



KNUTH-MORRIS-PRATT  
AND AHO-CORASICK

# KNUTH - MORRIS - PRATT (KMP)

# DEFINITION



- Knuth – Morris – Pratt (or KMP) algorithm is a algorithm that use to solve string matching problems, which is searching occurrences of “word” within the main “text – string”
- KMP consists of two following steps:
  - Preprocessing
  - Searching/Matching

# PROBLEM



- Given a main string and a pattern string, check whether the pattern occurs in the main string?
- For example:
  - The main string: “ABXABABABAA”
  - The word: “ABABAA”
  - YES, the pattern is occurs in the main string (from character 6<sup>th</sup>).

# MAIN IDEA OF KMP



- The idea behind KMP is to extend a suffix ending at position  $i$  to a suffix ending at position  $i+1$  while still matching the corresponding prefix, which improves efficiency by eliminating expensive string comparisons.

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

A	B	A	B	A	A
---	---	---	---	---	---

# NAIVE ALGORITHM

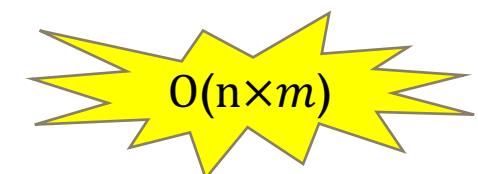
Main string:

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

Word:

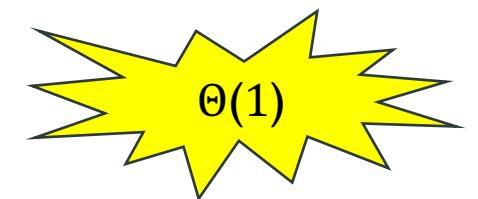
A	B	A	B	A	A
---	---	---	---	---	---

Time Complexity?



$$\Theta(n \times m)$$

Space Complexity?



$$\Theta(1)$$

# PREPROCESSING

- Now we need to preprocessing the pattern to enhance the time complexity  
→ Prefix function
- The prefix function for this string is defined as an array  $\pi$  of length  $m$ , where  $\pi[i]$  is the length of the longest proper prefix of the substring  $s[0..i]$  that is also a suffix of this substring.
- A proper prefix of a string is a prefix that is not equal to the string itself

$$\pi[i] = \max_{k=0\dots i} \{k : s[0\dots k-1] = s[i-(k-1)\dots i]\}$$

# PREPROCESSING

J      I

A	B	A	B	A	A
0					

# PREPROCESSING

J      I

A	B	A	B	A	A
0					

# PREPROCESSING

J      I

A	B	A	B	A	A
0	0				

# PREPROCESSING

J

I

A	B	A	B	A	A
---	---	---	---	---	---

0	0				
---	---	--	--	--	--

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0				

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I			

# PREPROCESSING

J

I

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I			
---	---	---	--	--	--

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I			

# PREPROCESSING

J

I

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2		
---	---	---	---	--	--

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2		

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2		

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	
---	---	---	---	---	--

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

I

A	B	A	B	A	A
0	0	I	2	3	

# PREPROCESSING

J

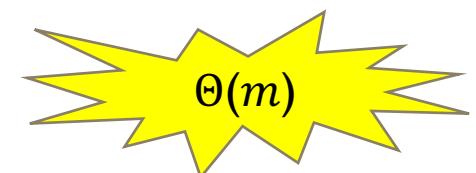
I

A	B	A	B	A	A
0	0	I	2	3	I

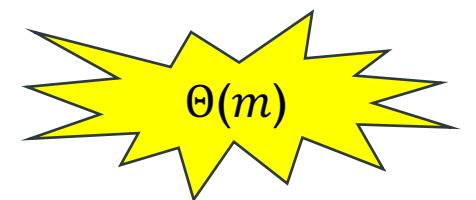
# PREPROCESSING

A	B	A	B	A	A
0	0	I	2	3	I

Time Complexity?



Space Complexity?



# STRING MATCHING

- After produce array  $\pi$ , we are ready to matching the main string now
  - Array  $\pi$  helps us not to recheck all pattern again
  - Array  $\pi$  helps us skip the overlap strings, hence, it will run linear time in the main string instead of go back to the next position of the current substring
- Reduce time complexity

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING

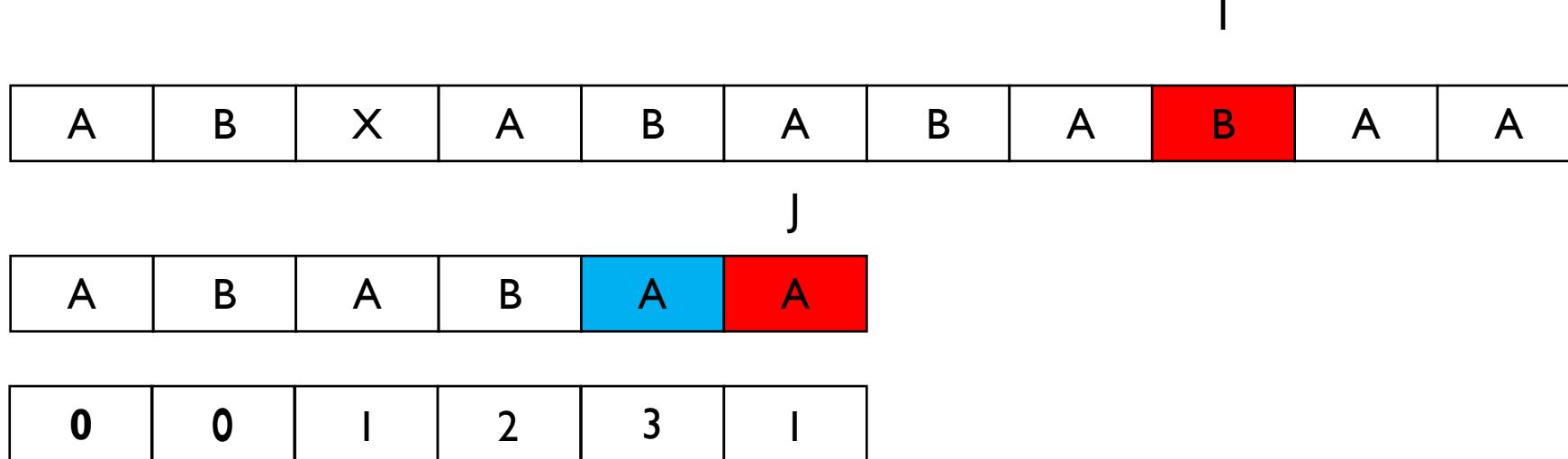
A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

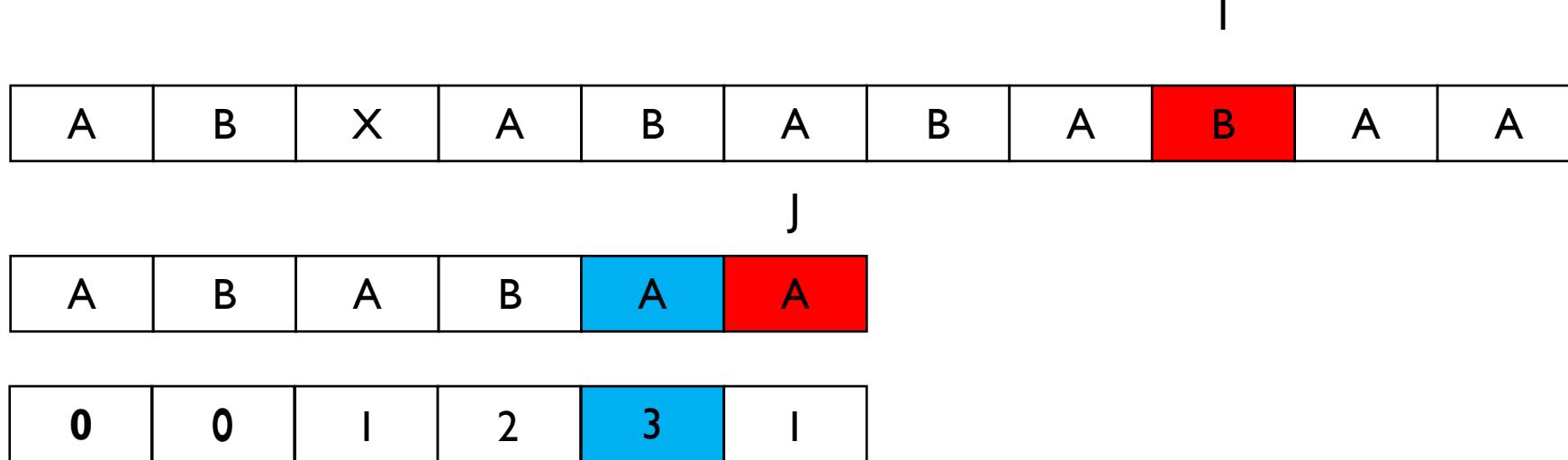
A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING



# STRING MATCHING



# STRING MATCHING

I

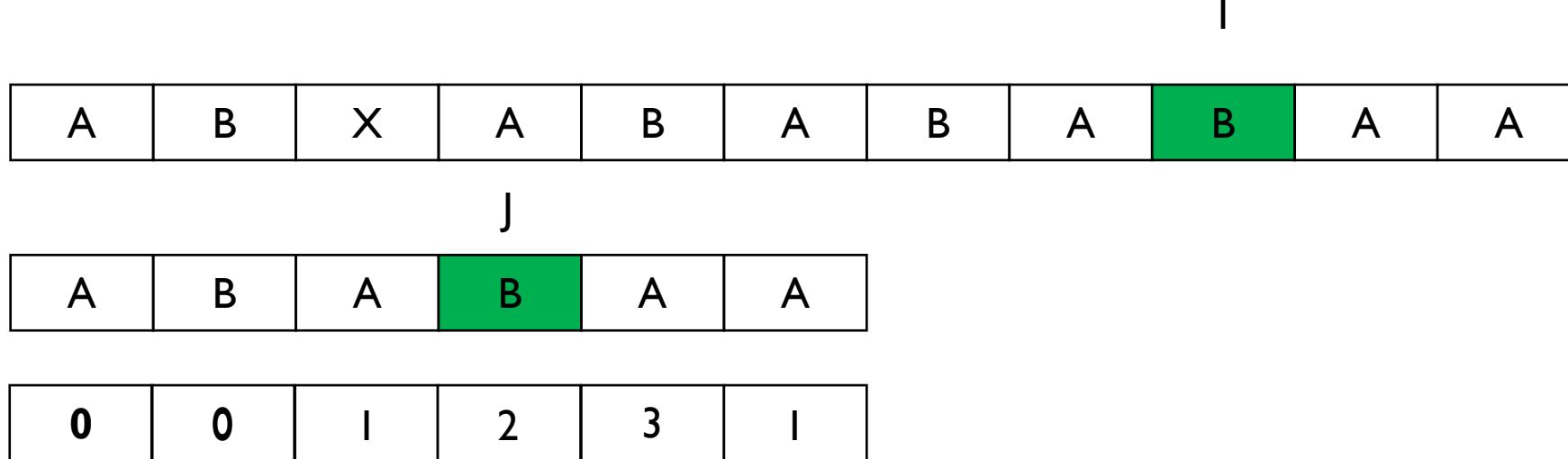
A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

# STRING MATCHING



# STRING MATCHING

I

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

J

A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

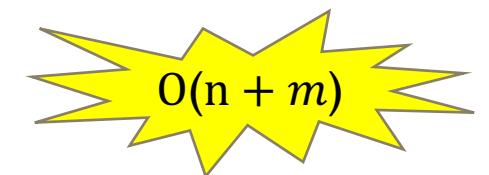
# STRING MATCHING

A	B	X	A	B	A	B	A	B	A	A
---	---	---	---	---	---	---	---	---	---	---

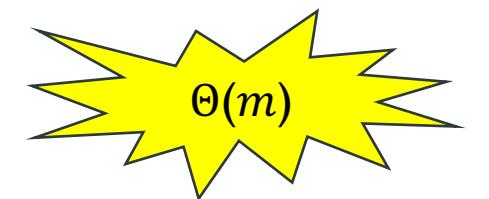
A	B	A	B	A	A
---	---	---	---	---	---

0	0	I	2	3	I
---	---	---	---	---	---

Time Complexity?



Space Complexity?



# AHO-CORASICK

# DEFINITION



- The Aho-Corasick algorithm is a dictionary-matching algorithm that can find all the occurrences of all sets of patterns within a given text in linear time.
- Aho – Corasick consists of two following steps:
  - Preprocessing
  - Searching/Matching

# PROBLEM



- Give a text and a set of multiple pattern strings, check how many pattern strings occur in the text?
- For example:
  - The Text: “DIDIDUADI”
  - The Pattern strings: “DI”, “DIDU”, “DIDI”, “DU”, “DUDUA”, “DUADI”
  - There are 5 pattern occur in the text: “DI”, “DIDU”, “DUDUA”, “DUADI”, “DU”.

# MAIN IDEA OF AHO- CORASICK

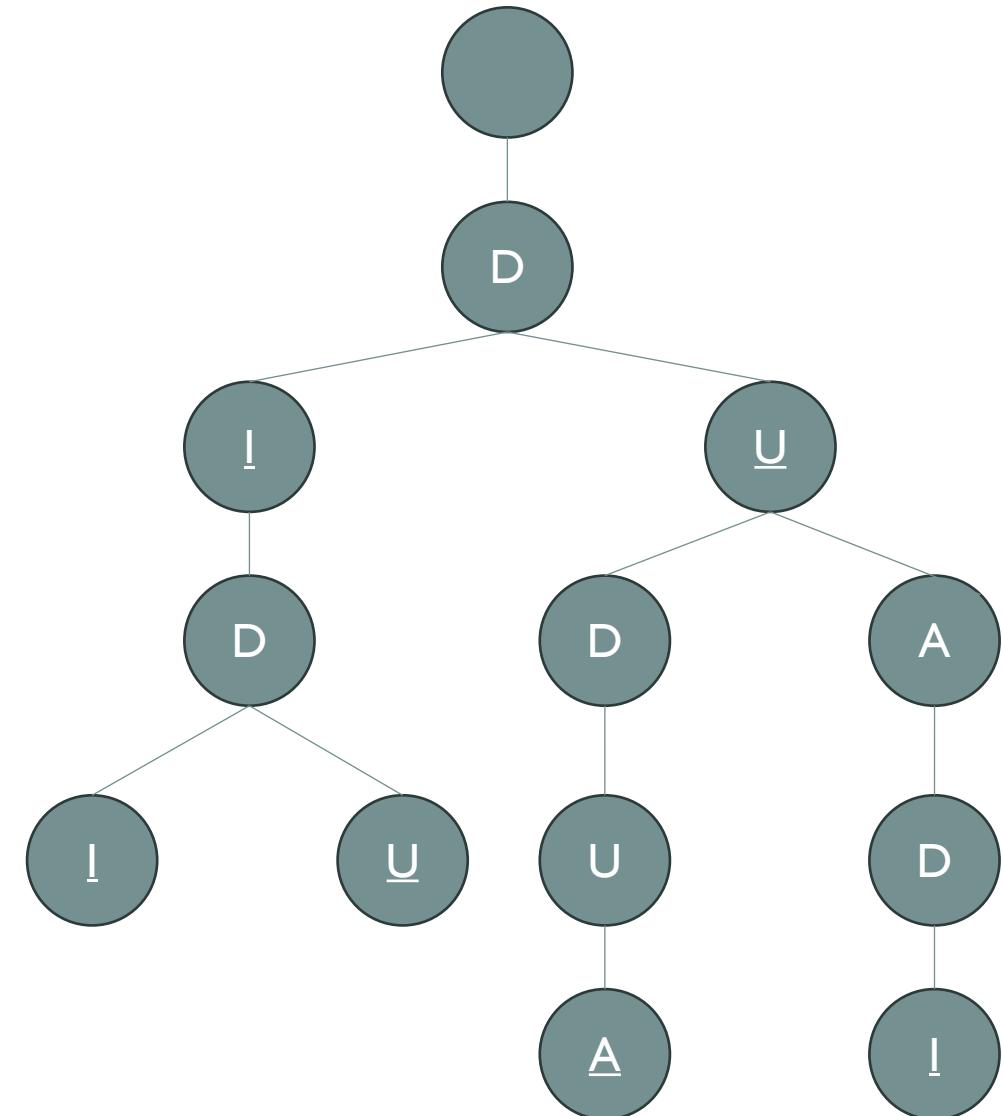


- Based on KMP algorithm's main idea, Aho-Corasick links the suffix ending at position  $i$  of the pattern to the prefix of another pattern string having the same as the suffix.
- So that, Aho-Corasick constructs trie structure to contain multiple pattern strings and each node of trie has a link pointing to the prefix that is similar to the suffix of current substring. If there aren't any prefix similar to the suffix, the link will point to the root of trie.
- The searching or matching process uses the link to link to the prefix when the mismatching occurs, so that we don't have to check the same substring many times.

# ABOUT TRIE

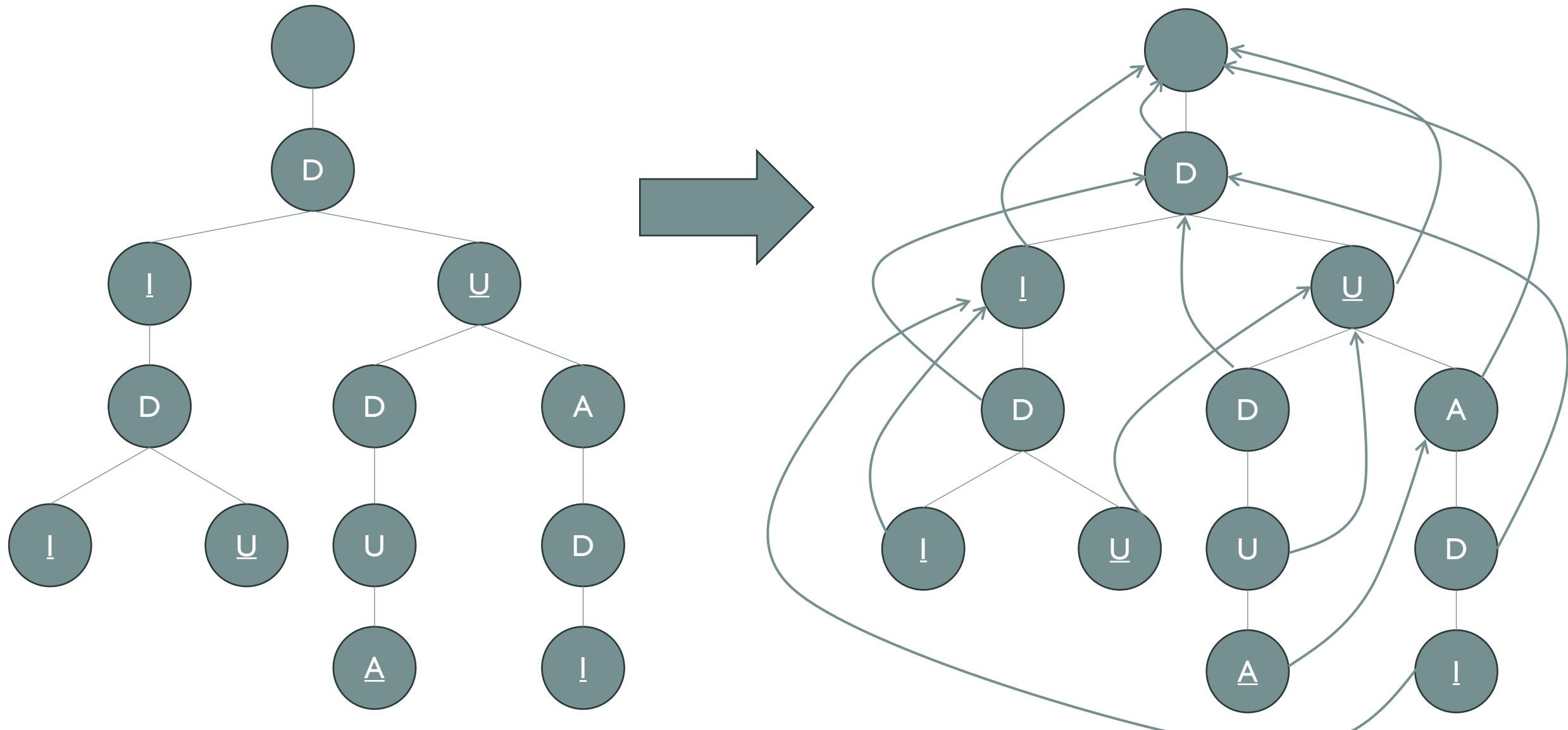
It's a tree data structure used for storage multi string. Every string having the same prefix has the same parents. Each node contains a character of a string

For example, this is a trie structure storages the pattern strings in the first example

