Control Systems HW3

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1.

(a)

```
sys_ss =

  A =
         x1
   x1   -10

  B =
        u1
   x1    1

  C =
        x1
   y1    1

  D =
        u1
   y1    0
```

(b)

```
sys_ss =

  A =
          x1     x2
   x1     -8   -2.5
   x2      2      0

  B =
       u1
   x1   2
   x2   0

  C =
          x1     x2
   y1   -1.5   -0.5

  D =
       u1
   y1   1
```

(c)

```
sys_ss =

  A =
          x1     x2     x3
   x1     -3   -1.5     -1
   x2      2      0      0
   x3      0    0.5      0

  B =
       u1
   x1   1
   x2   0
   x3   0

  C =
         x1    x2    x3
   y1     0   0.5     1

  D =
       u1
   y1   0
```

2.

(a)

```
sys_tf =

       1
  -------------
  s^2 - 8 s - 2
```

(b)

```
sys_tf =

         6 s - 10
  -----------------------
  s^3 + 6 s^2 - 21 s + 10
```

(c)

```
sys_tf =

      s - 2
  -------------
  s^2 + 2 s + 1
```

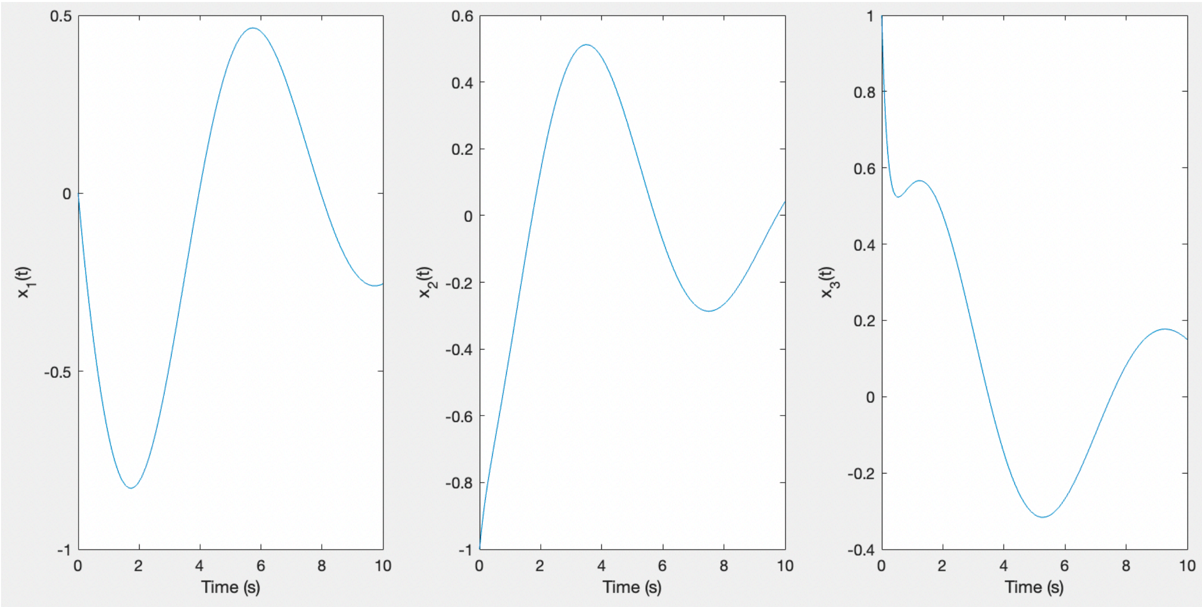
3.

