Introduction

Similar to greedy, dynamic programming are used when problems require minimisation and maximisation of an outcome. Hence dynamic programming algorithms are also **optimisation** problem (Dynamic programming shall be reference as DyP from now on, giggidy).

However, these problems have one unique difference to greedy. Greedy processes are done with no knowledge of previous decisions, but DyP is done with knowledge of *every* previous decisions. What does this mean you may ask. This will be made clear in the second example.

This document will explain the basics and the considerations of both these topics using question examples and a step-by-step worked-out thought process.

Dynamic Programming

What is it

In dynamic programming, unlike greedy, the best decisions are based on the previous decisions which each could have their own different outcome depending on what is decided. That sentence was very long and confusing, but lets explain with some examples.

Dynamic Programming: Weird Money and Change

Remember in greedy method, we had the money example:

```
One need to is given $16 in the least unit of currency. The units of currency
available being $10, $5 and $1. The greedy algorithm is simple: we start from
the highest unit of currency at each point of the transaction until you
completely pay off the $16.
```

But now, let's change up this problem and you will see why greedy is not the be all and end all.

```
Suppose that the problem is the same, $16 to give with the least units of
currency. However, this time, because of bad government planning, the available
currency is $10, $8, and $1.
```

Have you seen the problem yet?

If you were to apply greedy, you would give 1 x 10$ and 6 x $1, totaling 7 units of currency. But instead you could have given 2 x $8 currency which is clearly the least number of currency.

Oh no, you might be thinking: "Dynamic programming looks like its gonna be a pain in the ass, why did I do this course?". However, its actually not too tricky. All you have to do is 3 steps

1. Decide on the subproblem
2. Determine the base case(s)
3. Define the recursive case(s)

Yep, these are recursion problems.

Dynamic Programming: The Hungry Frog

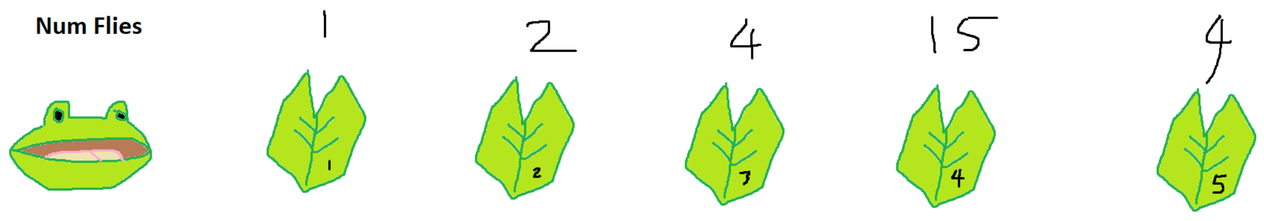
Let me demonstrate with simple example with my demonstration partner, *the frog*

```
Suppose you have a frog on a lilypad. In front of the frog, are a line of
lilypads, each with a number of flies avaialble to consume on them. The frog
want to consume the most number of flies of course, but can only jump forward 2
or 3 lilypads (I.E Lilypad 1 -> Lilypad 3 or Lilypad 4). There are N number of
lilypads. In what order, should the frog jump such that it consumes the most
flies?
```

```
Assumptions:
- You cannot jump backwards
- Frog starts at Lilypad 0, with 0 flies consumed
```

You might think, "Looks like greedy to me though." So let's test that hypthesis.

Suppose the number of flies are arranged in this order (Frog is on Lilypad 0):



Our greedy method is: "grab the most number of flies each time". So this would mean, between the two Lilypads we can pick, Lilypad 2 or Lilypad 3. Since Lilypad 3 has 4 flies, we go for that. Now we are on Lilypad 3, the next is Lilypad 5 or 6, and since Lilypad 5 has 4 flies compared to Lilypad 6's 2 flies, we jump to Lilypad 5. Total we consumed 8 flies.

Amazing, but you might have noticed you your frog just missed out on a fat buffet of 15 flies on Lilypad 4. But since you chose Lilypad 3, you could not jump to Lilypad 4 can eat 15 flies. You needed to make an ungreedy decision in the beginning to reap rewards later.