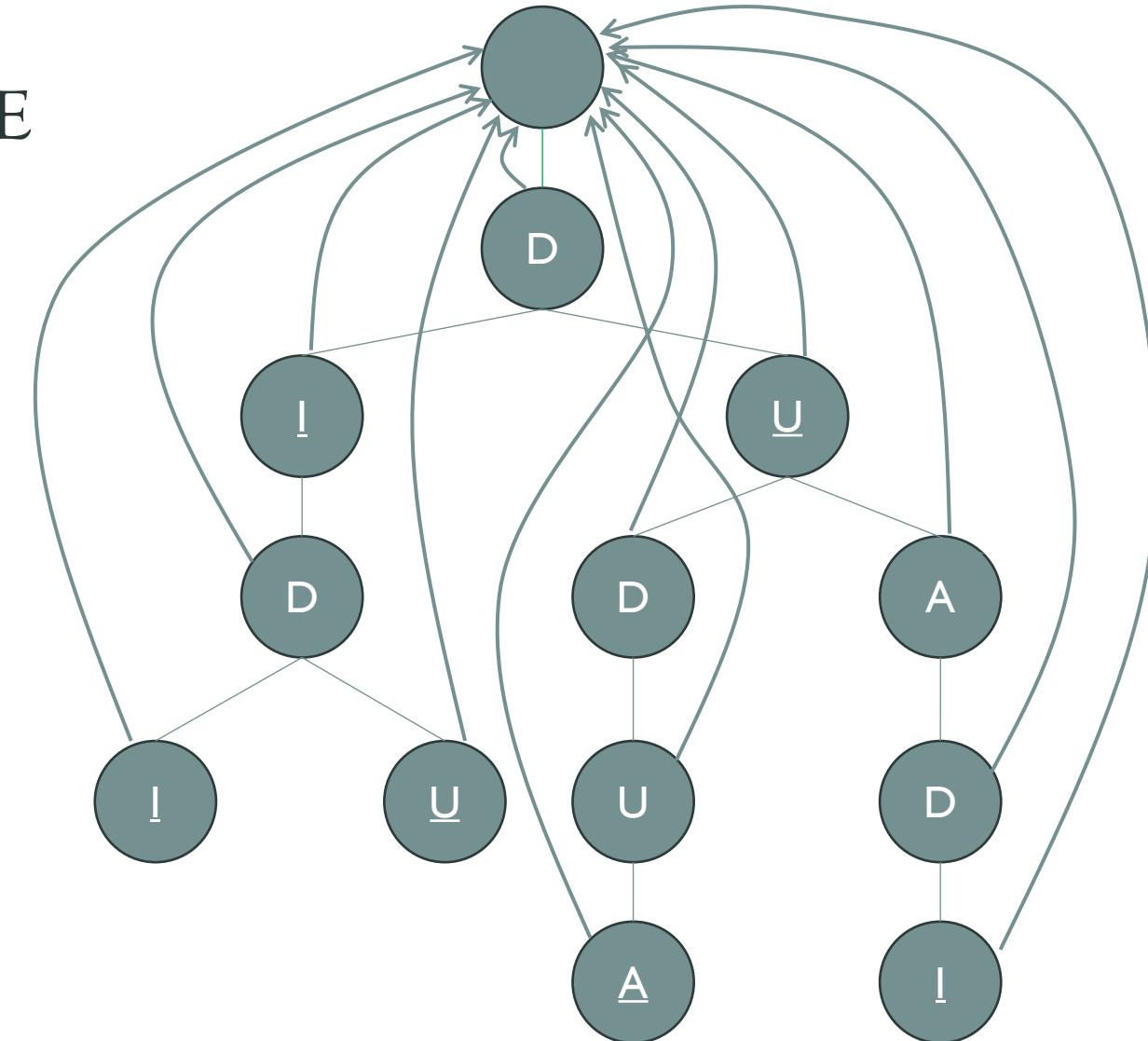


# PREPROCESSING

# LINKING THE NODE



STEP 1: CREATE A TRIE  
AND LINK ALL THE  
NODE TO THE ROOT

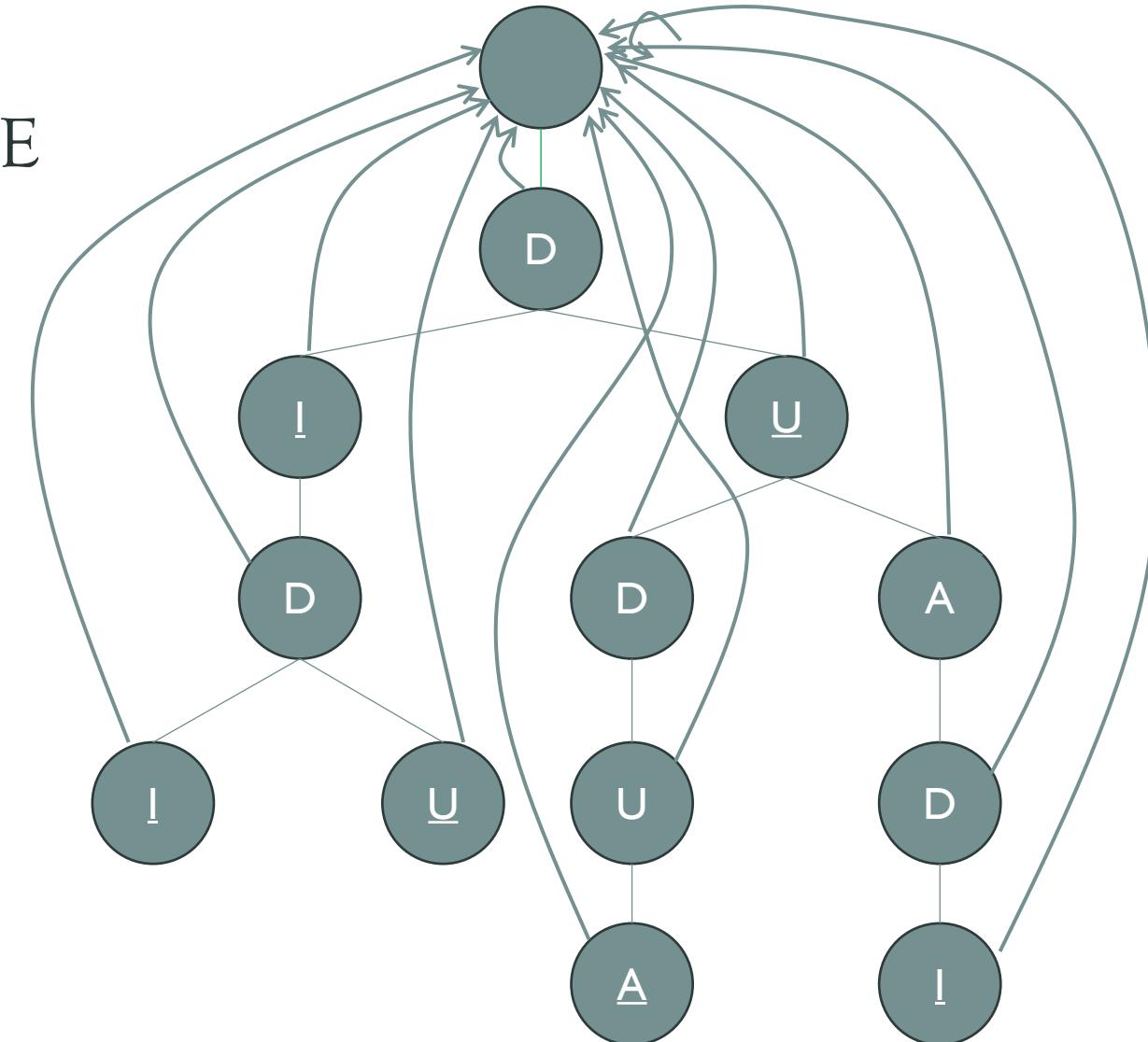


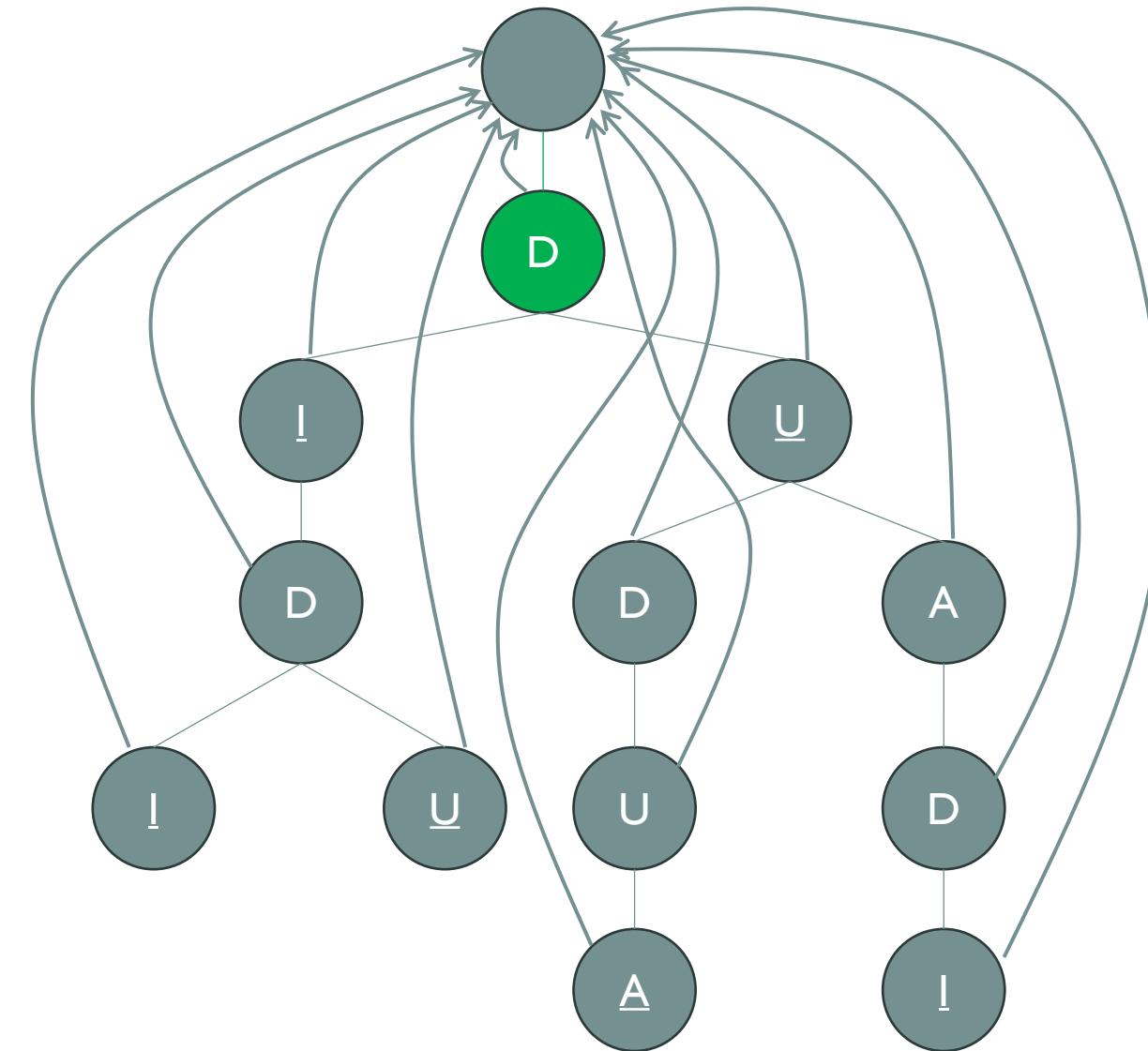
## STEP 2: RELINKING THE NODE

At node  $i$ , we call node which the link of node  $i$ 's parent points at is  $x$ .

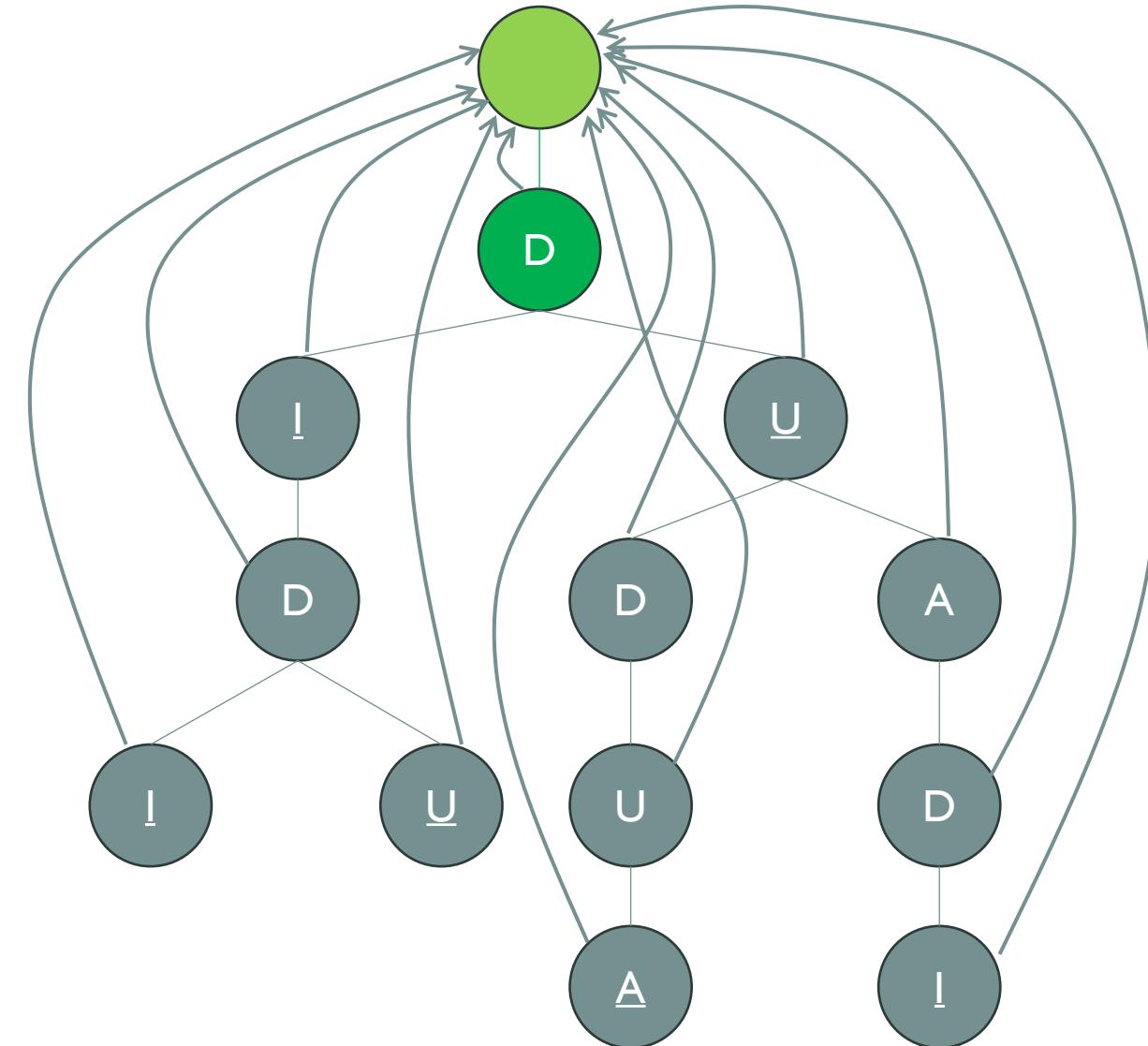
If  $x$  is the root, we do nothing.

If the child of node  $x$  is as same as node  $i$ ,  
the link of node  $i$  will point to node  $x$ 's child.  
Otherwise, we do nothing.

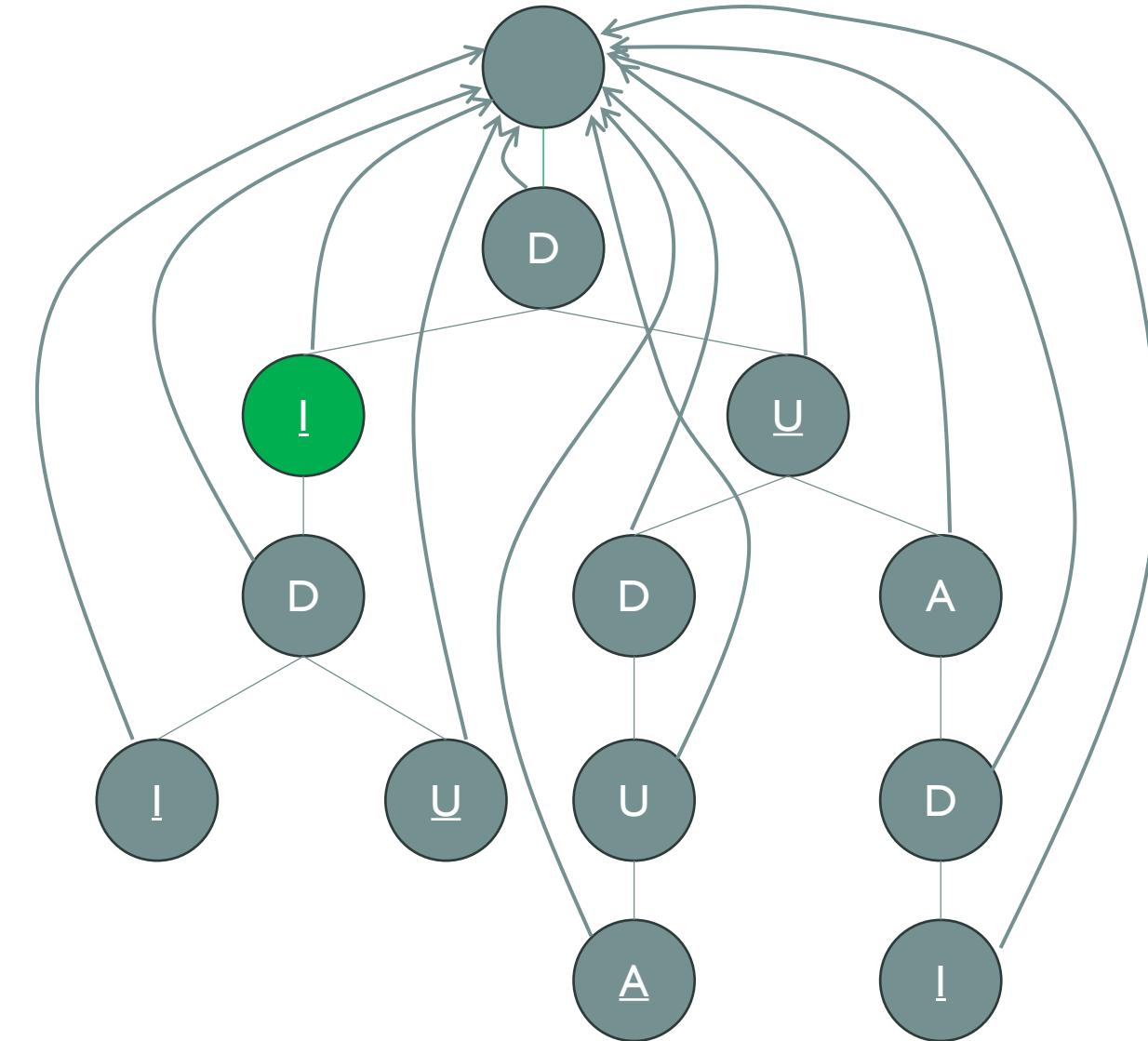




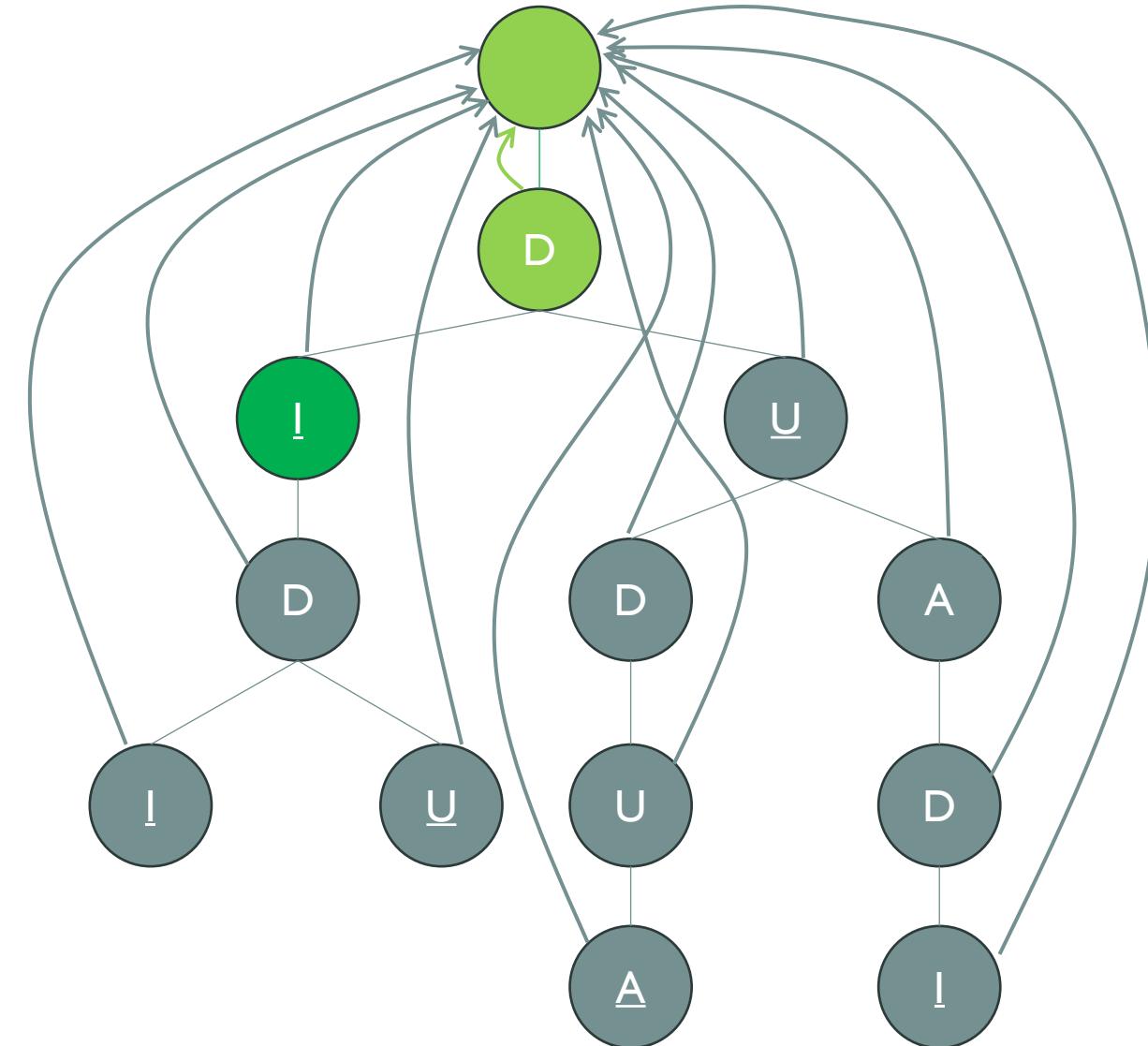
The parent of node  
'D' is a root.  
So we do nothing.



