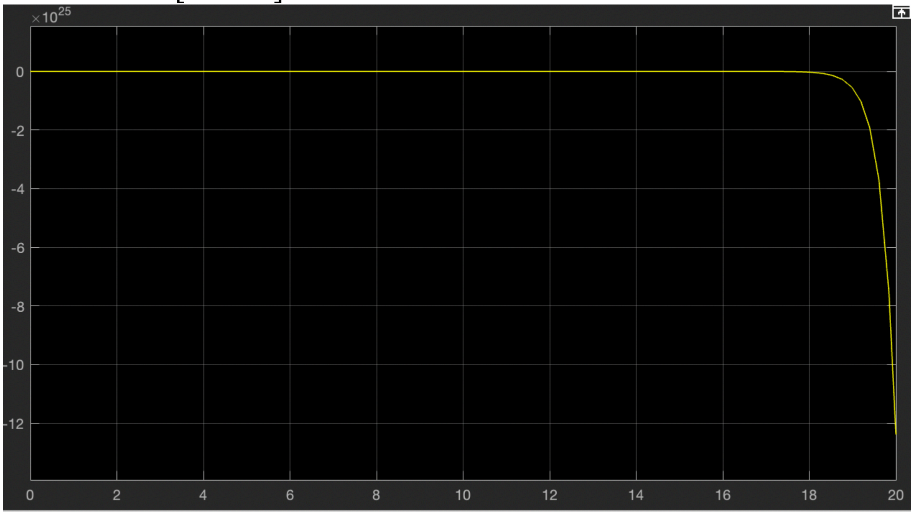
Control Systems HW4

107060011 電資院學士班大三 涂皓鈞

1. Simulation by Simulink with different K and C in time-domain

Assume that $g = 9.8\ m/s^2, l = 0.4\ m, m = 0.01\ kg$ and $M = 2.5\ kg$

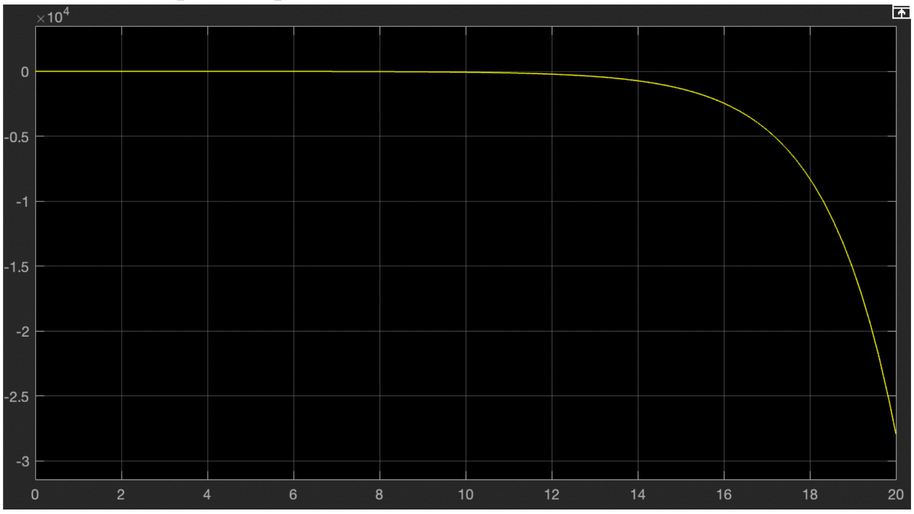
K = -15 C = [0 0 1 0]



