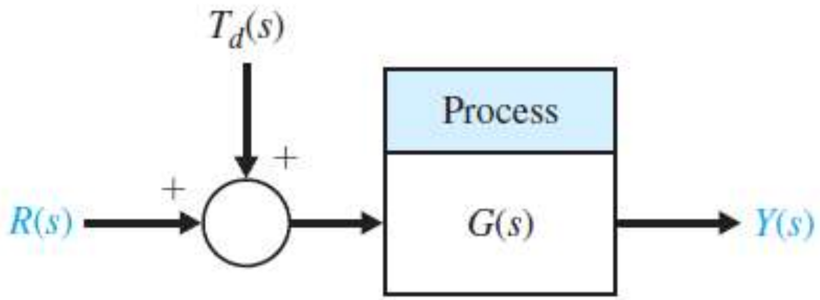
Chapter 4

Feedback Control System Characteristics

4.1 Introduction



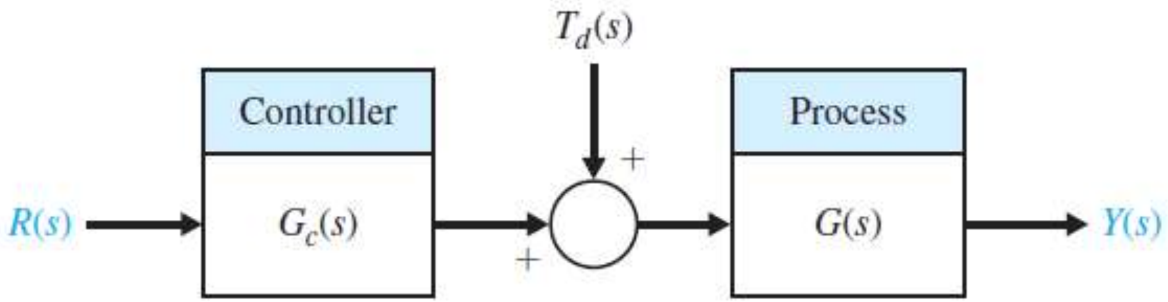
Open-loop system

- An open-loop system, or a system without feedback

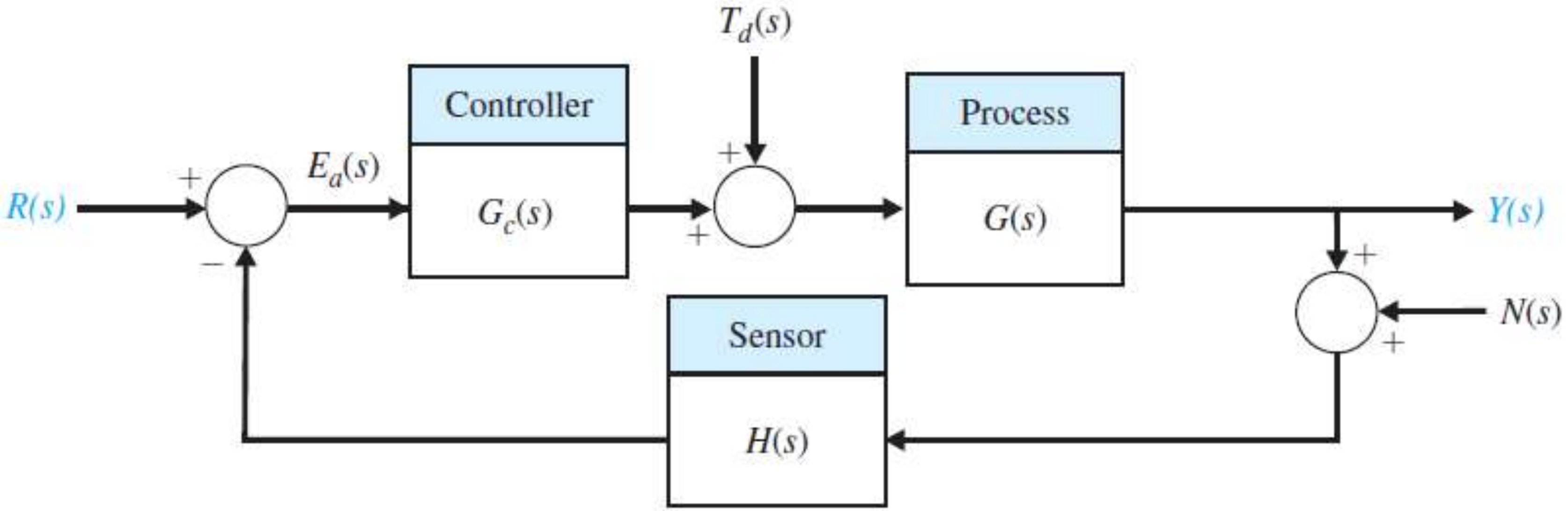
→disturbance directly influences the output

→the control system is highly sensitive to disturbances and variations in parameters of the process G(s)

→If the open-loop system does not provide a satisfactory response, then a suitable cascade controller $G_c(s)$ can be inserted preceding the process G(s)



