Chapter 10

Design of Feedback Control Systems

10.1 Introduction

- Adjust the system parameters in order to provide the desired system response.

→If it is not sufficient, we consider the structure of the system and redesign the system in order to obtain a suitable one.

- Design of a control system

→concerned with the arrangement, or the plan, of the system structure and the selection of suitable components and parameters.

- Compensation

→alteration or adjustment of a control system in order to provide a suitable performance

- Compensator (or controller)

→ an additional component that is inserted into a control system to compensate for a deficient performance

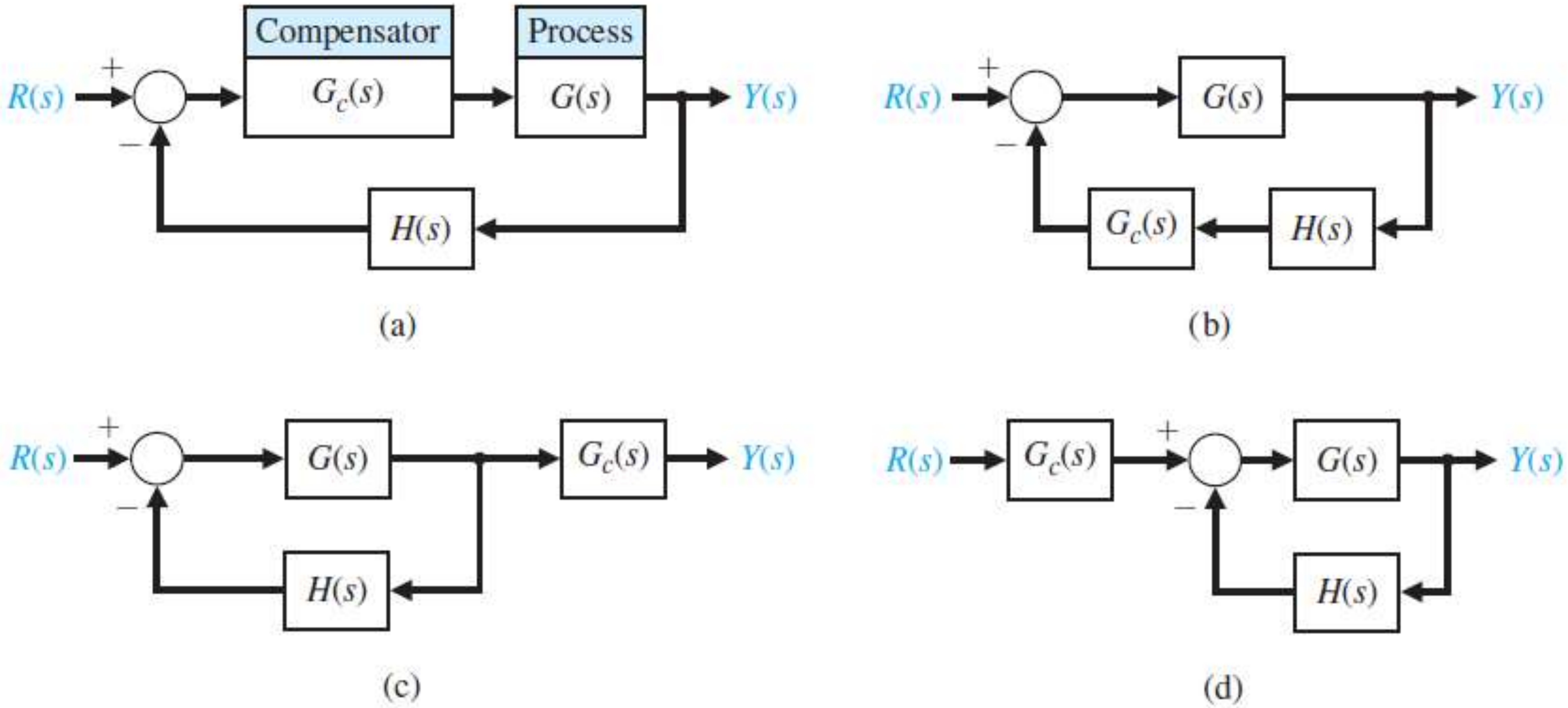


FIGURE 10.1
Types of compensation.
(a) Cascade compensation.
(b) Feedback compensation.
(c) Output, or load, compensation.
(d) Input compensation.

