The Zone–Tritone System:
A Unified Harmonic Framework in Modular Pitch Space

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Abstract

We present the Zone–Tritone System, a harmonic framework defined in the twelve-tone modular pitch space $\mathbb{Z}_{12}$. We show that the pitch set partitions naturally into two whole–tone zones corresponding to parity classes modulo 2. Whole–step motion preserves zone identity while semitone motion crosses zone boundaries. Tritones are shown to be order–two elements (i.e. involutions) lying fully within single zones, forming internal axes of harmonic gravity. Chromatic translation of a tritone produces dominant root motion descending by fourths. The melodic minor scale is proven to be dual–zone harmonic, containing two independent tritone anchors, and a general classification of harmonic systems by tritone rank is proposed. This yields a coherent mathematical account of harmonic gravity, chromaticism, and modal hybrid behavior.

Contents

1 Introduction

A central puzzle in tonal and post–tonal music theory is the interaction between chromatic symmetry and functional harmonic direction. Jazz practice demonstrates extensive use of tritone substitution, altered dominant sonorities, and melodic minor derived harmonies, yet a compact formal model explaining their internal relations has been lacking.

The Zone–Tritone System offers such a model. It is grounded in three structural principles:

Axiom 1. Whole–tone zones define harmonic color.

Axiom 2. Tritones define harmonic gravity.

Axiom 3. Half–steps define directional motion between zones.

We formalize these axioms in $\mathbb{Z}_{12}$ and derive several consequences.

2 Pitch Space

2.1 Definition of $\mathbb{Z}_{12}$

We represent pitch classes as elements of the additive cyclic group

$$\mathbb{Z}_{12} = \{0, 1, \ldots, 11\}$$

with arithmetic modulo 12.

We use the standard assignment

$$0 = C,\ 1 = C\sharp,\ 2 = D,\ 3 = E\flat,\ 4 = E,\ 5 = F,\ 6 = F\sharp,\ 7 = G,\ 8 = A\flat,\ 9 = A,\ 10 = B\flat,\ 11 = B.$$

3 Whole–Tone Zones

3.1 Definition

Define

$$Z_1 = \{0, 2, 4, 6, 8, 10\}, \quad Z_2 = \{1, 3, 5, 7, 9, 11\}.$$

[Zone Partition] Z_1 and Z_2 partition $\mathbb{Z}_{12}$.

Proof. Every integer mod 12 is congruent to either 0 or 1 mod 2, establishing disjointness and completeness. □

3.2 Zone Stability Under Whole Steps

If $x \in Z_i$, then $x \pm 2 \in Z_i$.

Proof. Parity is preserved. □

Thus whole–steps preserve harmonic color.

4 Half–Step Motion as Zone Crossing

If $x \in Z_1$ then $x \pm 1 \in Z_2$, and conversely.

Proof. Adding ± 1 flips parity. □

We interpret semitones as directional crossings between harmonic fields.

5 Tritones as Order–Two Elements

5.1 Definition

A tritone is the unordered pair

$$\tau = \{p,\ p + 6\}.$$

Every tritone lies within a single zone.

Proof. Since $6 \equiv 0 \pmod{2}$, adding six preserves parity. □

5.2 Involution Property

$$(p + 6) + 6 \equiv p \pmod{12}.$$

Thus tritones are involutions and therefore symmetry axes.

6 Chromatic Drift and the Cycle of Fourths

Let $\tau_n = \{p_n,\ p_n + 6\}$ and define

$$\tau_{n+1} = \{p_n - 1,\ p_n + 5\}.$$

Let R_n denote the root of the associated dominant chord.

Chromatic drift produces functional root motion by descending fourths:

$$R_{n+1} \equiv R_n - 5 \pmod{12}.$$

Proof. The tritone uniquely defines the third and seventh of a dominant chord. Their chromatic descent corresponds to functional resolution by a fourth. □

7 Dual–Zone Harmonic Systems

A pitch collection is dual–zone harmonic if it contains at least two distinct tritone anchors.

7.1 Melodic Minor

For C melodic minor,

$$S = \{0, 2, 3, 5, 7, 9, 11\}.$$

It contains:

$$\{11, 5\}, \quad \{3, 9\}.$$

Thus melodic minor is dual–zone harmonic.

8 Classification by Tritone Rank

We define the *tritone rank* of a collection as the number of distinct tritone axes contained within it.

System Type	Tritone Rank	Zone State
Major / Whole Tone	1	Single-zone
Melodic Minor / Altered	2	Dual-zone
Diminished	4	Symmetry cloud

This provides a structural classification of harmonic environments.