Open-loop control system



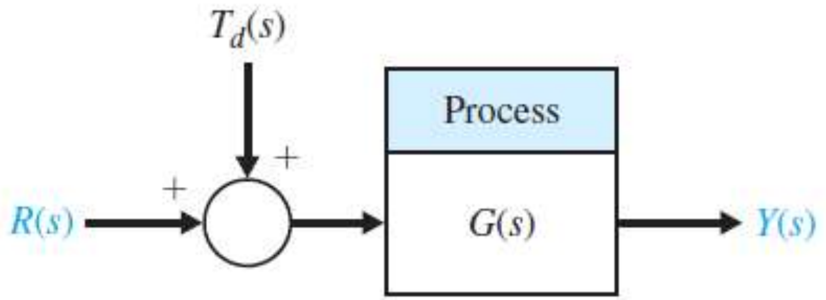
- A closed-loop system

→uses a measurement of the output signal

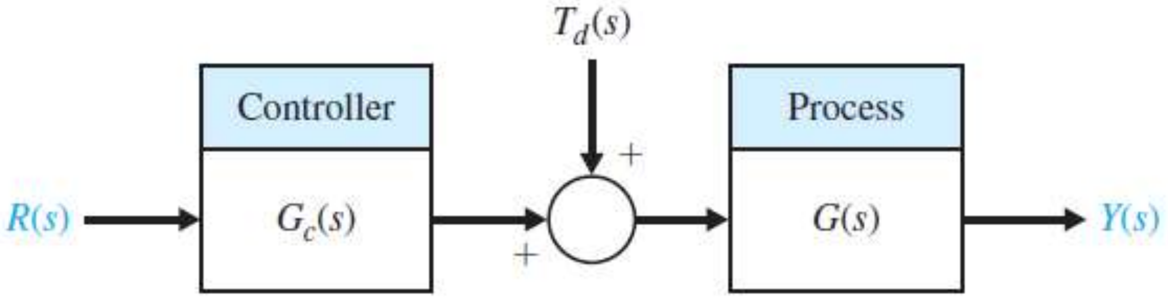
→compares the desired output with the measured signal

→generates an error signal accordingly

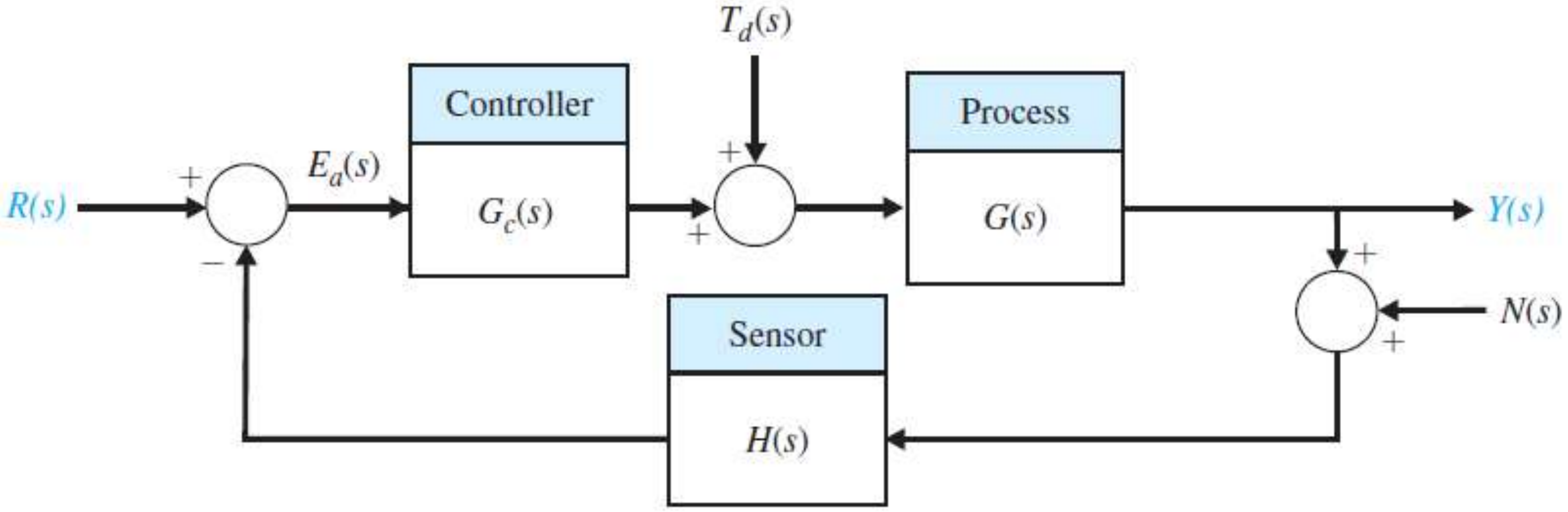
→(controller) uses the error signal to adjust the actuator.



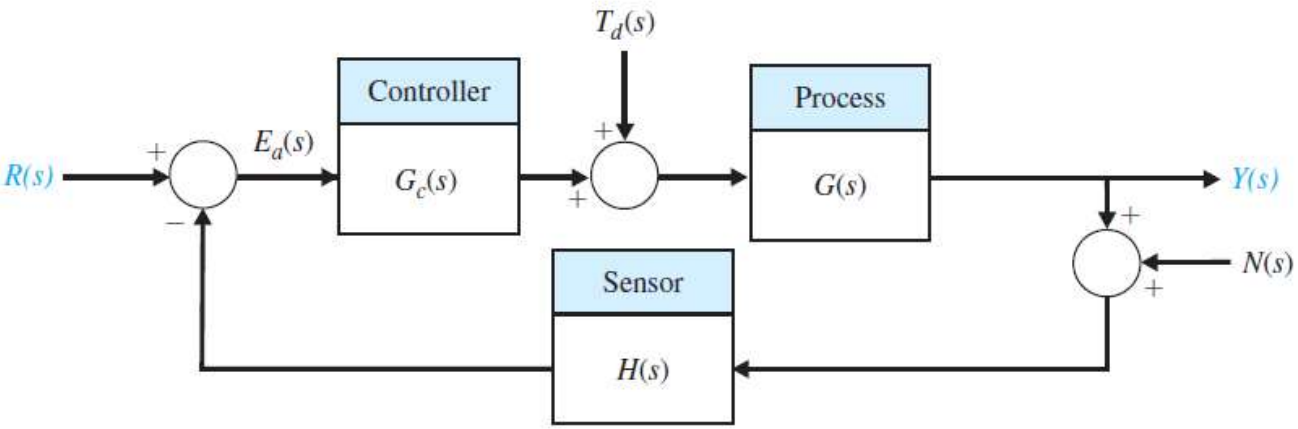
Open-loop system



Open-loop control system



Closed-loop control system



- advantages:

→decreased sensitivity of the system to variations in the parameters of the process G(s)

→improved rejection of the disturbances $T_d(s)$

→improved measurement noise attenuation N(s)