Transfer function:

```
     -1
----------
s^2 - 24.5
```

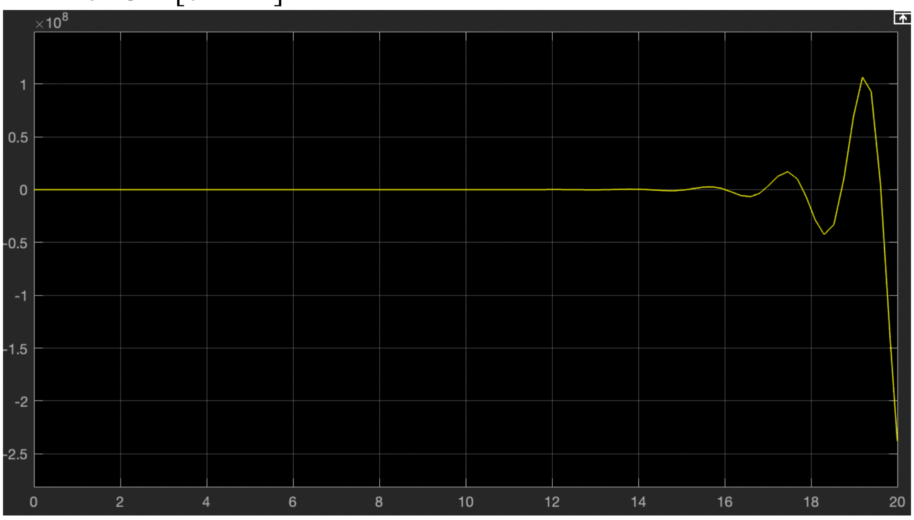
K = -15 C = [0 0 1 1]



Transfer function:

```
  -s - 1
----------
s^2 - 24.5
```

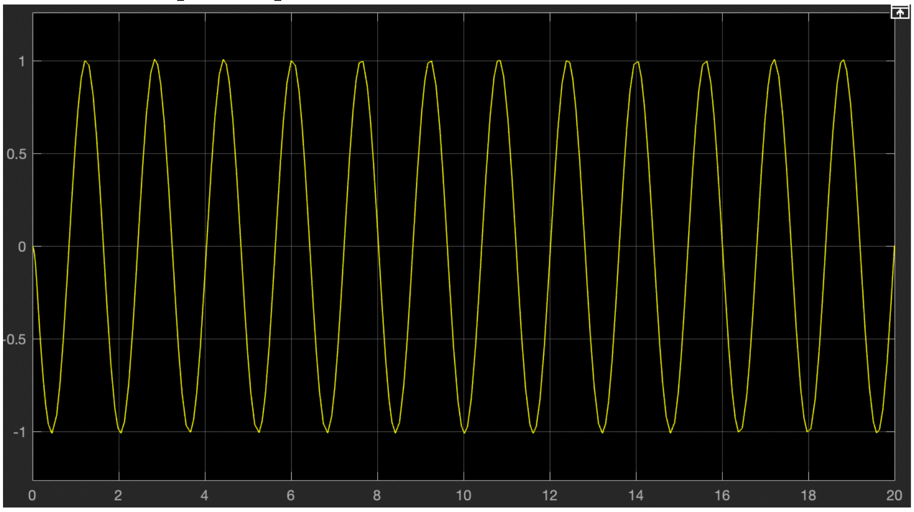
K = -15 C = [0 1 1 1]



Transfer function:

```
-0.6 s^2 - s - 9.761
--------------------
    s^3 - 24.5 s
```

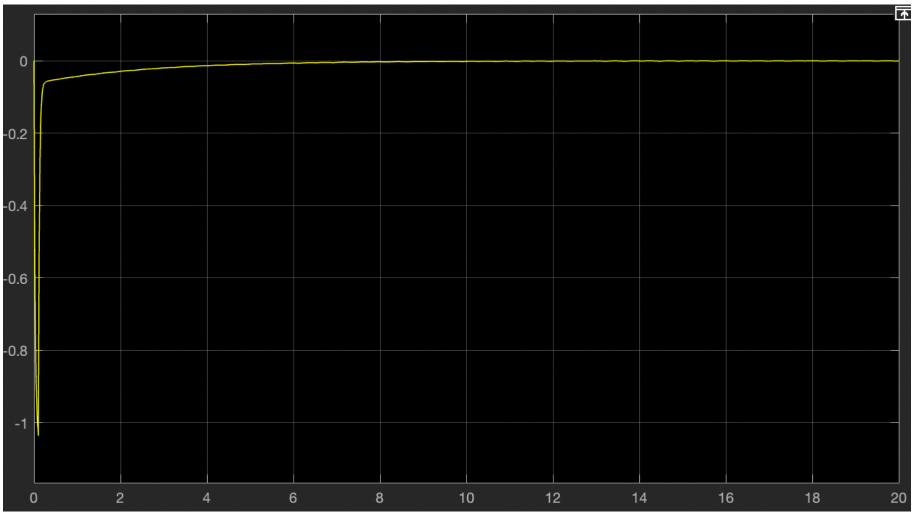
K = -40 C = [0 0 1 0]



