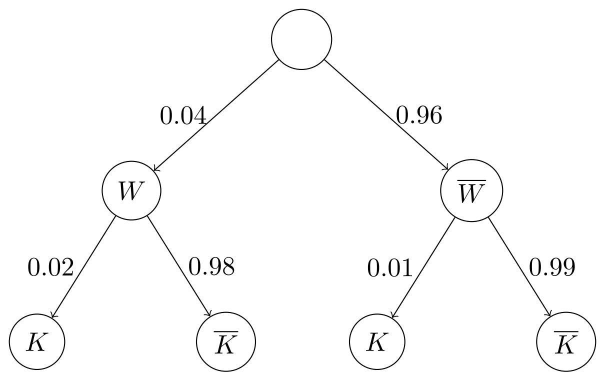
Aufgabe 1

a)

W = Werbung geschaut
K = Produkt gekauft



b)

$$\begin{aligned}
\mathbb{P}(K) &= \mathbb{P}((K \cap W) \cup (K \cap \overline{W})) \\
&= \mathbb{P}(K \cap W) + \mathbb{P}(K \cap \overline{W}) \\
&= 0.04 \cdot 0.02 + 0.96 \cdot 0.01 \\
&= 0.0104
\end{aligned}$$

c)

$$\begin{aligned}
\mathbb{P}(W|K) &= \frac{\mathbb{P}(W \cap K)}{\mathbb{P}(K)} \\
&= \frac{0.04 \cdot 0.02}{0.0104} \\
&= \frac{1}{13} \approx 0.07692
\end{aligned}$$

Aufgabe 2

a)

$$\int_{-\infty}^{\infty} f(x)\,dx \overset{!}{=} 1$$

$$\begin{aligned}
\Leftrightarrow \int_{-\infty}^{\infty} f(x)\,dx &= \int_{-3}^{0} a\cdot(3+x)\,dx + \int_{0}^{3} a\cdot(3-x)\,dx \\
&= \left[\frac{a}{2}x^2 + 3ax\right]_{-3}^{0} + \left[-\frac{a}{2}x^2 + 3ax\right]_{0}^{3} \\
&= -\frac{9a}{2} + 9a - \frac{9a}{2} + 9a
\end{aligned}$$

$$\begin{aligned}
9a &\overset{!}{=} 1 \\
\Leftrightarrow a &= \frac{1}{9}
\end{aligned}$$

b)

$$F(x) = \begin{cases} \frac{1}{18}x^2 + \frac{1}{3}x & \text{für } -3 \le x \le 0 \\ -\frac{1}{18}x^2 + \frac{1}{3}x & \text{für } 0 < x \le 3 \\ 0 & \text{sonst} \end{cases}$$

$F(x) = 0$ falls $x < -3$

$F(x) = 1$ falls $x > 3$

$F(x) = \int_{-3}^{x} \frac{1}{9}(3+t)\,dt = (*)$ falls $-3 \le x \le 0$

$F(x) = \frac{1}{2} + \int_{0}^{x} \frac{1}{9}(3-t)\,dt$ falls $0 \le x \le 3$ $= (\Delta)$

$$\begin{aligned}
(*) &= \left[\frac{1}{9}\left(3t + \frac{1}{2}t^2\right)\right]_{-3}^{x} \\
&= \frac{1}{9}\left(3x + \frac{1}{2}x^2\right) - \frac{1}{9}\left(-9 + \frac{1}{2}9\right) \\
&= \frac{1}{3}x + \frac{1}{18}x^2 + \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
(\Delta) &= \frac{1}{2} + \left[\frac{1}{3}t - \frac{1}{18}t^2\right]_{0}^{x} \\
&= \frac{1}{2} + \frac{1}{3}x - \frac{1}{18}x^2
\end{aligned}$$

c)

$\mathbb{E}(X) = 0$ aufgrund Symmetrie um 0.

$$\begin{aligned}
\mathbb{E}(X^2) &= \int_{-\infty}^{\infty} x^2 f(x)\,dx \\
&= \int_{-3}^{0} \frac{1}{9}x^3 + \frac{1}{3}x^2\,dx + \int_{0}^{3} -\frac{1}{9}x^3 + \frac{1}{3}x^2\,dx \\
&= \left[\frac{1}{36}x^4 + \frac{1}{9}x^3\right]_{-3}^{0} + \left[-\frac{1}{36}x^4 + \frac{1}{9}x^3\right]_{0}^{3} \\
&= -\frac{9}{4} + 3 - \frac{9}{4} + 3 \\
&= 1.5 \\
\mathrm{Var}(X) &= \mathbb{E}(X^2) - (\mathbb{E}(X))^2 \\
&= 1.5
\end{aligned}$$