→improved reduction of the steady-state error of the system

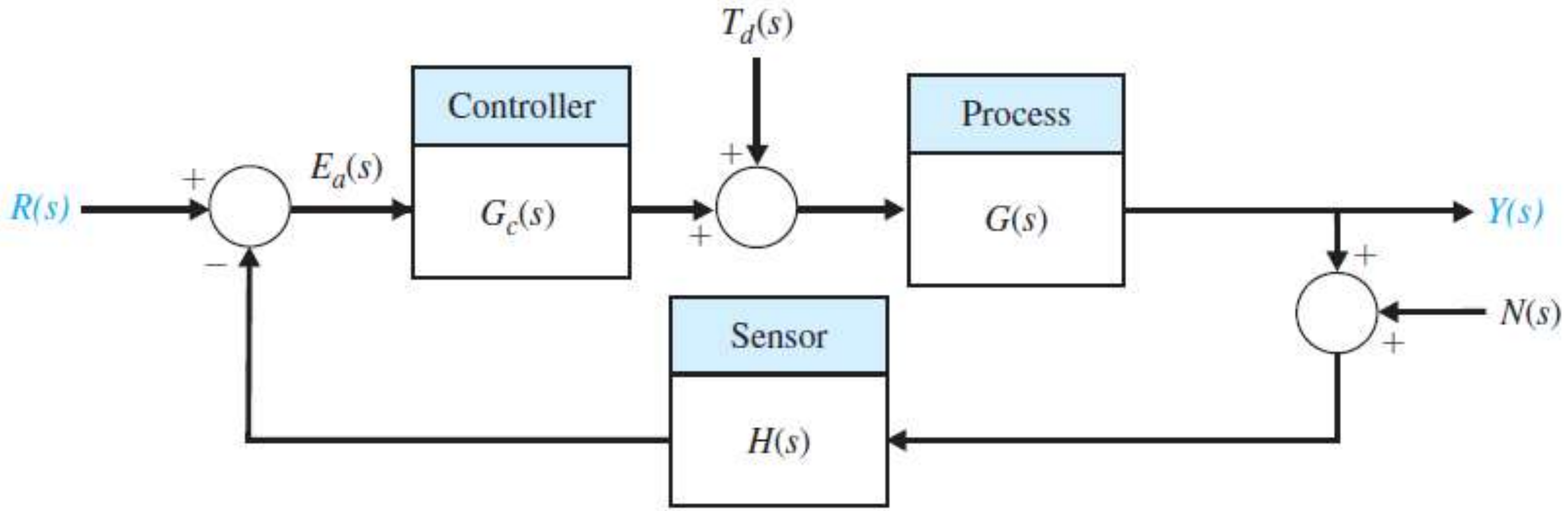
→easy control and adjustment of the transient response of the system.

- Disadvantages

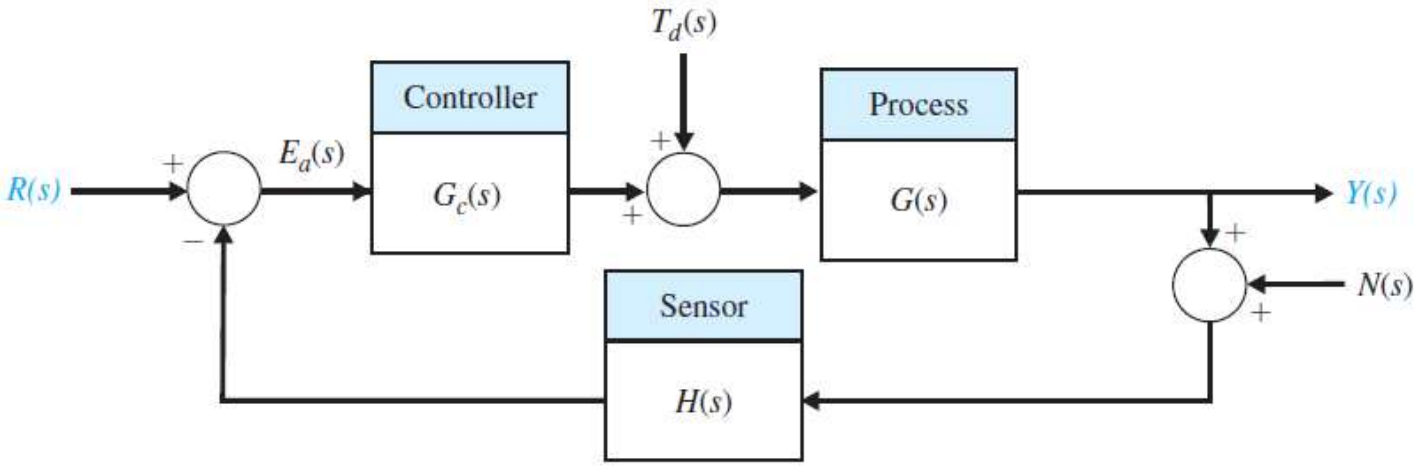
→higher cost

→increased system complexity

4.2 Error Signal Analysis



- Inputs: $R(s)$, $T_d(s)$, $N(s)$
- Output: $Y(s)$
- Tracking error: $E(s)=R(s)-Y(s)$ [assume $H(s)=1$]



$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

$$E(s) = \frac{1}{1 + G_c(s)G(s)}R(s) - \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

Define loop gain: $L(s) = G_c(s)G(s).$

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s).$$

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or $$E(s) = S(s)R(s) - S(s)G(s)T_d(s) + C(s)N(s).$$

$$S(s) = \frac{1}{1 + L(s)}$$ Sensitivity function (formally defined later)

$$C(s) = \frac{L(s)}{1 + L(s)}$$ Complementary sensitivity function

- S(s) and C(s) can be affected by controller $G_c(s)$
- S(s)+C(s)=1 (explains the term ``complementary'')
- Goal: E(s)→0; S(s) and C(s) are both small (is this possible given S(s)+C(s)=1 ?)

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s).$$

- Consider the magnitude $|L(j\omega)|$ over the range of frequencies ω
- Minimize E(s)

→L(s) large over the range of frequencies occupied by disturbances

→L(s) small over the range of frequencies occupied by measurement noise

- Solution

→ L(s) large at low frequencies and small at high frequencies (why?)

4.3 Sensitivity

- A process G(s) is subject to a changing environment, uncertainty in the exact values of the process parameters, and other factors that affect a control process
- A closed-loop system senses the change in the output due to the process changes and attempts to correct the output
- The sensitivity of a control system to parameter variations is of prime importance.

