Solution 1

(a) According to the slides in Lec2, the equation for ϵ is as follows:

$$\epsilon = g\sqrt{I^0}\epsilon_1 + \sqrt{(g^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2 \tag{1}$$

So, the variation δ^2 of ϵ is as follows:

$$\begin{aligned}\delta^2 &= \left(g\sqrt{I^0}\right)^2 + \left(\sqrt{(g^2\sigma_{2a}^2 + \sigma_{2b}^2)}\right)^2 &(2)\\ &= g^2 I^0 + g^2\sigma_{2a}^2 + \sigma_{2b}^2 &(3)\end{aligned}$$

(b) Let $I^0 = \frac{I^0}{k}, g = gk$, we can get a new equation for new image I:

$$I = gI^0 + gk\sqrt{\frac{I^0}{k}}\epsilon_1 + \sqrt{(g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2 \tag{4}$$

So, the zero-mean Gaussian noise ϵ is as follows:

$$\epsilon = g\sqrt{kI^0}\epsilon_1 + \sqrt{(g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2)}\epsilon_2 \tag{5}$$

And its variation δ^2 is:

$$\delta^2 = g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2 \tag{6}$$

(c) Because $I = \frac{\sum_{i=1}^{k} I_i}{k}$, and each $I_i = gI^0 + g\sqrt{kI^0}\epsilon_1 + \sqrt{g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2}\epsilon_2$.
So, the noise ϵ of I is:

$$\epsilon = \frac{1}{k}\left(\epsilon 1 + \epsilon 2 + \cdots + \epsilon k\right) \tag{7}$$

The variation δ^2 of ϵ is $\frac{1}{k^2}\left(\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_k^2\right)$, and $\delta_i^2 = g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2$. So δ^2 is:

$$\begin{aligned}\delta^2 &= \frac{1}{k^2}k\left(g^2kI_0 + g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2\right) &(8)\\ &= g^2I_0 + g^2k\sigma_{2a}^2 + \frac{1}{k}\sigma_{2b}^2 &(9)\end{aligned}$$

(d) I think a single shot with exposure time T is preferable. Because k shots with exposure time $\frac{T}{k}$ will produce more noises based on equation of image I, even though variation δ^2 of noise ϵ taking k shots is similar to a single shot (it is determined by k), seen in (3) and (9)

$$\epsilon = g\sqrt{I^0}\epsilon_1 + \sqrt{g^2\sigma_{2a}^2 + \sigma_{2b}^2}\epsilon_2 \tag{10}$$

$$\epsilon = g\sqrt{kI^0}\epsilon_1 + \sqrt{g^2k^2\sigma_{2a}^2 + \sigma_{2b}^2}\epsilon_2(kshots) \tag{11}$$

