1. a) $g(w)=\frac{1}{2}qw^2+rw+d$

$g'(w)=qw+r$. $g''(w)=q$.

b) $g(w)=-\cos(2\pi w^2)+w^2$

$g'(w)=\sin(2\pi w^2)4\pi w+2w$.

$g''(w)=\cos(2\pi w^2)(4\pi w)^2+\sin(2\pi w^2)4\pi+2$.

c) $g(w)=\sum_{p=1}^{P}\log(1+e^{-a_p w})$

$g'(w)=-\sum_{p=1}^{P}\frac{a_p}{1+e^{+a_p w}}$. $g''(w)=\sum_{p=1}^{P}\frac{e^{a_p w}}{(1+e^{a_p w})^2}a_p^2$

2. a) $g(\bar{w})=\frac{1}{2}\bar{w}^T\bar{Q}\bar{w}+\bar{r}^T\bar{w}+d$.

$\nabla g(\bar{w})=\bar{Q}\bar{w}+\bar{r}$

$\nabla^2 g(\bar{w})=\bar{Q}$.

b) $g(\bar{w})=-\cos(2\pi\bar{w}^T\bar{w})+\bar{w}^T\bar{w}$

$\nabla g(\bar{w})=\sin(2\pi\bar{w}^T\bar{w})4\pi\bar{w}+2\bar{w}$

$\nabla^2 g(\bar{w})=\cos(2\pi\bar{w}^T\bar{w})(4\pi)^2\bar{w}\bar{w}^T+2+\sin(2\pi\bar{w}^T\bar{w})4\pi$

c) $g(\bar{w})=\sum_{p=1}^{P}\log(1+e^{-\bar{a}_p^T\bar{w}})$

$\nabla g(\bar{w})=-\sum_{p=1}^{P}\frac{\bar{a}_p}{1+e^{\bar{a}_p^T\bar{w}}}$

$\nabla^2 g(\bar{w})=\sum_{p=1}^{P}\frac{e^{\bar{a}_p^T\bar{w}}}{(1+e^{\bar{a}_p^T\bar{w}})^2}\bar{a}_p\bar{a}_p^T$

5. To verify this problem, we just could prove that any cross-lines in the hyperplane perpendicular to the normal vector.

According to the infomation based on this problem,

~~$[g(\bar{v}), \nabla^T]$~~ Point $[\bar{v}, g(\bar{v})]$ is on the tangent hyperplane,

suppose point $[\bar{w}, \cancel{g}\, h(\bar{w}_1)]$ is on the tangent hyperplane.

So the Line $[h(\bar{w}_1)-g(\bar{v}), (h(\bar{w}_1)-g(\bar{v}))(\nabla g(\bar{v}))^{-1}]$ is on the plane.

$$[h(\bar{w}_1)-g(\bar{v}), (h(\bar{w}_1)-g(\bar{v}))(\nabla g(\bar{v})^{-1})]\begin{bmatrix}1\\-\nabla g(\bar{v})\end{bmatrix}=0.$$

we could set another line like this

So we verify the equation (2.3)