Closed-Loop Systems Reduce Sensitivity

- Closed-loop feedback control system can reduce the system's sensitivity

$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

$$T_d(s) = 0 \text{ and } N(s) = 0 \qquad G_c(s)G(s) \gg 1$$

$$\Rightarrow \quad Y(s) \cong R(s)$$

→The system is not sensitive to any variations happening to G(s)

→the effect of the variation of the parameters of the process is reduced

- Example in the previous slide did not illustrate the role of the sensitivity function. Let's take another look here.
- Suppose that the process undergoes a change such that the true plant model is $G(s) + \Delta G(s)$

consider the effect on the tracking error $E(s)$ due to $\Delta G(s)$

$$T_d(s) = N(s) = 0$$

$$E(s) + \Delta E(s) = \frac{1}{1 + G_c(s)(G(s) + \Delta G(s))} R(s).$$

$$\Delta E(s) = \frac{-G_c(s)\Delta G(s)}{(1 + G_c(s)G(s) + G_c(s)\Delta G(s))(1 + G_c(s)G(s))} R(s).$$

$$\Delta E(s) = \frac{-G_c(s)\Delta G(s)}{(1 + G_c(s)G(s) + G_c(s)\Delta G(s))(1 + G_c(s)G(s))} R(s).$$

Usual condition: $G_c(s)G(s) \gg G_c(s)\,\Delta G(s)$

$$\Delta E(s) \approx \frac{-G_c(s)\Delta G(s)}{(1 + L(s))^2} R(s).$$

For large $L(s)$, we have $1 + L(s) \approx L(s)$

$$\Delta E(s) \approx -\frac{1}{L(s)} \frac{\Delta G(s)}{G(s)} R(s).$$

Larger magnitude $L(s)$ translates into smaller changes in the tracking error

$$\Delta E(s) \approx -\frac{1}{L(s)} \frac{\Delta G(s)}{G(s)} R(s).$$

Percentage change

- System sensitivity

→ the ratio of the percentage change in the system transfer function to the percentage change of the process transfer function

$$T(s) = \frac{Y(s)}{R(s)}, \quad \Rightarrow \quad S = \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)}.$$

In the limit
(for small incremental changes)

$$\Rightarrow \quad S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}.$$

$$S = \frac{\partial T/T}{\partial G/G} = \frac{\partial \ln T}{\partial \ln G}.$$

- System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function (or parameter) for a small incremental change.
- Sensitivity of closed-loop system:

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}.$$

$$\Rightarrow S_G^T = \frac{\partial T}{\partial G}\cdot\frac{G}{T} = \frac{G_c}{(1 + G_cG)^2}\cdot\frac{G}{GG_c/(1 + G_cG)} \Rightarrow S_G^T = \frac{1}{1 + G_c(s)G(s)}.$$

- Sensitivity of open-loop system: $T(s)=G_c(s)G(s)$

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} =$$

What does this mean?

- Sensitivity with respect to a parameter

α is a parameter within the transfer function, $G(s)$

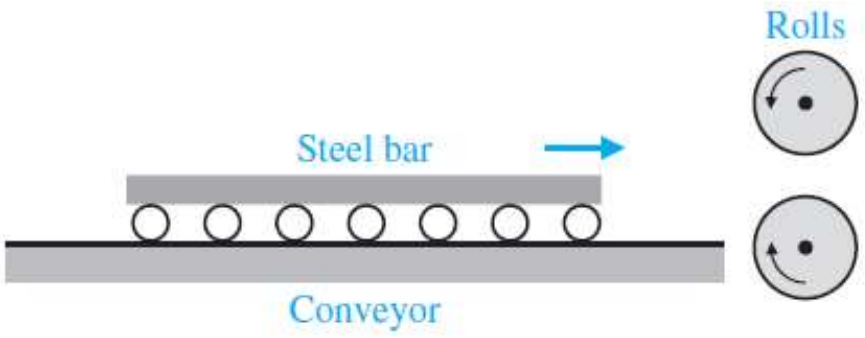
$$S_\alpha^T = S_G^T S_\alpha^G.$$

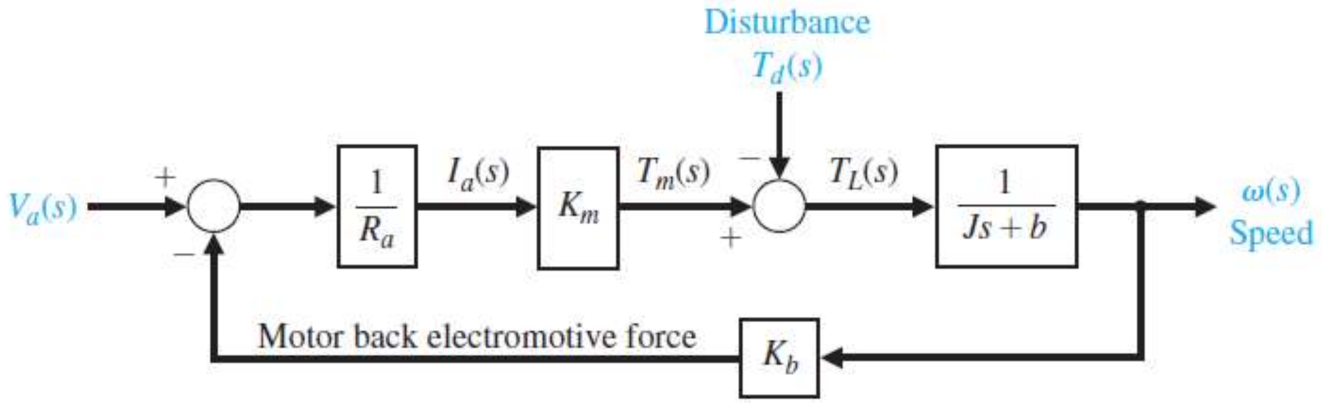
4.4 Disturbance

- Many control systems are subject to extraneous disturbance signals that cause the system to provide an inaccurate output
- Disturbance signal

→unwanted input signal that affects the output signal.

