Control System Homework 3

1.

CP3.1 Determine a state variable representation for the following transfer functions (without feedback) using the ss function:

(a) $G(s) = \dfrac{1}{s + 10}$

(b) $G(s) = \dfrac{s^2 + 5s + 3}{s^2 + 8s + 5}$

(c) $G(s) = \dfrac{s + 1}{s^3 + 3s^2 + 3s + 1}$

2.

CP3.2 Determine a transfer function representation for the following state variable models using the tf function:

(a) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 2 & 8 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = [1 \quad 0]$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 0 & 4 \\ 5 & 4 & -7 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{C} = [0 \;\; 1 \;\; 0]$

(c) $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C} = [-2 \quad 1].$

3.

CP3.4 Consider the system

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & -2 & -5 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}u(t),$$

$$y(t) = [1 \quad 0 \quad 0]\mathbf{x}(t).$$

(a) Using the tf function, determine the transfer function $Y(s)/U(s)$.

(b) Plot the response of the system to the initial condition $\mathbf{x}(0) = [0 \quad -1 \quad 1]^T$ for $0 \le t \le 10$.

(c) Compute the state transition matrix using the expm function, and determine $\mathbf{x}(t)$ at $t = 10$ for the initial condition given in part (b). Compare the result with the system response obtained in part (b).

4.

CP3.7 Consider the following system:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(t)$$

$$y(t) = [1 \quad 0]\mathbf{x}(t)$$

with

$$\mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Using the lsim function obtain and plot the system response (for $x_1(t)$ and $x_2(t)$) when $u(t) = 0$.

5.

CP3.8 Consider the state variable model with parameter K given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -K & -2 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t),$$

$$y(t) = [1 \quad 0 \quad 0]\mathbf{x}(t).$$

Plot the characteristic values of the system as a function of K in the range $0 \leq K \leq 100$. Determine that range of K for which all the characteristic values lie in the left half-plane.