Solution 2

Figure 1: Histogram equalization

Solution 3

(a) Gradient magnitude

Figure 2: gradient magnitudes

(b) NMS operation to optimize gradient magnitudes images

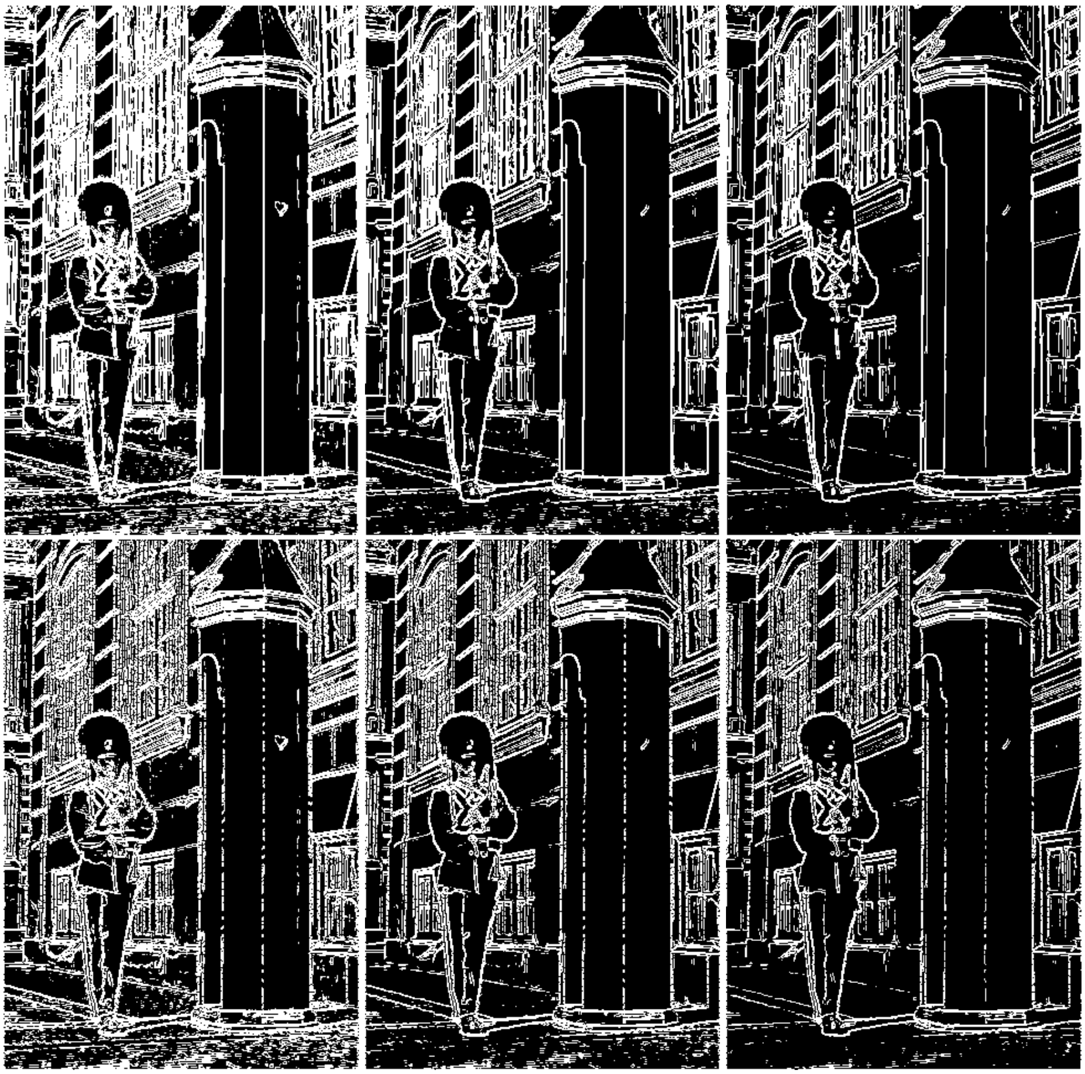

Figure 3: NMS (lower level three pics)

Solution 4

Bilateral Filtering

Figure 4: original image

Figure 5: $K = 9, \sigma_s = 2, \sigma_I = 0.5$; $K = 9, \sigma_s = 4, \sigma_I = 0.25$; $K = 9, \sigma_s = 16, \sigma_I = 0.125$; repeat 8 times $K = 9, \sigma_s = 2, \sigma_I = 0.125$

Figure 6: Original image with more noise

Figure 7: repeat 12 times $K = 9, \sigma_s = 8, \sigma_I = 0.125$

Solution 5

(a) Because F[u,v] is central symmetry image. So we can store half of F[u,v] image.
When both W_x, H_x are even, the image size will be $W_x/2 * H_x$
When both W_x, H_x are odd, the image size will be $(\lfloor W_x/2 \rfloor + 1) * H_x$
When W_x is odd, H_x is even, the image size will be $H_x/2 * W_x$
When H_x is odd, W_x is even, the image size will be $W_x/2 * H_x$

(b) Convolution in the Fourier Domain:

Figure 8: Convolution in the Fourier Domain

Solution 6

(a) Harr wavelet decomposition

Figure 9: Original image before Harr Wavelet Decompostion

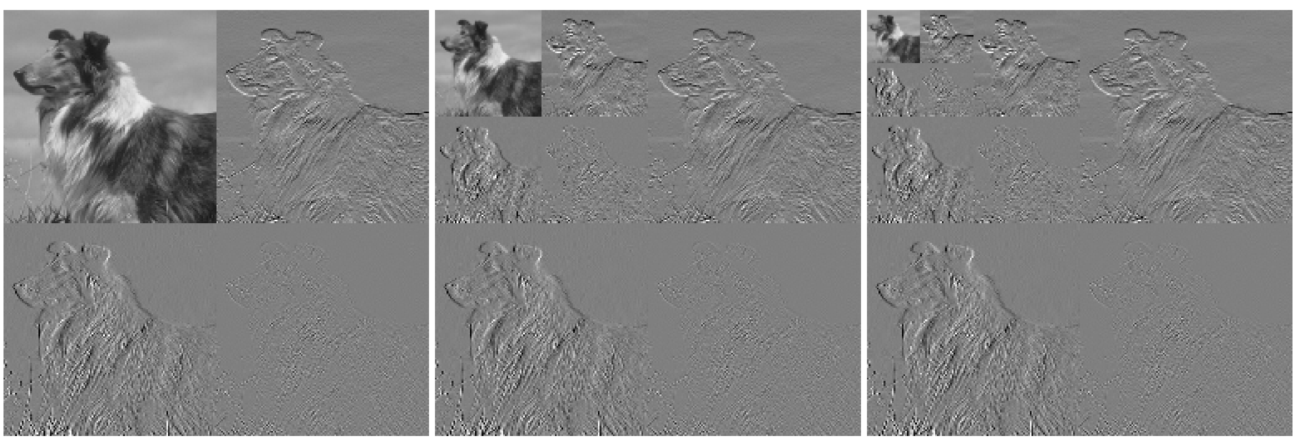

Figure 10: Image after Harr Wavelet Decompostion

(b) Image reconstruction from Harr wavelet decomposition

Figure 11: Image reconstruct from Harr Wavelet Decompostion

Information

This problem set took approximately 40 hours of effort.

I discussed this problem set with:

- Sijia Wang
- Chunyuan Li
- Likai Yan

I also got hints from the following sources:

- Wikipedia article on matrix calculus at https://en.wikipedia.org/wiki/Matrix_calculus
- Read array operation from http://cn.mathworks.com/help/matlab/ref/circshift.html?s_tid=gn_loc_drop
- Read numpy tutorial from http://cs231n.github.io/python-numpy-tutorial/
- Study complex number from https://www.khanacademy.org/math/algebra2/introduction-to-complex-numbers-algebra-2/the-complex-numbers-algebra-2/v/complex-number-intro
- Get mathematical functions from https://docs.scipy.org/doc/numpy/reference/routines.math.html
- Study how to use LaTeX from https://www.latex-tutorial.com/tutorials/
- Study the process of Bilateral filter from http://blog.csdn.net/abcjennifer/article/details/7616663
- Understand Fourier Transform from http://blog.jobbole.com/70549/
- Study Fourier Transform in Imaging process from http://m.blog.csdn.net/abcjennifer/article/details/7622228