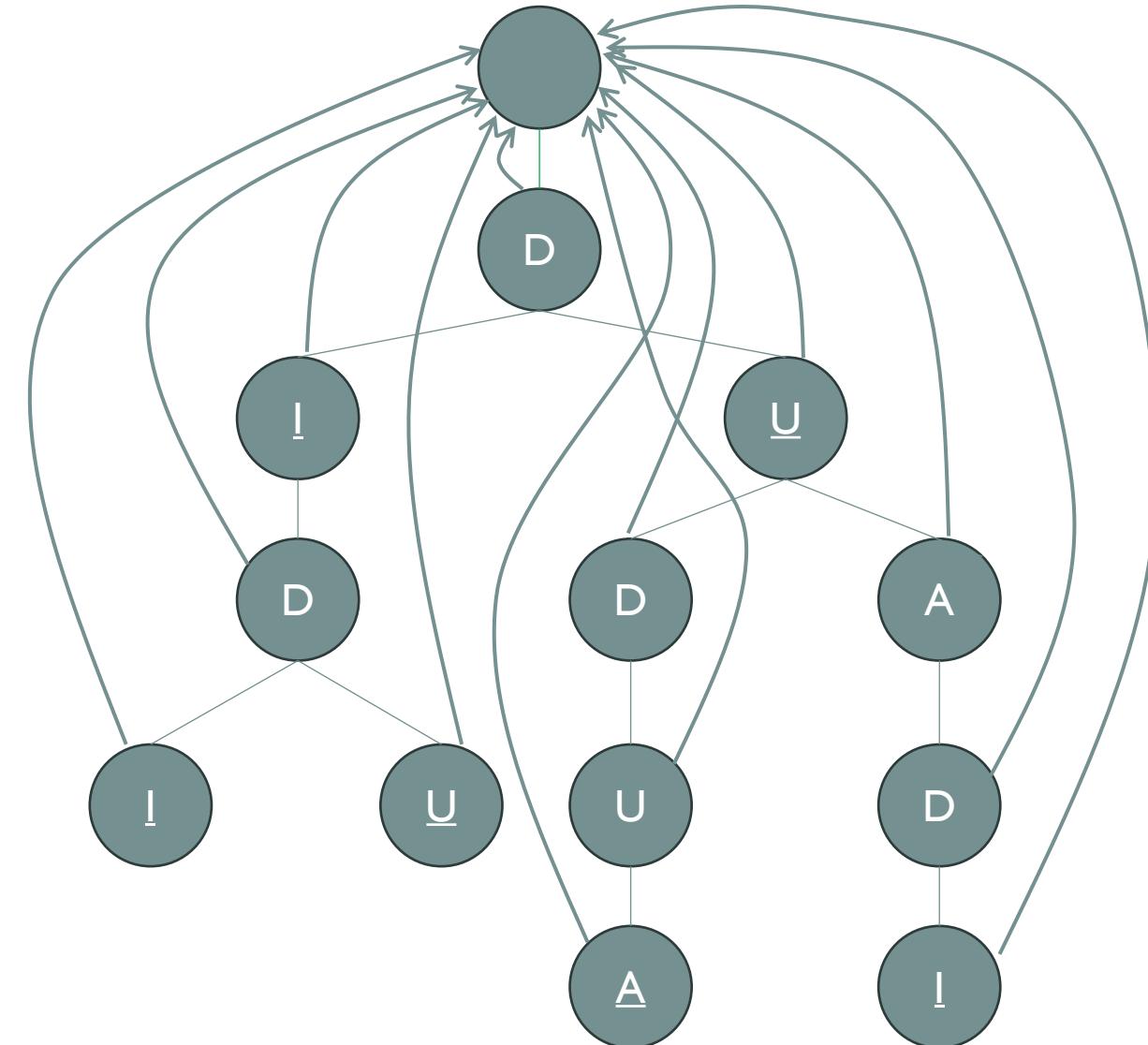
Move to Node 'I'



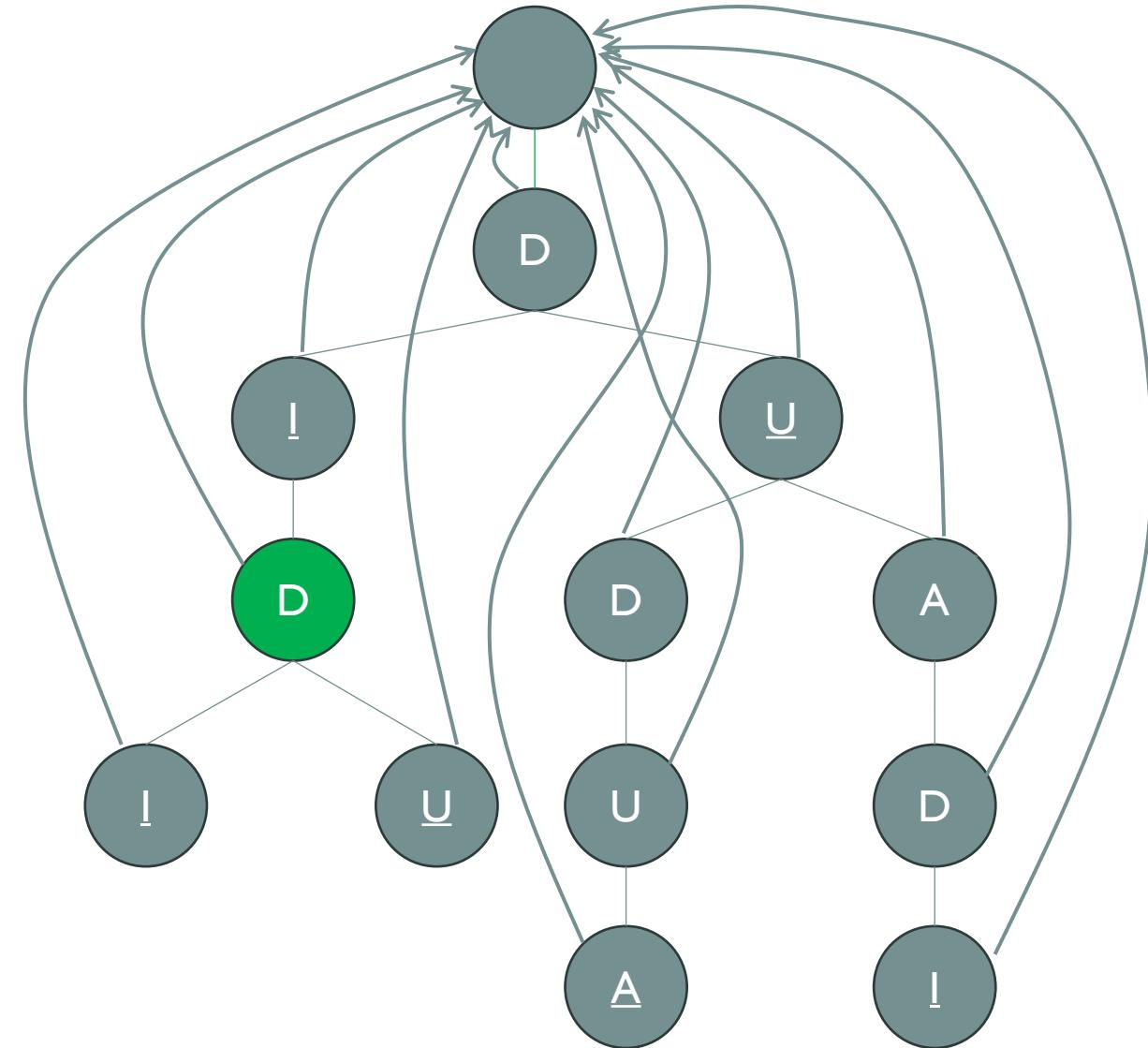
The parent of node 'I' is node 'D' and the link of that node point to the root.  
The root don't have child as same node 'I' so we do nothing



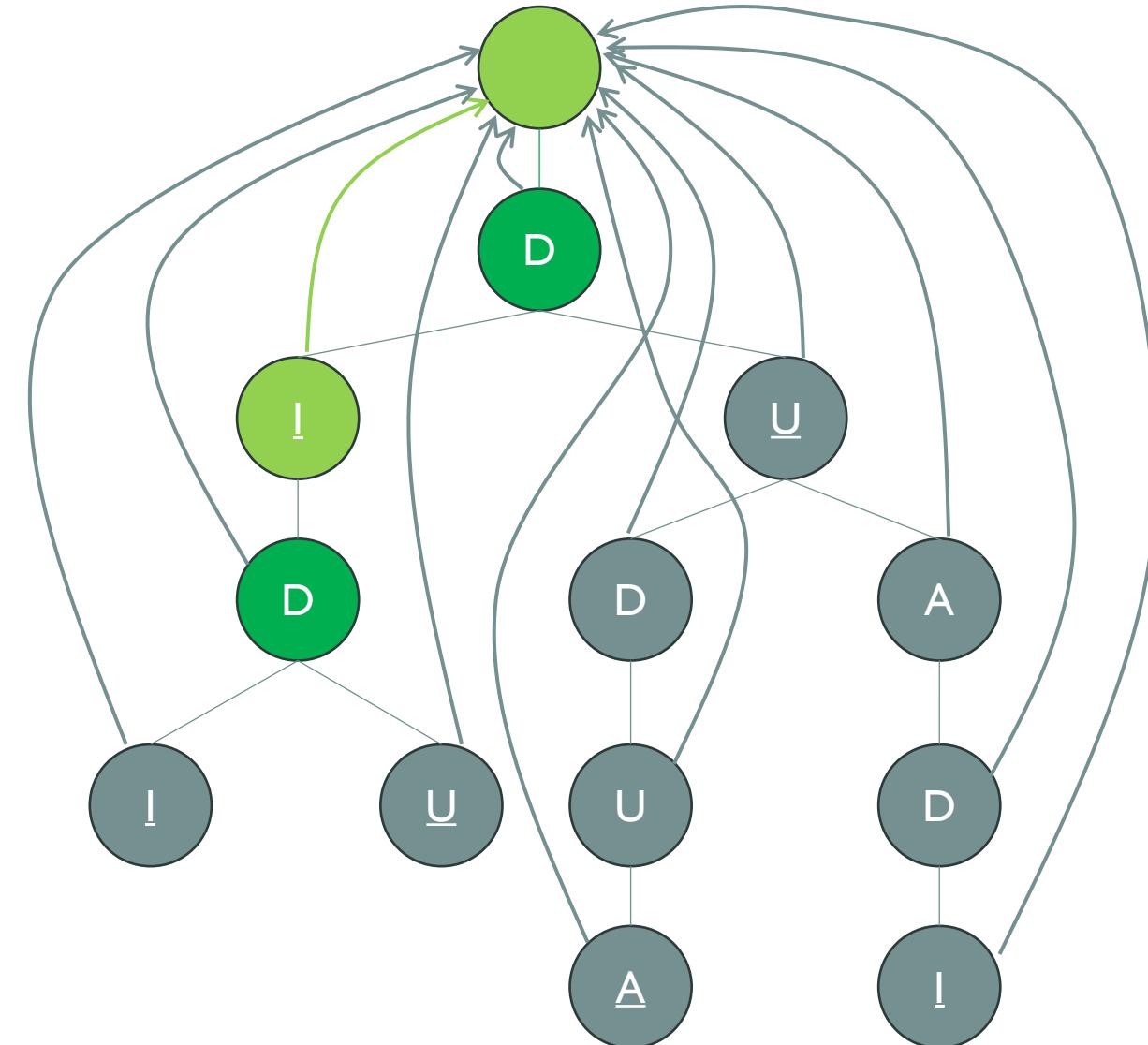
The parent of node 'I' is node 'D' and the link of that points to the root.  
The root don't have child as same node 'I' so we do nothing



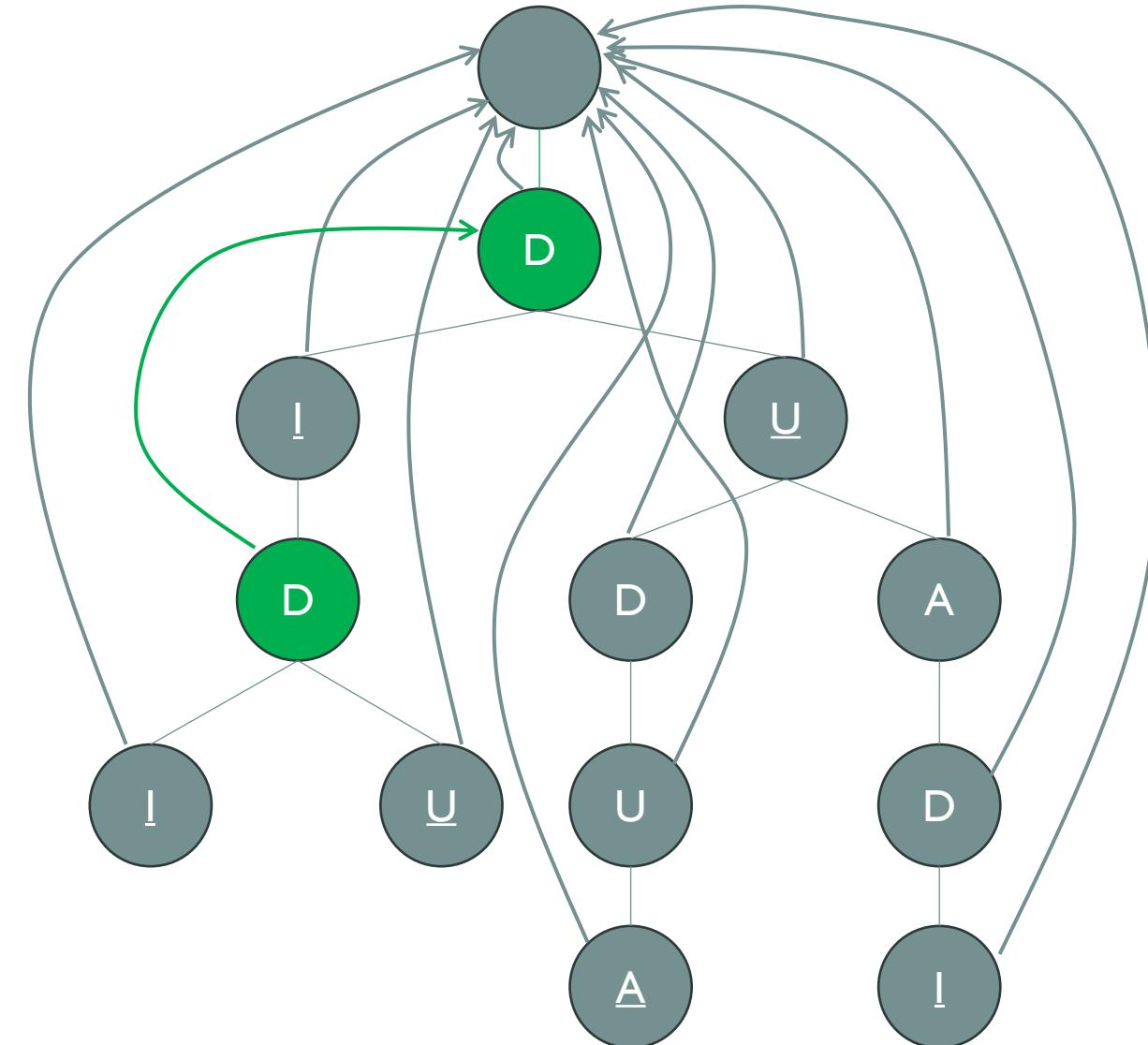
Move to node 'D'



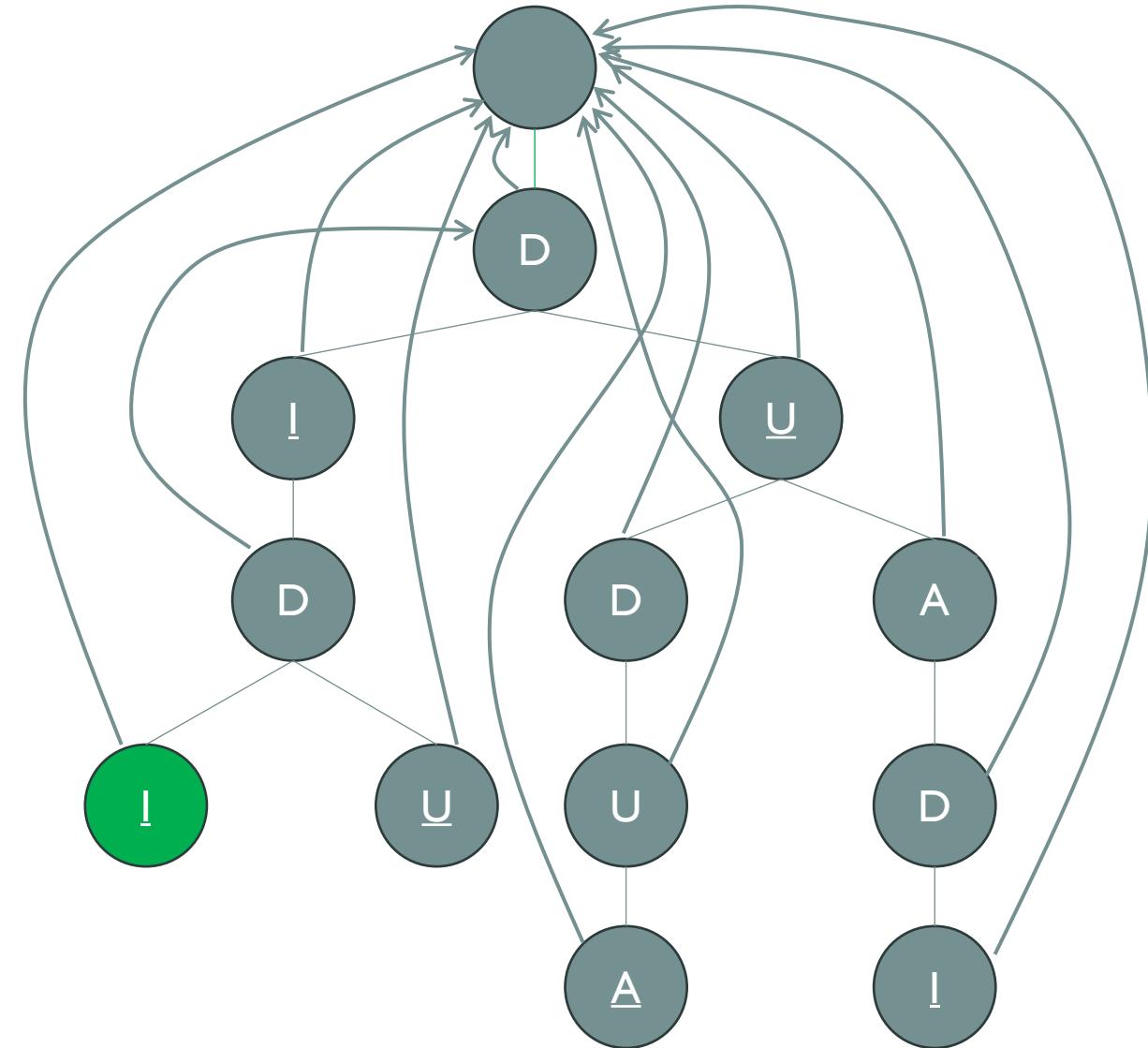
The parent of node 'D' is node 'I' and the link of that points to the root.  
The root has node 'D' as its child node so the link of node 'D' will point to that child node.