Transfer function:

```
    -1
----------
s^2 - 24.5
```

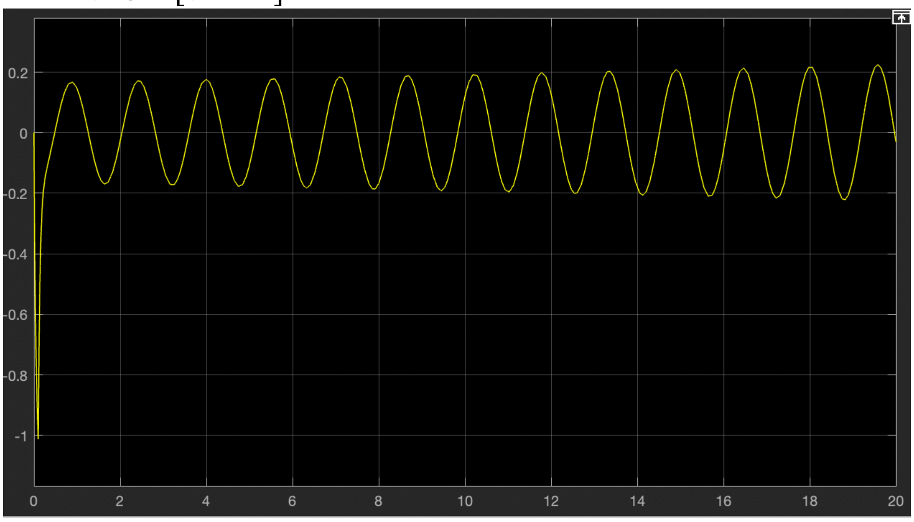
K = -40 C = [0 0 1 1]



Transfer function:

```
  -s - 1
----------
s^2 - 24.5
```

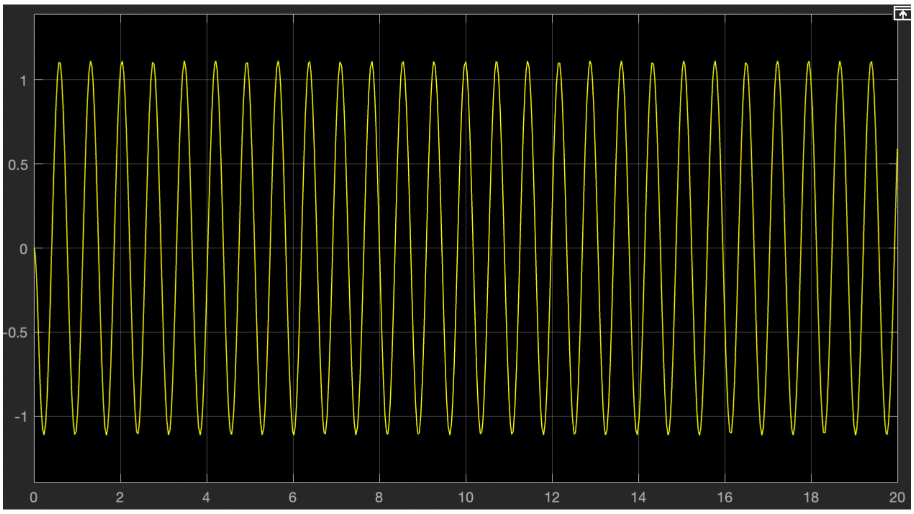
K = -40 C = [0 1 1 1]



Transfer function:

```
-0.6 s^2 - s - 9.761
--------------------
    s^3 - 24.5 s
```

