Solution 1

(a) Assume a point $P = [X, Y, Z, 1]^T$ in 3D world, P_1, P_2 are corresponded points in images of camera1 and camera 2. $P_1 = [x, y, 1]^T, P_2 = [x', y, 1]^T$. So we have:

$$P_1 \sim K[I|\bar{0}]P \tag{1}$$

$$P_2 \sim K[I|t]P \tag{2}$$

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$t = [-t_x, 0, 0]^T \tag{4}$$

The homogeneous coordinate of $P_1 and P_2$ are as following:

$$P_1 = \begin{bmatrix} Xf \\ Yf \\ Z \end{bmatrix} P_2 = \begin{bmatrix} Xf - t_x f \\ Yf \\ Z \end{bmatrix} \tag{5}$$

We get $x = \frac{XF}{Z}, x' = \frac{Xf - t_x f}{Z}$. Therefore:

$$d = x - x' = \frac{XF}{Z} - \frac{Xf - t_x f}{Z} = \frac{f t_x}{Z} \tag{6}$$

(b) Based on the equation $X\alpha + Y\beta + Z\gamma = k$(where $X = \frac{Zx}{f}, Y = \frac{Zy}{f}$, because $x = \frac{Xf}{Z}$ and $y = \frac{Yf}{Z}$ according to equation (5)), we can get the following equations:

$$rZ = k - \frac{x\alpha}{f}Z - \frac{y\beta}{f}Z \Rightarrow Z = \frac{k}{\gamma + \frac{x\alpha}{f} + \frac{y\beta}{f}} \tag{7}$$

Based on equation (6) and (7), $d = ax + by + c$ can be represented by:

$$d = \frac{t_x}{k}(x\alpha + y\beta + f\gamma) \tag{8}$$

(c) The homogeneous coordinate of $P_1 and P_2$ are as following:

$$P_1 = \begin{bmatrix} Xf \\ Yf \\ Z \end{bmatrix} P_2 = \begin{bmatrix} Xf \\ Yf \\ Z - t_z \end{bmatrix} \tag{9}$$

Therefore, we can get x', y' as following:

$$x' = \frac{xZ}{Z - t_z} \tag{10}$$

$$y' = \frac{yZ}{Z - t_z} \tag{11}$$

The set of possible (x', y') that could match to (x, y) is:

$$\frac{x'}{y'} = \frac{x}{y} \tag{12}$$

Solution 2

(a) buildcv

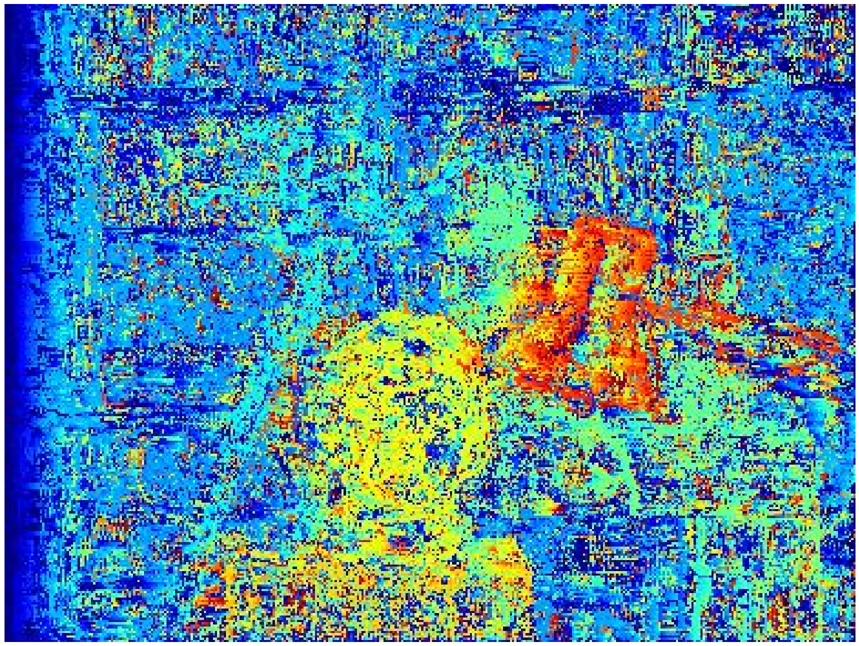

Figure 1: Disparity Image by buildcv

(b) bfilt

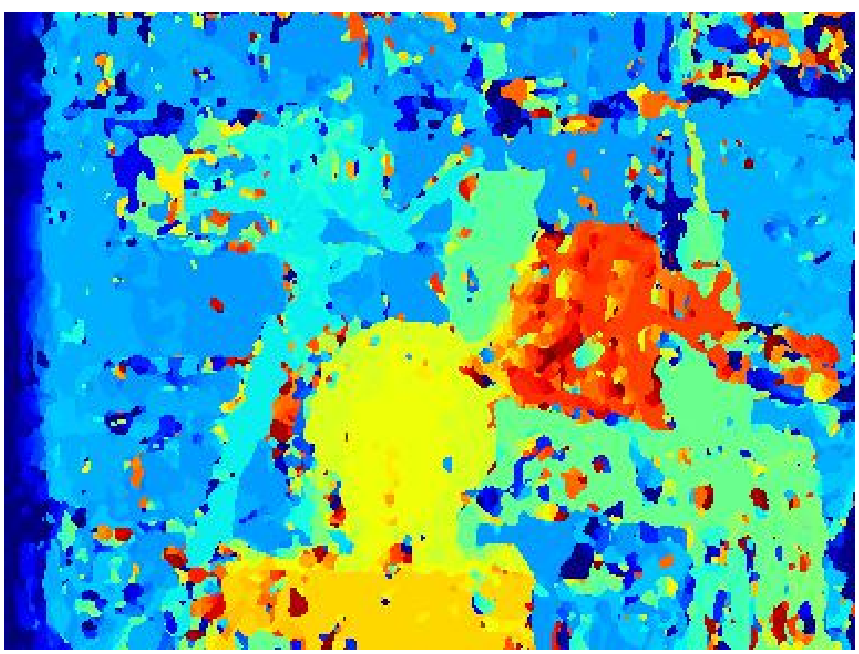

Figure 2: Smoothed cost volume using Bilateral filtering

Solution 3

(a) viterbilr

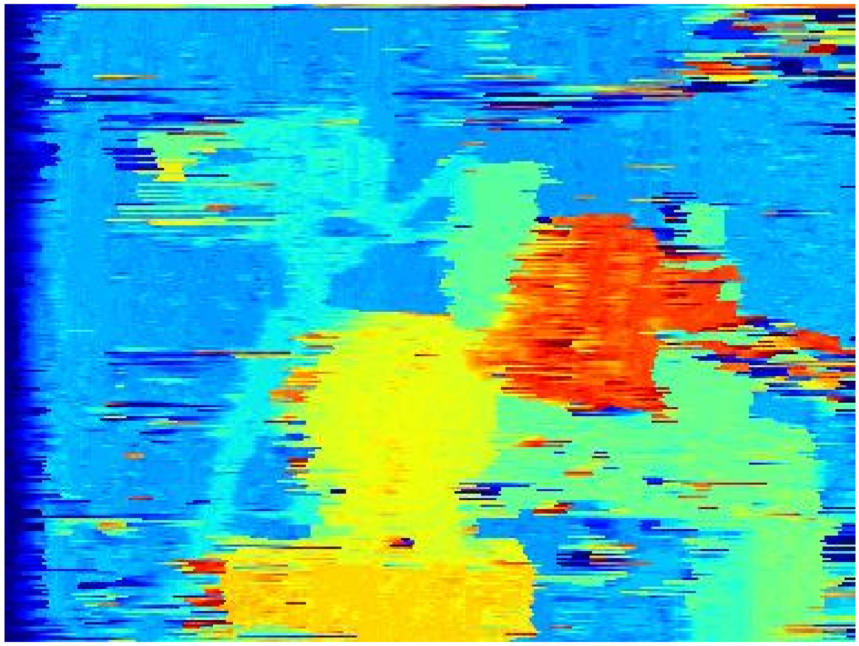

Figure 3: Smoothed cost volume using forward-backward algorithm

(b) SGM

