Theory of Computation, Fall 2022
Assignment 3 (Due October 10 Monday 10:00 am)

Q1. Consider the NFA M in Figure 1. Construct a regular expression R such that $L(R) = L(N)$. You should strictly follow the algorithm we used in the class, and show all the intermediate steps. More precisely, you should first convert N into an equivalent NFA that satisfies certain conditions, and then eliminate state q_1 and q_2 in order.

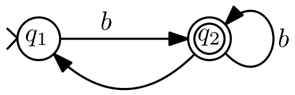

Figure 1: M

Q2. Are the following statement true of false? Briefly explain your answer.

(a) Every subset of a regular language is regular.

(b) The union of a finite number of regular languages must be regular.

(c) Languages that contain only a finite number of strings must be regular.

(d) The union of an infinite number of regular languages must be regular.

(e) The intersection of an infinite number of regular languages must be regular.
(Hint: Think about the De Morgan's Laws)

(f) Let A and B be two languages. If A is regular, and $A \cup B$ is regular, then B must be regular.

(g) Let A and B be two languages such that $A \cap B = \emptyset$. If A is regular, and $A \cup B$ is regular, then B must be regular.

Q3. Let M be a DFA. Let p be the number of states in M. Is the following statement true of false? Briefly explain your answer.

> If $L(M)$ has a string w with $|w| \geq p$, then $L(M)$ must be infinite (That is, L(M) contains an infinite number of strings).

Q4. Use pumping theorem to show that the language $\{ww : w \in \{a, b\}^*\}$ is not regular.

Q5. Show that the language $\{0^m 1^n : m \neq n\}$ is not regular. (Hint: you may find that pumping theorem does not work well in this case. Try the closure property.)