METHODS OF MATHEMATICAL PHYSICS SEMINAR

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1. Description of Mathematical Physics and the Seminar

Mathematical physics is an interdisciplinary science which, on the basis of the fundamental (mostly phenomenological) laws of physics, uses mathematical methods to study process evolving in material media. Its purpose is to formulate equations describing a process to within a reasonable degree of idealization (ie., disregarding details that are not essential for its qualitative and quantitative essences), to develop methods for solution of the resulting problem, and to analyze the qualitative and quantitative properties of the solutions. In this latter respect mathematical physics borders on numerical analysis and experimental natural science.

In this seminar we shall restrict our attention to phenomena of the "macro" world—more precisely, to processes evolving in continuous media.

This seminar would probably consist of 8 sessions, telling the story of some naive analytical methods of mathematical physics. The main textbook we use, *Methods of Mathematical Physics* by Qiao Gu, focuses on analytical tools (ie., series, integral transforms, eigenfunctions, special functions, etc.). But actually these methods are far below from modern mathematical physics. Here goes an typical example.

In quantum mechanics, an important case of heat equation is Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi(x,t) := \hat{H}\Psi,$$

which describes the motion of particles in probability aspect. The method solving the general heat equation shows little about the specific properties of the solution since it's an integral form where it's nearly impossible to get rid of the integral sign by simple calculation. In other words, the integral seems hiding the detailed picture of the solution away from us. Then physicists apply separation of variables and derive satisfactory solutions for some simple cases. When spherical coordinates and series are introduced, physicists derive a reduced equation only including angular terms. The solutions are called spherical harmonics. But these harmonics are not entirely what the experiment shows, which says there ought to be more solutions! A more structural method, algebra, yields these extra solutions and predicts the intrinsic physical observable for particles - quantum angular momentum, also known as "spins". We are now able to calculate the periodic table using these spins and construct complicated many-particle system. This system is feasible but complicated. Physicists invent a kind of more “entangled” representation of the system equivalent to the previous one. The two representation are connected by Clebsch-Gordan coefficients. For two-particle system, the coefficients form matrices which are sometimes orthogonal. But when the number of particles increases, there would be high-dimensional arrays. Its operations have to be redefined but all the computations are still feasible by complicated group theory.

This example shows that mathematical methods for physics are not simple and easy as we might assume. But we don’t need to know so much at first since other advanced methods require deep mathematics. Another example is for minimal energy problem. Think about heating a homogenous cube wire! Minutes later, its shape would change into one that keep the energy of the system at the lowest level. The energy-minimized shape however is asymmetric. This problem gets its mathematical solution recently. Minkowski flow and strong algebraic topology are used in the proof.

To be noticed, our attention in this semester will be simply restricted on partial differential equations with applied solutions and two special functions.

2. Syllabus

The main reference of this seminar is *Methods of Mathematical Physics* by Qiao Gu. Several adjustments will be taken depending on the evaluation of the seminar.

Table 1: Syllabus

Lec #	**Topics**	**Chapters**
1	ODE Review Vector Differential Operator Laplace Operator	Chap 1
	Fourier Integral Convolution	Chap 2
	Fourier Transformation Dirac Delta Function	Chap 3
	Introduction to Laplace’s Eq., Heat Eq., Wave Eq. Physical Interpretation Transport Eq. & its Solution	Chap 5
2	Laplace’s Eq. & Poisson Eq. Homogeneous Eq. & Nonhomogeneous Eq. Initial & Boundary Value Problems Green Function & Its Integral	Chap 12

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Table 1 – Continued from previous page

Lec #	**Topics**	**Chapters**
	Geometric Restriction for the Boundary	
3 & 4	Heat Eq. & Wave Eq.	Chap 10
5 & 6	Separation of Variables	Chap 6 & 7
	Methods of Eigenfunction	Chap 8 & 9
	Laplace Transformation Integral Transformation	Chap 4 Chap 11
7 & 8	Bessel Function	Chap 13
	Legendre Function	Chap 14

3. Recommended Readings

Table 2: Recommended Readings

Level	**Title**	**Author**	**Feature**
Basic	Methods of Mathematical Physics	Qiao Gu	Analysis
	Partial Differential Equations	Lawrence Evans	Qualitative Theory
	Partial Differential Equations: an Introduction	Walter Strauss	Low Dimension, Specific Cases
Advanced	Methods of Mathematical Physics I & II	R. Courant & D. Hilbert	Classical, Complete, Hard
	Mathematical Physics: a Modern Introduction to its Foundations	Sadri Hassani	Algebra, Def-Prop-Thm-Proof
	Mathematical for Physics: a Guided Tour for Graduate Students	Micheal Stone & Paul Goldbart	Modern, Topology, Physics

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