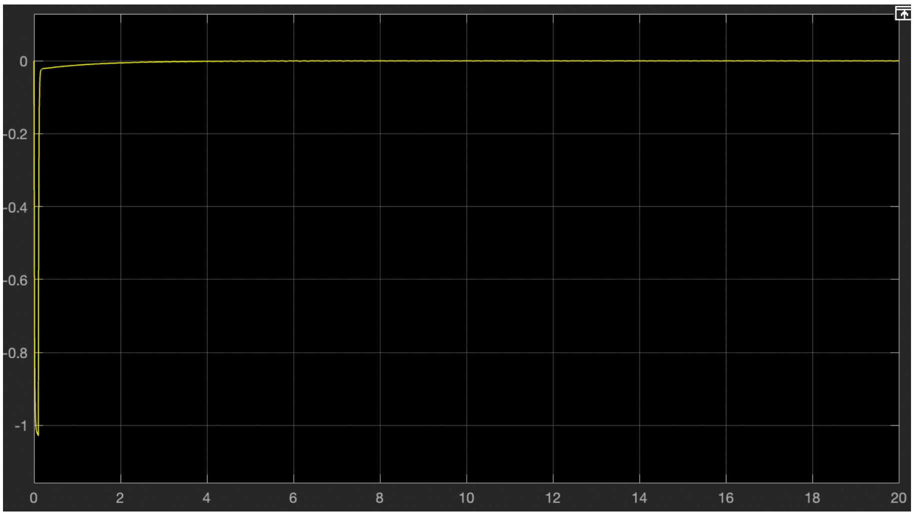
K = -100 C = [0 0 1 0]



Transfer function:

```
    -1
----------
s^2 - 24.5
```

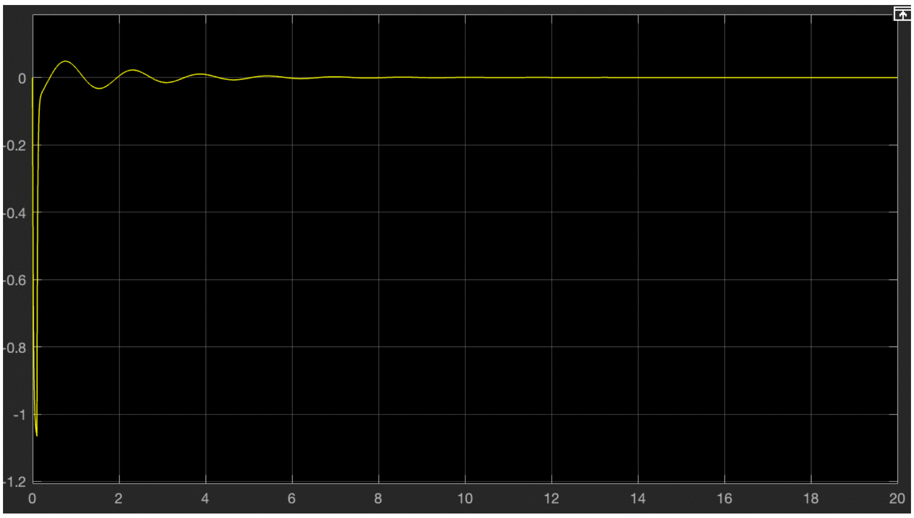
K = -100 C = [0 0 1 1]



Transfer function:

```
  -s - 1
----------
s^2 - 24.5
```

K = -100 C = [0 1 1 1]



Transfer function:

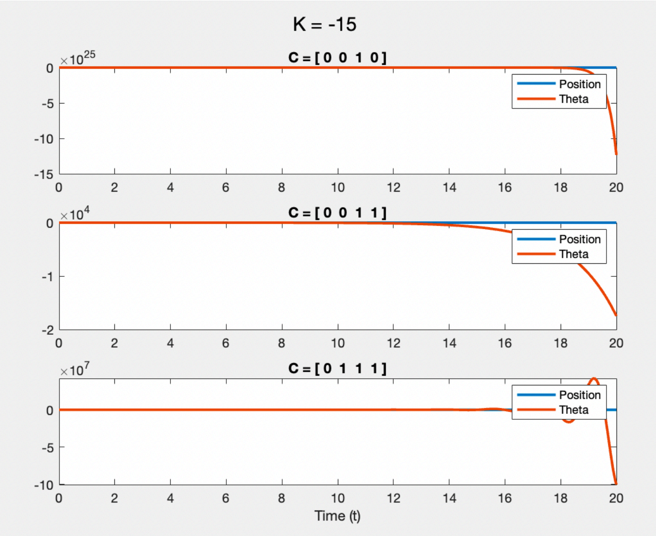
```
-0.6 s^2 - s - 9.761
--------------------
    s^3 - 24.5 s
```

