A NEW COUNTEREXAMPLE TO NGUYEN'S CONJECTURE ON SURFACE FIBRATION[†]

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ABSTRACT. Suppose $f : S \to \mathbb{P}^1$ is a fibration of genus g with 3 singular fibers, and two of the fibers are semistable. In 1998, Nguyen conjectured in [Ng] that such kind of fibration does not exist for $g \geq 2$. But in 2013, Cheng Gong, Xin Lu, and Sheng-Li Tan found a counterexample to Nguyen's conjecture for $g = 2$ in [GLT]. Note that such kind of fibration shows strong arithmetic properties, and as such the counterexamples are important, but rare in fact. In this paper, a new counterexample to Nguyen's conjecture for $g = 2$ is constructed.

KEYWORDS. Algebraic surface; Fibration; Singular fiber.

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