Now, before you go check the census date to drop the course, let's think for a moment how to solve this problem. How can we ensure that for each Lilypad we are making the most optimal solution for future. Let's look at our 3 steps:

Step 1: The subproblem

So instead of saying what is the optimal amount of flies that can be consumed overall, lets just ask a smaller question:

```
What is the optimal amount of flies I can eat up to each lilypad?
```

Let's think about it:

- Lilypad 1: We can ignore Lilypad 1 since we can't get to it
- Lilypad 2: We can get 2 flies
- Lilypad 3: We can get 4 flies
- Lilypad 4: We can consume 15 + 2 = 17 flies suppose that we jump from Lilypad 2
- Lilypad 5: We can consume either 6 flies if we jumped from Lilypad 2 or 8 flies if we jumped from Lilypad 3, so the optimal amount of flies is 8.

But how do we figure out all the optimal case at each lilypad? How do we check? Exactly: Recursion

Step 2: The base case

Right now the idea of recursion still probably haven't clicked yet, so bare with me, I'm getting there. Trust me.

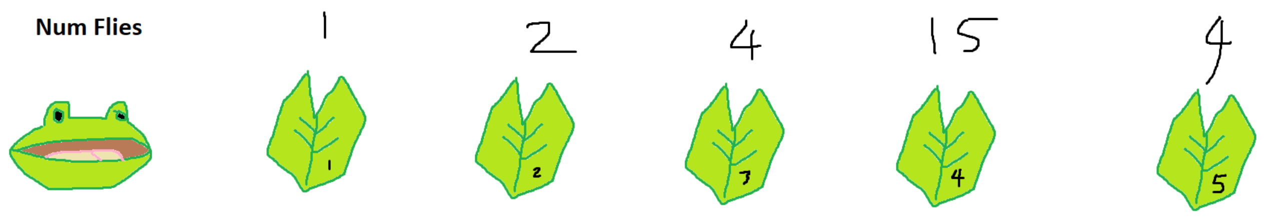
With any recursion we need a base case, since the frog is always starting on the Lilypad 0, where there are 0 flies. The base case is:

```
Lilypad(0) = 0
```

Step 3: The recursive case

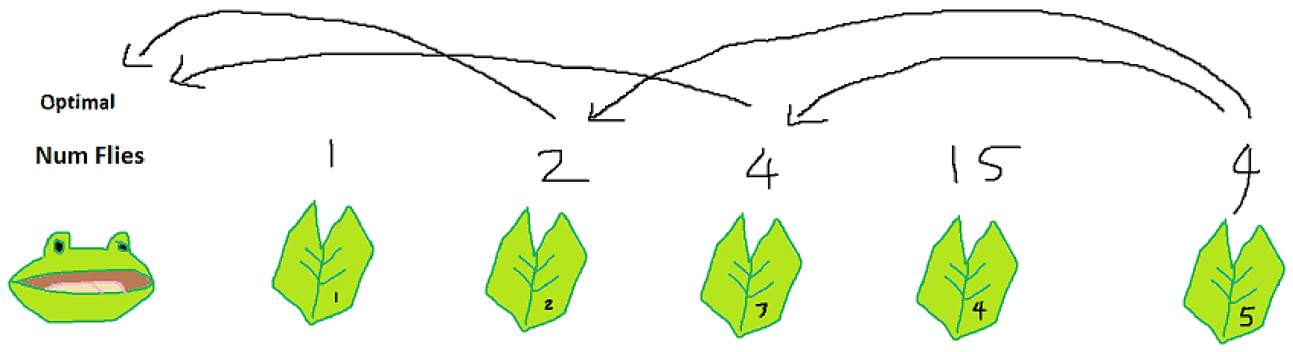
Here, you will finally see the build up with been edging towards.

Let's ask a simple question, what is the optimal number of flies we can eat at Lilypad 5?

