

$$17. \text{a) } g(\bar{w}) = \log(1 + e^{\bar{w}^T \bar{w}})$$

$$\nabla g(\bar{w}) = \frac{1}{1 + e^{\bar{w}^T \bar{w}}} \cdot e^{\bar{w}^T \bar{w}} \cdot 2\bar{w} = \bar{w}$$

$$\bar{w} = \bar{0}$$

$$\text{Since } \bar{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}, \quad \begin{matrix} w_1 = 0 \\ w_2 = 0. \end{matrix}$$

See figure $g(\bar{w})$, we could conclude $g(\bar{w})$ is convex and the stationary point is a global minimum.

c) d) see figure.

d). Compared with the global minimum of $g(\bar{w})$, $\bar{w}^0 = 4 \cdot \bar{1}_{\text{N} \times 1}$ is a large number.

So, $\log(1+e^t) \approx t$. the second order Taylor series approximation is:

$$h(\bar{w}) = w_0 + \bar{w} - \bar{w}_0 = \bar{w}$$

For Newton method, $\nabla h(\bar{w}) = 0 \rightarrow \bar{w}'$ is the stationary point of $h(\bar{w})$ is also the stationary point of $g(\bar{w})$, so the situation makes sense.

3.3

$$a) g(\bar{w}) = \sum_{p=1}^P \left(\frac{\bar{w}^T \tilde{x}_p \tilde{x}_p^T \bar{w}}{\tilde{x}_p^T \tilde{x}_p} - 2 \tilde{x}_p^T \bar{w} y_p + y_p^2 \right) = \frac{1}{2} \bar{w}^T \left(2 \sum_{p=1}^P \tilde{x}_p \tilde{x}_p^T \right) \bar{w} - \sum_{p=1}^P \tilde{x}_p^T \bar{w} y_p + \sum_{p=1}^P y_p^2$$

$$\bar{Q} = 2 \sum_{p=1}^P \tilde{x}_p \tilde{x}_p^T, \quad \bar{r} = \left[-2 \sum_{p=1}^P \tilde{x}_p^T y_p \right]^T = -2 \sum_{p=1}^P \tilde{x}_p^T y_p = \sum_{p=1}^P y_p^2$$

$$b) \nabla g(\bar{w}) = \sum_{p=1}^P 2 \tilde{x}_p (\tilde{x}_p^T \bar{w} - y_p) \quad \nabla^2 g(\bar{w}) = 2 \sum_{p=1}^P \tilde{x}_p \tilde{x}_p^T = \bar{Q}$$

$$c) \nabla g(\bar{w}) = \bar{Q} \bar{w} + \bar{r} \quad \nabla^2 g(\bar{w}) = \bar{Q}$$

$$\nabla^2 g(\bar{w}^0) \bar{w}' = \nabla^2 g(\bar{w}^0) \bar{w}^0 - \nabla g(\bar{w}^0)$$

$$\bar{Q} \bar{w}' = \bar{Q} \bar{w}^0 - \bar{Q} \bar{w}^0 - \bar{r}$$

$$\bar{Q} \bar{w}' = -\bar{r}$$

$$\text{So } \left(\sum_{p=1}^P \tilde{x}_p \tilde{x}_p^T \right) \bar{w} = \sum_{p=1}^P \tilde{x}_p y_p$$



3.10.

$$a) z(t) = \frac{1}{1+e^{-t}} \quad z^{-1}(z(t)) = \log\left(\frac{1}{1 - \frac{1}{1+e^{-t}}}\right) = \log(e^t) = t.$$

$$b) z(b + \pi_p w) \approx y_p \quad p = 1 \dots P \quad (3.24)$$

$$z^{-1}(z(b + \pi_p w)) = b + \pi_p w \approx \log\left(\frac{y_p}{1 - y_p}\right) \quad p = 1 \dots P$$

c)

