

## ✔ Congratulations! You passed!

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1. Consider the following five events:

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1. Correctly guessing a random 128-bit AES key on the first try.
2. Winning a lottery with 1 million contestants (the probability is  $1/10^6$ ).
3. Winning a lottery with 1 million contestants 5 times in a row (the probability is  $(1/10^6)^5$ ).
4. Winning a lottery with 1 million contestants 6 times in a row.
5. Winning a lottery with 1 million contestants 7 times in a row.

What is the order of these events from most likely to least likely?

- ☐ 2, 4, 3, 1, 5
- ☒ 2, 3, 4, 1, 5
- ☐ 2, 3, 5, 4, 1
- ☐ 3, 2, 5, 4, 1

✔ Correct

- The probability of event (1) is  $1/2^{128}$ .
- The probability of event (5) is  $1/(10^6)^7$  which is about  $1/2^{139}$ . Therefore, event (5) is the least likely.
- The probability of event (4) is  $1/(10^6)^6$  which is about  $1/2^{119.5}$  which is more likely than event (1).
- The remaining events are all more likely than event (4).

2. Suppose that using commodity hardware it is possible to build a computer

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for about \$200 that can brute force about 1 billion AES keys per second.

Suppose an organization wants to run an exhaustive search for a single

128-bit AES key and was willing to spend 4 trillion dollars to buy these

machines (this is more than the annual US federal budget). How long would

it take the organization to brute force this single 128-bit AES key with

these machines? Ignore additional costs such as power and maintenance.

- ☐ More than a million years but less than a billion ( $10^9$ ) years
- ☐ More than a day but less than a week
- ☐ More than a 100 years but less than a million years

- ☒ More than a billion ( $10^9$ ) years
- ☐ More than a month but less than a year

☒ **Correct**

The answer is about 540 billion years.

- # machines =  $4 \cdot 10^{12} / 200 = 2 \cdot 10^{10}$
- # keys processed per sec =  $10^9 \cdot (2 \cdot 10^{10}) = 2 \cdot 10^{19}$
- # seconds =  $2^{128} / (2 \cdot 10^{19}) = 1.7 \cdot 10^{19}$

This many seconds is about 540 billion years.

3. Let  $F : \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}^n$  be a secure PRF (i.e. a PRF where the key space, input space, and output space are all  $\{0, 1\}^n$ ) and say  $n = 128$ .

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Which of the following is a secure PRF (there is more than one correct answer):

☒  $F'((k_1, k_2), x) = \begin{cases} F(k_1, x) & \text{when } x \neq 0^n \\ k_2 & \text{otherwise} \end{cases}$

☒ **Correct**

Correct. A distinguisher for  $F'$  gives a distinguisher for  $F$ .

☐  $F'(k, x) = F(k, x) \parallel 0$

(here  $\parallel$  denotes concatenation)

☒  $F'((k_1, k_2), x) = F(k_1, x) \parallel F(k_2, x)$  (here  $\parallel$  denotes concatenation)

☒ **Correct**

Correct. A distinguisher for  $F'$  gives a distinguisher for  $F$ .

☐  $F'(k, x) = F(k, x) \oplus F(k, x \oplus 1^n)$

☐  $F'(k, x) = k \oplus x$

☒  $F'(k, x) = F(k, x)[0, \dots, n-2]$

(i.e.,  $F'(k, x)$  drops the last bit of  $F(k, x)$ )

☒ **Correct**

Correct. A distinguisher for  $F'$  gives a distinguisher for  $F$ .

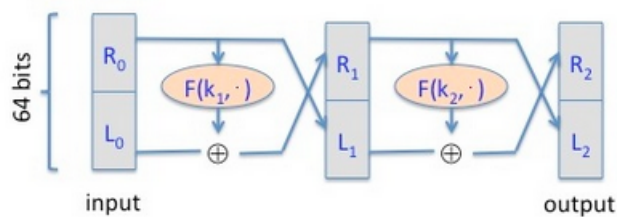
4. Recall that the Luby-Rackoff theorem discussed in [The Data Encryption Standard lecture](#) states that applying a **three** round Feistel network to a secure PRF gives a secure block cipher. Let's see what goes wrong if we only use a **two** round Feistel.

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Let  $F : K \times \{0, 1\}^{32} \rightarrow \{0, 1\}^{32}$  be a secure PRF.

Recall that a 2-round Feistel defines the following PRP

$F_2 : K^2 \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$ :



Here  $R_0$  is the right 32 bits of the 64-bit input and  $L_0$  is the left 32 bits.

One of the following lines is the output of this PRP  $F_2$  using a random key, while the other three are the output of a truly random permutation  $f : \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$ . All 64-bit outputs are encoded as 16 hex characters.

Can you say which is the output of the PRP? Note that since you are able to distinguish the output of  $F_2$  from random,  $F_2$  is not a secure block cipher, which is what we wanted to show.

**Hint:** First argue that there is a detectable pattern in the xor of  $F_2(\cdot, 0^{64})$  and  $F_2(\cdot, 1^{32}0^{32})$ . Then try to detect this pattern in the given outputs.

☒ On input  $0^{64}$  the output is "290b6e3a 39155d6f".

On input  $1^{32}0^{32}$  the output is "d6f491c5 b645c008".

☐ On input  $0^{64}$  the output is "5f67abaf 5210722b".

On input  $1^{32}0^{32}$  the output is "bbe033c0 0bc9330e".

☐ On input  $0^{64}$  the output is "9d1a4f78 cb28d863".

