Theory of Computation, Fall 2022
Assignment 12 (Due December 19 Monday 10:00am)

Q1. According to the recursive definition we presented in the class, what is the value of $div(m, 0)$?

Q2. Let $f : \mathcal{N} \to \mathcal{N}$ be a primitive recursive function. Define $F : \mathcal{N} \to \mathcal{N}$ to be

$$F(n) = f(f(\ldots f(n)\ldots))$$

where there are n compositions. For example, $F(0) = f(0)$ and $F(1) = f(f(1))$. Show that F is primitive recursive.

Q3. Show that for any $k \geq 2$, the following function is primitive recursive.

$$\varphi_k(n_1, \ldots, n_k) = max\{n_1, \ldots, n_k\}$$

for any $n_1, \ldots, n_k \in \mathcal{N}$.

Q4. Prove that the function $h_p(n)$ we presented in the class is primitive recursive.