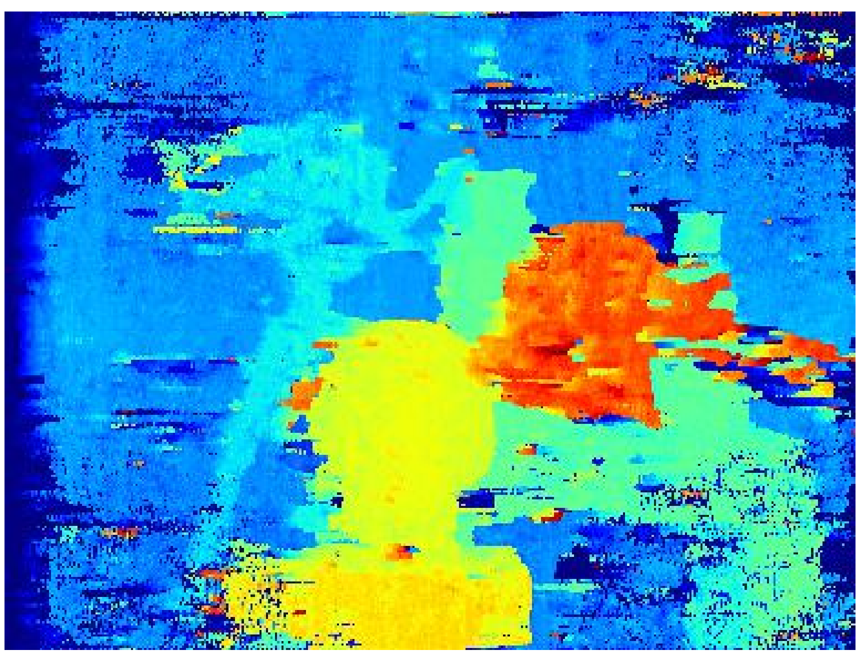

Figure 4: Smoothed cost volume using SGM

Solution 4

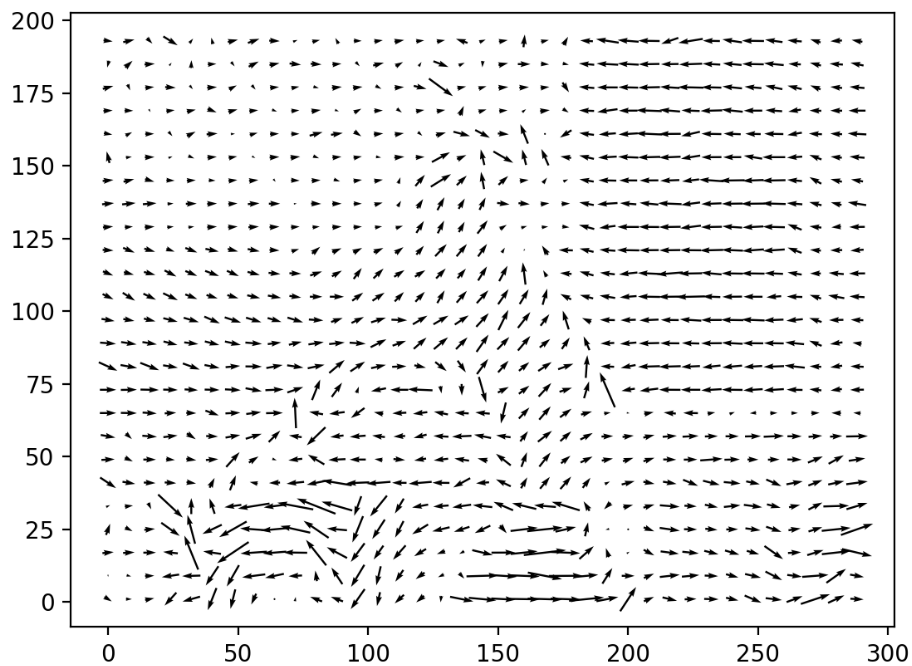

Figure 5: Optical Flow by Lucas Kanade Method

Information

This problem set took approximately 30 hours of effort.

I discussed this problem set with:

- Sijia Wang
- Chunyuan Li
- Jiarui Xing

I also got hints from the following sources:

- lecture ppts