

MATLAB

Simulink - PID

Problem 1

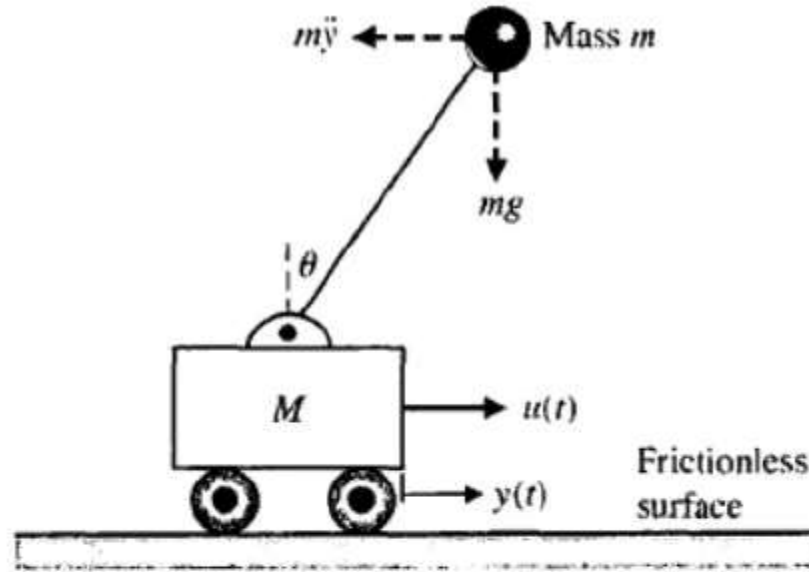


Figure 1 A cart and an inverted pendulum.

Force (horizontal):

$$(M + m)\ddot{y}(t) = u(t) - (mL\ddot{\theta}(t) \cos \theta(t) - mL\dot{\theta}(t)^2 \sin \theta(t))$$

Torque:

$$mL^2\ddot{\theta}(t) = mgL \sin \theta(t) - mL\ddot{y}(t) \cos \theta(t)$$

Assume that $M=100\text{Kg}$, $m=10\text{Kg}$, $L=1\text{m}$, $g=9.81\text{m/s}^2$ and $G_c(s) = K_P + \frac{K_I}{s} + sK_D$,

*This is a guideline for Problem 1.

*The equations are not exactly the same, but same concepts

1) Use ode45 to simulate the output response in time-domain.

$$\ddot{y} = -\frac{mg}{M}\theta + \frac{1}{M}u$$

$$\ddot{\theta} = \frac{(M+m)g}{ML}\theta - \frac{1}{ML}u$$

$$x_1 = y, x_2 = \dot{y}, x_3 = \int \theta dt, x_4 = \theta, x_5 = \dot{\theta}$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{mg}{M}x_4 + \frac{1}{M}u$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}u$$

$$\text{PID control } U(s) = G_c(s)(R(s) - \theta(s))$$

$$u(t) = -K_P\theta(t) - K_I \int \theta(t)dt - K_D \frac{d\theta(t)}{dt} = -K_P x_4 - K_I x_3 - K_D x_5$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{mg}{M}x_4 + \frac{1}{M}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = x_5$$

$$\dot{x}_5 = \frac{(M+m)g}{ML}x_4 - \frac{1}{ML}(-K_P x_4 - K_I x_3 - K_D x_5)$$

$$x_{10} = 0, x_{20} = 0, x_{30} = 0, x_{40} = 1, x_{50} = 0$$

q(s) can be obtained by (sl-A).

$$q(s) = MLs^3 - K_D s^2 - [(M+m)g + K_P]s - K_I$$

Problem 1 - Reference Code

```
clear all
clc
% Parameters:
m=10;
M=100;
L=1;
g=9.81;

% PID tuning
KP = ???
KI = ???
KD = ???

% Define reference signal
r = 0;
fun = @(t,x) [x(2); -m*g*x(4)/M; x(4); x(5); (M+m)*g*x(4)/(M*L)] + (-KP*x(4)-KI*x(3)-KD*x(5))*[0; 1/M; 0; 0; -1/(M*L)];
T = linspace(0, 15, 1e3);
x0 = [0; 0; 0; 1; 0];
[t, Y] = ode45(@(t,x) fun(t,x), T, x0);

figure
%%%%%%%%plot Yourself%%%%%%%%%%
xlabel('Time [s]')
ylabel('Angular rotation [rad]')

syms x;
digits(6)
eqn = M*L*x^3-KD*x^2-((M+m)*g+KP)*x-KI == 0;
s = solve(eqn,x);
vpa(s)
```

Problem 2

2) Use Simulink to simulate the output response in s-domain.