10.2 Approaches to System Design

- Performance of a control system can be described in terms of

→ time-domain or frequency-domain performance measures

- Time-domain approach

→ specs: peak time, P.O., settling time for a step input; steady-state errors for several test signal inputs and disturbance inputs

→ specs can be defined in terms of the desirable location of the poles and zeros of the closed-loop transfer function

→ a compensator to alter the locus of the roots as the parameter is varied

→ root locus methods

- Frequency-domain approach

→ specs: resonant peak and resonant frequency of the closed-loop system, gain margin, phase margin,

→ The design of the compensator is developed in terms of the frequency response as portrayed on the Nyquist plot, the Bode plot, or the Nichols chart.

→Bode plot is preferred because a cascade transfer function is readily accounted for by adding the frequency response of the network

- The design of a system is concerned with the alteration of the frequency response (Bode plot) or the root locus of the system in order to obtain a suitable system performance.
- When possible, one way to improve the performance of a control system is to alter the process itself.

→ the process is often unalterable or has been altered as much as possible and still results in unsatisfactory performance.

→addition of compensators becomes imperative for improving the performance of the system.

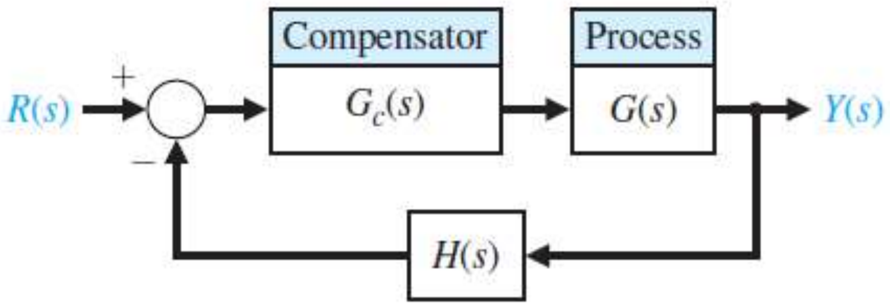
Assumptions and Outline

- The process has been improved as much as possible and that the G(s) representing the process is unalterable

→A compensator must be used:

1. Phase-lead design using Bode plot (sec. 10.4)
2. Phase-lead design using root locus (sec. 10.5)
3. Phase-lead design using root locus (sec. 10.7)
4. Phase-lag design using Bode plot (sec. 10.8)

10.3 Cascade Compensators

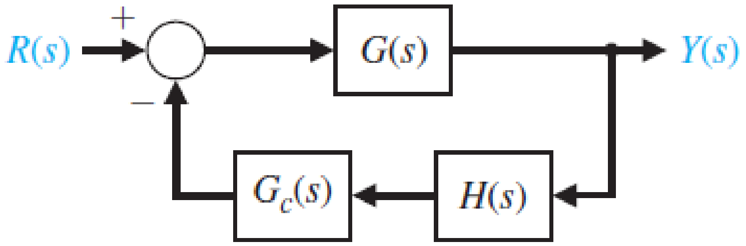


- Consider the design of a cascade compensator

→*L(s) = Gc(s)G(s)H(s)*

→Compensator Gc(s) can be chosen to alter either the shape of the root locus or the frequency response

$$G_c(s) = \frac{K\prod_{i=1}^{M}(s + z_i)}{\prod_{j=1}^{n}(s + p_j)}.$$



→judicious selection of the poles and zeros

- To illustrate the properties, we consider a first-order compensator.