→can be effectively reduced by a closed-loop system





The loading effect can be approximated by a step change of disturbance torque.

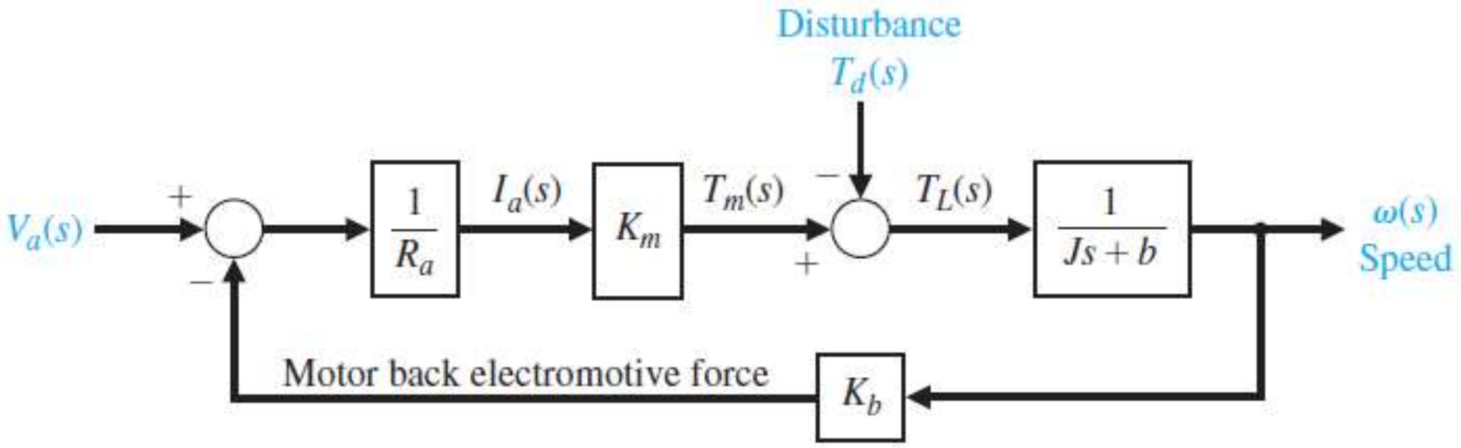


FIGURE 4.8
Open-loop speed control system (without tachometer feedback).

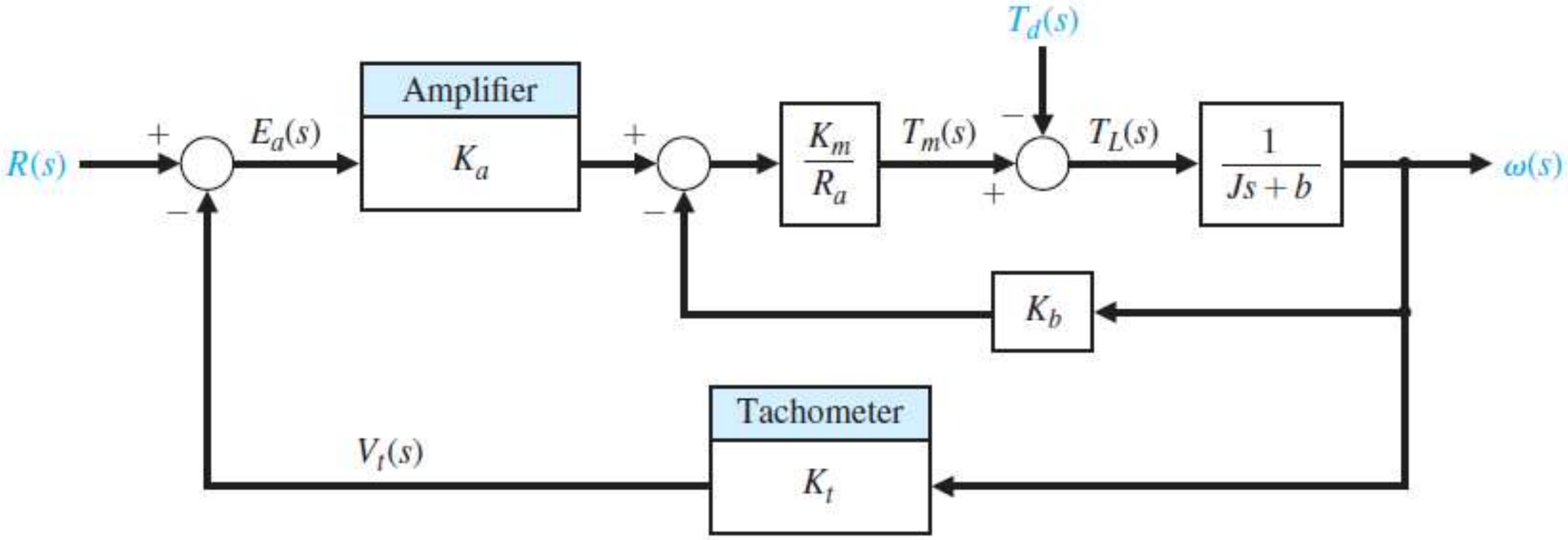
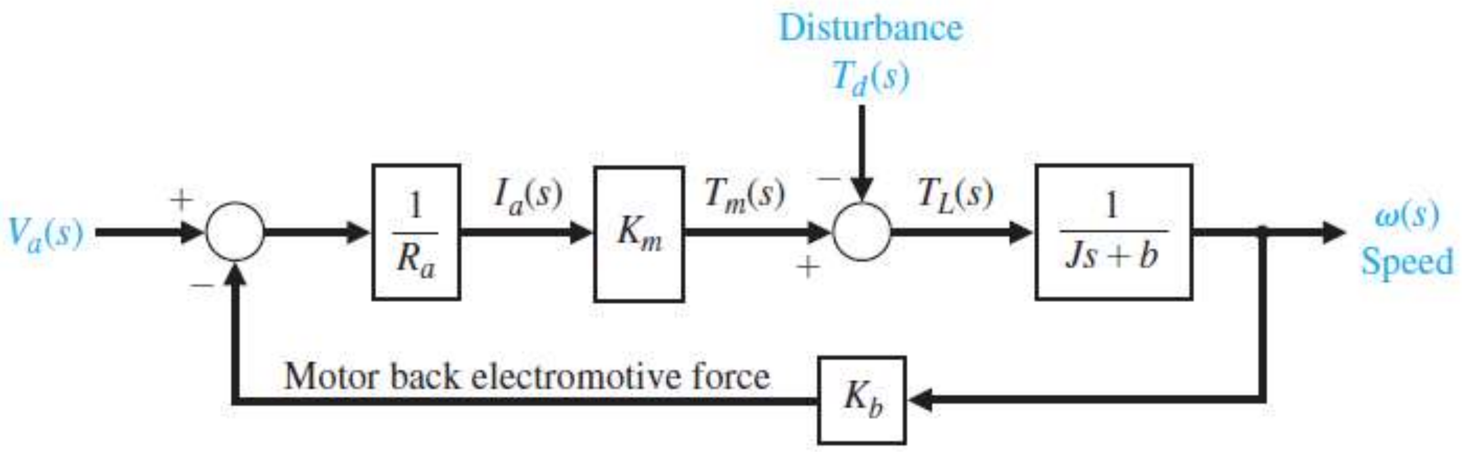