Aufgabe 3

$$
\begin{aligned}
L_n(a,b) &= \prod_{i=1}^{n} f_{(a,b)}(x_i) \\
&= \prod_{i=1}^{n} a \cdot b \cdot (1+bx_i)^{-(a+1)} \\
\ln(L_n(a,b)) &= \sum_{i=1}^{n} \ln(a \cdot b \cdot (1+bx_i)^{-(a+1)}) \\
&= \sum_{i=1}^{n} (\ln(a) + \ln(b) - (a+1)\ln(1+bx_i)) \\
&= n \cdot \ln(a) + n \cdot \ln(b) - (a+1) \sum_{i=1}^{n} \ln(1+bx_i) \\
\frac{\partial}{\partial a} \ln(L_n(a,b)) &= n \cdot \frac{1}{a} - \sum_{i=1}^{n} \ln(1+bx_i) \overset{!}{=} 0 \\
\Leftrightarrow \frac{n}{a} &= \sum_{i=1}^{n} \ln(1+bx_i) \\
\Leftrightarrow a &= \frac{n}{\sum_{i=1}^{n} \ln(1+bx_i)} \\
\frac{\partial}{\partial^2 a} \ln(L_n(a,b)) &= -\frac{n}{a^2}
\end{aligned}
$$

Für alle $a > 0$ ist die 2. partielle Ableitung der Log-Likelihood-Funktion negativ.

$$a = \frac{n}{\sum_{i=1}^{n} \ln(1+bx_i)} > 0$$

maximiert somit die Log-Likelihood-Funktion.

Aufgabe 4

a)

$A :=$ „Spielfigur verlässt vollbesetztes Haus“
$X_i :=$ Augenzahl im i-ten Wurf

$$
\begin{aligned}
\mathbb{P}(A) &= \mathbb{P}(X_1 = 6 \cup X_2 = 6 \cup X_3 = 6) \\
&= \mathbb{P}(X_1 = 6) + \mathbb{P}(X_2 = 6) + \mathbb{P}(X_3 = 6) \\
&= \frac{1}{6} + \frac{5}{6} \cdot \frac{1}{6} + \frac{5}{6} \cdot \frac{5}{6} \cdot \frac{1}{6} \\
&= \frac{91}{216} \approx 0.4213
\end{aligned}
$$

b)

$B :=$ „Spielfigur verlässt nicht vollbesetztes Haus“

$$\begin{aligned}
\mathbb{P}(B) &= \mathbb{P}(X_1 \neq 6 \cap X_2 \neq 6 \cap X_3 \neq 6) \\
&= 1 - \mathbb{P}(A) \\
&= \frac{125}{216} \approx 0.5783 \\
\mathbb{P}(\text{„Spielfigur verlässt nach 3. Runde das Haus“}) &= \mathbb{P}(B)^2 \cdot \mathbb{P}(A) \\
&\approx 0.141
\end{aligned}$$

c)

$$\begin{aligned}
\mathbb{P}(\text{„3 von 4 Spielern verlassen das Haus“}) &= \binom{4}{3} \cdot \mathbb{P}(A)^3 \cdot \mathbb{P}(B) \\
&\approx 0.1731
\end{aligned}$$

Aufgabe 5

$X_i \sim \mathcal{N}(3, 0.2^2)$
$Y_i \sim \mathcal{N}(1.5, 0.1^2)$

$$\begin{aligned}
Z &:= \sum_{i=1}^{4} X_i + \sum_{i=1}^{9} Y_i \\
Z &\sim \mathcal{N}(\sum_{i=1}^{4} 3 + \sum_{i=1}^{9} 1.5, \sum_{i=1}^{4} 0.2^2 + \sum_{i=1}^{9} 0.1^2) \\
&= \mathcal{N}(25.5, 0.25) \\
\mathbb{P}(Z > 26) &= 1 - \mathbb{P}(Z \leq 26) \\
&= 1 - \mathbb{P}(\frac{Z - 25.5}{0.5} \leq \frac{0.5}{0.5}) \\
&= 1 - \Phi(1) \\
&\approx 0.1587
\end{aligned}$$