Assume that $\theta(t) \rightarrow 0^\circ \Rightarrow \sin \theta(t) \approx \theta(t), \cos \theta(t) \approx 1, \dot{\theta}(t) \approx 0,$

$$(M + m)\ddot{y}(t) + mL\ddot{\theta}(t) = u(t)$$

$$\ddot{y}(t) + L\ddot{\theta}(t) - g\theta(t) = 0$$

$$G(s) = \frac{-1/ML}{s^2 - \frac{(M+m)g}{ML}}$$

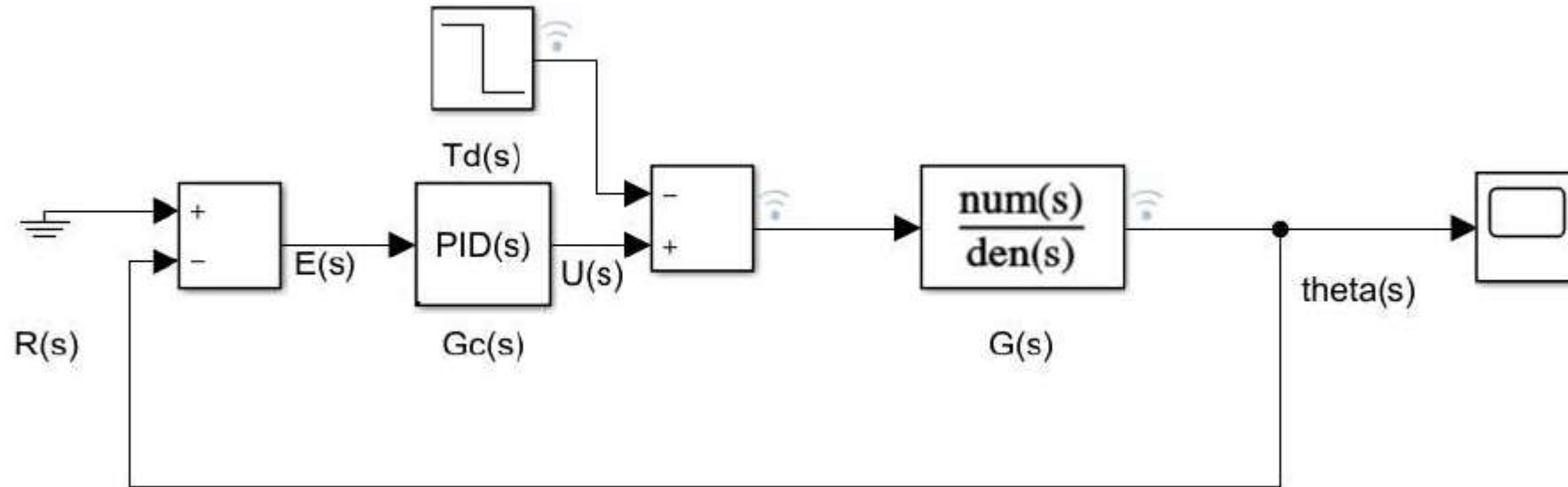
Applying PID control to the inverted pendulum.

Assume that $T_d(s) = 1(T_d(t) = \delta(t))$

Create Block Diagram

You will need the following function blocks:

- > PID Controller
- > Add
- > Ground
- > Step
- > Transfer Fcn
- > Scope (see output result)



Set parameters

Block Parameters: PID Controller

PID 1dof (mask) (link)

This block implements continuous- and discrete-time PID control algorithms and includes advanced features such as anti-windup, external reset, and signal tracking. You can tune the PID gains automatically using the 'Tune...' button (requires Simulink Control Design).

Controller: Form:

Time domain:

☒ Continuous-time

☐ Discrete-time

Discrete-time settings

Sample time (-1 for inherited):

Compensator formula

$$P + I \frac{1}{s} + D \frac{N}{1 + N \frac{1}{s}}$$

Main Initialization Output Saturation Data Types State Attributes

Controller parameters

Source:

Proportional (P):

Integral (I):

Derivative (D):

☒ Use filtered derivative

Filter coefficient (N):

Automated tuning

Select tuning method:

☒ Enable zero-crossing detection

Block Parameters: Transfer Fcn

Transfer Fcn

The numerator coefficient can be a vector or matrix expression. The denominator coefficient must be a vector. The output width equals the number of rows in the numerator coefficient. You should specify the coefficients in descending order of powers of s.