→can then be extended to higher-order compensators, for example, by cascading several first-order compensators

General Design Guideline

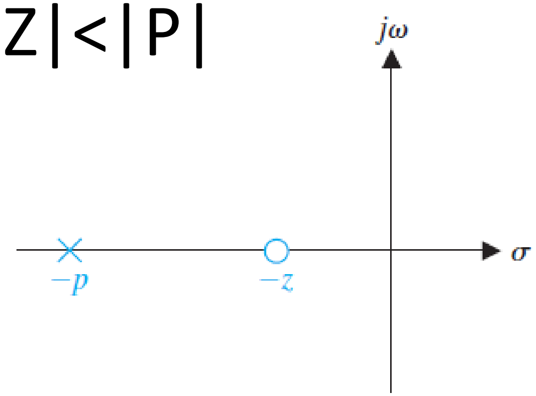
- A compensator Gc(s) is used with a process G(s) so that

1. the overall loop gain can be set to satisfy the steady-state error requirement
2. the system dynamics can be adjusted favorably without affecting the steady-state error

Phase-Lead Compensator

$$\boxed{G_c(s) = \frac{K(s+z)}{s+p}.}$$

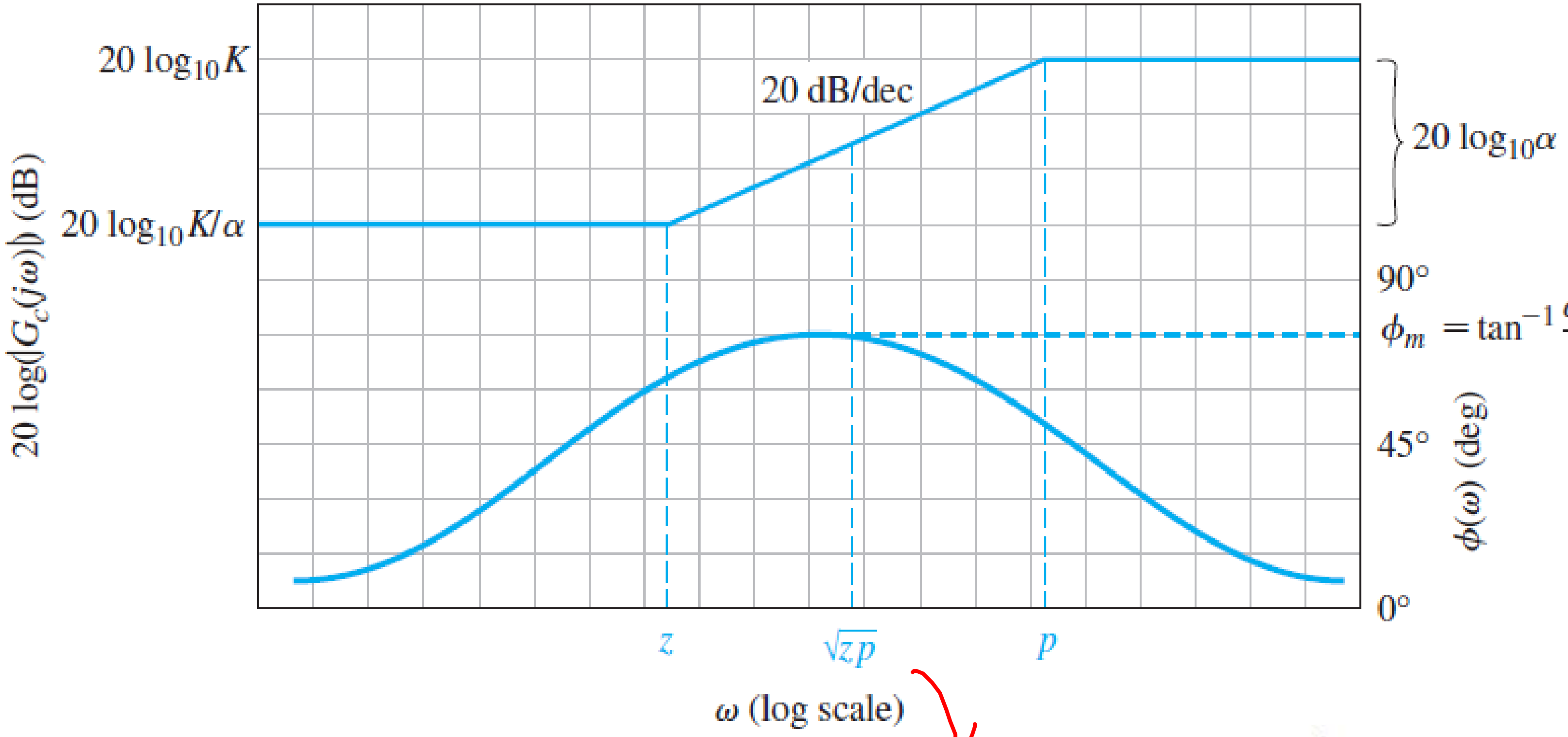
- |Z|<|P|