9 Worked Musical Examples

In this section we illustrate the Zone–Tritone framework with concrete harmonic progressions drawn from common-practice jazz vocabulary. For clarity, we present chord symbols and pitch–class analysis; engraved staff notation may be added in figures as needed.

9.1 Example 1: Classical II–V–I in C Major

Consider the progression

$$\text{Dm7} \to \text{G7} \to \text{Cmaj7}.$$

A common voicing in pitch classes is:

$$\begin{aligned} \text{Dm7} &: \{2,5,9,0\} \quad (D,F,A,C), \\ \text{G7} &: \{7,11,2,5\} \quad (G,B,D,F), \\ \text{Cmaj7} &: \{0,4,7,11\} \quad (C,E,G,B). \end{aligned}$$

The tritone anchor of G7 is

$$\tau = \{11,5\} = \{B,F\}.$$

Both tones lie in the same zone:

$$11 \in Z_2, \quad 5 \in Z_2.$$

The voice–leading Dm7 → G7 → Cmaj7 is therefore interpreted as:

1. Zone background: Z_2 stabilizes the tritone $\{11,5\}$.
2. Chromatic drift is not yet engaged; the tritone is static.
3. Resolution occurs when $\{11,5\}$ moves to $\{0,4,7\}$ (root and third of Cmaj7), collapsing the gravity axis into a stable major triad.

In practice, melodic material outlining $\{11,5\}$ over G7 aligns the listener's ear with the active gravity axis.

9.2 Example 2: Tritone Substitution

Now replace G7 by its tritone substitute:

$$\text{Dm7} \to \text{D}\flat 7 \to \text{Cmaj7}.$$

A possible voicing:

$$\text{D}\flat 7 : \{1,5,9,11\} \quad (D\flat, F, A\flat, B).$$

The tritone anchor of D♭7 is again

$$\tau' = \{11,5\} = \{B,F\}.$$

Thus both G7 and D♭7 share the same tritone anchor and therefore the same gravity axis, even though their roots differ by a tritone.

Within the Zone–Tritone System this is seen as an *anchor exchange* at the root level while the internal axis remains invariant.

9.3 Example 3: Chromatic Dominant Chain

Consider the chromatic sequence

$$\mathrm{G7} \to \mathrm{C7} \to \mathrm{F7} \to \mathrm{B}\flat 7.$$

Using tritone anchors:

$$\begin{aligned}
\mathrm{G7} &: \{11, 5\} = \{B, F\}, \\
\mathrm{C7} &: \{10, 4\} = \{B\flat, E\}, \\
\mathrm{F7} &: \{9, 3\} = \{A, E\flat\}, \\
\mathrm{B}\flat 7 &: \{8, 2\} = \{A\flat, D\}.
\end{aligned}$$

Each step is a chromatic translation of the tritone by -1:

$$\{11, 5\} \to \{10, 4\} \to \{9, 3\} \to \{8, 2\}.$$

By Theorem 9, this induces root motion by descending fourths:

$$7 \to 0 \to 5 \to 10 \quad (\mathrm{G} \to \mathrm{C} \to \mathrm{F} \to \mathrm{B}\flat).$$

From a perceptual standpoint, the listener experiences a coherent sequence of gravitational reorientations, despite the surface chromaticism.

9.4 Example 4: Melodic Minor and Dual–Zone Harmony

Consider C melodic minor over a Cmi(maj7) sonority:

$$\mathrm{Cmi(maj7)} : \{0, 3, 7, 11\} \quad (C, E\flat, G, B).$$

The scale

$$S = \{0, 2, 3, 5, 7, 9, 11\}$$

contains two tritone anchors:

$$\tau_1 = \{11, 5\} = \{B, F\}, \quad \tau_2 = \{3, 9\} = \{E\flat, A\}.$$

Improvisation that alternately emphasizes $\{11, 5\}$ and $\{3, 9\}$ engages two independent gravity axes. This realizes the dual–zone nature of melodic minor and explains the characteristic mixture of modal stability and directed motion associated with this sound.

9.5 Example 5: Minor II–V and Tritone Neighbors

A typical minor II–V in C minor is

$$\mathrm{Dm7}\flat 5 \to \mathrm{G7alt} \to \mathrm{Cmi}.$$

If G7alt is realized with a tritone anchor $\{11, 5\}$ and extensive altered tensions from the C altered scale, the Zone–Tritone model interprets the alterations as controlled excursions within the dual–zone environment, while the underlying gravity remains tied to the tritone axis.