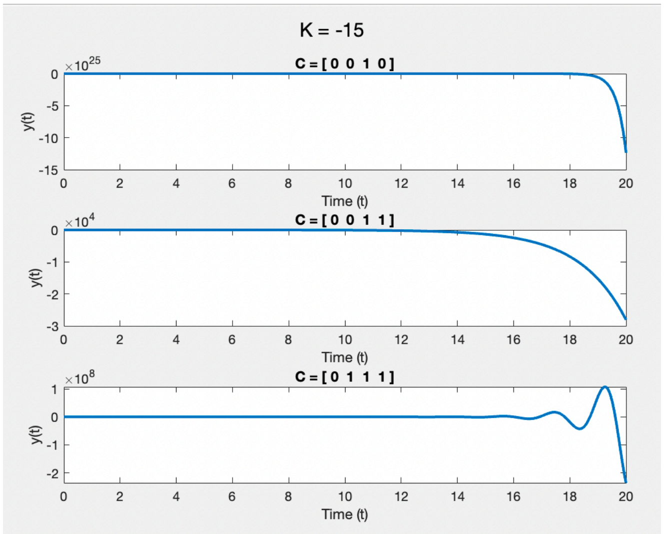
2. Simulation by ode45 with different K and C in time-domain

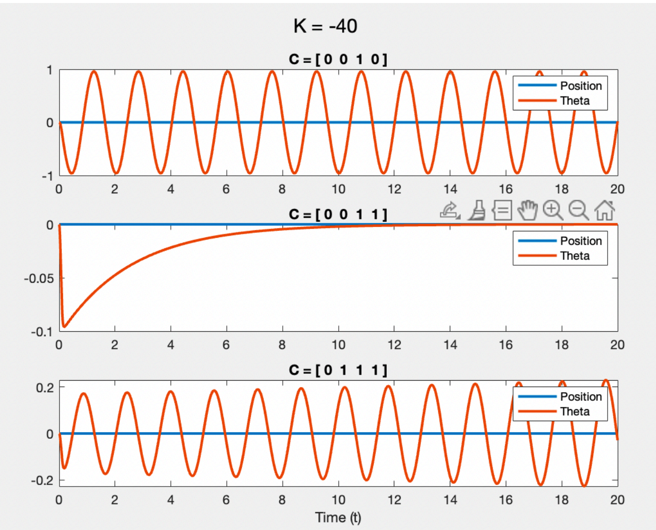
Assume that $g = 9.8\ m/s^2, l = 0.4\ m, m = 0.01\ kg$ and $M = 2.5\ kg$ (The same as part 1)