FIGURE 4.9
Closed-loop speed tachometer control system.

Tracking Error of Open-Loop System

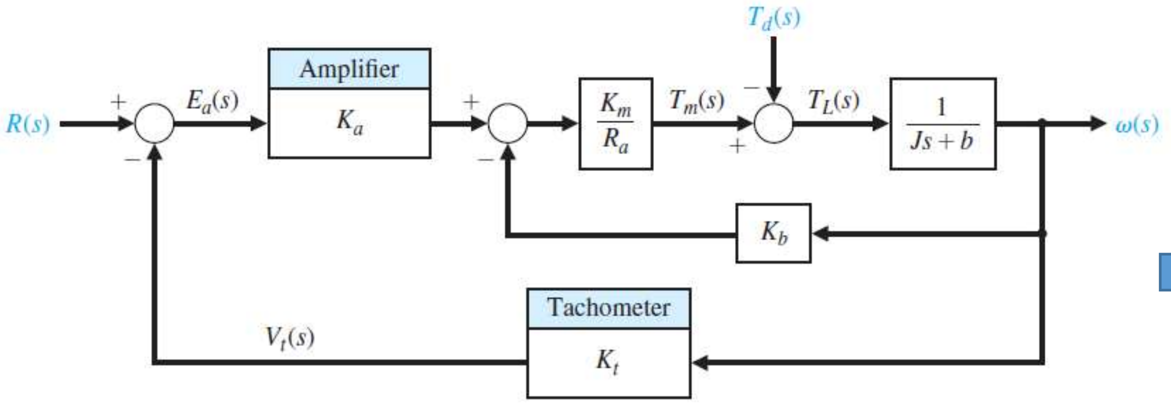


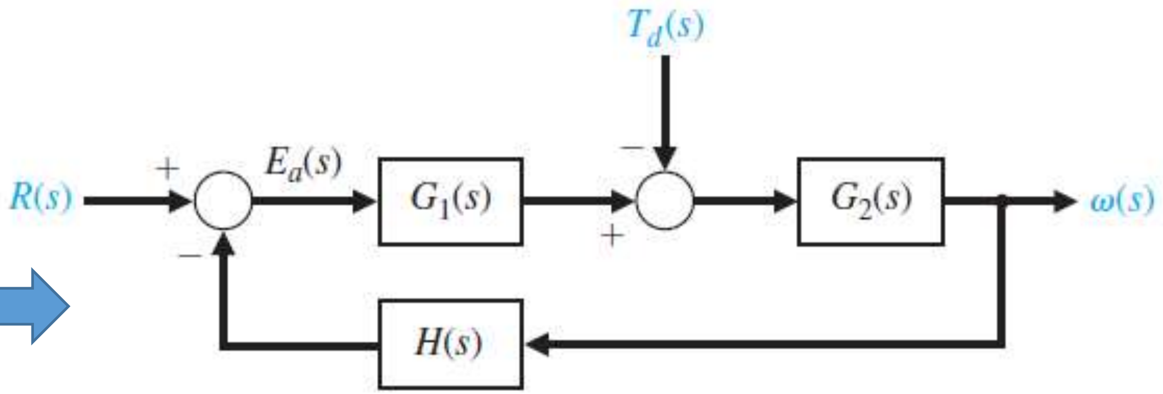
$$E(s) = -\omega(s) = \frac{1}{Js + b + K_m K_b / R_a} T_d(s).$$

Error cannot be reduced

"Life can only be improved by a closed-loop mind," Wei-Yu Chiu, philosopher

Tracking Error of Closed-Loop System





$$E(s) = -\omega(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} T_d(s).$$

if $G_1 G_2 H(s)$ is much greater than 1 over the range of s,

$$E(s) \approx \frac{1}{G_1(s)H(s)} T_d(s).$$

Error can be reduced by controller

Measurement Noise Attenuation—E(s)/N(s)

$$E(s) = \frac{1}{1 + L(s)}R(s) - \frac{G(s)}{1 + L(s)}T_d(s) + \frac{L(s)}{1 + L(s)}N(s).$$

When $R(s) = T_d(s) = 0$,

$$E(s) = C(s)N(s) = \frac{L(s)}{1 + L(s)}N(s).$$

- Noise attenuation

→a small loop gain over the frequencies associated with the expected noise signals.