Chromatic approach dominants (e.g. A♭7 preceding G7alt) are treated as intermediate states in a gravity chain, generated via tritone drift.

These examples illustrate that a wide range of jazz progressions can be seen as specific realizations of zone stability, zone crossing, and tritone drift, rather than as ad hoc chord–scale pairings.

10 A Markov Model of Harmonic Gravity

The linear recurrences derived in the Zone–Tritone System admit a natural probabilistic generalization. Instead of treating harmonic motion as strictly deterministic, we model it as a Markov process constrained by zone structure and tritone anchors.

10.1 State Space

We define a discrete state space $\mathcal{S}$ whose elements may be chosen at different levels of resolution:

(i) *Root states:* $\mathcal{S}_R = \mathbb{Z}_{12}$, where each state corresponds to a dominant root.

(ii) *Axis states:* $\mathcal{S}_\tau = \{\tau_1, \ldots, \tau_6\}$, the six tritone anchors.

(iii) *Hybrid states:* pairs (R, τ) with τ consistent with the dominant built on R.

For simplicity we first consider $\mathcal{S}_R$.

10.2 Transition Kernel

A first–order Markov chain on $\mathcal{S}_R$ is specified by a stochastic matrix $P = (p_{ij})$ where

$$p_{ij} = \Pr(R_{n+1} = j \mid R_n = i).$$

In a purely functional idealization, we might set

$$p_{i,j} = \begin{cases} 1 & j \equiv i - 5 \pmod{12}, \\ 0 & \text{otherwise}, \end{cases}$$

thus recovering the strict cycle of fourths.

In practice, we allow a distribution concentrated around the functional target:

$$p_{i,j} = \begin{cases} \alpha & j \equiv i - 5 \pmod{12} \\ \beta & j \equiv i \pmod{12} \\ \gamma & j \equiv i + 2 \pmod{12} \\ \delta & \text{otherwise}, \end{cases}$$

with $\alpha > \beta \geq \gamma \geq \delta \geq 0$ and rows normalized to sum to 1. Here α encodes functional resolution, while β, γ, δ encode local prolongation and neighboring motions.

10.3 Zone–Conditioned Transitions

We may refine the model by conditioning on zone structure. Let $Z : \mathbb{Z}_{12} \to \{0, 1\}$ be the zone map.

Define

$$p_{i \to j} = f\big(Z(i), Z(j), C(i, j)\big)$$

where $C(i, j)$ indicates zone crossing. For example, weights may be assigned so that:

- transitions preserving the active tritone axis but changing the root within a zone are given moderate probability,
- transitions implementing chromatic tritone drift (and thus fourth motion) are given highest probability,
- arbitrary jumps that disrupt both axis and zone receive low probability.

This yields a Markov chain whose high–probability paths correspond to musically idiomatic motion.

10.4 Axis–Based Markov Chain

Alternatively, we may define a chain on the six tritone anchors $\mathcal{S}_\tau$ by viewing chromatic drift as the primary transition:

$$q_{kl} = \begin{cases} 1 & \tau_l = T_{\pm 1}(\tau_k) \\ 0 & \text{otherwise,} \end{cases}$$

in the idealized case, or with softened probabilities in the empirical case.

This highlights the role of tritone movement as the principal driver of functional reorientation.

10.5 Applications

Such Markov models have several applications:

1. *Generative harmony:* sampling from the chain to produce idiomatic dominant progressions consistent with the Zone–Tritone grammar.
2. *Style analysis:* estimating transition matrices from corpora of jazz standards or improvisations and comparing empirical statistics with theoretically predicted structure.
3. *Pedagogical tools:* visualizing likely next states in a gravity chain as a student explores chord sequences.

In all cases, the probabilistic model is constrained by the underlying deterministic structure of zones, tritones, and chromatic drift.

11 Methods: Estimating Gravity Transition Matrices from Jazz Repertoire

To empirically validate the probabilistic gravity model introduced above, we estimate transition matrices from annotated jazz harmony corpora. The procedure described here allows reproducibility and comparison across datasets and styles.

11.1 Corpus Preparation

Lead sheets, harmonic analyses, or symbolic encodings (e.g. MusicXML, Humdrum, iRealPro, or hand-annotated chord charts) are converted into ordered sequences of chord roots:

$$(R_1, R_2, \ldots, R_N), \qquad R_i \in \mathbb{Z}_{12}.$$

Secondary dominants, tritone substitutions, backdoor dominants, and altered dominants are normalized to dominant-function labels by taking the pitch class of the chord root modulo 12. Non-functional chords (e.g. sustained modal sonorities) may either be excluded or treated as self-transitions, depending on the study design.

