Computer Networks Homework # 1

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Define the geodesic (short path) distance between two nodes as the minimum number of hops from one node to the other. Define the diameter of a network as the maximum geodesic distance among all the pairs of two nodes. Define the degree of a node as the number of links connected to that node.

1. If the diameter of a network with 100 nodes is 1, what is the minimum number of links in this network?

   Ans: It represents that each node has link with the other, so **the minimum required links are** $\binom{100}{2} = 4950$.

2. If the diameter of a network with 100 nodes is 2, what is the minimum number of links in this network?

   Ans: The network is just like Fig. 1 shown as right. The other 99 nodes surrounds and are connected to the center node respectively. Thus, **the minimum number of links are 99**.

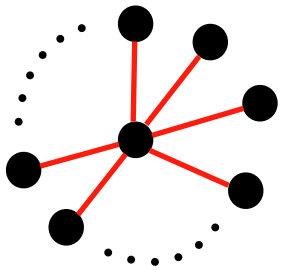

Fig. 1

3. For a network of 100 nodes, if the degree of every node is at most 2, what is the minimum diameter of that network?

   Ans: The network is just like Fig. 2 shown as right. All nodes are connected as a ring, and the degree of all the node is 2. Thus, **the minimum number of links are** $\lfloor \frac{100}{2} \rfloor = 50$.

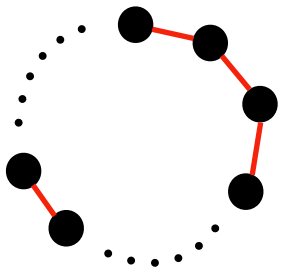

Fig. 2

4. For a network of 100 nodes, if the degree of every node is at most 3, is it possible that the diameter of this network is not greater than 5?

   Ans: It is not possible. In graph theory, the degree-diameter problem is to find the largest size of the graph $n_{d,k}$ in terms of the diameter k and the greatest degree d of the graph. (The size of the graph, namely, the number of nodes). The size of the graph $n_{d,k}$ is bounded above by Moore bound $M_{d,k}$, which gives that

$$M_{d,k} = 1 + d\sum_{i=0}^{k-1}(d-1)^i$$

Thus, $n_{3,5} \leq M_{3,5} = 1 + 3\sum_{i=0}^{4}(3-1)^i = 94 < 100$ for greatest degree $d = 3$ and the diameter $k = 5$, which indicates that it is not possible that the diameter of this network is not greater than 5 under such conditions.

(Reference for Moore bound formula: https://en.wikipedia.org/wiki/Moore_grapha)