On input  $1^{32}0^{32}$  the output is "75e5e3ea 773ec3e6".

☐ On input  $0^{64}$  the output is "7b50baab 07640c3d".

On input  $1^{32}0^{32}$  the output is "ac343a22 cea46d60".

☒ **Correct**

Observe that the two round Feistel has the property that

the left of  $F(\cdot, 0^{64}) \oplus F(\cdot, 1^{32}0^{32})$  is  $1^{32}$ .

The two outputs in this answer are the only ones with this property.

5. Nonce-based CBC. Recall that in [Lecture 4.4](#) we said that if one wants to use CBC encryption with a non-random unique nonce then the nonce must first be encrypted with an **independent** PRP key and the result then used as the CBC IV.

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Let's see what goes wrong if one encrypts the nonce with the **same** PRP key as the key used for CBC encryption.

Let  $F : K \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$  be a secure PRP with, say,  $\ell = 128$ . Let  $n$  be a nonce and suppose one encrypts a message  $m$  by first computing  $IV = F(k, n)$  and then using this IV in CBC encryption using  $F(k, \cdot)$ . Note that the same key  $k$  is used for computing the IV and for CBC encryption. We show that the resulting system is not nonce-based CPA secure.

The attacker begins by asking for the encryption of the two block message  $m = (0^\ell, 0^\ell)$  with nonce  $n = 0^\ell$ . It receives back a two block ciphertext  $(c_0, c_1)$ . Observe that by definition of CBC we know that  $c_1 = F(k, c_0)$ .

Next, the attacker asks for the encryption of the one block message  $m_1 = c_0 \oplus c_1$  with nonce  $n = c_0$ . It receives back a one block ciphertext  $c'_0$ .

What relation holds between  $c_0, c_1, c'_0$ ? Note that this relation lets the adversary win the nonce-based CPA game with advantage 1.

- ☒  $c_1 = c'_0$
- ☐  $c'_0 = c_0 \oplus 1^\ell$
- ☐  $c_0 = c_1 \oplus c'_0$
- ☐  $c_1 = c_0$

☒ **Correct**

This follows from the definition of CBC with an encrypted nonce as defined in the question.

6. Let  $m$  be a message consisting of  $\ell$  AES blocks

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(say  $\ell = 100$ ). Alice encrypts  $m$  using CBC mode and transmits the resulting ciphertext to Bob. Due to a network error, ciphertext block number  $\ell/2$  is corrupted during transmission.

All other ciphertext blocks are transmitted and received correctly.

Once Bob decrypts the received ciphertext, how many plaintext blocks will be corrupted?

- ☐  $\ell/2$
- ☐ 3
- ☒ 2
- ☐ 0
- ☐  $\ell$

☒ **Correct**

Take a look at the CBC decryption circuit. Each ciphertext blocks affects only the current plaintext block and the next.

7. Let  $m$  be a message consisting of  $\ell$  AES blocks (say  $\ell = 100$ ). Alice encrypts  $m$  using randomized counter mode and

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transmits the resulting ciphertext to Bob. Due to a network error

transmits the resulting ciphertext to Bob. Due to a network error,

ciphertext block number  $\ell/2$  is corrupted during transmission.

All other ciphertext blocks are transmitted and received correctly.

Once Bob decrypts the received ciphertext, how many plaintext blocks will be corrupted?

☐  $1 + \ell/2$

☐ 0

☐  $\ell/2$

☒ 1

☐ 3

☒ **Correct**

Take a look at the counter mode decryption circuit. Each ciphertext block affects only the current plaintext block.

8. Recall that encryption systems do not fully hide the **length** of

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transmitted messages. Leaking the length of web requests has been used to eavesdrop on encrypted HTTPS traffic to a number of

web sites, such as tax preparation sites, Google searches, and healthcare sites.

Suppose an attacker intercepts a packet where he knows that the packet payload is encrypted using AES in CBC mode with a random IV. The encrypted packet payload is 128 bytes. Which of the following messages is plausibly the decryption of the payload:

☐ 'The significance of this general conjecture, assuming its truth, is easy to see. It means that it may be feasible to design ciphers that are effectively unbreakable.'

☒ 'In this letter I make some remarks on a general principle relevant to enciphering in general and my machine.'

☐ 'If qualified opinions incline to believe in the exponential conjecture, then I think we cannot afford not to make use of it.'

☐ 'The most direct computation would be for the enemy to try all  $2^r$  possible keys, one by one.'

☒ **Correct**

The length of the string is 107 bytes, which after padding becomes 112 bytes,

and after prepending the IV becomes 128 bytes.

9. Let  $R := \{0, 1\}^4$  and consider the following PRF  $F : R^5 \times R \rightarrow R$  defined as follows:

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$$F(k, x) := \begin{cases} t = k[0] \\ \text{for } i=1 \text{ to } 4 \text{ do} \\ \quad \text{if } (x[i-1] == 1) \quad t = t \oplus k[i] \\ \text{output } t \end{cases}$$

That is, the key is  $k = (k[0], k[1], k[2], k[3], k[4])$  in  $R^5$  and the function at, for example, 0101 is defined as  $F(k, 0101) = k[0] \oplus k[2] \oplus k[4]$ .

For a random key  $k$  unknown to you, you learn that

$$F(k, 0110) = 0011 \text{ and } F(k, 0101) = 1010 \text{ and } F(k, 1110) = 0110 .$$

What is the value of  $F(k, 1101)$ ? Note that since you are able to predict the function at a new point, this PRF is insecure.

1111

✓ Correct