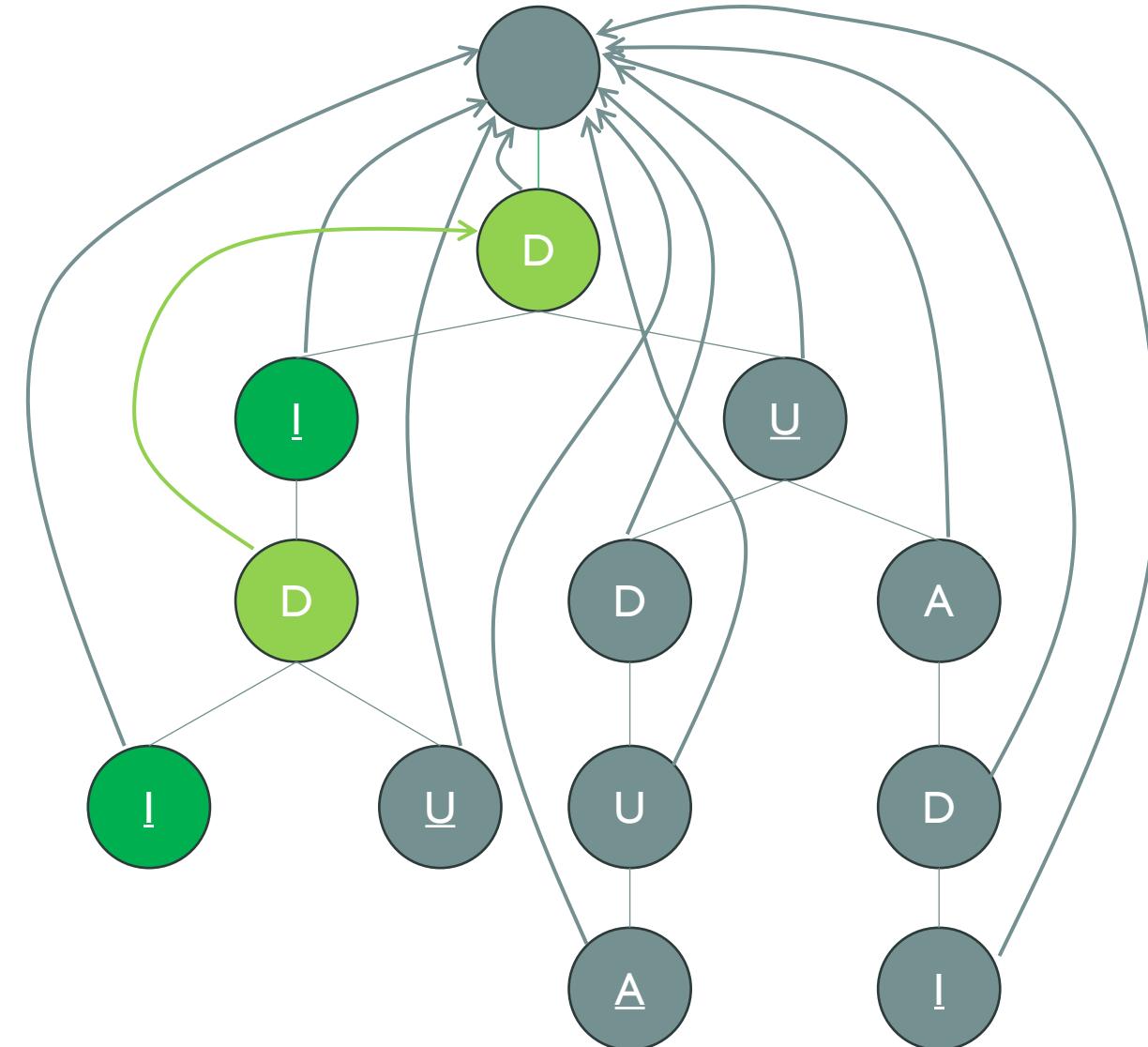
The parent of node 'D' is node 'I' and the link of that points to the root.  
The root has node 'D' as its child node so the link of node 'D' will point to that child node.



Move to node 'I'

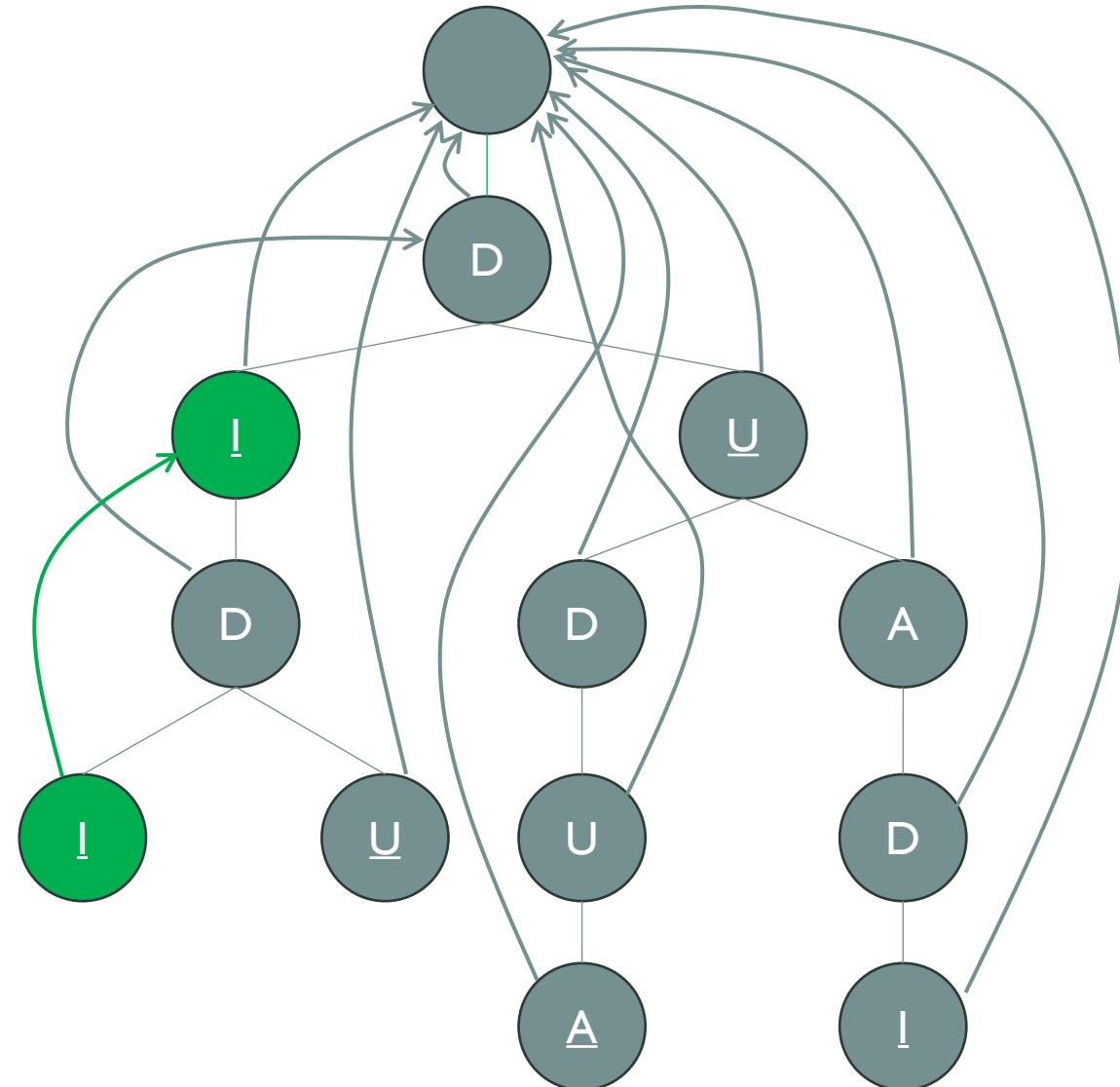


The parent of node 'I' is node 'D' and the link of that points to another node 'D'. That node 'D' has node 'I' as its child node so the link of node 'I' will point to that child node.

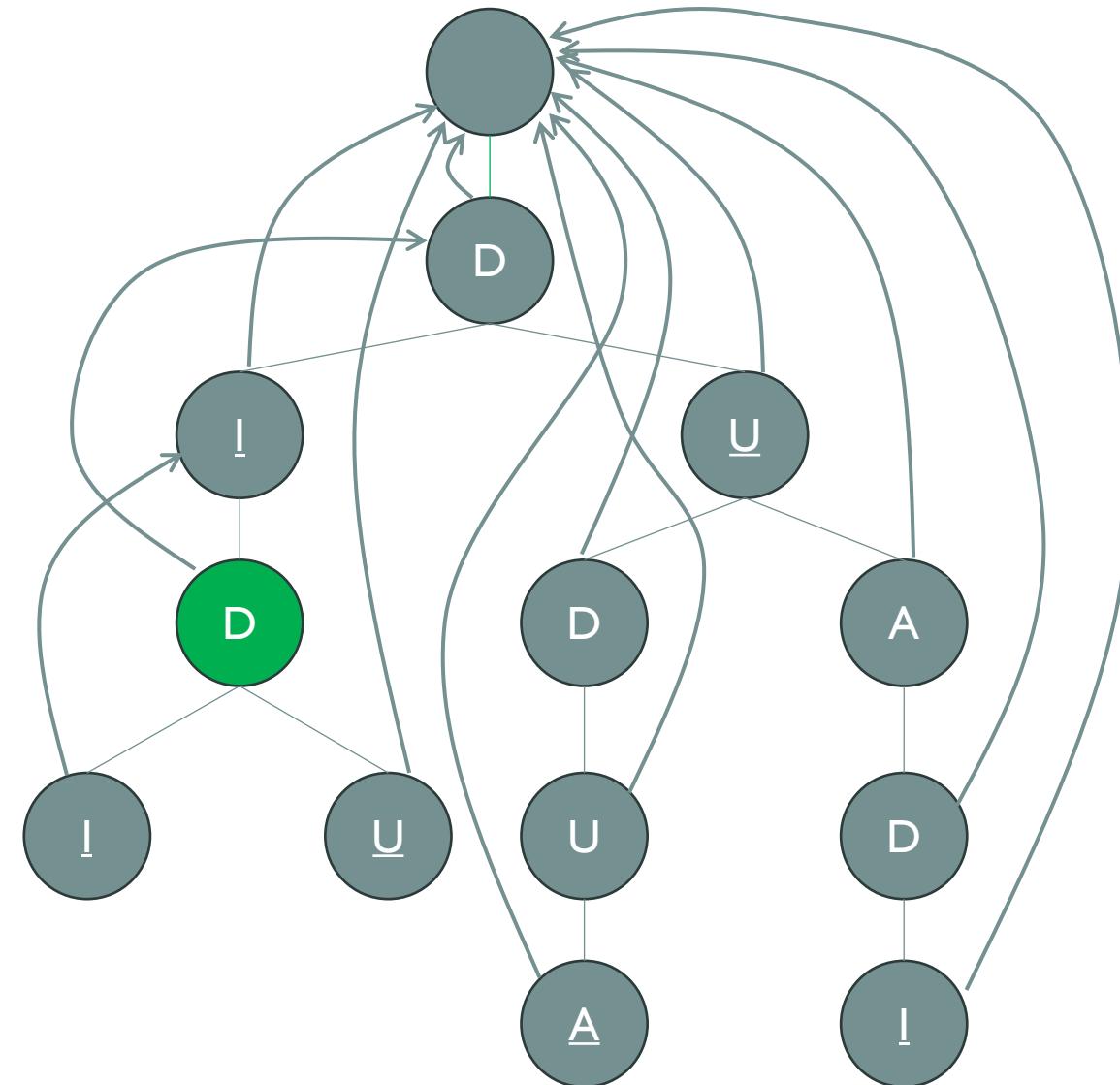


The parent of node 'I' is node 'D' and the link of that node points to another node 'D'.

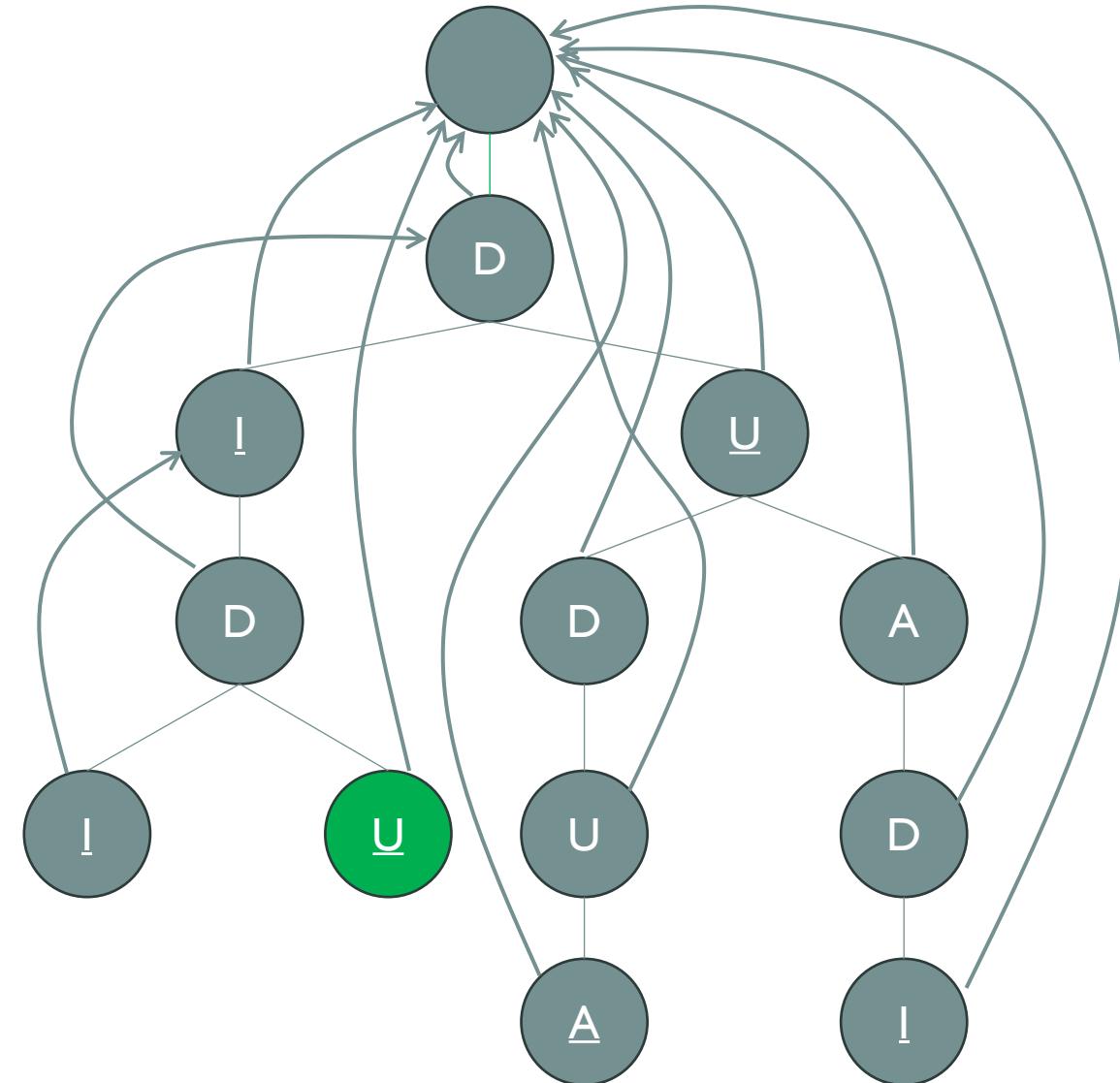
That node 'D' has node 'I' as its child node so the link of node 'I' will point to that child node.



Node 'I' doesn't have any child node so we move back to node 'D' and move to another 'D' 's child node

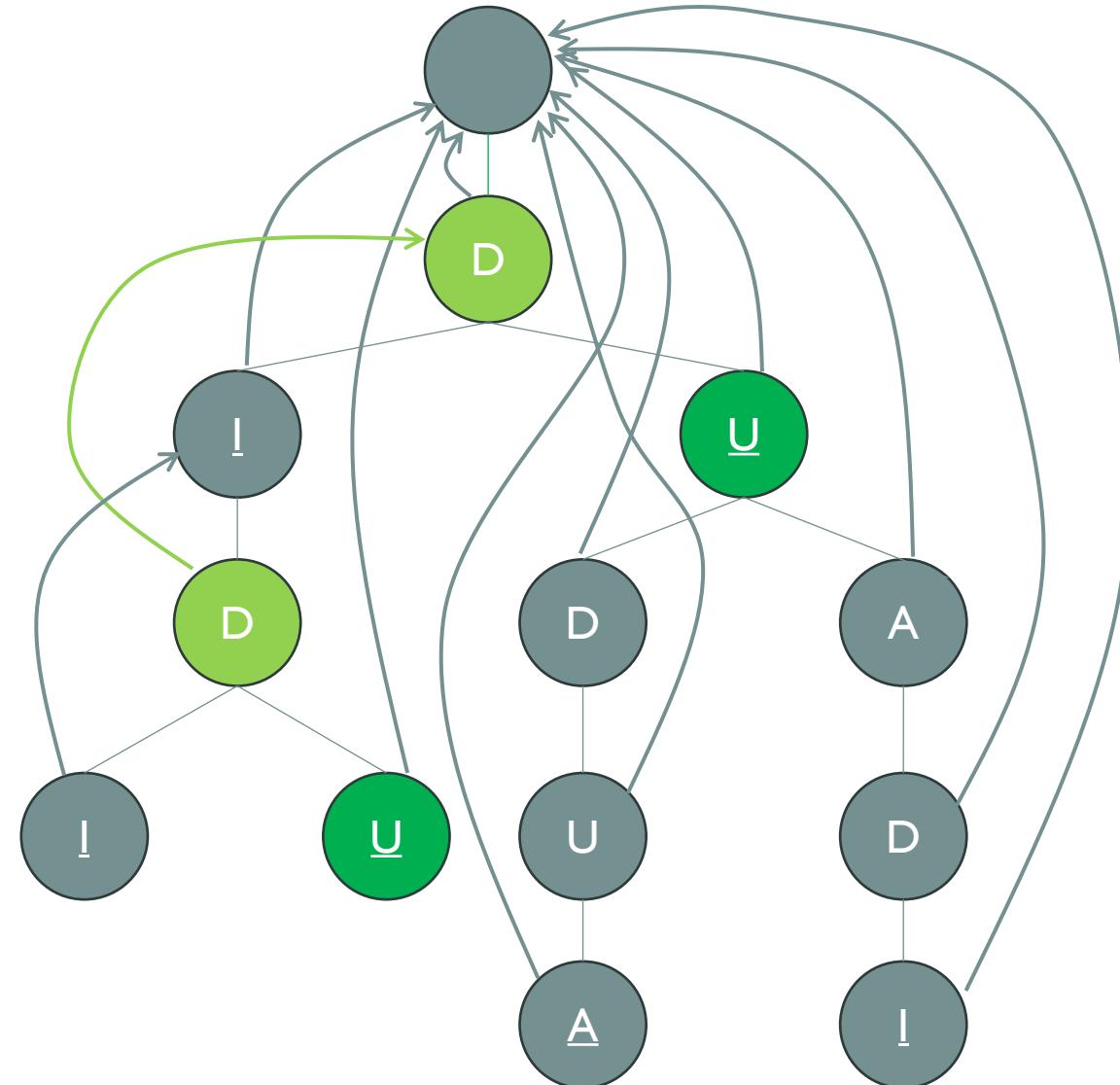


Node 'I' doesn't have any child node so we move back to node 'D' and move to another 'D' 's child node



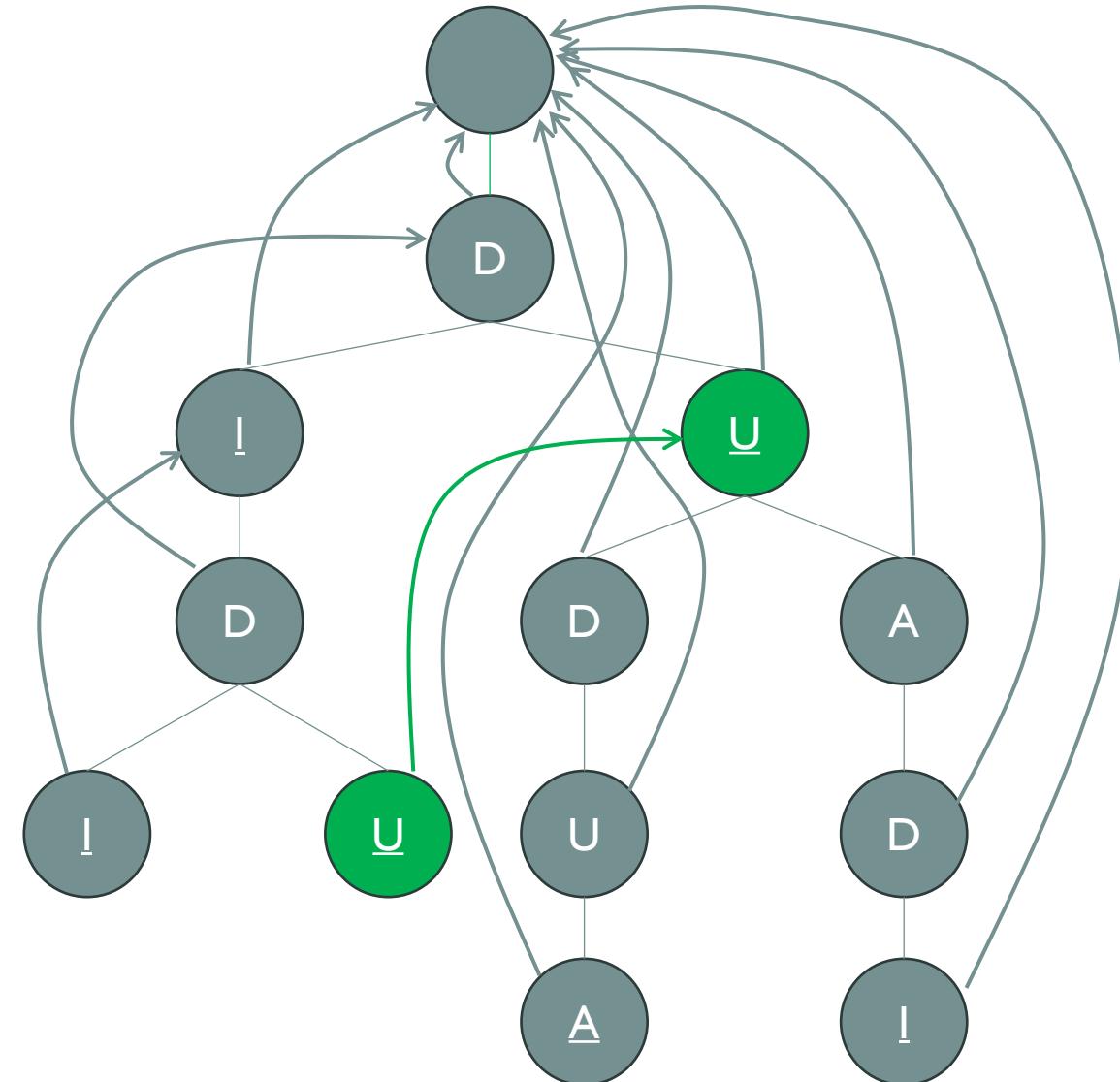
The parent of node 'U' is node 'D' and the link of that node points to another node 'D'.

