Simulation

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LIF model

$$
\begin{aligned}
C\dot{V_i} &= -g_{L,i}(V_i - V_L) + I_{syn,i} + I_{ext,i} \quad V_i < V_{th,i} \\
V_i &= V_{rest} \qquad t \in [t_k^i, t_k^i + T_{ref}]
\end{aligned} \tag{1}
$$

where:

- V_i: Membrane potential
- C: Capacitor
- $g_{L,i}$: Leaky conductance
- V_L: Leaky reversal potential
- $I_{syn,i}$: Synchronizing current
- $I_{ext,i}$: Input current
- $V_{th,i}$: Threshold voltage
- V_{rest}: Rest potential
- t: Time Stamp
- t_k^i: The kth spiking time of neuron $i0$
- T_{ref}: Refractory period

Synaptic Model

Considering AMPA, NAMPA, GABAa, GABAb as transmitters,

$$
\begin{aligned}
I_{syn,i} &= I_{AMPA,i} + I_{NAMPA,i} + I_{GABAa,i} + I_{GABAb,i} \\
I_{u,i} &= -g_{u,i}(V_i - V_i^u)J_{u,i} \\
\dot{J_{u,i}} &= -\frac{J_{u,i}}{\tau_i^u} + \sum_{k,j} w_{ij}\delta(t - t_k^i)
\end{aligned} \tag{2}
$$

where

- $u = AMPA, NAMPA, GABAa, GABAb$
- $g_{u,i}$: Transmitter conductance
- V_i^u: Transmitter reversal potential
- $J_{u,i}$: Transmitter concentration
- w_{ij}: Weights of connectivity

Autoregressive model

Calcium signal model is described as

$$
\begin{aligned}
c_{t,i} &= \sum_{q=1}^{p} \lambda_i c_{t-q,i} + s_{t,i} \\
y_{t,i} &= \alpha(c_{t,i} + b) + v_{t,i} \quad v_{t,i} \sim \mathcal{N}(0, \sigma_i^2)
\end{aligned} \tag{3}
$$

where:

- $c_{t,i}$: Calcium concentration
- $s_{t,i}$: Spike number
- $y_{t,i}$: Observed calcium
- α: Amplitude
- b: Baseline
- $v_{t,i}$: White noise

Ensemble Kalman Filter (EnKF)

Model

$$\begin{aligned} x_k &= f(x_{k-1}, u_k) + w_k \\ y_k &= h(x_k) + v_k \end{aligned} \tag{4}$$

where:

- $Q = E(w_k^T w_k)$
- $R = E(v_k^T v_k)$
- $P_{k|k} = cov(x_k - \hat{x}_{k|k})$
- $P_{k|k-1} = cov(x_k - \hat{x}_{k|k-1})$

Sampling

Sampling $(x^1, x^2, \ldots, x^N) = MultivariateNormal(x_0, P, N)$

where,

- P is the initial error covariance matrix
- N is the sampling number

Predict

prior state estimation

$$\begin{aligned} x^i_{k|k-1} &= f(x^i_{k-1|k-1}, u_k) \\ \hat{x}_{k|k-1} &= \sum_{i=1}^{N} x^i_{k|k-1} \end{aligned} \tag{5-1}$$

prior covariance matrix estimation

error

$$\begin{aligned} e_{k|k-1} &= x_k - \hat{x}_{k|k-1} \\ &= f(x_{k-1}, u_k) + w_k - f(\hat{x}_{k-1|k-1}, u_k) \end{aligned} \tag{5-2}$$

$$
\begin{aligned}
P_{k|k-1} &= cov(e_{k|k-1}) \\
&= cov(f(x_{k-1}, u_k) - f(\hat{x}_{k-1|k-1}, u_k) + w_k) \\
&= cov(f(x_{k-1}, u_k) - f(\hat{x}_{k-1|k-1}, u_k)) + Q \\
&\approx \frac{1}{N-1} \sum_{i=1}^{N} cov(\hat{x}^i_{k|k-1} - \hat{x}_{k|k-1}) + Q
\end{aligned}
\tag{5-3}
$$

Update

Prior observed value

$$
\begin{aligned}
\hat{z}^i_{k|k-1} &= h(\hat{x}^i_{k|k-1}) \\
\hat{z}_{k|k-1} &= \frac{1}{N} \sum_{i=1}^{N} \hat{z}^i_{k|k-1}
\end{aligned}
\tag{6-1}
$$

System Uncertainty

Residual

$$
\begin{aligned}
y_{k|k-1} &= z_k - \hat{z}_{k|k-1} \\
&= h(x_{k-1}) + v_k - h(\hat{x}_{k|k-1})
\end{aligned}
\tag{6-2}
$$

$$
\begin{aligned}
S_k &= cov(y_{k|k-1}) \\
&= cov(h(x_{k-1}) + v_k - h(\hat{x}_{k|k-1})) \\
&= cov(h(x_{k-1}) - h(\hat{x}_{k|k-1})) + R \\
&\approx \frac{1}{N-1} \sum_{i=1}^{N} cov(\hat{z}^i_{k|k-1} - \hat{z}_{k|k-1}) + R
\end{aligned}
\tag{6-3}
$$

Cross covariance matrix

$$
\begin{aligned}
M_k &= cov(e_{k|k-1}, y_{k|k-1}) \\
&\approx \frac{1}{N-1} \sum_{i=1}^{N} cov(\hat{x}^i_{k|k-1} - \hat{x}_{k|k-1}, \hat{z}^i_{k|k-1} - \hat{z}_{k|k-1})
\end{aligned}
\tag{6-4}
$$

Kalman gain

$$
K_k = M_k S_k^{-1}
\tag{6-5}
$$

Posterior estimation of x

Sample $(z^1, z^2, \ldots, z^N) = MultivariateNormal(z, R, N)$

$$
\begin{aligned}
\hat{x}^i_{k|k} &= \hat{x}^i_{k|k-1} + K_k(z^i - \hat{z}^i_{k|k-1}) \\
\hat{x}_{k|k} &= \frac{1}{N} \sum_{i=1}^{N} \hat{x}^i_{k|k}
\end{aligned}
\tag{6-6}
$$

Posterior estimation of P

$$
\begin{aligned}
P_{k|k} &= cov(x_k - \hat{x}_{k|k}) \\
&= cov(x_k - \hat{x}_{k|k-1} - K_k(z^i - \hat{z}^i_{k|k-1})) \\
&\approx P_{k|k-1} - K_k S_k K_k
\end{aligned}
\tag{6-7}
$$