA. What sequence of configurations does the machine go through on input aab?



Q2. Are the following statements true of false? No explanation is required.

(a) Let w be a string. The number of distinct symbols that appear in w must be finite.

(b) Let L be a language. The number of distinct symbols that appear in L must be finite.

(c) Every DFA accepts one and only one string.

(d) Every DFA accepts one and only one language.

(e) If $(q_1, w_1) \vdash_M^* (q_2, w_2)$ and $(q_2, w_2) \vdash_M (q_3, w_3)$, then $(q_1, w_1) \vdash_M^* (q_3, w_3)$.

(f) If a language is finite, it must be regular.

(g) Let L be a language over some alphabet Σ. L is also a language over any alphabet that is a superset of Σ.

Q3. Prove that the following languages are regular.

(a) the set of all binary strings that end with 00.

(b) $\{w \in \{0,1\}^* : w$ starts with 1 and is a multiple of 3 when interpreted as a binary integer$\}$ (Hint: use states to maintain the remainder of number read so far, when divided by 3.)

Q4. Let $M = (K, \Sigma, \delta, s, F)$ be a DFA. Let Σ' be some superset of Σ. Extend the input alphabet of M to Σ' without changing the language it accepts. In other words, you need to construct a DFA $M' = (K', \Sigma', \delta', s', F')$ such that $L(M') = L(M)$.

Q5. Let A and B be two regular languages over some alphabet Σ. Define

$$A \cap B = \{w : w \in A \wedge w \in B\}.$$

Show that $A \cap B$ is also regular. (Hint: think about how we proved $A \cup B$ is regular in class.)

Q6. Let A and B be two regular languages over some alphabet Σ. Let $\#$ be a symbol not in Σ. Consider the following language.

$$L = \{w_1 \# w_2 : w_1 \in A \wedge w_2 \in B\}$$

Show that L is also regular.

Q7. Let A be a regular language over Σ. Consider the following language.

$$\bar{A} = \{w \in \Sigma^* : w \notin A\}$$

Show that $\bar{A}$ is regular.