11.2 State Encoding

Two levels of representation are supported:

1. **Root state model.** Each state corresponds to a dominant root $R \in \mathbb{Z}_{12}$.
2. **Axis state model.** Each state corresponds to one of the six tritone anchors

   $$\tau_k = \{p_k, p_k + 6\}.$$

   The anchor associated with a dominant chord is taken to be its $(3, 7)$ pair.

Both representations yield Markov chains

$$X_1, X_2, \ldots, X_N$$

on a finite state space.

11.3 Counting Transitions

For each adjacent pair (X_i, X_{i+1}) we increment a count

$$c_{ab} \leftarrow c_{ab} + 1 \qquad \text{whenever } X_i = a,\ X_{i+1} = b.$$

This yields a raw transition count matrix

$$C = (c_{ab}).$$

To avoid zero-probability issues, Laplace smoothing may optionally be applied:

$$\tilde{c}_{ab} = c_{ab} + \lambda, \qquad \lambda \geq 0.$$

11.4 Normalizing to a Stochastic Matrix

Row-wise normalization yields the empirical stochastic matrix

$$p_{ab} = \frac{\tilde{c}_{ab}}{\sum_b \tilde{c}_{ab}}.$$

Each row therefore encodes the conditional probability

$$p_{ab} = \Pr(X_{n+1} = b \mid X_n = a).$$

11.5 Zone–Conditioned Analysis

To assess the influence of whole–tone structure, each transition is also classified by the zone map

$$Z : \mathbb{Z}_{12} \to \{0, 1\}.$$

This yields empirical counts for:

- Zone-stable transitions ($Z(a) = Z(b)$),
- Zone-crossing transitions ($Z(a) \neq Z(b)$),
- Chromatic tritone drift transitions,

$$\tau \to T_{\pm 1}(\tau),$$

- Functional fourth motion

$$R \to R - 5 \pmod{12}.$$

These categories allow statistical comparison between theoretical predictions and observed practice.

11.6 Goodness-of-Fit and Statistical Comparison

Let P_{theory} denote the idealized transition model and $P_{\text{empirical}}$ the corpus-estimated matrix.

Similarity may be quantified using:

- Kullback–Leibler divergence,

$$D_{\mathrm{KL}}(P_{\text{empirical}} \parallel P_{\text{theory}}),$$

- Matrix norm differences,
- Eigenstructure comparison,
- Stationary distribution analysis.

Significant concentration of probability mass around descending-fourth motion and chromatic tritone drift supports the claim that empirical jazz harmony conforms to Zone–Tritone gravity dynamics.

11.7 Reproducibility

All code, corpora, transition matrices, and statistical summaries should be versioned and archived. We recommend symbolic encodings and open-source analytics scripts to enable transparent replication and extension of this work.

12 Group–Theoretic Interpretation

12.1 Pitch Space as a Cyclic Group

The pitch set $\mathbb{Z}_{12}$ forms a cyclic group under addition modulo 12. We emphasize that all harmonic transformations in the Zone–Tritone System are endomorphisms of this group.

12.2 Parity Cosets as Whole–Tone Zones

Define the subgroup

$$2\mathbb{Z}_{12} = \{0, 2, 4, 6, 8, 10\}.$$

Then $Z_1 = 2\mathbb{Z}_{12}$ and $Z_2 = 1 + 2\mathbb{Z}_{12}$ are the two cosets.

Whole–tone zones are precisely the cosets of the even–step subgroup.

Proof. Every element belongs to exactly one residue class modulo 2. □

Thus zone membership is a group–theoretic parity property.

12.3 Tritones as Order–Two Elements

Consider the map

$$\iota(x) = x + 6.$$

ι is an involution:

$$\iota \circ \iota = \mathrm{id}.$$

Proof. $(x + 6) + 6 \equiv x \pmod{12}$. □

Hence every tritone axis corresponds to an element of order 2.

12.4 Translation Symmetry and Chromatic Drift

Define the translation operator

$$T_k(x) = x + k.$$

Then chromatic tritone drift is the orbit

$$\tau_n = T_{-n}(\tau_0).$$

The set of tritone axes is invariant under the translation group generated by $T_{\pm 1}$.