- |Z|<<|P|

→ A differentiator

$$G_c(s) \approx \frac{K}{p}s.$$

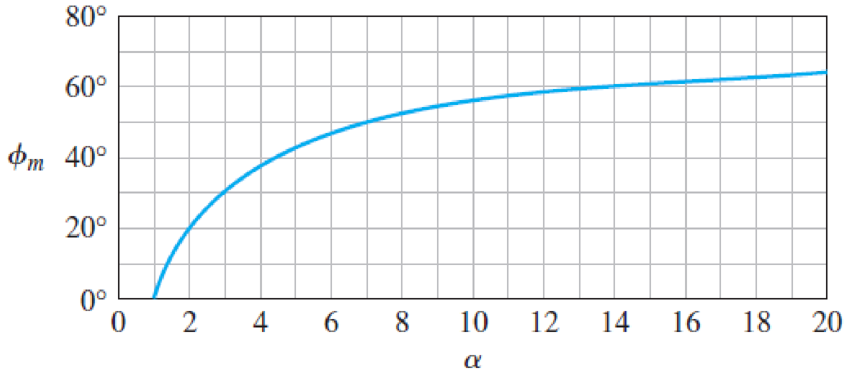


$$\phi_m = \tan^{-1}\frac{\alpha - 1}{2\sqrt{\alpha}}$$

$$\sin\phi_m = \frac{\alpha - 1}{\alpha + 1}.$$

$$\omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}.$$

$$\boxed{G_c(s) = \frac{K(1 + \alpha\tau s)}{\alpha(1 + \tau s)},}$$



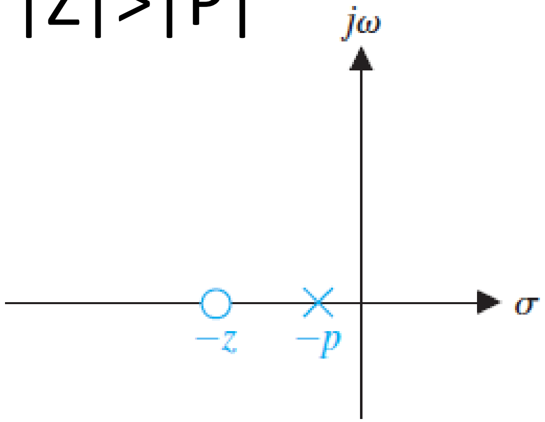
$\tau = 1/p$ and $p = \alpha z$ (or $\alpha = p/z > 1$)

Physical constraint: maximum phase angle <70°

What if a larger angle is required?

