The Zone–Tritone System: A Formal Harmonic Framework in Modular Pitch Space

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Abstract

We formalize the Zone–Tritone System as a harmonic framework expressed in the modular pitch space $\mathbb{Z}_{12}$. Zones are defined as residue classes modulo 2, tritones as elements separated by $6 \bmod 12$, and chromatic drift as translation operators on tritone pairs. We prove that whole–step motion preserves zone identity, half–step motion transfers energy between zones, and tritone drift generates perfect fourth root motion. We also show that the melodic minor scale is dual–zone harmonic due to the simultaneous presence of two tritone anchors. This yields a mathematically coherent account of dominant function, chromaticism, and harmonic gravity.

1 Pitch Space and Modular Arithmetic

We represent the twelve pitch classes of equal temperament as the additive group

$$\mathbb{Z}_{12} = \{0, 1, 2, \ldots, 11\}$$

under addition modulo 12.

A pitch class $p \in \mathbb{Z}_{12}$ corresponds canonically to:

$$0 = C,\ 1 = C\sharp,\ 2 = D,\ 3 = E\flat,\ 4 = E,\ 5 = F,\ 6 = F\sharp,\ 7 = G,\ 8 = A\flat,\ 9 = A,\ 10 = B\flat,\ 11 = B.$$

2 Whole–Tone Zones

Define the two whole–tone zones as the residue classes modulo 2:

$$Z_1 = \{0, 2, 4, 6, 8, 10\}, \qquad Z_2 = \{1, 3, 5, 7, 9, 11\}.$$

These satisfy:

$$Z_1 \cup Z_2 = \mathbb{Z}_{12}, \qquad Z_1 \cap Z_2 = \emptyset.$$

Proposition 2.1. *If $x \in Z_i$, then $x \pm 2 \in Z_i$.*

Proof. If $x \equiv r \pmod 2$ then $x \pm 2 \equiv r \pmod 2$. Thus zone membership is preserved under addition of whole steps. □

We therefore interpret whole steps as *zone–stabilizing motion*.

3 Half–Step Motion

A semitone is the transformation

$$x \mapsto x \pm 1 \pmod{12}.$$

Proposition 3.1. *If $x \in Z_1$ then $x \pm 1 \in Z_2$, and conversely.*

Proof. If $x \equiv 0 \pmod 2$ then $x \pm 1 \equiv 1 \pmod 2$, and vice–versa. □

We therefore interpret semitones as *zone–crossing transformations*.

4 Tritones as Internal Zone Axes

A tritone is defined as an unordered pair

$$\tau = \{p,\ p + 6 \pmod{12}\}.$$

Proposition 4.1. *If $p \in Z_i$ then $p + 6 \in Z_i$.*

Proof. Since $6 \equiv 0 \pmod 2$, adding 6 preserves parity. □

Thus every tritone belongs entirely to one zone and functions as an internal axis of harmonic gravity.

5 Chromatic Tritone Drift and the Cycle of Fourths

Let a tritone be denoted:

$$\tau_n = \{p_n,\ p_n + 6\}.$$

Define chromatic drift by:

$$\tau_{n+1} = \{p_n - 1,\ p_n + 6 - 1\}.$$

Let $G(\tau)$ denote the root of the dominant defined by τ.

Theorem 5.1. *Chromatic drift of a tritone induces a root motion of descending fourths:*

$$R_{n+1} \equiv R_n - 5 \pmod{12}.$$

Proof. Two pitch classes separated by 6 semitones determine the third and seventh of a dominant chord. Shifting the pair down by a semitone preserves their interval while lowering both voices. In tonal convention, semitone descent of the third–seventh pair corresponds to root descent by a perfect fourth, i.e. $-5 \bmod 12$. □

6 Dual–Zone Structure of the Melodic Minor Scale

Let $S \subset \mathbb{Z}_{12}$ denote a scale. For C melodic minor:

$$S = \{0, 2, 3, 5, 7, 9, 11\}.$$

Proposition 6.1. *S contains two tritone pairs:*

$$\tau_1 = \{11, 5\}, \qquad \tau_2 = \{3, 9\}.$$

Proof. Direct verification shows each unordered pair differs by $6 \bmod 12$ and belongs to S. $\square$

Definition 6.2. A pitch collection is said to be *dual–zone harmonic* if it contains at least two distinct tritone anchors.

Corollary 6.3. *The melodic minor scale is dual–zone harmonic.*

A Proof Appendix for the Zone–Tritone Canon

A.1 Axiom 1: Zones Define Color

Whole–tone zones are closed under whole–step motion.

A.2 Axiom 2: Tritones Define Gravity

Every tritone lies fully within a single zone.

A.3 Axiom 3: Half–Steps Define Motion

Semitone motion changes zone membership.

A.4 Axiom 4: Chromatic Drift Produces Fourth Cycles

Chromatic translation of a tritone produces dominant roots descending by fourths.

A.5 Axiom 5: Melodic Minor as a Dual–Zone Hybrid

The melodic minor scale contains two independent tritone anchors.