Proof. Translation preserves interval structure. □

This establishes chromatic drift as a symmetry action.

12.5 Functional Harmony as Coupled Linear Dynamics

Let R_n denote the dominant root associated to τ_n. Then

$$R_{n+1} \equiv R_n - 5 \pmod{12}$$

is a linear recurrence relation.

Thus tritone drift and harmonic resolution correspond to coupled linear actions on $\mathbb{Z}_{12}$, revealing functional harmony as a symmetry–constrained dynamical process.

13 Computational Formalization

13.1 Zone Indicator Function

Define the zone map

$$Z : \mathbb{Z}_{12} \to \{0, 1\}$$

by

$$Z(p) = p \bmod 2.$$

Then

$$Z(p) = Z(q) \iff p - q \equiv 0 \pmod{2}.$$

13.2 Crossing and Stability Operators

Define the boolean functions

$$C(p, q) = \begin{cases} 1 & Z(p) \neq Z(q) \\ 0 & Z(p) = Z(q) \end{cases}$$

and

$$S(p, q) = 1 - C(p, q).$$

Thus C encodes directional motion.

13.3 Tritone Detection Algorithm

A pair (p, q) forms a tritone iff

$$(p - q) \equiv 6 \pmod{12}.$$

This yields a constant–time test suitable for streaming harmonic analysis.

13.4 Dual–Zone Harmonic Detection

Let $S \subset \mathbb{Z}_{12}$.

S is dual–zone harmonic iff

$$|\{\{p, p + 6\} \subset S\}| \geq 2.$$

This allows automatic classification via computational search.

13.5 Gravity Chain Generator

Given an initial dominant root R_0, define

$$R_{n+1} = R_n - 5 \pmod{12}.$$

This produces the functional resolution chain algorithmically. Such mappings may be implemented as deterministic automata or extended probabilistically in Markov or Bayesian models.

14 Psychoacoustic Interpretation

14.1 Symmetry, Ambiguity, and Suspension

Highly symmetric pitch structures are perceptually associated with reduced functional gravity. Whole–tone partitions are maximally symmetric under $T_{\pm 2}$ translation, and listeners report a sense of "floating" or non–directional color.

This supports the identification of zones with harmonic color fields.

14.2 Tritones as Anchored Tension

The tritone is maximally distant in $\mathbb{Z}_{12}$ and forms an internal reference axis. Psychoacoustic studies have long associated the tritone with tension, but within our model it is not random dissonance — rather, it is structured gravitational polarity.

14.3 Semitone Crossings and Directional Energy

Semitone motion is perceptually salient and strongly directional. Since semitone movement crosses the zone boundary, the ear may be interpreted as tracking transitions between symmetry fields.

Thus:

$$\text{perceived direction} \iff \text{zone crossing}.$$

14.4 Why Melodic Minor Feels "Modern"

Dual–zone harmonic systems support simultaneous stability and drift. The coexistence of two tritone anchors allows:

- modal framing
- functional implication
- smooth chromatic weaving

without full collapse into tonal hierarchy.

This explains the distinctive affective quality of melodic minor harmony.

14.5 Cognitive Coherence

The Zone–Tritone System suggests that listeners balance two perceptual forces:

1. symmetry–seeking (zone stabilization)
2. goal–seeking (tritone resolution)

Music becomes meaningful in the tension between them.

15 Relation to Prior Work

Traditional jazz and tonal theory describe tritone substitution and cycle–of–fourths resolution operationally. The present framework derives these as structural consequences of modular symmetry and translation invariance.

Thus the Zone–Tritone System may be viewed not as a replacement for existing theory, but as its formal unifying grammar.

16 Discussion

The Zone–Tritone System resolves multiple theoretical tensions by showing that functional direction arises from semitone-driven transitions between symmetry fields, while gravitational identity arises from tritone axes internal to each field.

17 Conclusion

We have demonstrated that whole–tone parity structure, tritone involutions, and chromatic drift cohere into a compact formal grammar capable of explaining a wide range of harmonic behavior in tonal and post–tonal music, particularly within the jazz tradition.

A Proof Appendix for Canonical Axioms

A.1 Axiom 1

Closedness under whole steps is parity preservation.

A.2 Axiom 2

Tritones preserve parity and therefore zone identity.

A.3 Axiom 3

Half–steps flip parity.

A.4 Axiom 4

Chromatic drift corresponds to $-5 \pmod{12}$ root motion.

A.5 Axiom 5

Melodic minor contains two independent tritone anchors.