Measurement Noise Attenuation—Y(s)/N(s)

- For noise attenuation, we examined the relationship between E(s) and N(s). We can also examine the relationship between Y(s) and N(s)

$$Y(s) = \frac{-G_c(s)G(s)}{1 + G_c(s)G(s)}N(s),$$

- For large loop gain,

$$Y(s) \simeq -N(s),$$

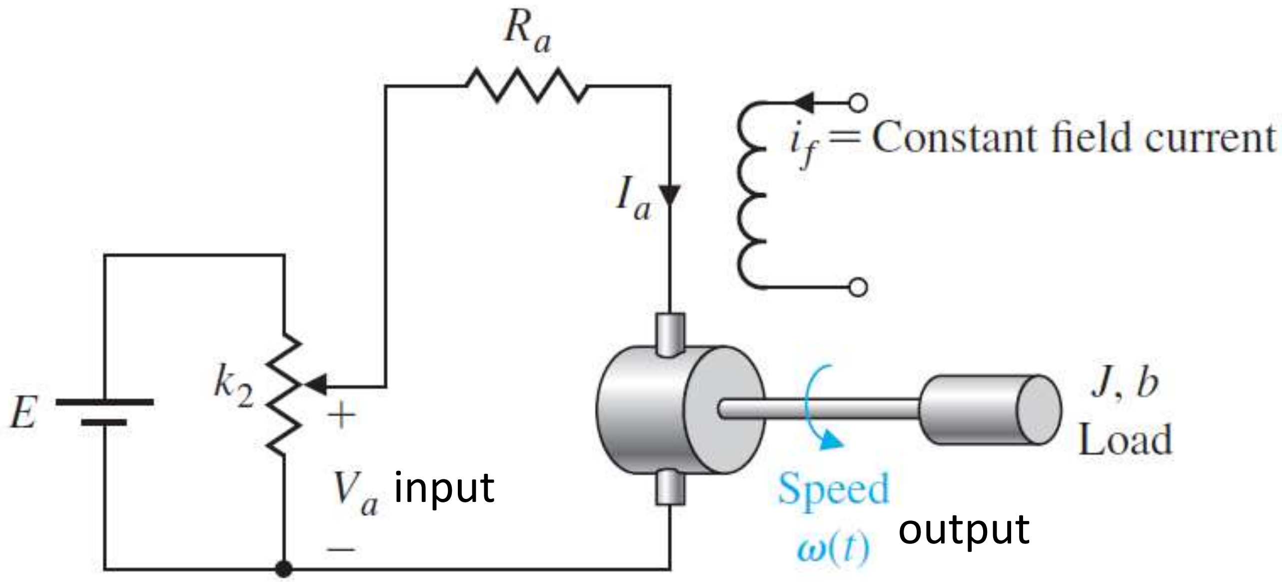
→ smaller loop gain leads to measurement noise attetuation

4.5 Control of Transient Response

- Transient response

→response of a system as a function of time before steady-state

- Transient response often must be adjusted until it is satisfactory

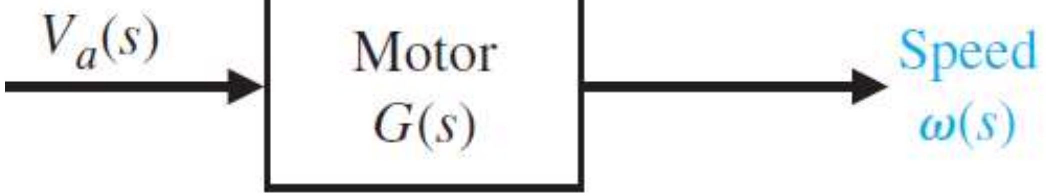


TF of the open-loop system

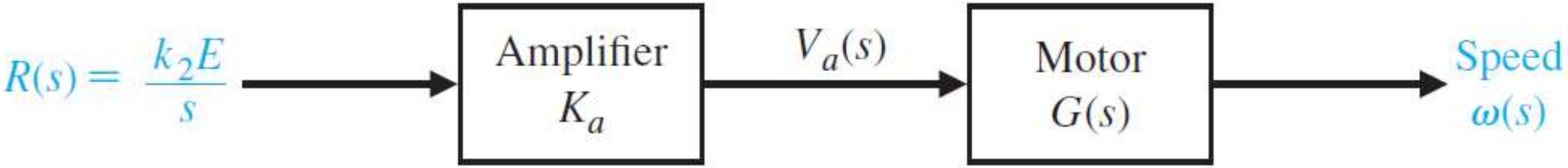
$$\frac{\omega(s)}{V_a(s)} = G(s) = \frac{K_1}{\tau_1 s + 1},$$

$$K_1 = \frac{K_m}{R_a b + K_b K_m} \quad \text{and} \quad \tau_1 = \frac{R_a J}{R_a b + K_b K_m}.$$