Aufgabe 6

a)

$$
\begin{aligned}
\mathbb{E}(X) &= 4 \cdot 0.4 + 6 \cdot 0.2 + 8 \cdot 0.4 \\
&= 6 \\
\mathbb{E}(X^2) &= 4^2 \cdot 0.4 + 6^2 \cdot 0.2 + 8^2 \cdot 0.4 \\
&= 39.2 \\
\mathrm{Var}(X) &= \mathbb{E}(X^2) - (\mathbb{E}(X))^2 \\
&= 39.2 - 6^2 \\
&= 3.2
\end{aligned}
$$

b) $S_{320} := \sum_{i=1}^{320} X_i \underset{\text{approx}}{\sim} W$

$$
\begin{aligned}
\mathbb{P}(S_{320} > 1952) &= 1 - \mathbb{P}(S_{320} \leq 1952) \\
&= 1 - \mathbb{P}\left(\frac{S_{320} - 320 \cdot 6}{\sqrt{3.2 \cdot 320}} \leq \frac{1952 - 320 \cdot 6}{\sqrt{3.2 \cdot 320}} \right) \\
&= 1 - \mathbb{P}\left(\frac{S_{320} - 1920}{32} \leq 1 \right) \\
\text{ZGWS} \quad &\approx 1 - \Phi(1) \\
&\approx 1 - 0.8413 = 0.1587
\end{aligned}
$$

Aufgabe 7

a)

$$
\begin{aligned}
\mathbb{E}(T_n) &= \mathbb{E}(\frac{2}{n} \cdot \sum_{i=1}^{n} X_i) \\
&= \frac{2}{n} \sum_{i=1}^{n} \mathbb{E}(X_i) \\
&= \frac{2}{n} \sum_{i=1}^{n} \frac{\theta}{2} \\
&= \frac{2}{n} \cdot n \cdot \frac{\theta}{2} \\
&= \theta
\end{aligned}
$$

b)

$$\begin{aligned}\mathrm{Var}(T_n) &= \mathrm{Var}\left(\frac{2}{n}\cdot\sum_{i=1}^{n} X_i\right)\\ &= \frac{4}{n^2}\sum_{i=1}^{n}\mathrm{Var}(X_i)\\ &= \frac{4}{n}\cdot\frac{\theta^2}{12}\\ &= \frac{\theta^2}{3n}\end{aligned}$$

Aufgabe 8

a)

Klasse	h_j	r_j	Höhe
$(0;1]$	5	$\frac{5}{16} = 0.3125$	$\frac{0.3125}{1} = 0.3125$
$(1;2]$	3	$\frac{3}{16} = 0.1875$	$\frac{0.1875}{1} = 0.1875$
$(2;3]$	3	$\frac{3}{16} = 0.1875$	$\frac{0.1875}{1} = 0.1875$
$(3;5]$	2	$\frac{2}{16} = 0.125$	$\frac{0.125}{2} = 0.0625$
$(5;8]$	3	$\frac{3}{16} = 0.1875$	$\frac{0.1875}{3} = 0.0625$

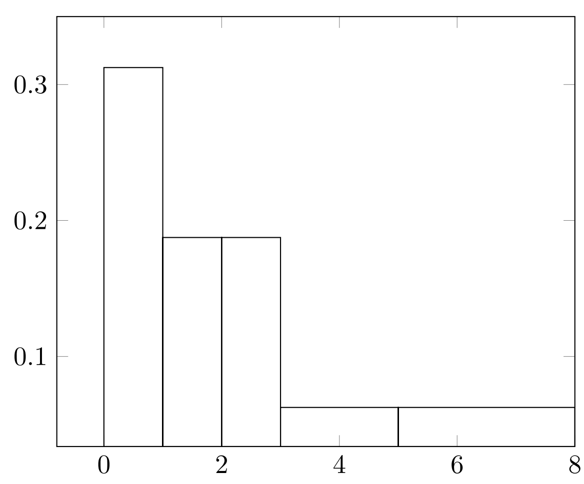


Abbildung 1: Histogramm

b)

$$x_{0.5} = \frac{1.82 + 2.36}{2} = 2.09$$
$$x_{0.8} = 4.51$$

c)

Klasse	relativ, kumuliert
$(0; 1]$	0.3125
$(1; 2]$	0.5
$(2; 3]$	0.6875
$(3; 5]$	0.8125
$(5; 8]$	1

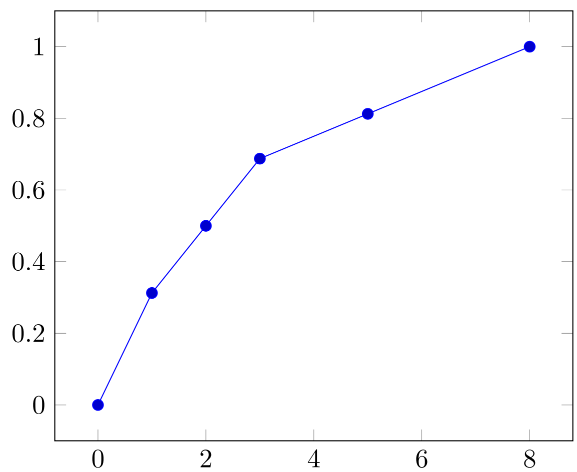


Abbildung 2: empirische Verteilungsfunktion