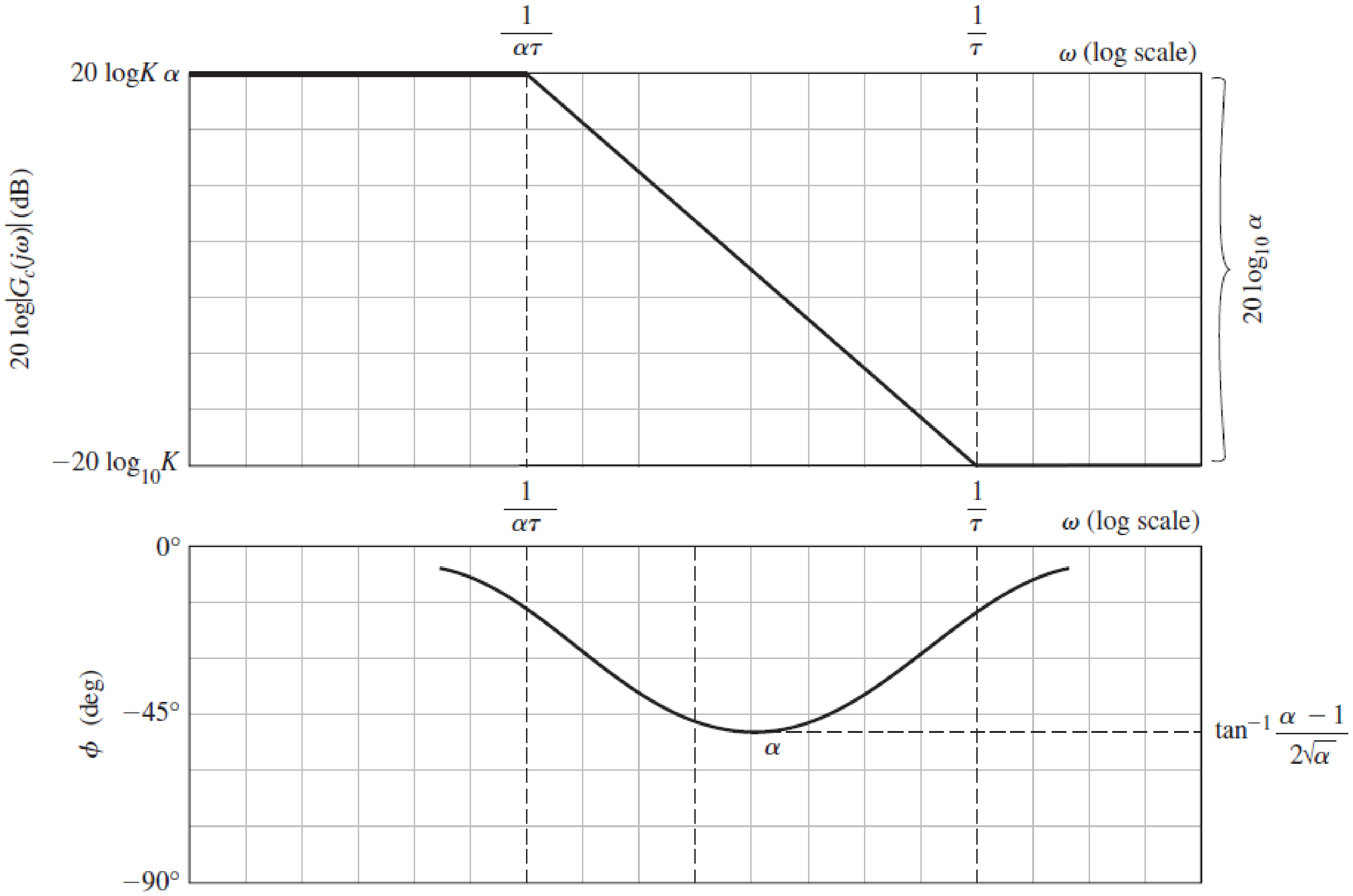
Phase-Lag Compensator

- |Z|>|P|



$$G_c(s) = K\alpha \frac{1 + \tau s}{1 + \alpha\tau s},$$

$\tau = 1/z$ and $\alpha = z/p > 1.$



Note:

α = p/z for phase-lead compensator

α = z/p for phase-lag compensator

Purposes of Phase-Lead and Phase-Lag Compensators

- Phase-lead Compensation

1) Reshape the root locus (root locus)
2) Add a phase margin (Bode diagram)

- Phase-lag Compensation (not to provide a phase-lag angle)

1) Reduce the steady-state error (or increase the error constant) without affecting the root locus (Root locus method)
2) Add a phase margin (Bode diagram) how?