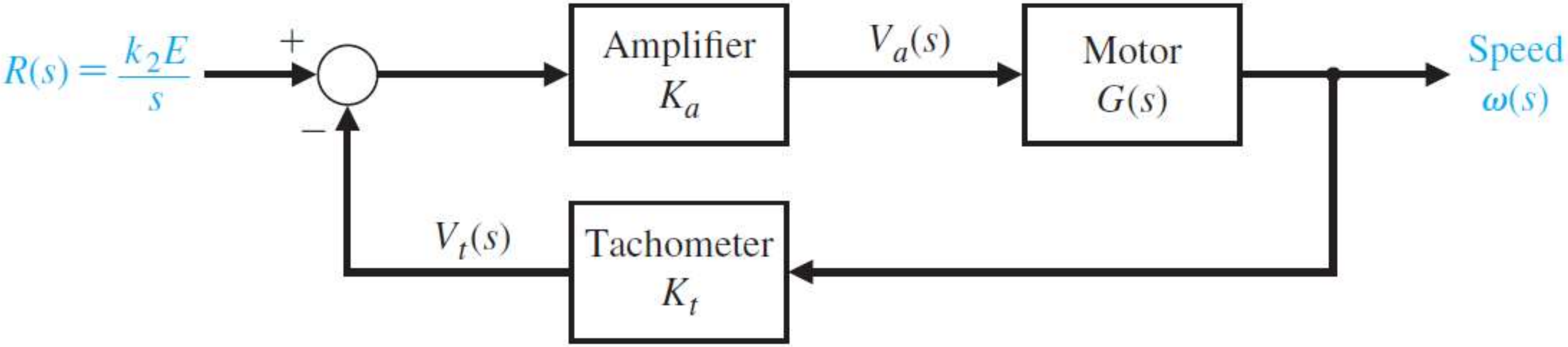
Open-loop system



Open-loop speed control system



Closed-loop speed control system



- Consider step command

$$R(s) = \frac{k_2 E}{s},$$

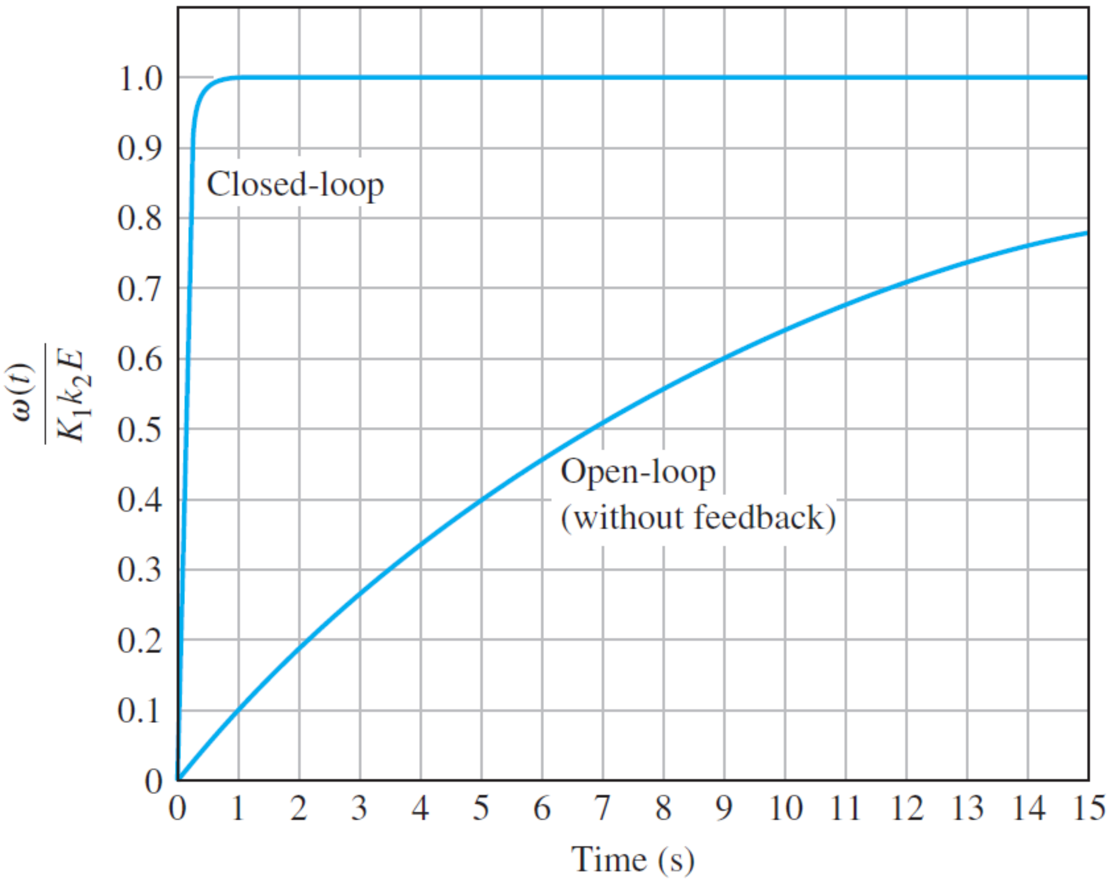
- Output of open-loop system

$$\omega(t) = K_a K_1 (k_2 E)(1 - e^{-t/\tau_1}) \qquad 1/\tau_1 = 0.10$$

- Output of closed-loop system

$$\omega(t) \approx \frac{1}{K_t}(k_2 E)\left[1 - \exp\left(\frac{-(K_a K_t K_1)t}{\tau_1}\right)\right]. \qquad (K_a K_t K_1)/\tau_1 = 10$$

The response of the open-loop and closed-loop speed control system



4.6 Steady-State Error

- Steady-state (SS) error

→the error after the transient response has decayed, leaving only the continuous response

- SS error of open-loop system

$$E_0(s) = R(s) - Y(s) = (1 - G_c(s)G(s))R(s)$$

- SS error of closed-loop system

$$E_c(s) = \frac{1}{1 + G_c(s)G(s)}R(s).$$

- SS error of open-loop system

$$e_o(\infty) = \lim_{s \to 0} s(1 - G_c(s)G(s))\left(\frac{1}{s}\right) = \lim_{s \to 0} (1 - G_c(s)G(s))$$
$$= 1 - G_c(0)G(0).$$

- SS error of closed-loop system

$$e_c(\infty) = \lim_{s \to 0} s\left(\frac{1}{1 + G_c(s)G(s)}\right)\left(\frac{1}{s}\right) = \frac{1}{1 + G_c(0)G(0)}.$$

- DC loop gain $L(0) = G_c(0)G(0)$ >1 in general
- For large DC gain

→Open-loop control system has large SS error

→Closed-loop control system has small SS error

Why not set DC gain to 1?

$$\begin{aligned} e_o(\infty) &= \lim_{s \to 0} s(1 - G_c(s)G(s))\left(\frac{1}{s}\right) = \lim_{s \to 0} (1 - G_c(s)G(s)) \\ &= 1 - G_c(0)G(0). \end{aligned}$$

- No SS error for open-loop system if $G_c(0)G(0) = 1$

4.7 Cost of Feedback

- An increased number of components and complexity in the system
- loss of gain

→L(s) from in open-loop reduces to L(s)/(1+L(s)) in closed-loop

- Introduction of the possibility of instability

→even when the open-loop system is stable, the closed-loop system may not be always stable

- Addition of feedback to dynamic systems causes more challenges for the designer

- We want output Y(s) to equal the reference input R(s)

→Why not set $G_c(s)G(s) = 1$ in open-loop?

1. Process G(s) represents a real process and possesses dynamics that may not appear directly in the transfer function.
2. The parameter in *G*(*s)* may be uncertain or vary with time.

→We cannot perfectly set $G_c(s)G(s) = 1$

Summary

- Closed-loop control systems allow us to

→reduce sensitivity

→reject disturbances

→attenuate measurement noise

→Improve transient response and reduce SS error

- Separation of disturbances at low frequencies and measurement noise at high frequencies

→ high/low loop gain at low/high frequencies