(a)

```
sys_tf =

            1
  ---------------------
  s^3 + 5 s^2 + 2 s + 3
```

(b)

Response of the system:



$x_1(t)$ $x_2(t)$ $x_3(t)$

$x_1(10)$, $x_2(10)$ & $x_3(10)$ are obtained from the response:

```
x10_sim =

   -0.2545
    0.0418
    0.1500
```

(c)

State transition matrix:

```
Phi =

    0.0853    0.3183    0.0637
   -0.1911   -0.0421   -0.0003
    0.0010   -0.1905   -0.0405
```

$x_1(10)$, $x_2(10)$ & $x_3(10)$ are obtained from the transition matrix:

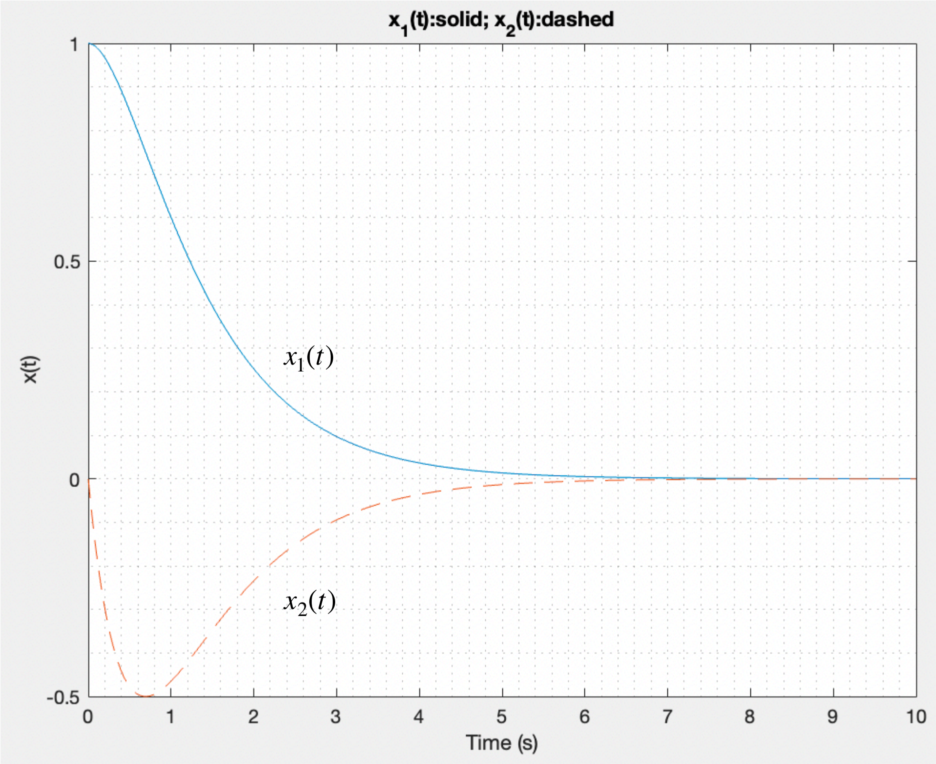
```
x10_phi =

   -0.2545
    0.0418
    0.1500
```

Comment:

The results are the same and do make sense. Part (b) acquires $x(t)$ from the basic plot of the transfer function, while part (c) uses the solution as below:

$$x(t) = \phi(t)x(0) + \int_0^t \phi(t-\tau)Bu(t)d\tau$$

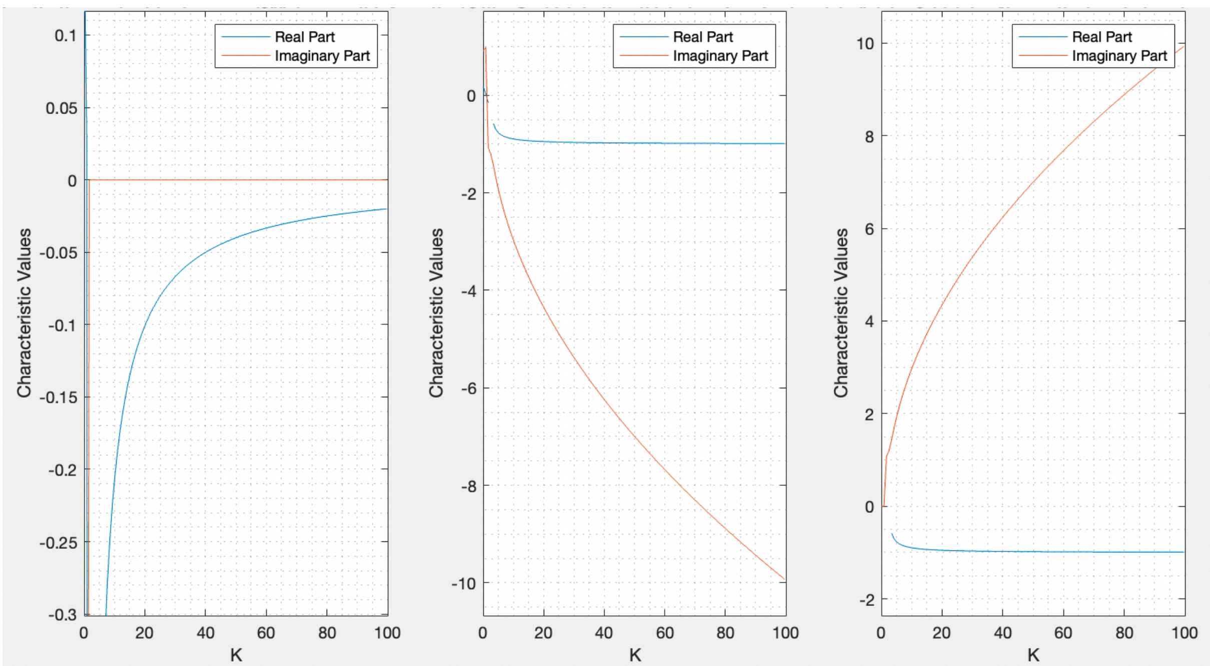
4.



5.

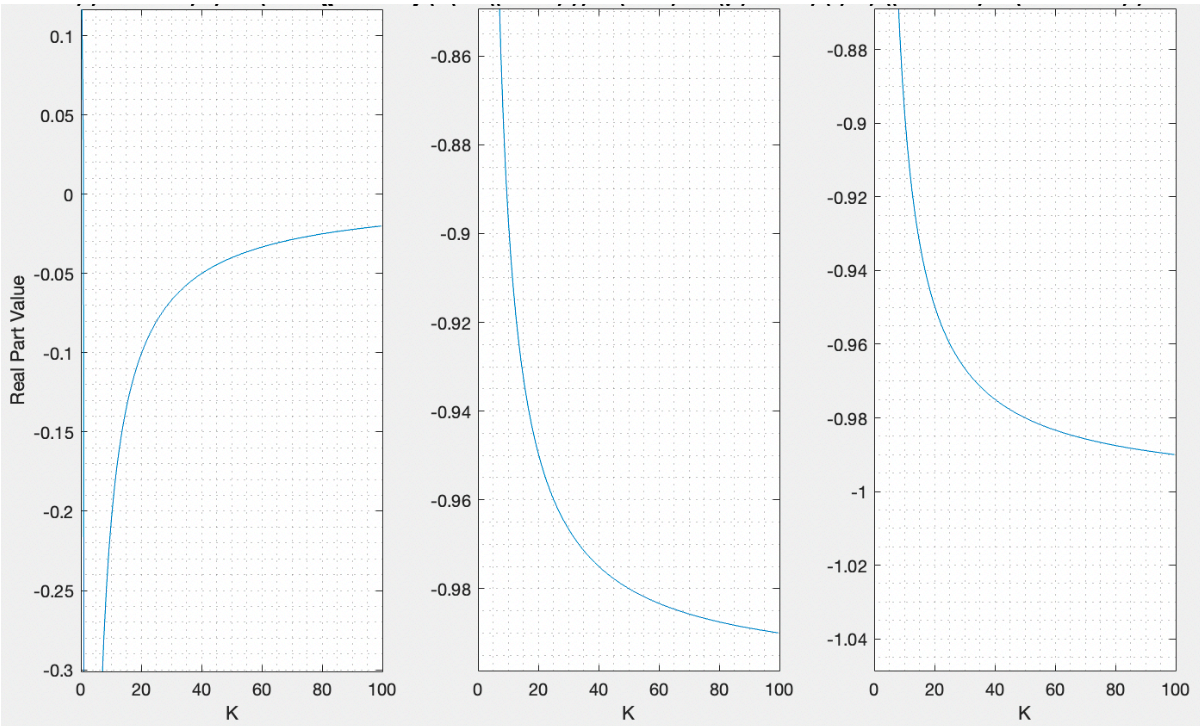
We know that eigenvalues of a matrix stand for solutions of the transfer function poles. By avoiding **positive real part of eigenvalues** from matrix A (dimension: 3x3 →3 eigenvalues) happening within specific range of K, we can make sure the all the characteristic values are in the left half-plane. The following shows the plot of the characteristic values to K :

