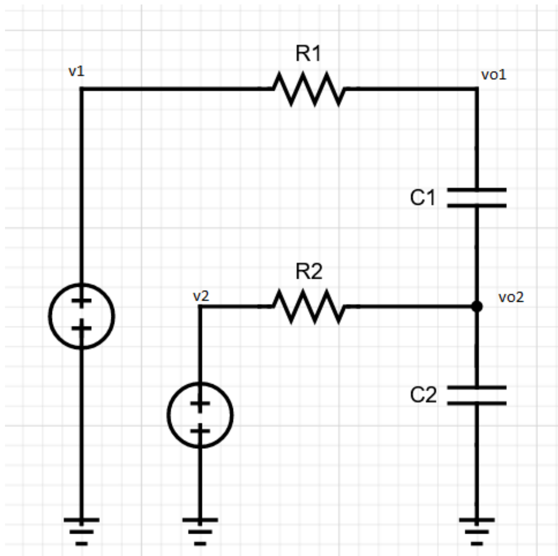
ECE550: Project - Dr. Raich - Due March 20, 2023

The project will be done in groups of two. Regard this project as a take home exam: you should not collaborate with others (except for your partner to this project). You are required to use Matlab and attach code and simulation results as part of your project. In your answers, provide a clear theoretical explanation to your results.

Consider the following system:



Consider the state vector as $\boldsymbol{x} = [v_{c1}(t), v_{c2}(t)]^T$, i.e., the voltage across capacitor 1 and the voltage across capacitor 2. The input is defined as $\boldsymbol{u} = [v_1(t), v_2(t)]^T$ and the output is defined as $\boldsymbol{y} = [v_{o1}(t), v_{o2}(t)]^T$.

Questions hereon refer to the system presented above.

~~1.~~ (Ch 2) Derive the state and observation equations for this problem given by

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)\\ \boldsymbol{y}(t) &= \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t).\end{aligned} \tag{1}$$

Show that the aforementioned matrices are:

$$\boldsymbol{A}_{n\times n} = -\frac{1}{R_1C_1}\begin{bmatrix} 1 & 1 \\ \frac{C_1}{C_2} & \frac{C_1}{C_2}(1+\frac{R_1}{R_2}) \end{bmatrix}, \quad \boldsymbol{B} = \frac{1}{R_1C_1}\begin{bmatrix} 1 & 0 \\ \frac{C_1}{C_2} & \frac{C_1}{C_2}\frac{R_1}{R_2} \end{bmatrix},$$
$$\boldsymbol{C} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

For the rest of the project proceed with the following values of the parameters of the system: $R_2 = \frac{1}{3}R_1$ and $C_2 = 4C_1$. For all the analytic expressions, feel free to replace $R_1C_1 = T$. For any numerical simulation, use $T = 1[\text{ms}]$.

~~2.~~ (Ch 20) Simulation: To simulate the continuous-time system in (1), we would like to use the second approximation taught in class, in which a continuous time-domain state equation is set as a difference equation, i.e.,

$$\begin{aligned}\boldsymbol{x}[k+1] &= \boldsymbol{A}'\boldsymbol{x}[k] + \boldsymbol{B}'\boldsymbol{u}[k]\\ \boldsymbol{y}[k] &= \boldsymbol{C}'\boldsymbol{x}[k] + \boldsymbol{D}'\boldsymbol{u}[k],\end{aligned} \tag{2}$$

where $\boldsymbol{A}' = \Phi((k+1)\Delta, k\Delta)$ and $\boldsymbol{B}' = \int_{k\Delta}^{(k+1)\Delta} \Phi((k+1)\Delta, \sigma)B(\sigma)d\sigma$, $\boldsymbol{C}' = \boldsymbol{C}$, and $\boldsymbol{D}' = \boldsymbol{D}$, and Δ is the sampling period such that $\boldsymbol{x}[k] = \boldsymbol{x}(k\Delta)$, $\boldsymbol{u}[k] = \boldsymbol{u}(k\Delta)$, and $\boldsymbol{y}[k] = \boldsymbol{y}(k\Delta)$. Select $\Delta = \frac{1}{100}[\text{ms}]$.

~~(a)~~ Our goal is to verify the approximation by comparing the state vector $\boldsymbol{x}(t)$ and the output $\boldsymbol{y}(t)$ (for $t_0 = 0 \leq t \leq 10$[ms]) to their theoretical values. Start by obtaining the state and output using simulations. Here $t_0 = 0$ and $\boldsymbol{x}(t_0) = [0, 0]^T$. Consider two cases: (i) $u(t) = [4, 1]^T \mu(t)$ for $t \geq 0$ and (ii) $u(t) = [4, 1]^T \cos(2\pi t/T)\mu(t)$. We define the step function $\mu(t)$ as $\mu(t) = 1$ for $t \geq 0$ and $\mu(t) = 0$ for $t < 0$.

~~(b)~~ Plot $\boldsymbol{x}(t)$ and $\boldsymbol{y}(t)$ for each of the cases.

3. (Ch 3-5) Derive the solution for each of the two cases in Q2, using the using the eigendecomposition of $\boldsymbol{A}$ and compare (by plotting) the solution you obtained analytically and the solution obtained by simulations. Theoretical values of the solution should be obtained analytically by hand.

4. (Ch 6-7,12) Stability:

   (a) Is the system uniformly stable? (prove) Uniformly exponentially stable? (prove) Uniformly asymptotically stable? (prove)

   (b) For $Q = I_{n \times n}$, check Lyapunov stability criteria for uniform stability and for uniform exponential stability.

   (c) Is the system uniformly bounded-input bounded-output?

5. For our system,

   (a) (Ch 9,13) Is the system controllable on [0,10ms]?

   (b) (Bonus 10%) Design an input $u(t)$ for $0 \leq t \leq 10ms$, that for $\boldsymbol{x}(0) = [0, 0]^T$ (defined earlier) yields $\boldsymbol{x}(10ms) = [1, 1]^T$.

6. Is the system observable on [0,10ms]?