Provide an attenuation!

10.4-10.8, and 10.10

Videos helpful for understanding control system designs

- What are Lead Lag Compensators? An Introduction.

https://www.youtube.com/watch?v=vXwOzDs5xKY&list=PLUMWjy5jgHK1NC52DXXrriwihVrYZKqjk&index=33

- Designing a Lead Compensator with Root Locus

https://www.youtube.com/watch?v=vXwOzDs5xKY&list=PLUMWjy5jgHK1NC52DXXrriwihVrYZKqjk&index=34

- Designing a Lead Compensator with Bode Plot

https://www.youtube.com/watch?v=vXwOzDs5xKY&list=PLUMWjy5jgHK1NC52DXXrriwihVrYZKqjk&index=35

- Designing a Lag Compensator with Root Locus

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- Designing a Lag Compensator with Bode Plot

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Cascade Compensation: Phase-Lead + RL

- Reshape the root locus

$$L(s) = K\left(\frac{s+z}{s+p}\right)G(s) = G_c(s)G(s) \qquad (1)$$

a) Place z to the left of the first two real poles of G(s).

b) Select p to the left of z.

c) Evaluate K = f(p, z) where f represents the relationship that meets the requirement on the error constant if any; otherwise, choose K to meet the specs.

$$\zeta \approx 0.01\phi_{\mathrm{pm}} \text{ for } \zeta \le 0.7 \text{ and } \phi_{\mathrm{pm}} \text{ in degrees.}$$

Cascade Compensation: Phase-Lead + Bode

- Add a phase margin (Bode diagram method)

$$L(s) = K\frac{p}{z}\left(\frac{s+z}{s+p}\right)G(s) = G_c(s)G(s) \qquad (2)$$

a) Select loop gain K to meet the steady-steady error (or error constants);

b) Select additional phase $\phi_m < 70^\circ$ to be added to the phase margin of $KG(\mathrm{j}\omega)$

c) Determine phase center ω_m satisfying
$|KG(\mathrm{j}\omega_m)| = -10\log_{10}\alpha$ dB where $\alpha = (1+\sin(\phi_m))/(1-\sin(\phi_m))$
with ϕ_m in radians

d) Evaluate $p = \omega_m\alpha^{0.5}$ and $\mathrm{z} = p/\alpha$

Cascade Compensation: Phase-Lag + RL

- Reduce the steady-state error (or increase the error constant) without affecting the root locus where the loop transfer function is defined in (1).

a) Determine the loop gain K to meet the specs, excluding the one for the error constant.
b) Determine the pole--zero ratio $\alpha = z/p$ to achieve the error constant.
c) Place z close to origin so that the shape of the root locus is only changed slightly.
d) Evaluate $p = \frac{z}{\alpha}$.

Cascade Compensation: Phase-Lag + Bode

- Add a phase margin where loop transfer function is defined in (2).

a) Select loop gain K to meet requirements of the steady-state error or error constant.

b) Select the new gain crossover frequency ω_c that results in the desired margin (allowing for a small amount of safety, e.g., 5°)

c) Determine the corresponding magnitude attenuation M dB to make ω_c the new crossover frequency.

d) Evaluate

$$z = \omega_c/10,\ \alpha = 10^{M/20},\ \text{and}\ p = z/\alpha.$$

Cascade Compensation: PI Controller + RL

- Increase the system type by 1

$$L(s) = K(\frac{s+z}{s})G(s) = G_c(s)G(s)$$

a) Place z (root locus is pulled to the right if z is located to the left and vice versa).
b) Select K to meet specs, excluding the one related to error constant.

Input Compensation: Prefilter

- Reduce the effect of the zero introduced by a phase-lead or PI compensator to the closed-loop transfer function. The effect includes P.O., rise time, and settling time.

$$G_c(s) = \frac{p}{s+p}.$$