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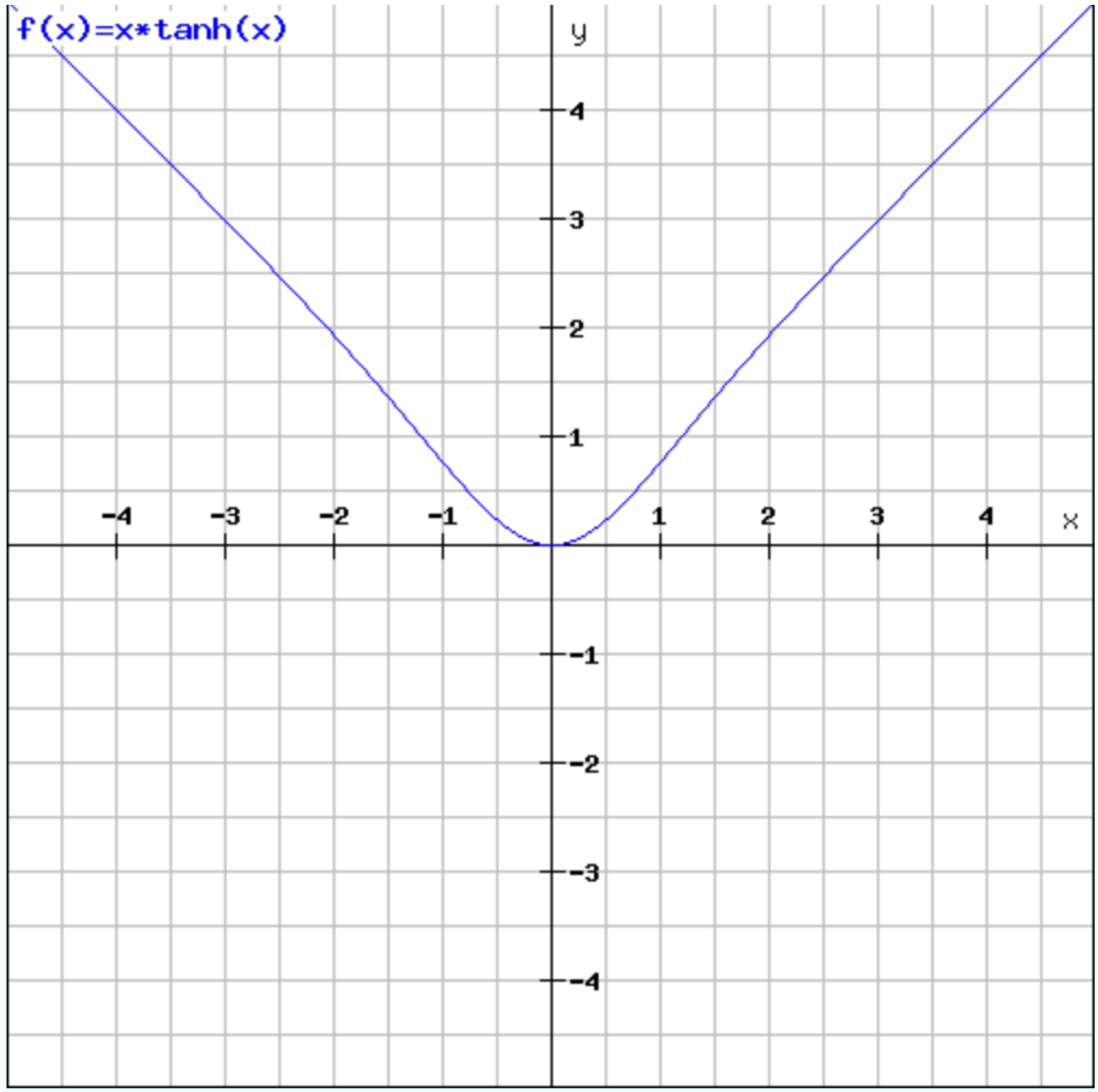
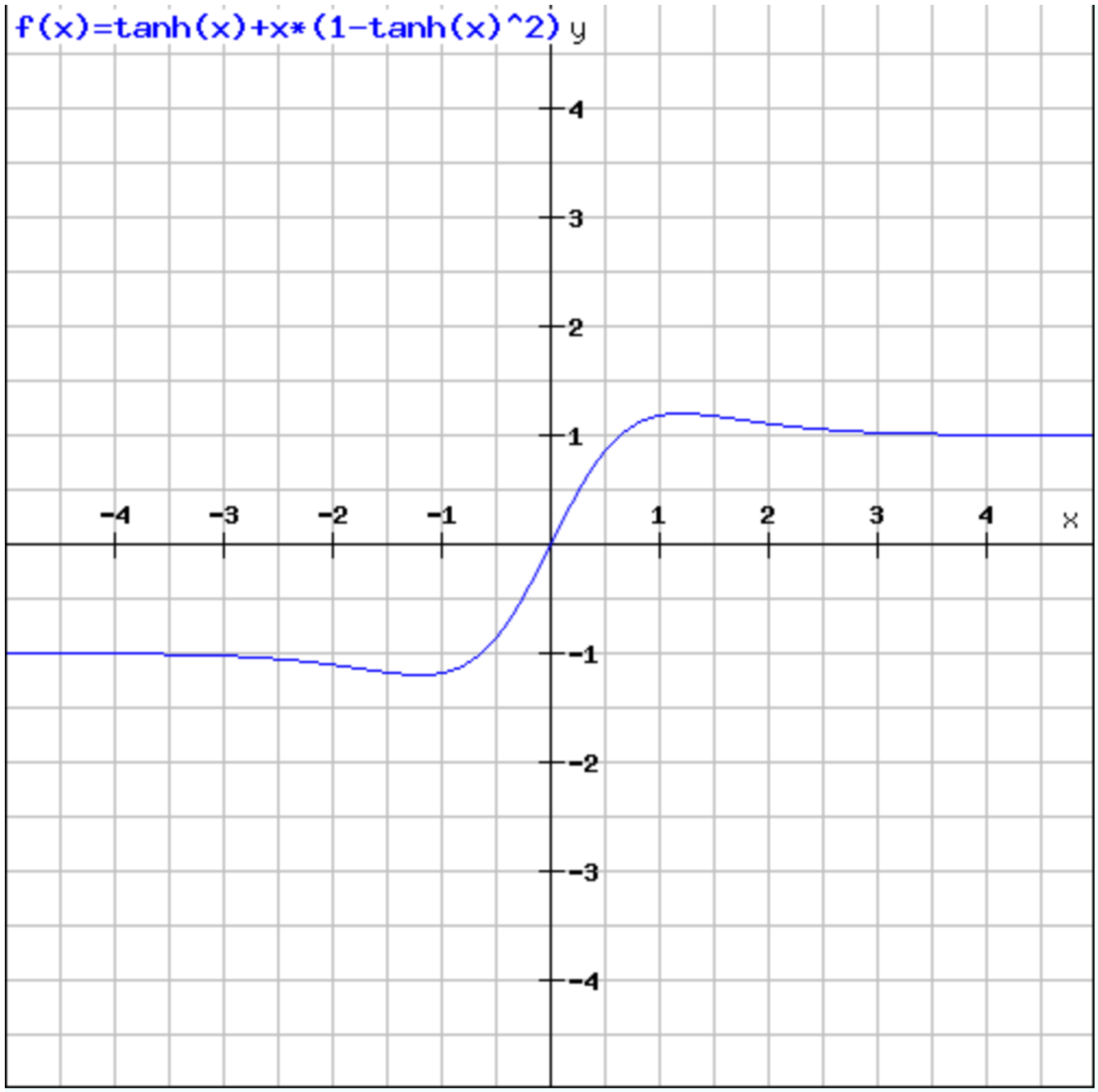
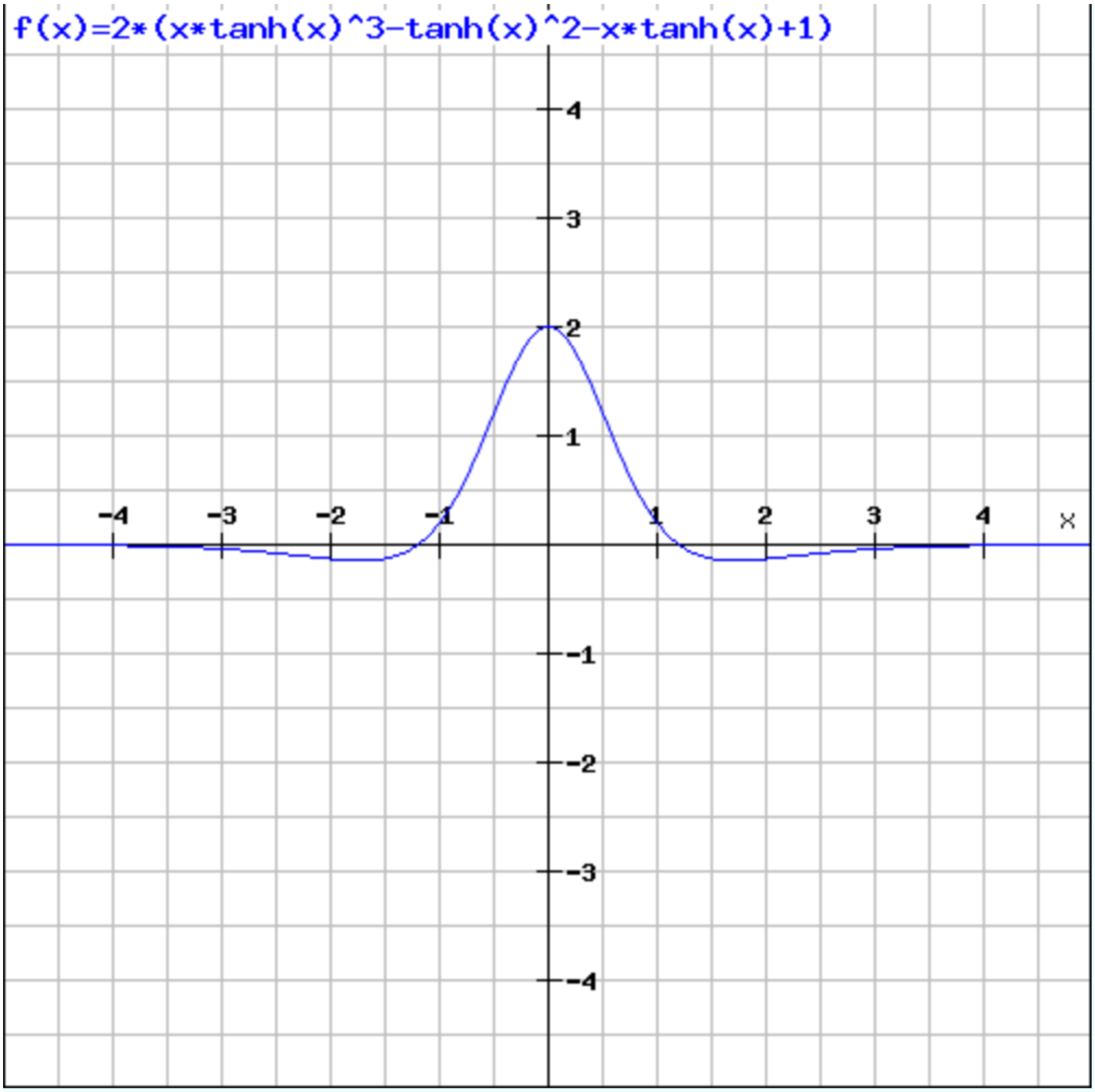

f(x)=x*tanh(x)
y
4
3
2
1
-4
-3
-2
-1
1
2
3
4
x
-1
-2
-3
-4



f(x)=tanh(x)+x*(1-tanh(x)^2)
y
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2
3
4
x


f(x)=2*(x*tanh(x)^3-tanh(x)^2-x*tanh(x)+1)
4
3
2
1
-1
-2
-3
-4
-4
-3
-2
-1
1
2
3
4
x