Theory of Computation, Fall 2022
Assignment 2 (Due September 26 Monday 10:00 am)

Q1. Let M be an arbitrary NFA. Let M' be the NFA obtained from M by exchanging the role of final and non-final states. Is it always true that $L(M) \cap L(M') = \emptyset$? If not, give a counterexample.

Q2. Let M be the NFA in Figure 1a. Someone constructs another NFA M' as in Figure 1b and claims that $L(M') = (L(M))^*$. Prove that he\she is wrong. (Hint: It suffices to find a string that is in $L(M')$ but not in $(L(M))^*$).
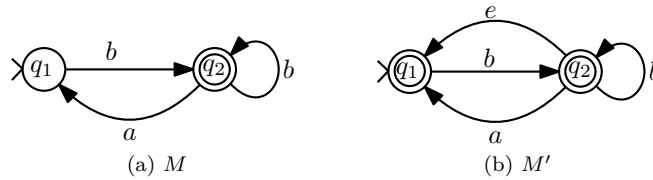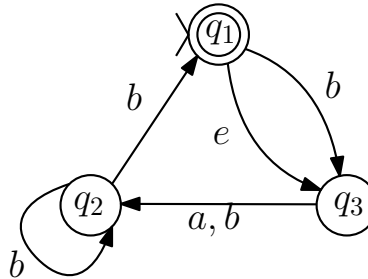

Figure 1: Two NFAs

Q3. Let $L = \{w \in \{a, b, c\} : |w| \geq 1$ and the last symbol of w has appeared at least twice in $w\}$. Construct a NFA to accept L. Your NFA should have no more than 5 states.

Q4. Convert the following NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the initial state.

Q5. For any two regular expressions R_1 and R_2, we say $R_1 = R_2$ if $L(R_1) = L(R_2)$. Let R be a regular expression. Are the following statements true or false? Provide counterexamples for false statements.

    (a) $R \cup \emptyset = R$

    (b) $R\emptyset = R$

    (c) $R \cup \emptyset^* = R$

    (d) $R\emptyset^* = R$

Q6. Write a regular expression for the language

$$\{w \in \{a, b\}^* : \text{the number of } b\text{'s in } w \text{ is divisible by } 3\}.$$

Q7. Construct a NFA that accepts $(ab \cup aba)^*$.