Parameters

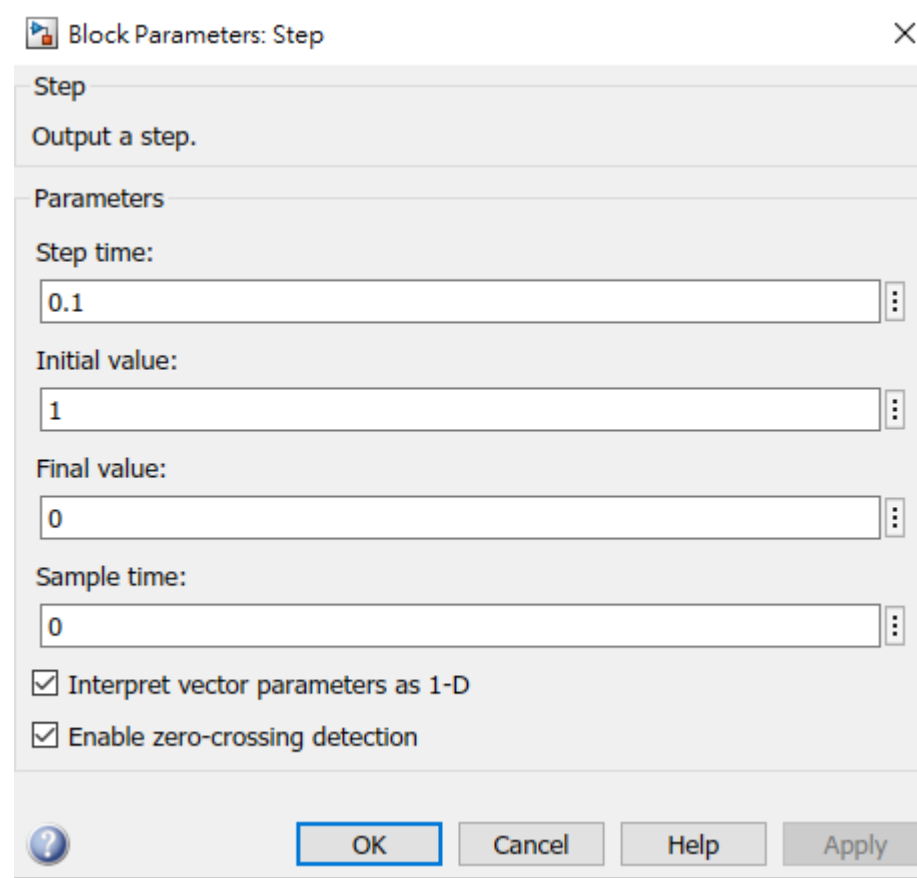
Numerator coefficients:

Denominator coefficients:

Absolute tolerance:

State Name: (e.g., 'position')

Set Parameters (Cont.)



A screenshot of the 'Block Parameters: Step' dialog box. The dialog has a title bar with a question mark icon, the text 'Block Parameters: Step', and a close button. The main area is divided into sections: 'Step' with the description 'Output a step.', 'Parameters' with four input fields, and two checked checkboxes at the bottom. The input fields are for 'Step time' (0.1), 'Initial value' (1), 'Final value' (0), and 'Sample time' (0). Each field has a three-dot menu icon to its right. The checkboxes are 'Interpret vector parameters as 1-D' and 'Enable zero-crossing detection'. At the bottom are buttons for '?', 'OK', 'Cancel', 'Help', and 'Apply'.

Block Parameters: Step

Step

Output a step.

Parameters

Step time:

0.1

Initial value:

1

Final value:

0

Sample time:

0

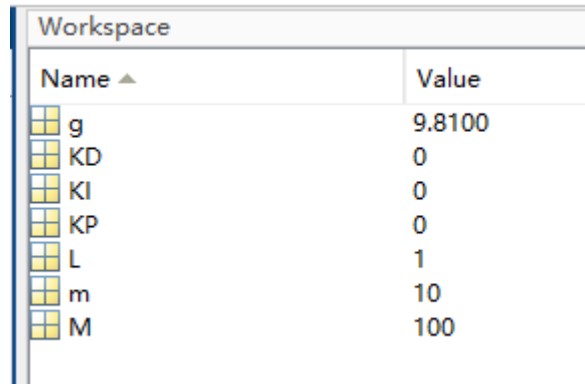
☒ Interpret vector parameters as 1-D

☒ Enable zero-crossing detection

? OK Cancel Help Apply

How to run Simulink with matlab parameter

- In parameter file, choose the parameter K_D , K_I , and K_P
- Next, Run the parameter.m file, in order to save the data in workspace



The screenshot shows the MATLAB Workspace window with a table of variables and their values. The variables are g, KD, KI, KP, L, m, and M. The values are 9.8100, 0, 0, 0, 1, 10, and 100 respectively. Each variable name has a small yellow icon to its left.

Name ▲	Value
g	9.8100
KD	0
KI	0
KP	0
L	1
m	10
M	100

- After run the parameter.m, you can press run in Simulink