Root1 Root2 Root3



Only the real part of the three roots with $0 \leq K \leq 100$:

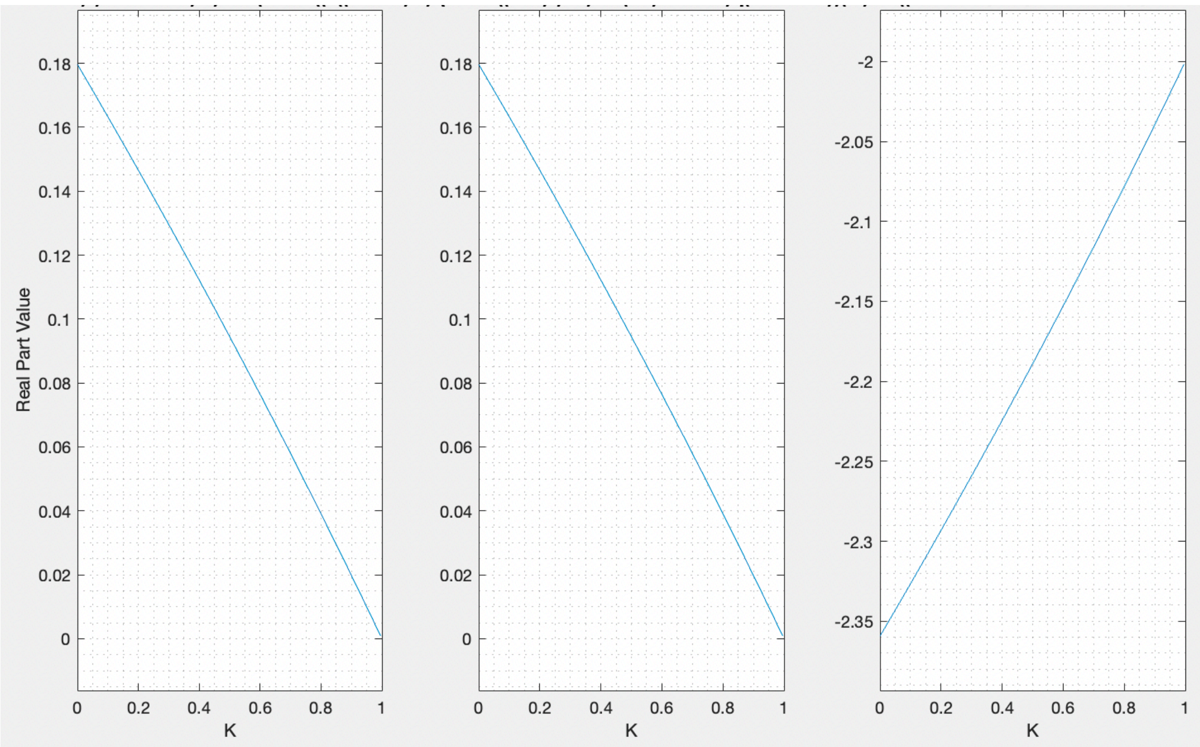
Root1 Root2 Root3



From the above plot, we can see that the positive real part of the root happens in root1 when K approaches to 0. Let's focus on $0 \leq K \leq 1$ next.

Only the real part of the three roots with $0 \leq K \leq 1$:

Root1 Root2 Root3



From the graph shown as above, we can find that the real part of root1 and root2 have positive values when $K \leq 1$. **We conclude that all the characteristic values are in the left half-plane only when** $1 < K \leq 100$**.**