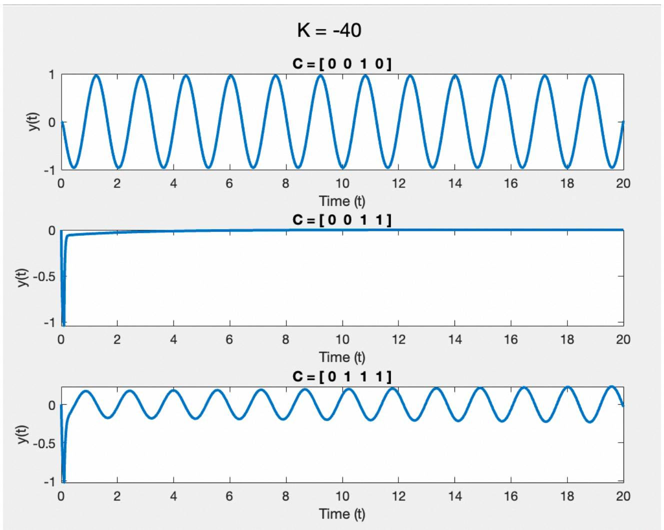
Position & θ vs. Time

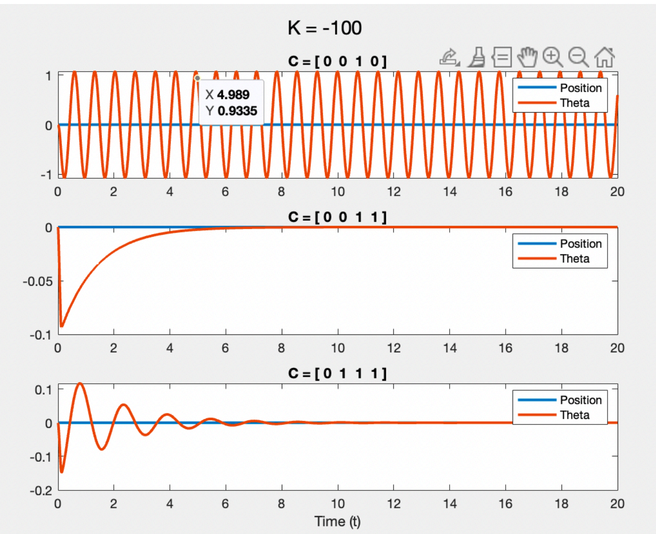
Output Response

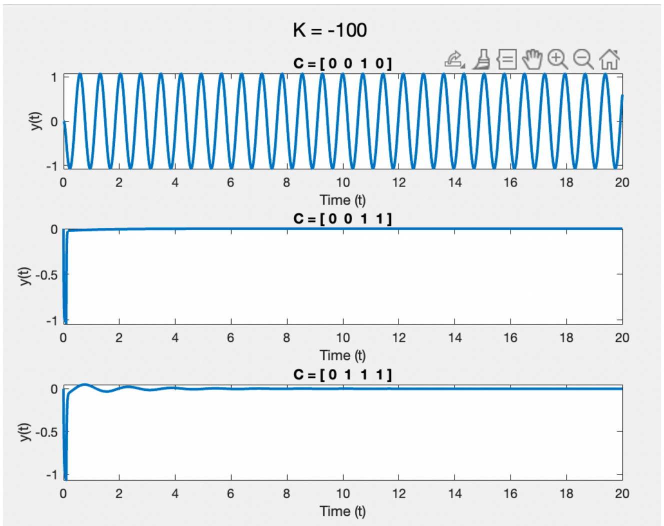












3. Analysis

The equilibrium condition of the inverted pendulum control system is $\theta(t) = 0$ and $d\theta(t)/dt = 0$. What we could do for stabilizing the system is to adjust $Gc(s)$ and observe the relations between the number of inputs and the output response since the state space model gives that

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

, where $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g/l & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ -1/(Ml) \end{bmatrix}$, $\begin{matrix} C_1 = [0 \quad 0 \quad 1 \quad 0] \\ C_2 = [0 \quad 0 \quad 1 \quad 1], \\ C_3 = [0 \quad 1 \quad 1 \quad 1] \end{matrix}$ $D = [0]$ and

assume that $g = 9.8\ m/s^2, l = 0.4\ m, m = 0.01\ kg$ and $M = 2.5\ kg$.

First, given a unity control system, we know that the bigger loop gain (to increase K) reflects the lower sensitivity, also meaning that it is faster to go to its steady state but higher overshoot will be occurred. Focusing on the $C_1 = [0 \quad 0 \quad 1 \quad 0]$, from the simulation of Simulink and ode45, we can see that The system goes to its steady state more quickly from $K = -15$ to -100, it oscillates though. This unstable situation is undesirable. Next, observing $C_2 = [0 \quad 0 \quad 1 \quad 1]$, this case is more stable for lower K and its steady state error is zero, but its percent overshoot is serious. As for $C_3 = [0 \quad 1 \quad 1 \quad 1]$, it needs bigger K to make its steady state error be zero with the comparison to C_2. From the three case above, we can observe the relation between the number of inputs to system output response.

It is obvious to see that the simulation by Simulink and ode45 in time-domain can work and basically come to the same results. This homework verifies the correctness of the inverse Laplace transform from s-domain to time-domain and gives us a clearer image about steady state error and sensitivity of the control system mentioned in class and the textbook.