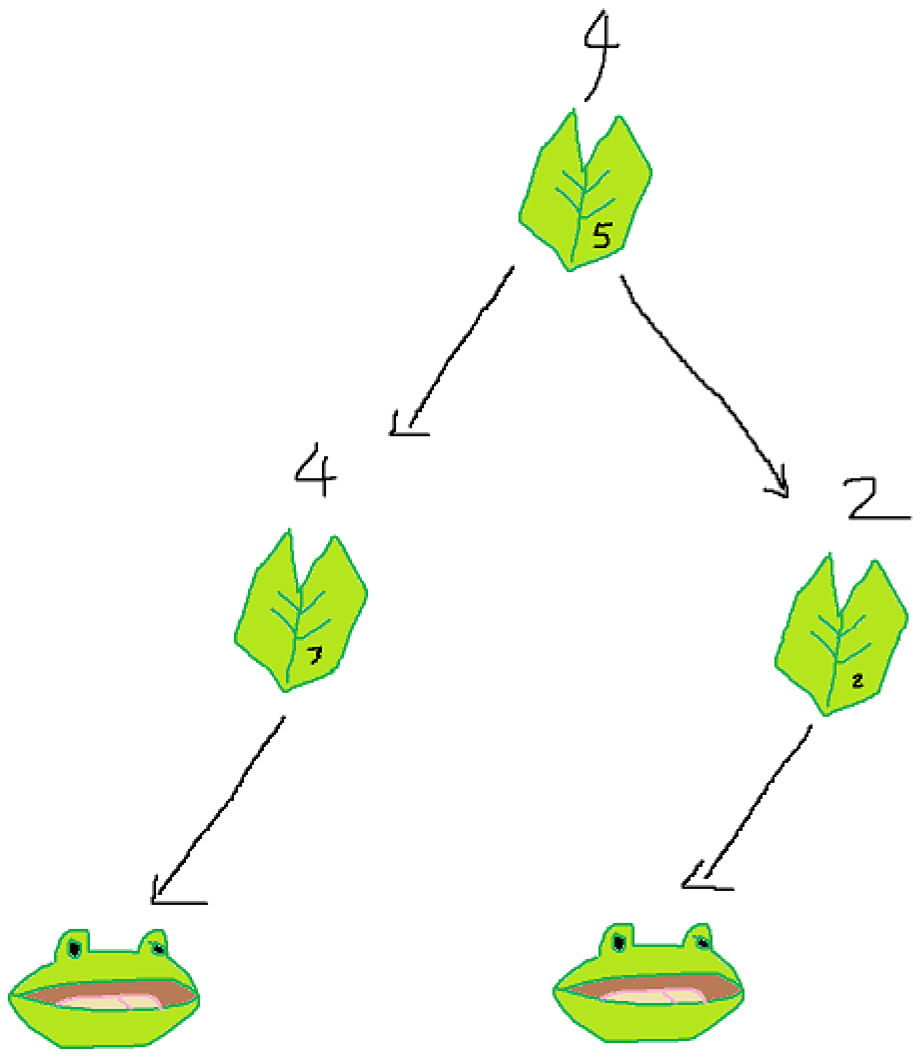


Well, it depends, we could have jumped from either Lilypad 2 or Lilypad 3. Okay, but which of these two is the better option, well obviously Lilypad 3, since it has more flies.

To visualise the previous statement:



or you could do it like this



Now do you see what I mean?

You can find the optimal amount of flies by generating using the previous Lilypad optimal flies. Obviously, at this few lilypads, its a bit redundant, but if are dealing with a larger amount of lilypad, it becomes more apparent.

If you do the same recursive tree structure for every Lilypad, then you will find the most optimal amount of flies you can eat up to each one!

Conclusion

So in words, your solution is:

```
Find the most optimal number of flies up to lilypad 0, by adding the most
optimal of the two Lilypad before which can jump from. Store the recurrsive call
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back order of lilypad index to find the route to lead to the most optimal
lilypad
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In maths land, we will say:

$$\mathrm{opt}(i) = f_i + \max\{\mathrm{opt}(i-2), \mathrm{opt}(i-3)\}$$

Where i is the lilypad index

Notes:

- There can always be more base cases, depending or your circumstance
- The recursive cases are the hardest to reason. My advice: When in doubt, draw a tree.
- There can be a large number of recursive cases, it won't always so neatly become a binary tree.