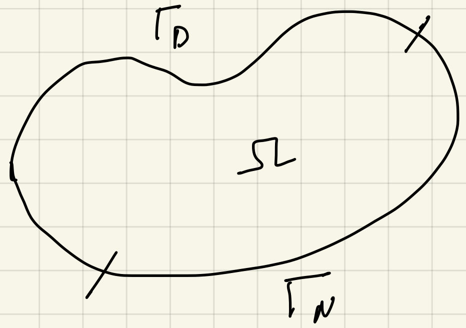


Strong form

$$\begin{cases} -\operatorname{div}\left(C : \varepsilon(\vec{u}) \right) = 0 & \text{in } \Omega \\ \vec{u} = 0 & \text{on } \Gamma_D \\ \left(C : e(\vec{u}) \right) \cdot \vec{n} = \vec{g} & \text{on } \Gamma_N \end{cases}$$

* δ_{ij} is the kronecker delta function

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases}$$

where,

$$C_{ijkl} := \lambda \delta_{ij} \delta_{kl} + \mu \left(\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right)$$

$$\mu = \frac{E}{2(1+\nu)}, \quad \lambda = \frac{E \nu}{(1-2\nu)(1+\nu)}$$

Weak formulation

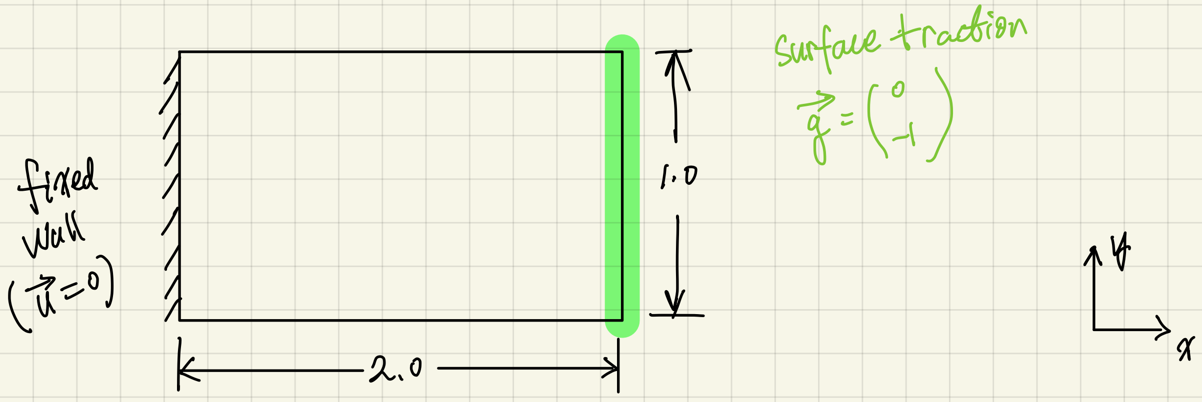
$$-\int_{\Omega} \left(\varepsilon(\vec{u}) : C \right) : \varepsilon(\vec{\tilde{u}}) \, d\Omega + \int_{\Gamma_N} \vec{g} \cdot \vec{\tilde{u}} \, d\Gamma = 0 \quad \forall \vec{\tilde{u}} \in \bar{U}$$

Tensor form $\longrightarrow$ Vector form

i.e. $\Omega \subset \mathbb{R}^d$, $d = 2$

$$\left(\varepsilon(\vec{u}) : C \right) : \varepsilon(\vec{\tilde{u}}) = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} \\ \frac{\partial u_2}{\partial x_2} \\ \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \end{bmatrix}^T \begin{bmatrix} \lambda + 2\mu & \lambda & 0 \\ \lambda & \lambda + 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix} \begin{bmatrix} \frac{\partial \tilde{u}_1}{\partial x_1} \\ \frac{\partial \tilde{u}_2}{\partial x_2} \\ \frac{\partial \tilde{u}_1}{\partial x_2} + \frac{\partial \tilde{u}_2}{\partial x_1} \end{bmatrix}$$

Numerical Example



$E = 1.0$

$\nu = 0.3$