That node 'D' has node 'U' as its child node so the link of node 'U' will point to that child node.

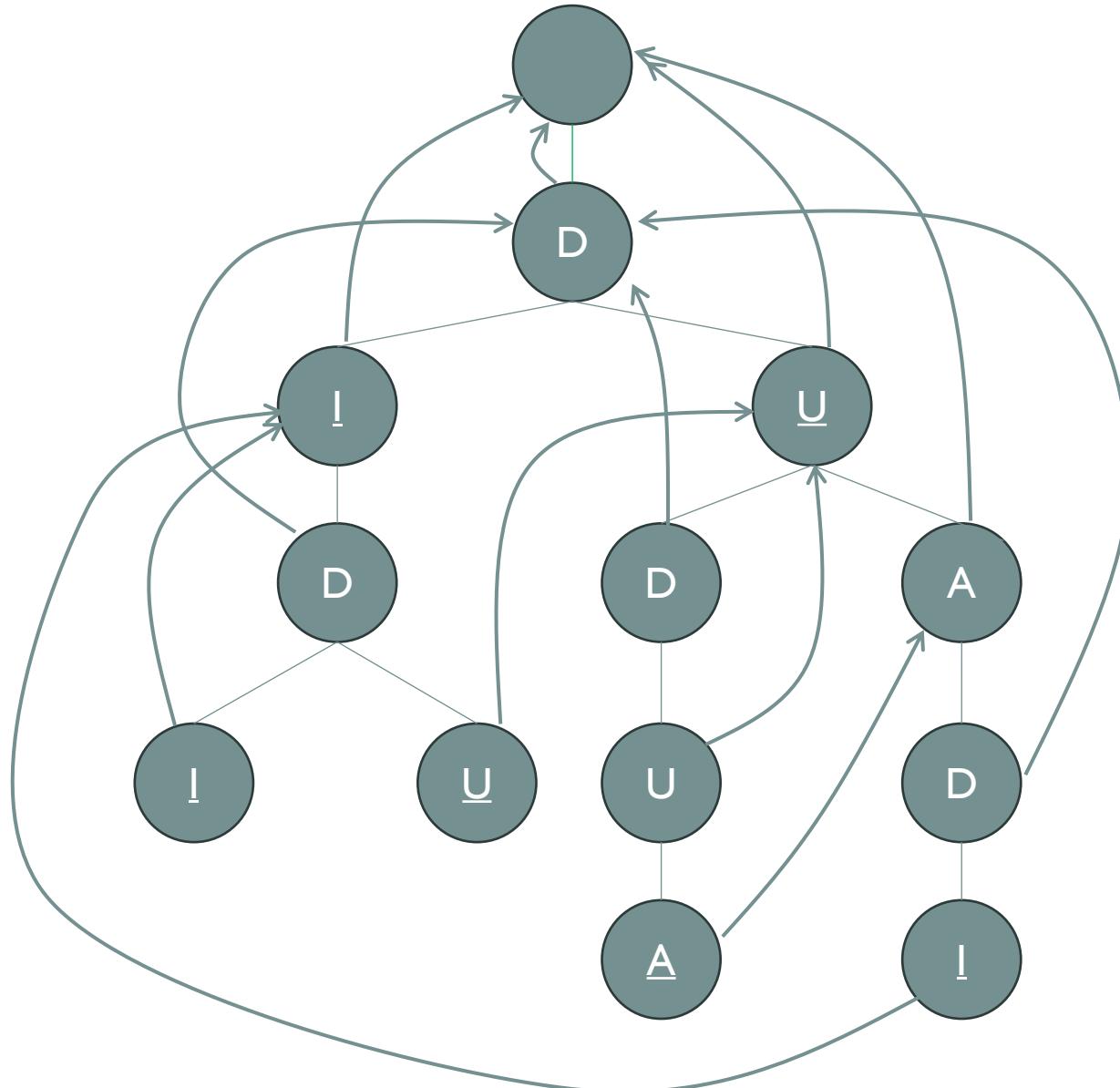


The parent of node 'U' is node 'D' and the link of that node point to another node 'D'.

That node 'D' has node 'U' as its child node so the link of node 'U' will point to that node.



Continue with other nodes,  
we will have a trie tree with  
links like this picture



# SEARCHING / MATCHING PROCESS

## SEARCHING/ MATCHING PROCESS

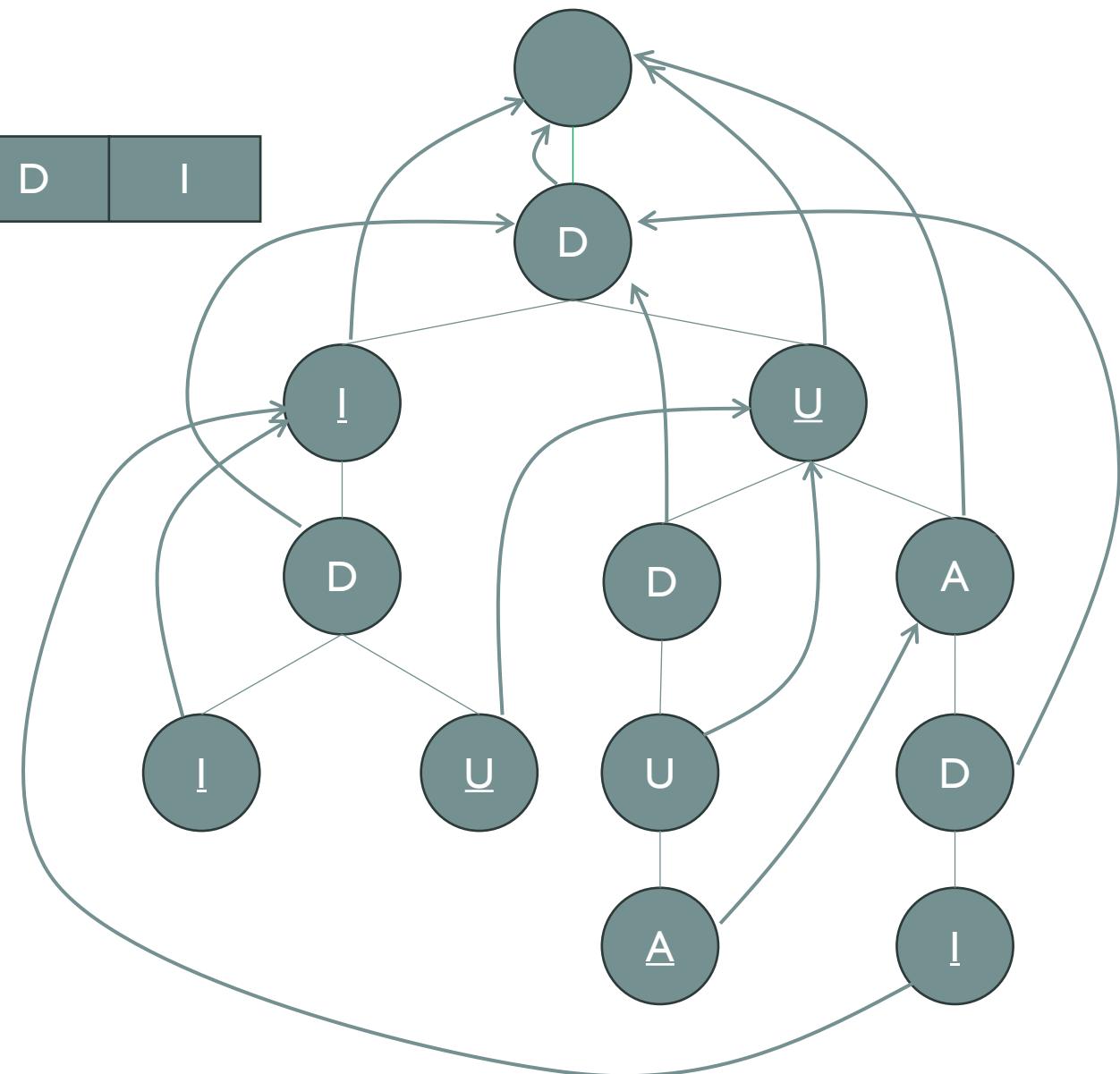
STEP 1: At the  $i$ th position of the text, if the current node has a child node as same as that character, we travel through that child node. Otherwise, we trace back to other prefix by using the link to check other child node until there is a child node as same at the character.

STEP 2: After travel to that node, we use the link to trace back to all the prefix to assure all the pattern have been checked.

The text: “DIDUDUADI”

D	I	D	U	D	U	A	D	I
---	---	---	---	---	---	---	---	---

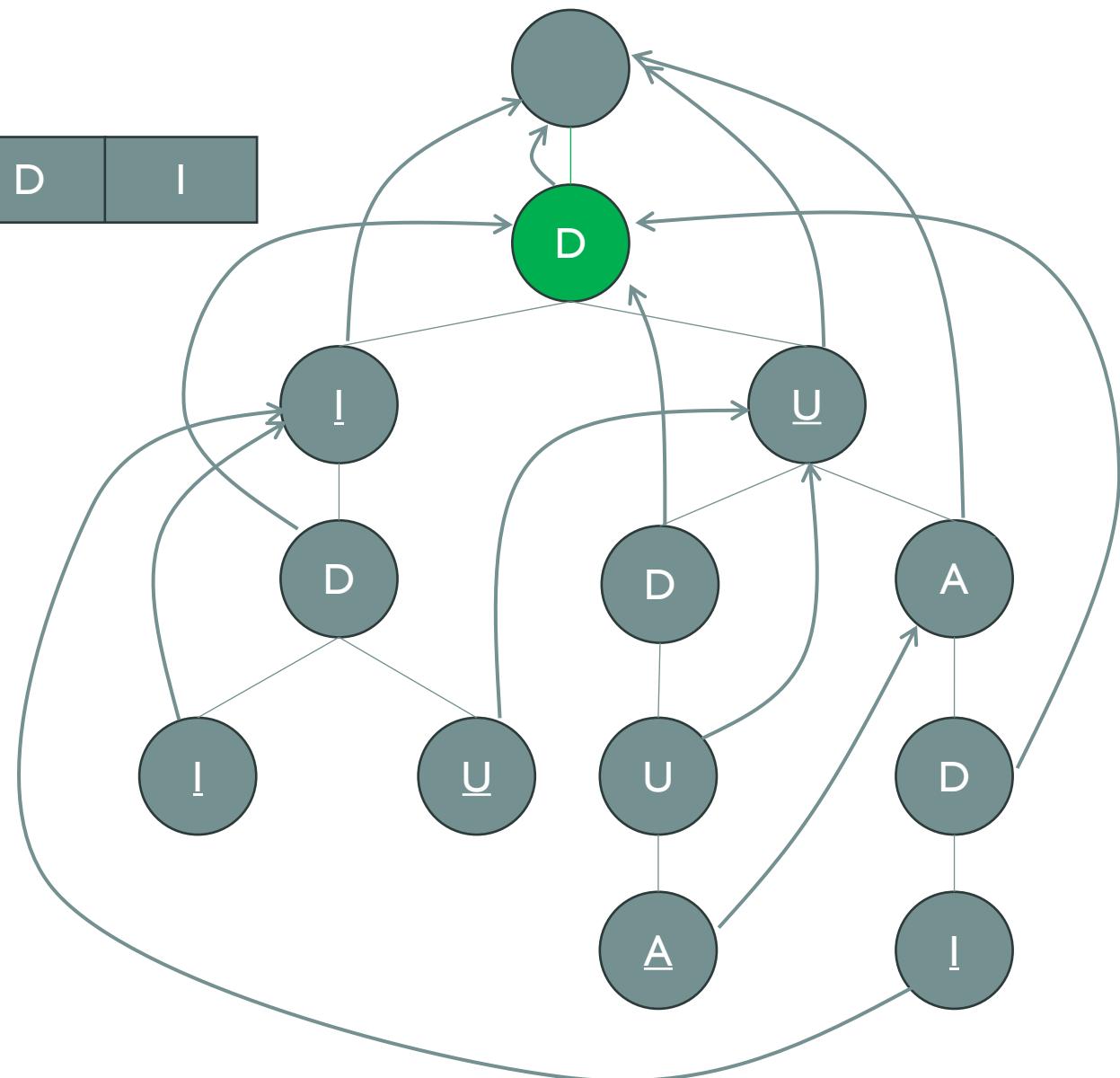
The pattern strings occur in the text:



The text: “DIDUDUADI”

D	I	D	U	D	U	A	D	I
---	---	---	---	---	---	---	---	---

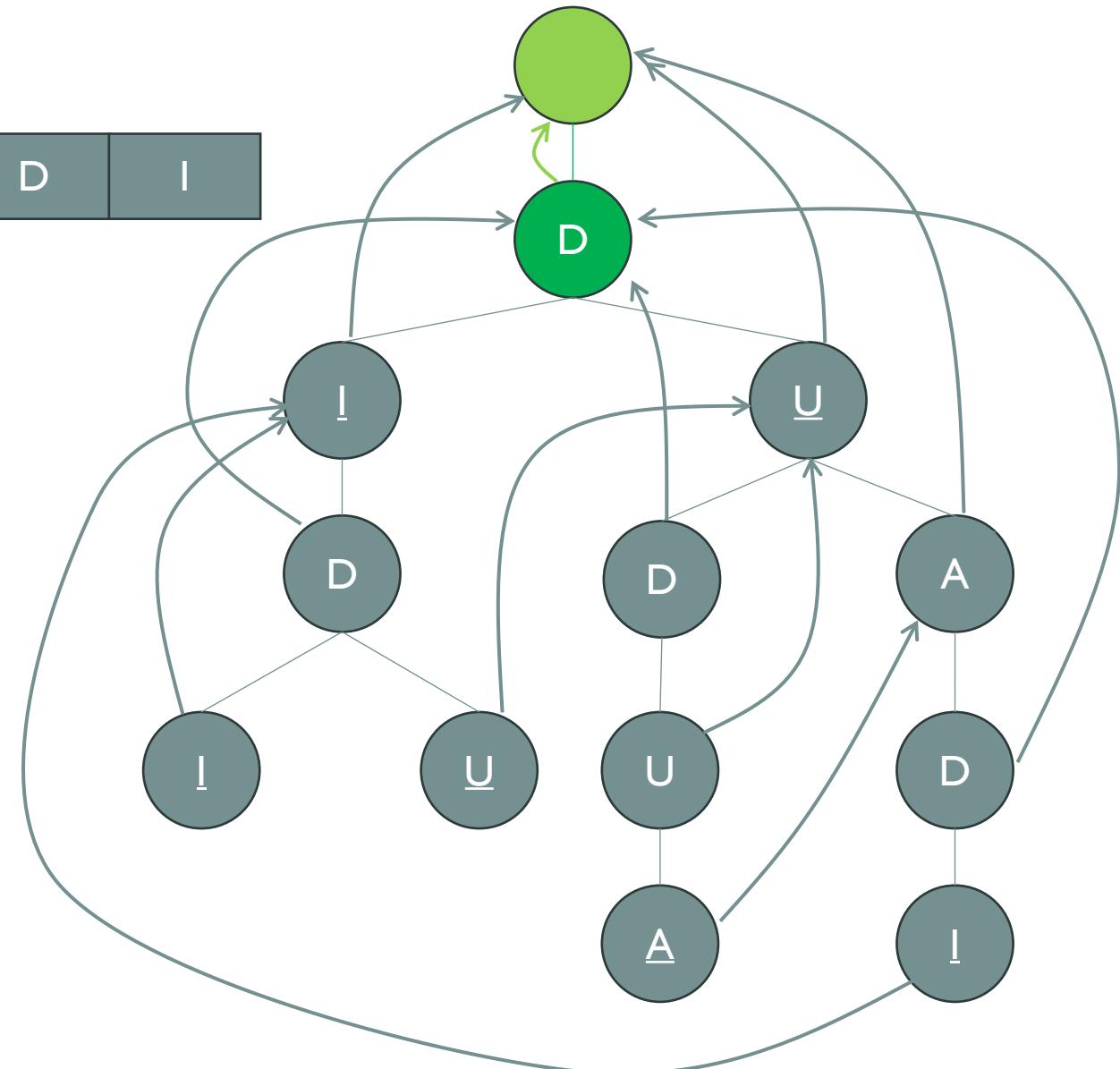
The pattern strings occur in the text:



The text: “DIDUDUADI”

D	I	D	U	D	U	A	D	I
---	---	---	---	---	---	---	---	---

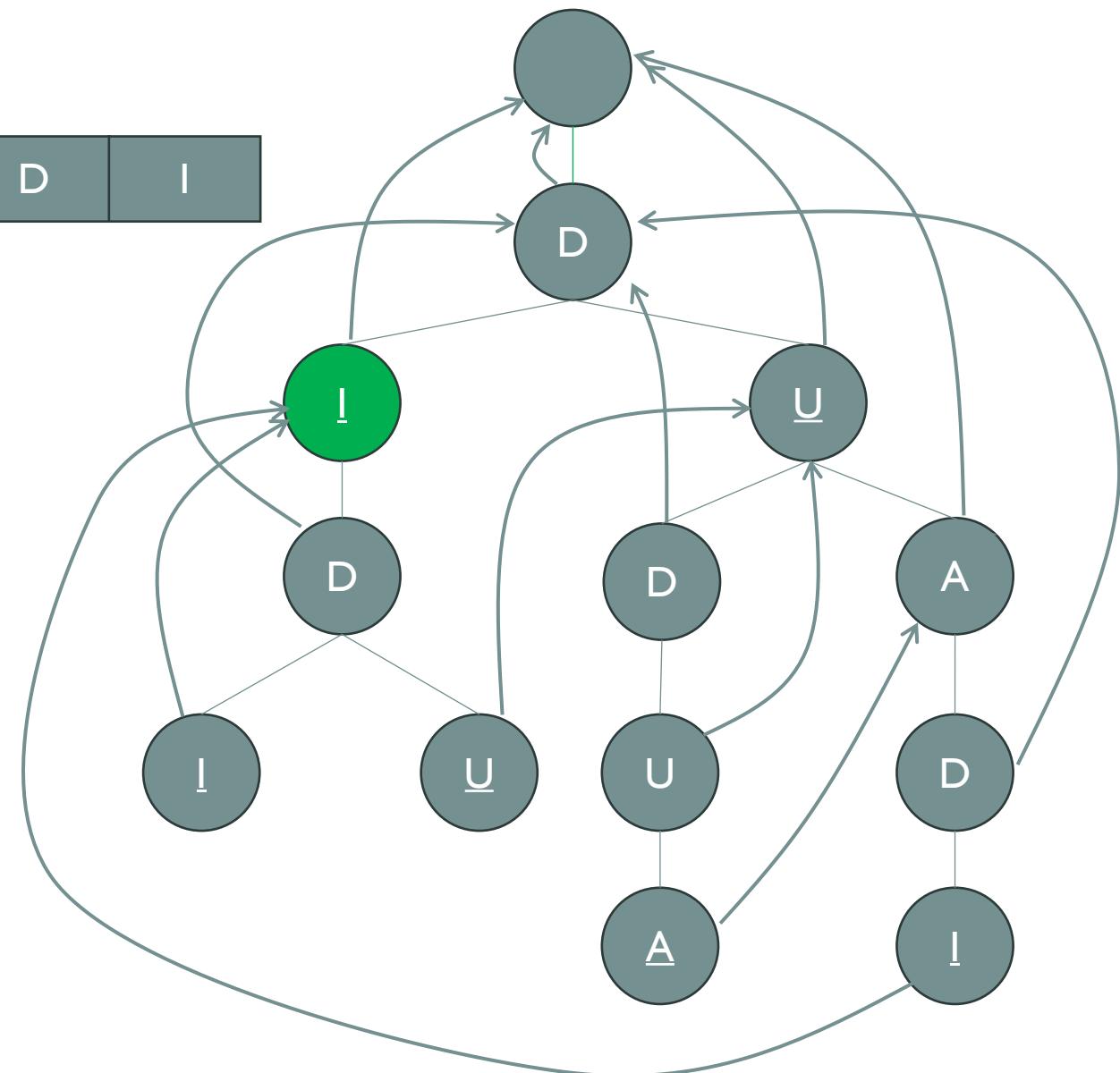
The pattern strings occur in the text:



The text: “DIDUDUADI”



The pattern strings occur in the text:

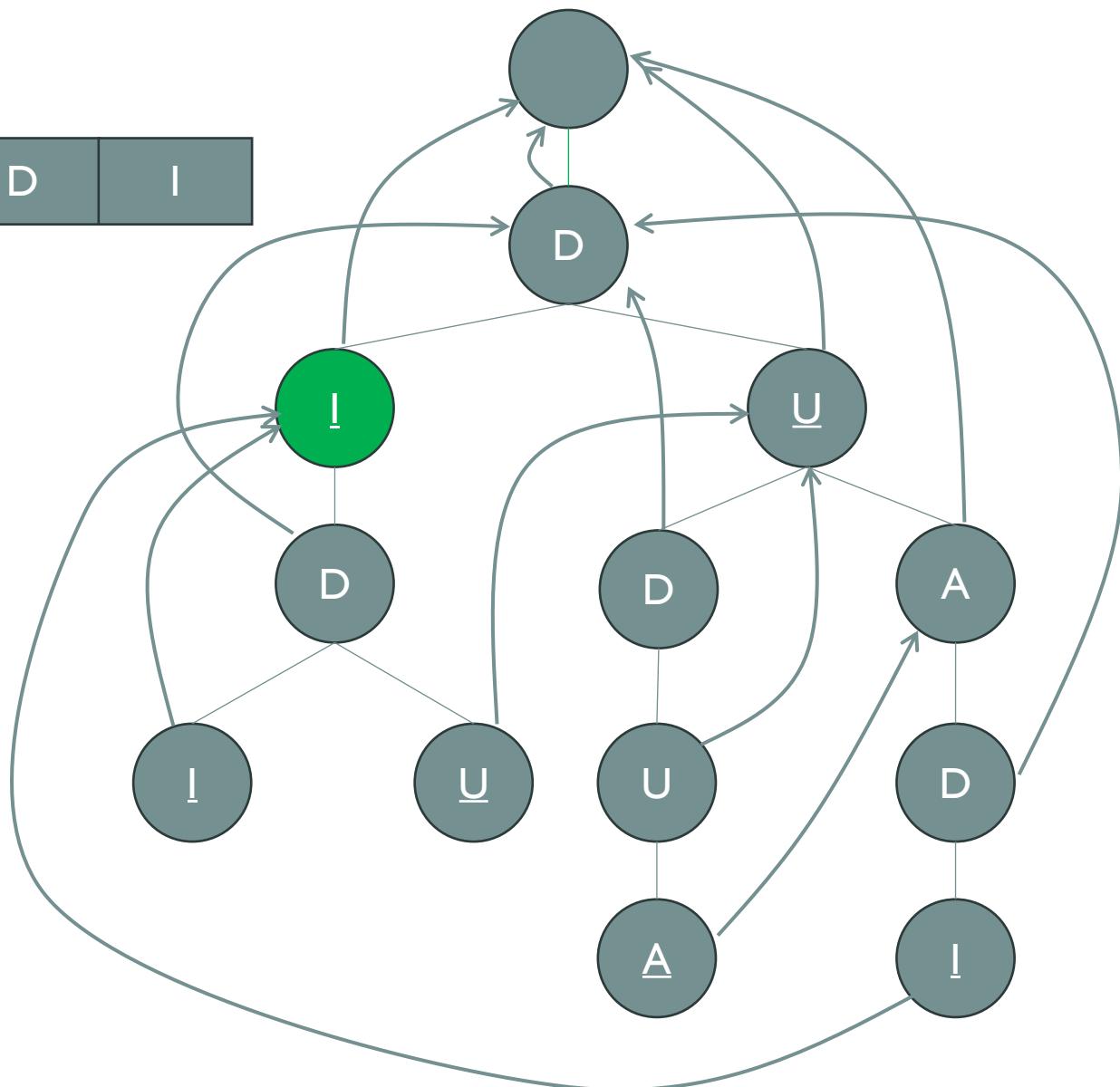


# The text: “DIDUDUADI”



# The pattern strings occur in the text:

DI

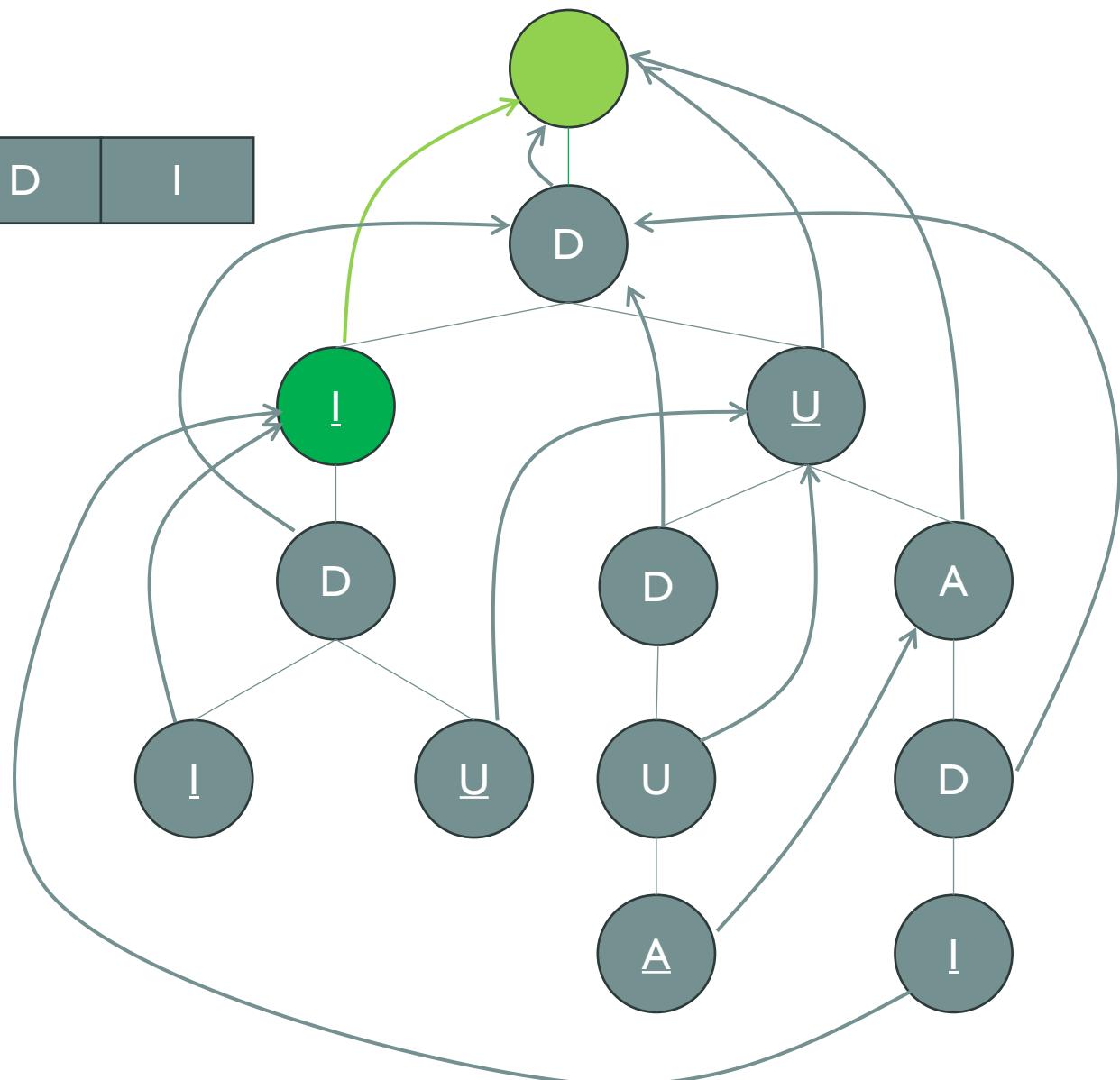


The text: “DIDUDUADI”



The pattern strings occur in the text:

DI

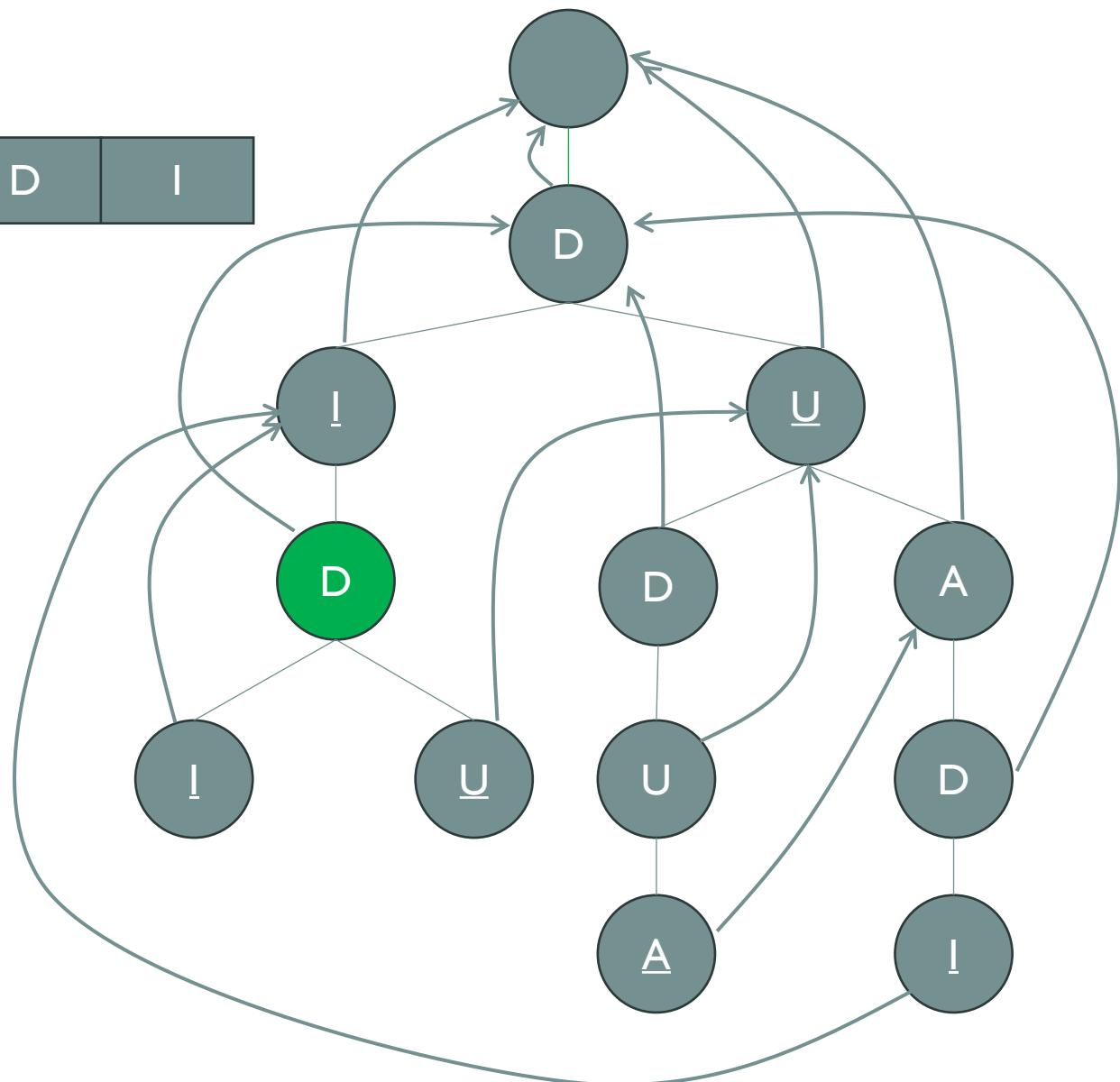


The text: “DIDUDUADI”



The pattern strings occur in the text:

DI

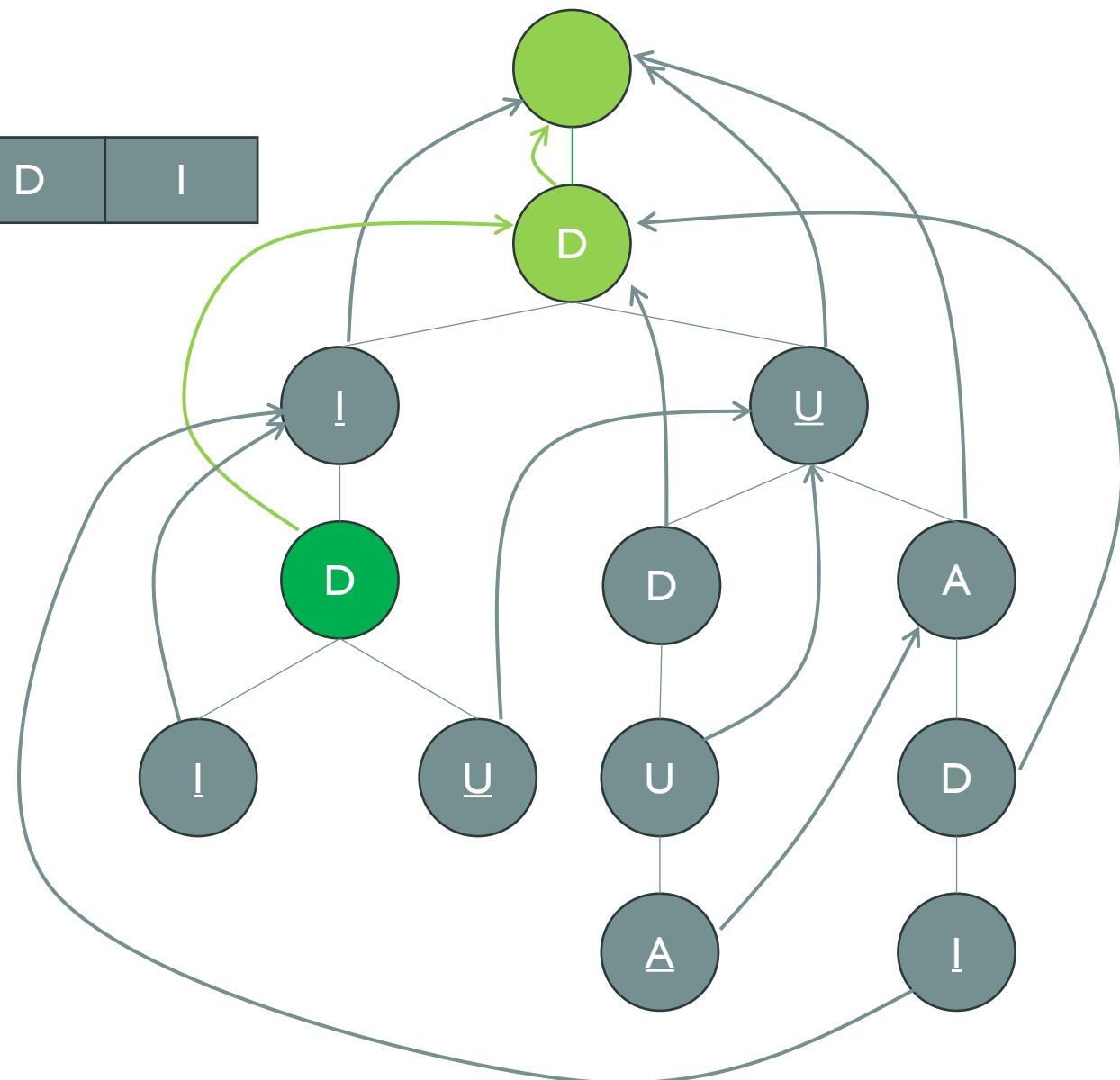


The text: “DIDUDUADI”



The pattern strings occur in the text:

DI

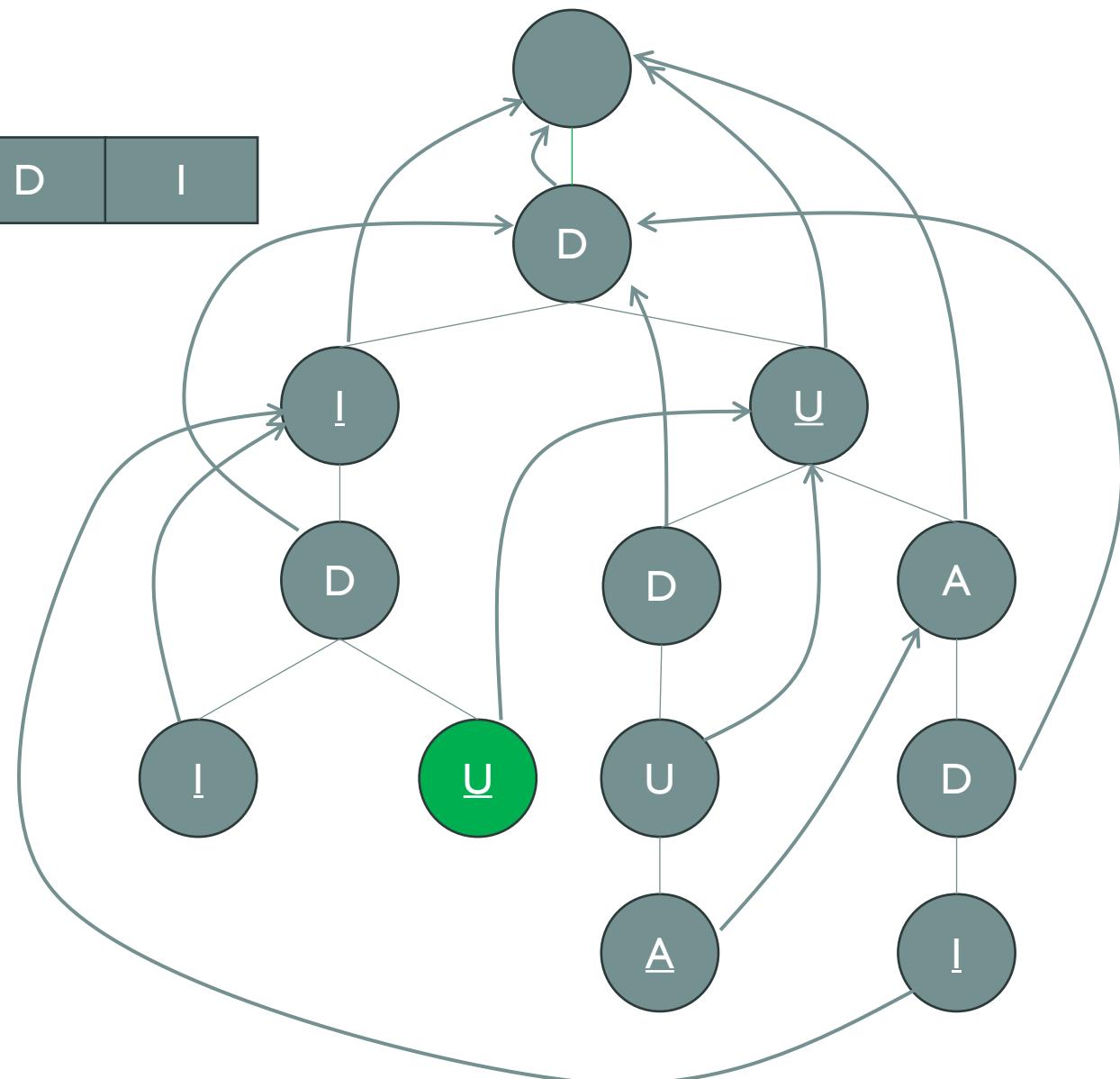


The text: “DIDUDUADI”



The pattern strings occur in the text:

DI



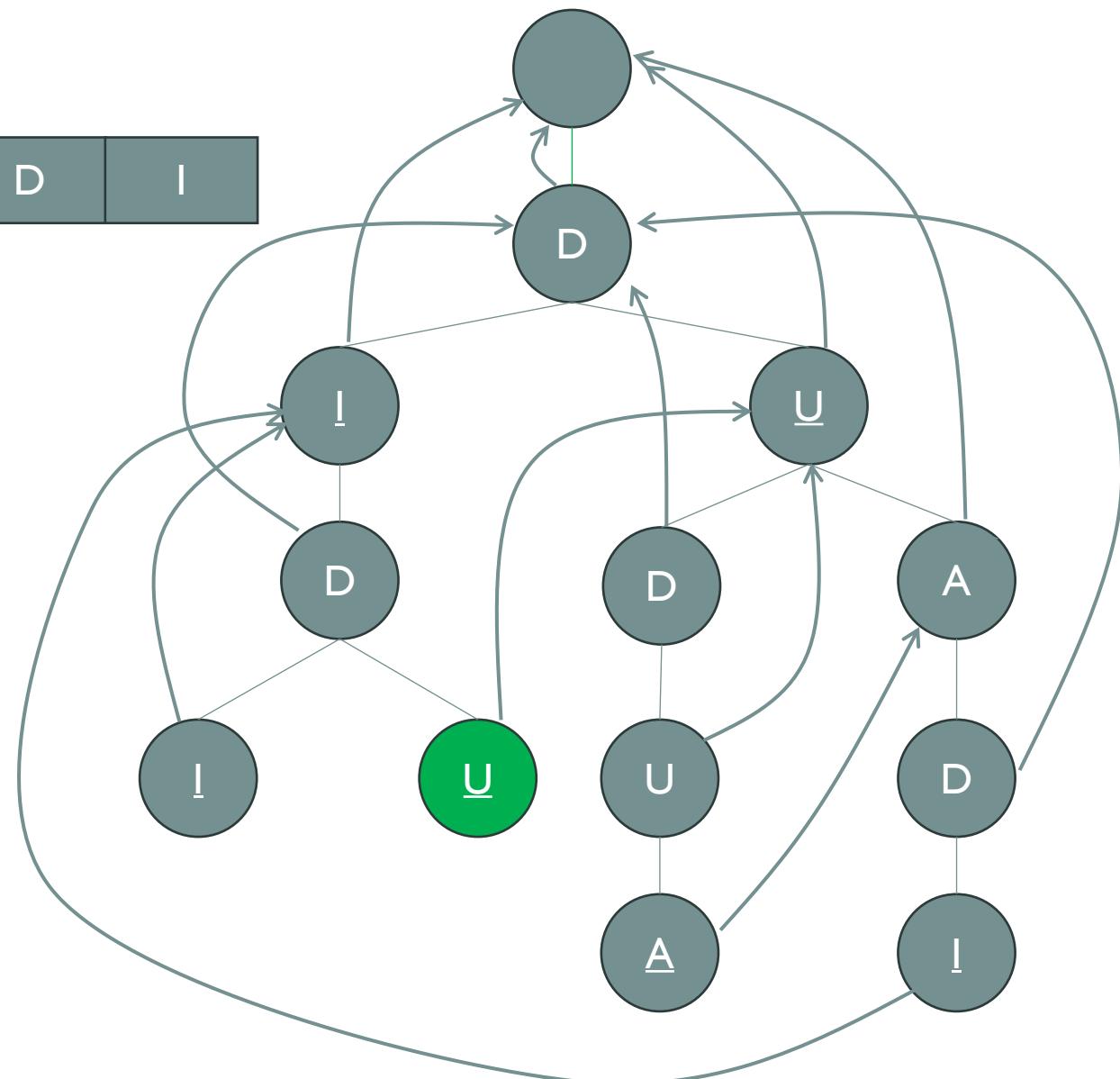
The text: “DIDUDUADI”



The pattern strings occur in the text:

DI

DIDU



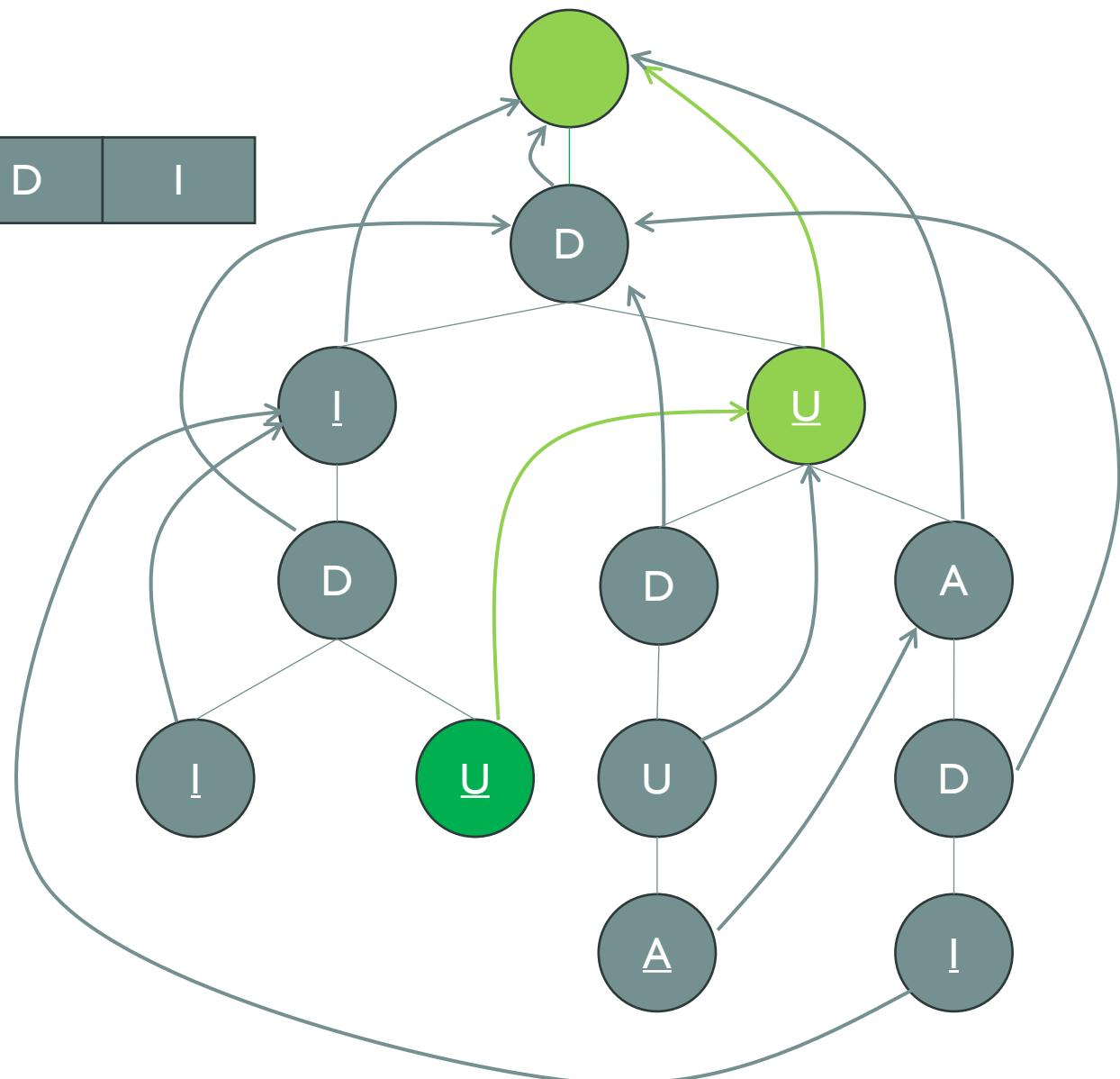
The text: “DIDUDUADI”



The pattern strings occur in the text:

DI

DIDU



The text: “DIDUDUADI”

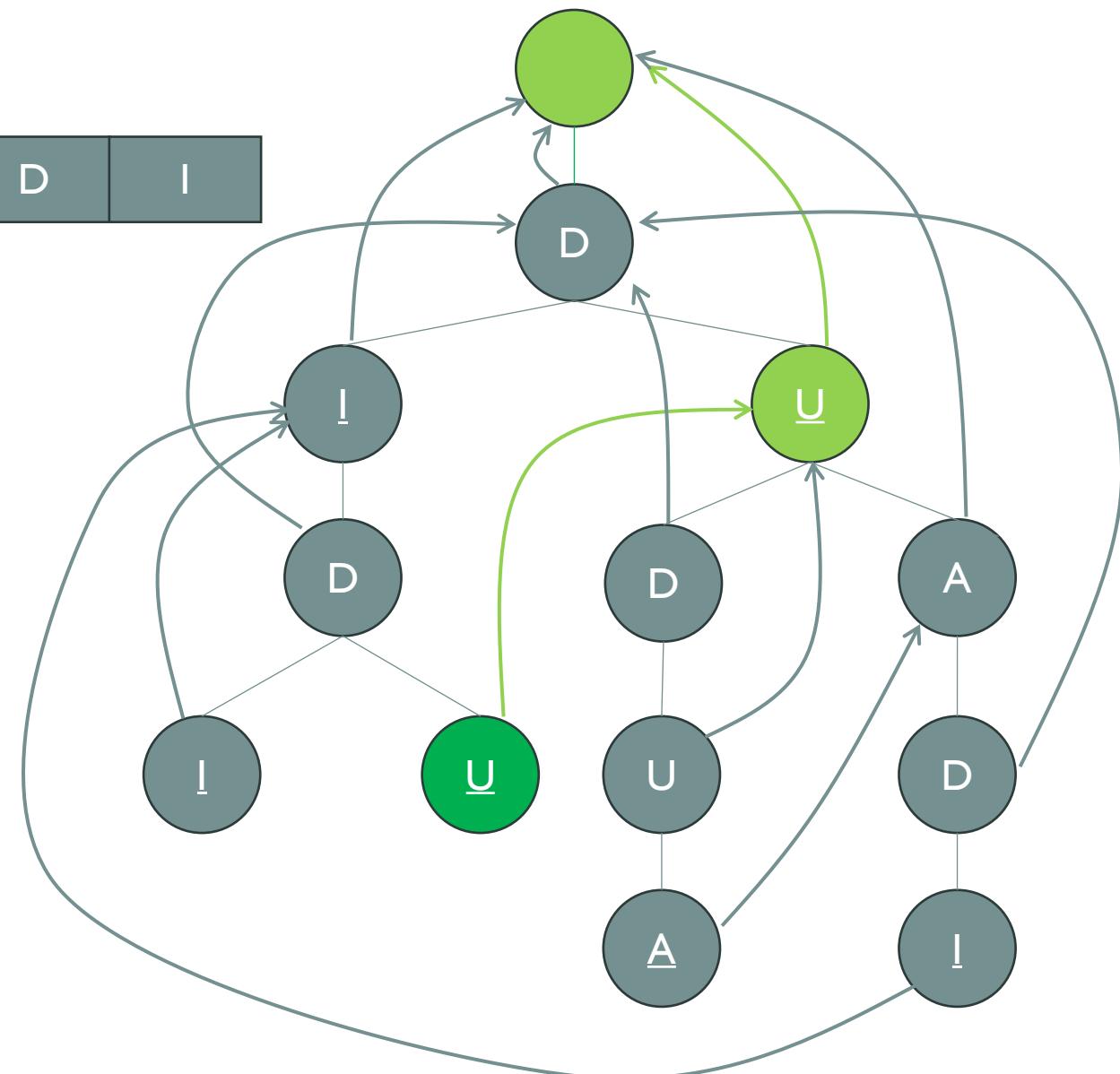


The pattern strings occur in the text:

DI

DIDU

DU



The text: “DIDUDUADI”

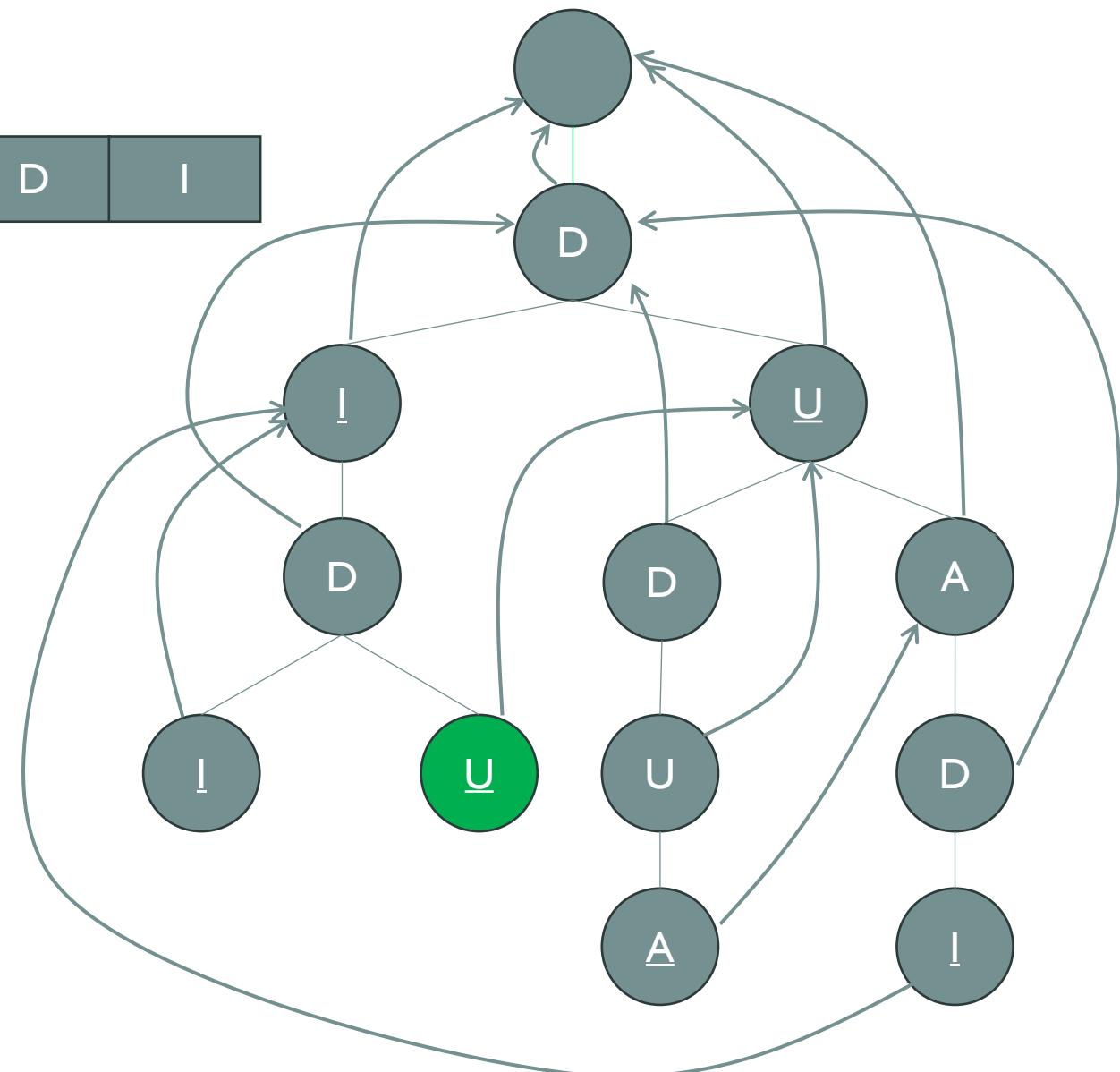


The pattern strings occur in the text:

DI

DIDU

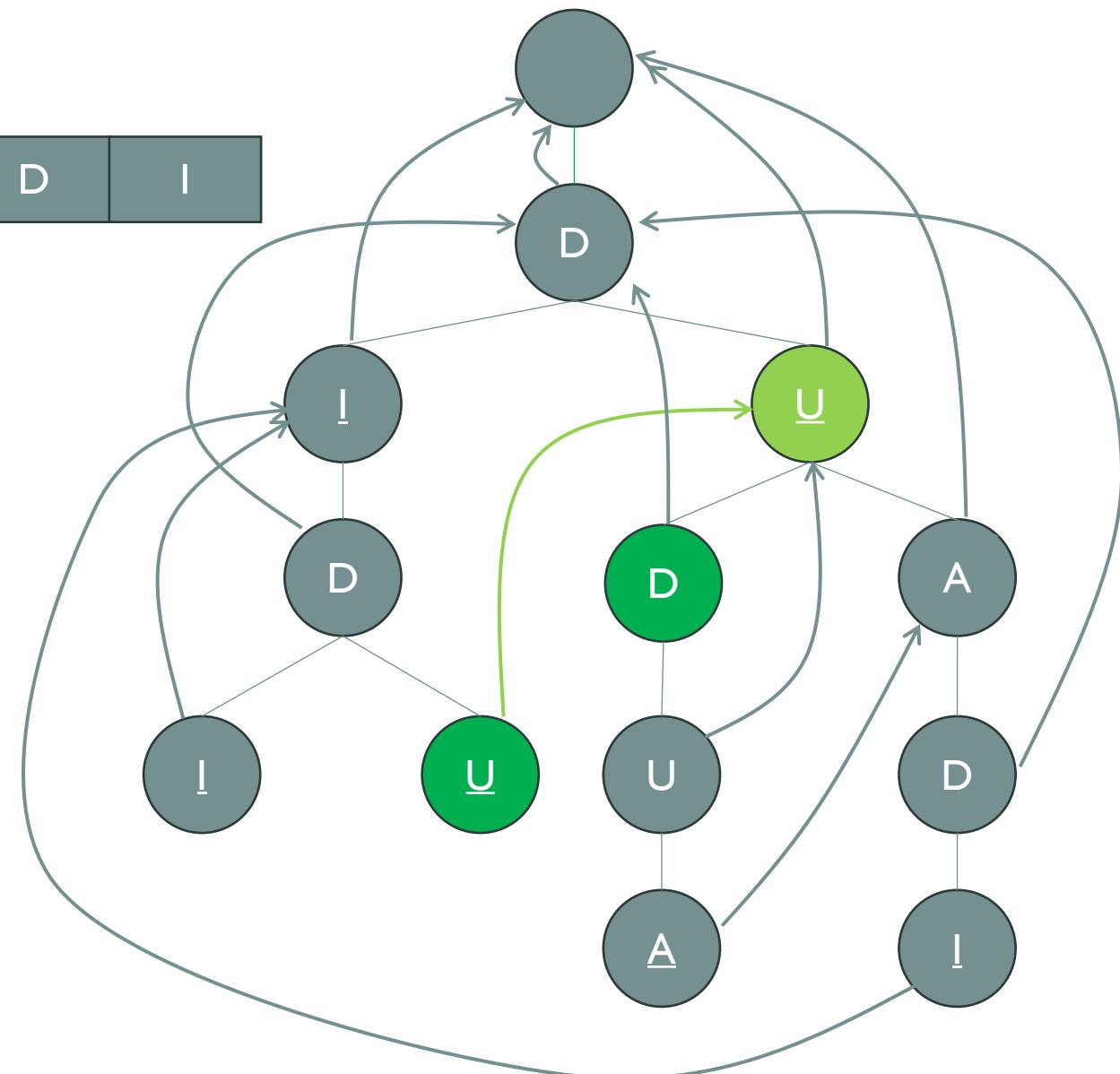
DU



# The text: “DIDUDUADI”



# The pattern strings occur in the text:



The text: “DIDUDUADI”

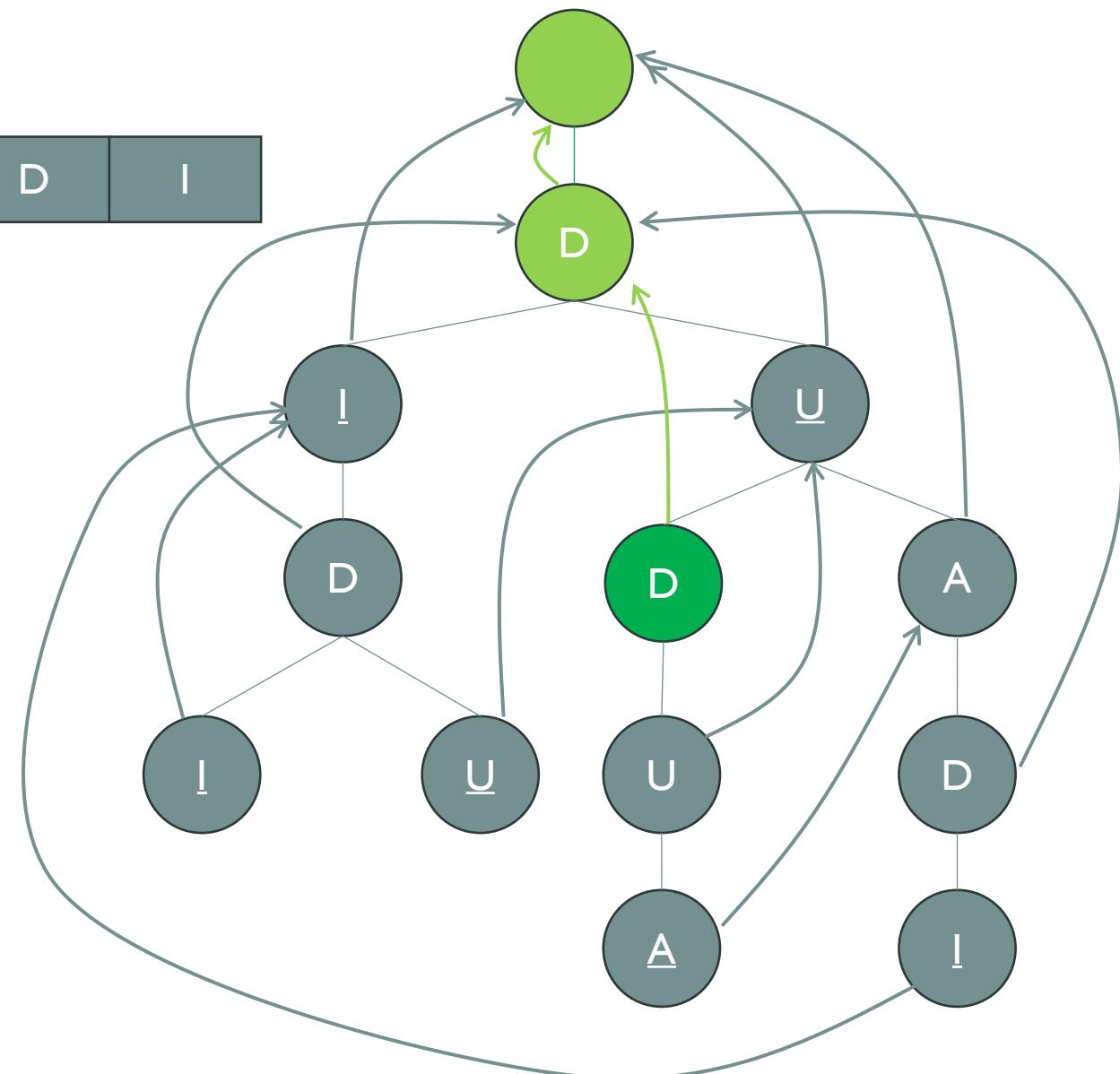


The pattern strings occur in the text:

DI

DIDU

DU



The text: “DIDUDUADI”

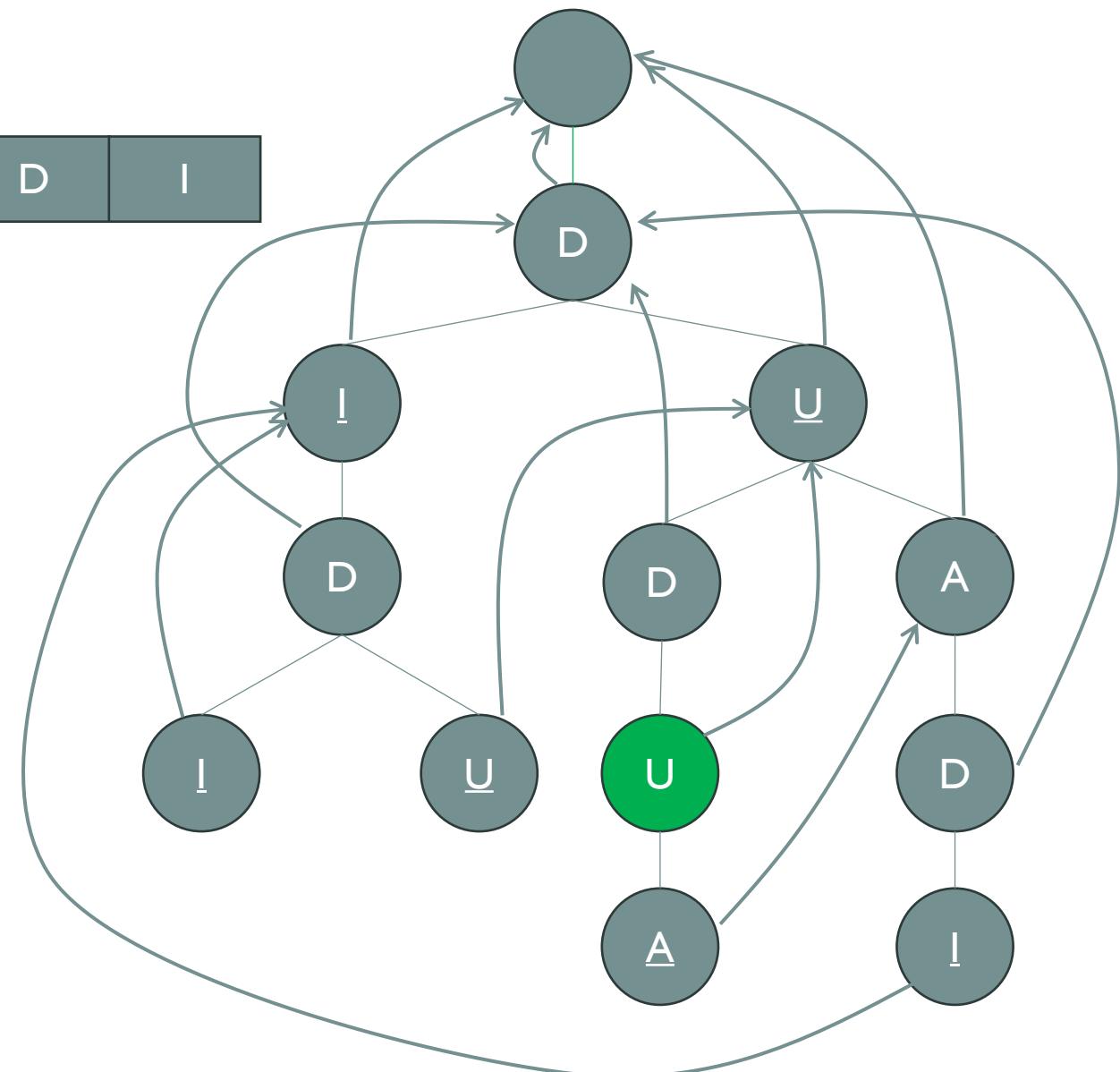


The pattern strings occur in the text:

DI

DIDU

DU



The text: “DIDUDUADI”

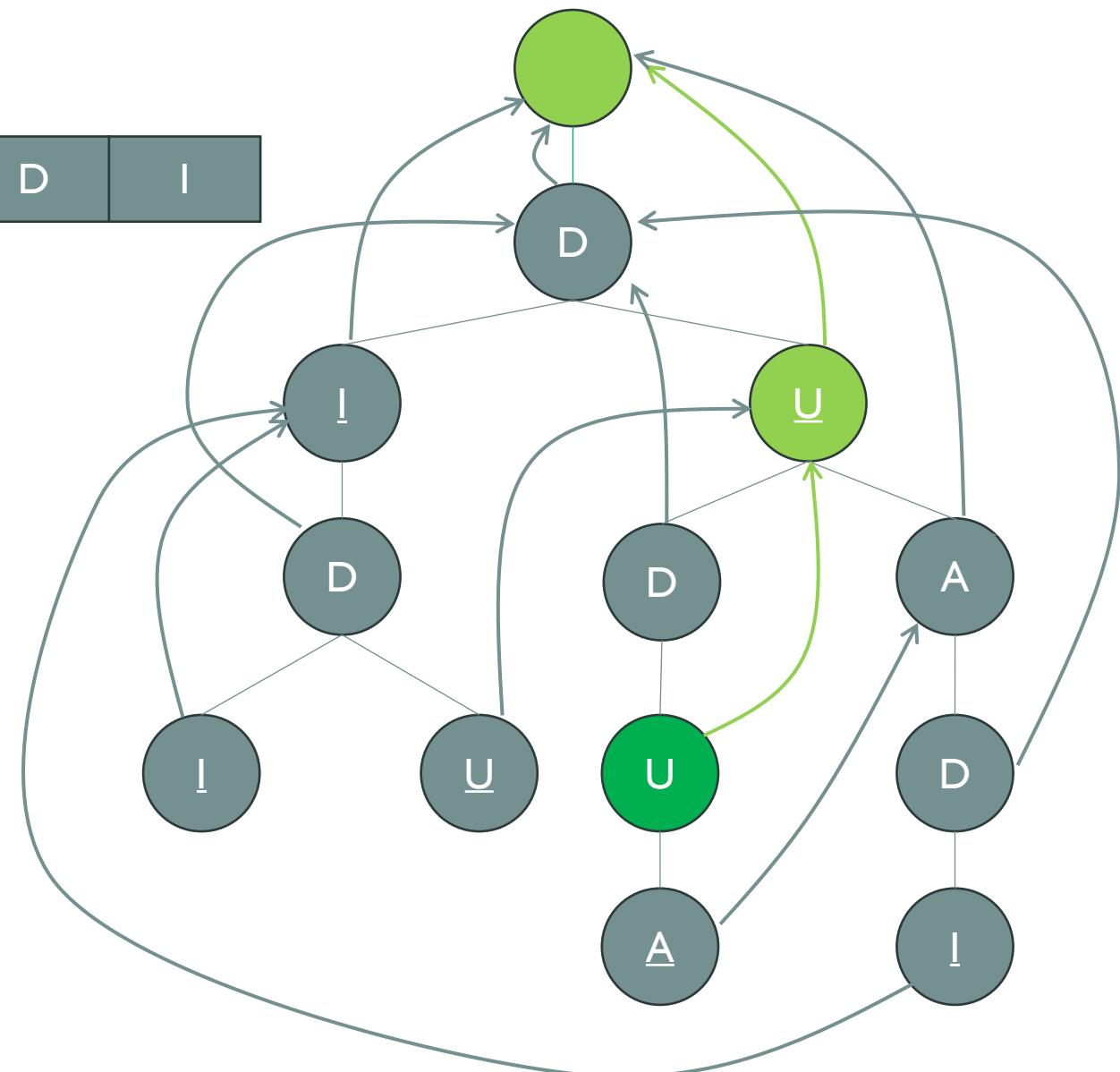


The pattern strings occur in the text:

DI

DIDU

DU



# The text: “DIDUDUADI”



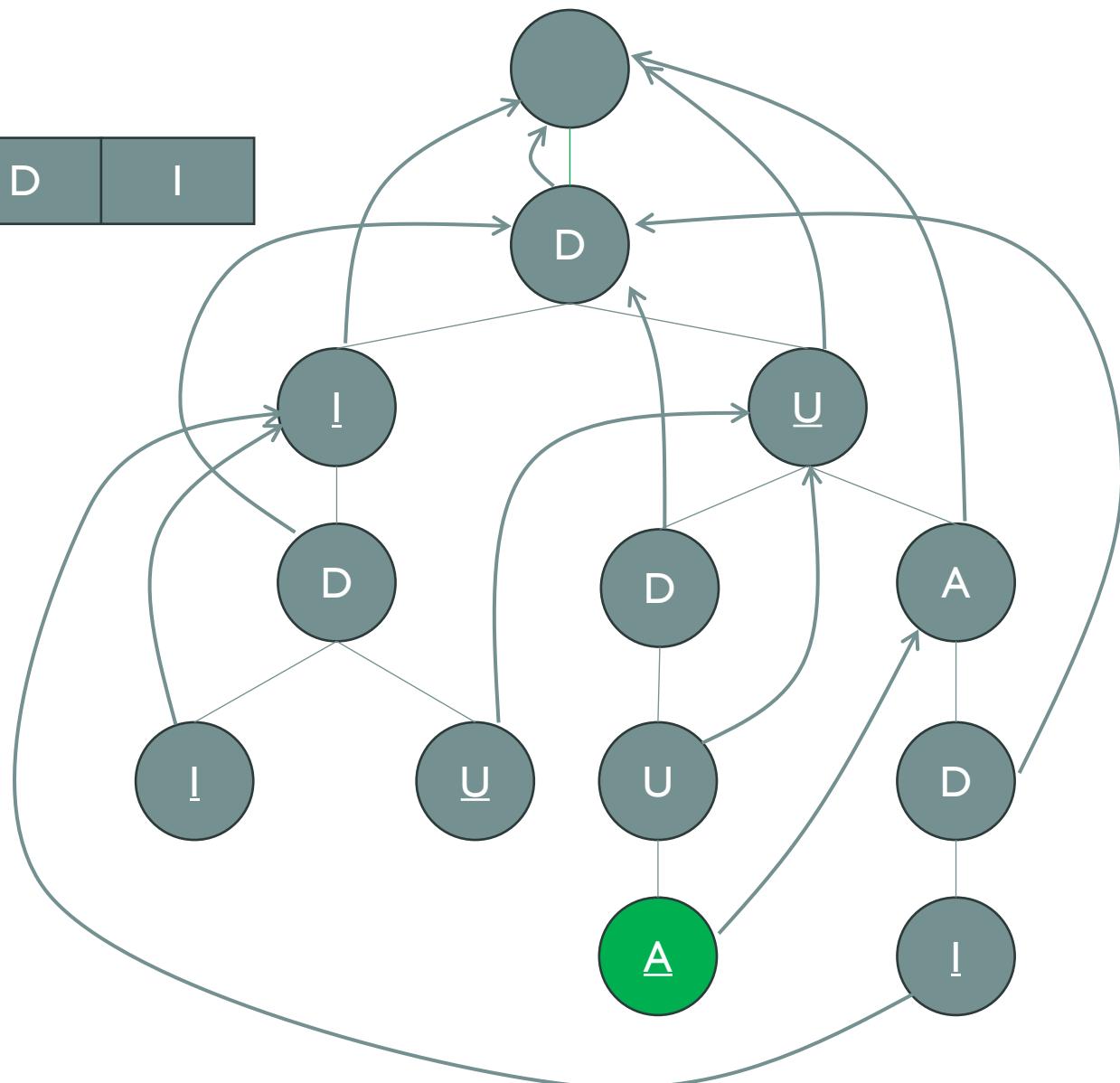
# The pattern strings occur in the text:

DI

DIDU

DU

DUDUA



The text: “DIDUDUADI”



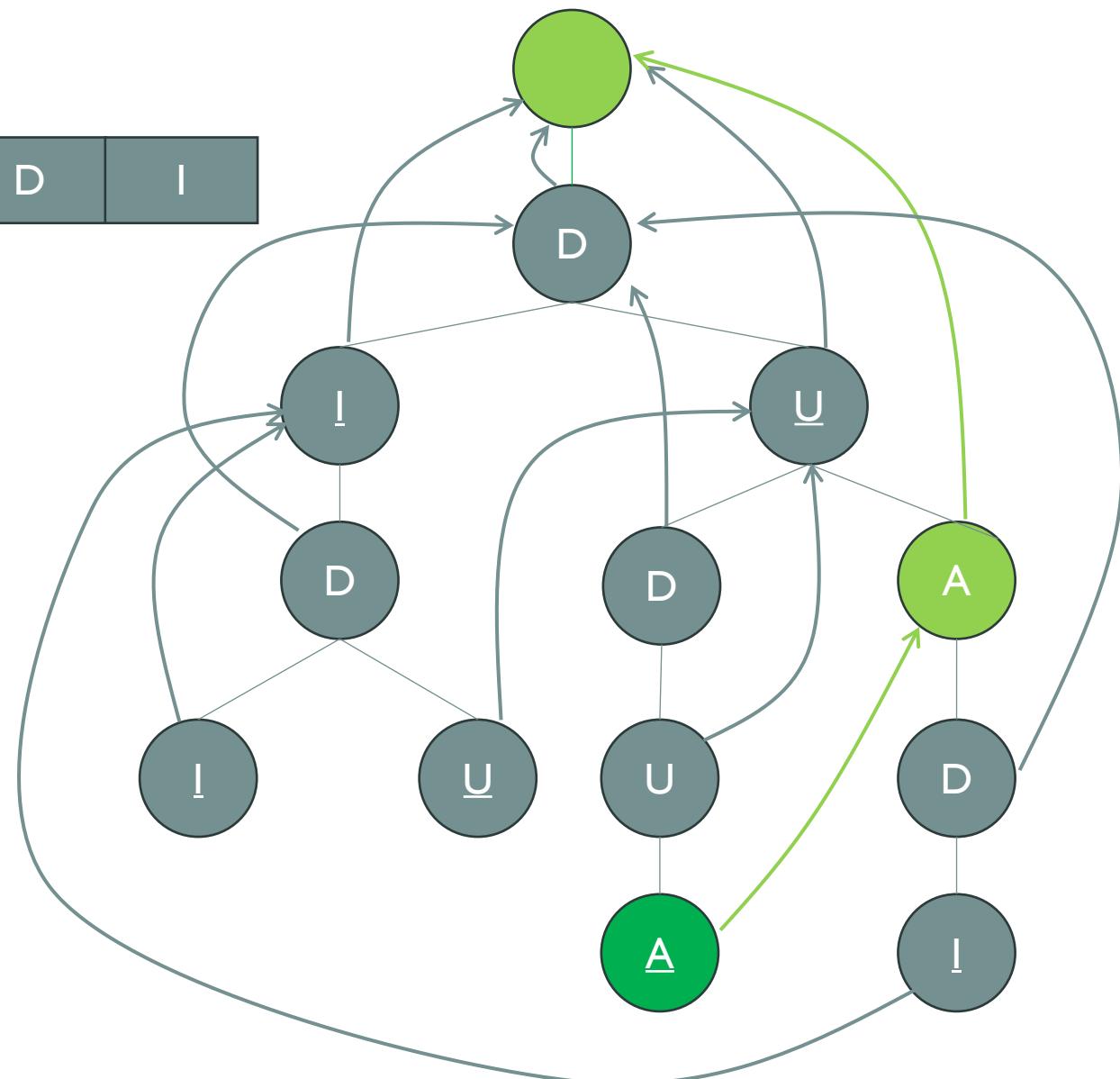
The pattern strings occur in the text:

DI

DIDU

DU

DUDUA



# The text: “DIDUDUADI”



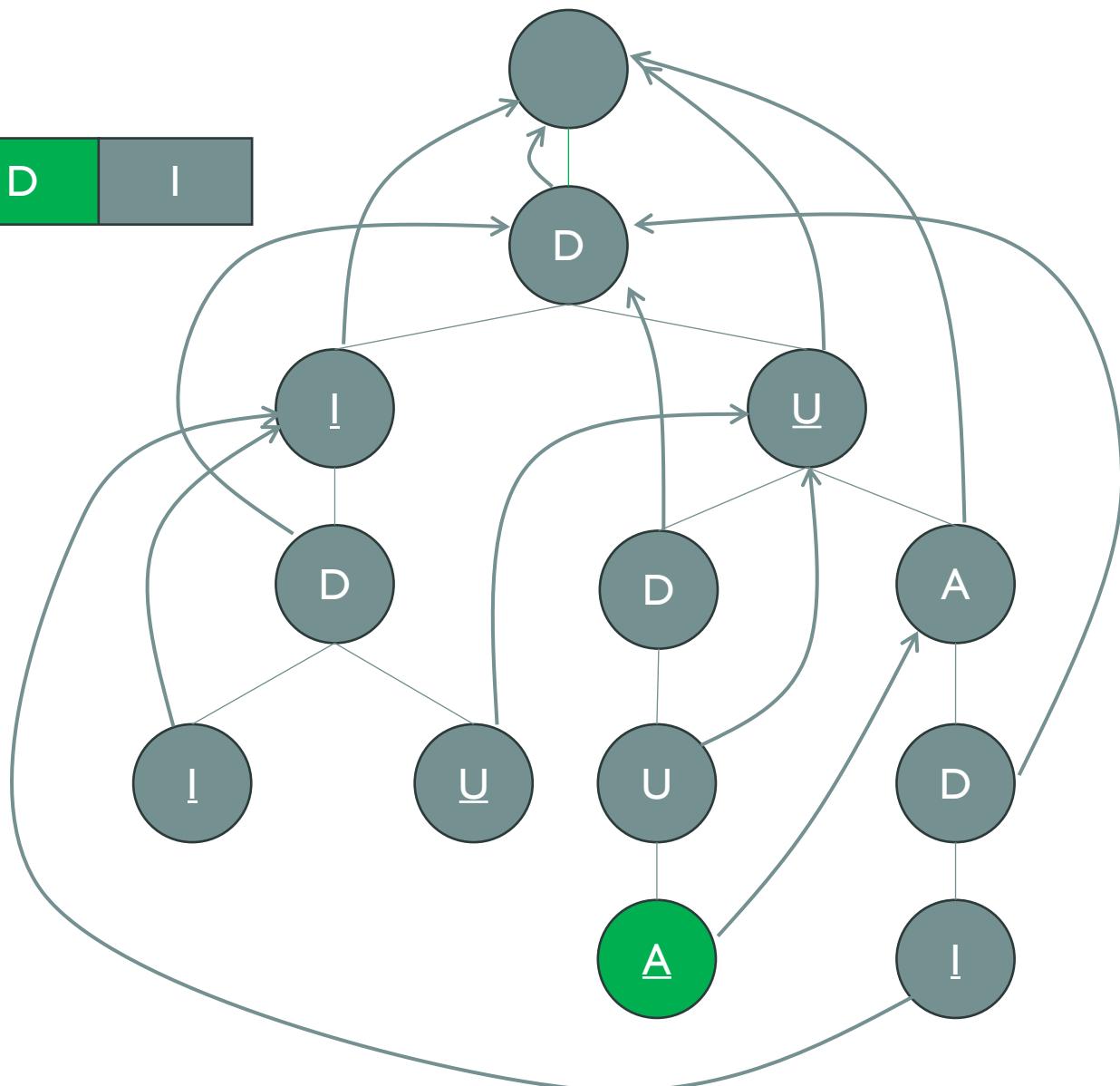
# The pattern strings occur in the text:

DI

DIDU

DU

DUDUA



# The text: “DIDUDUADI”



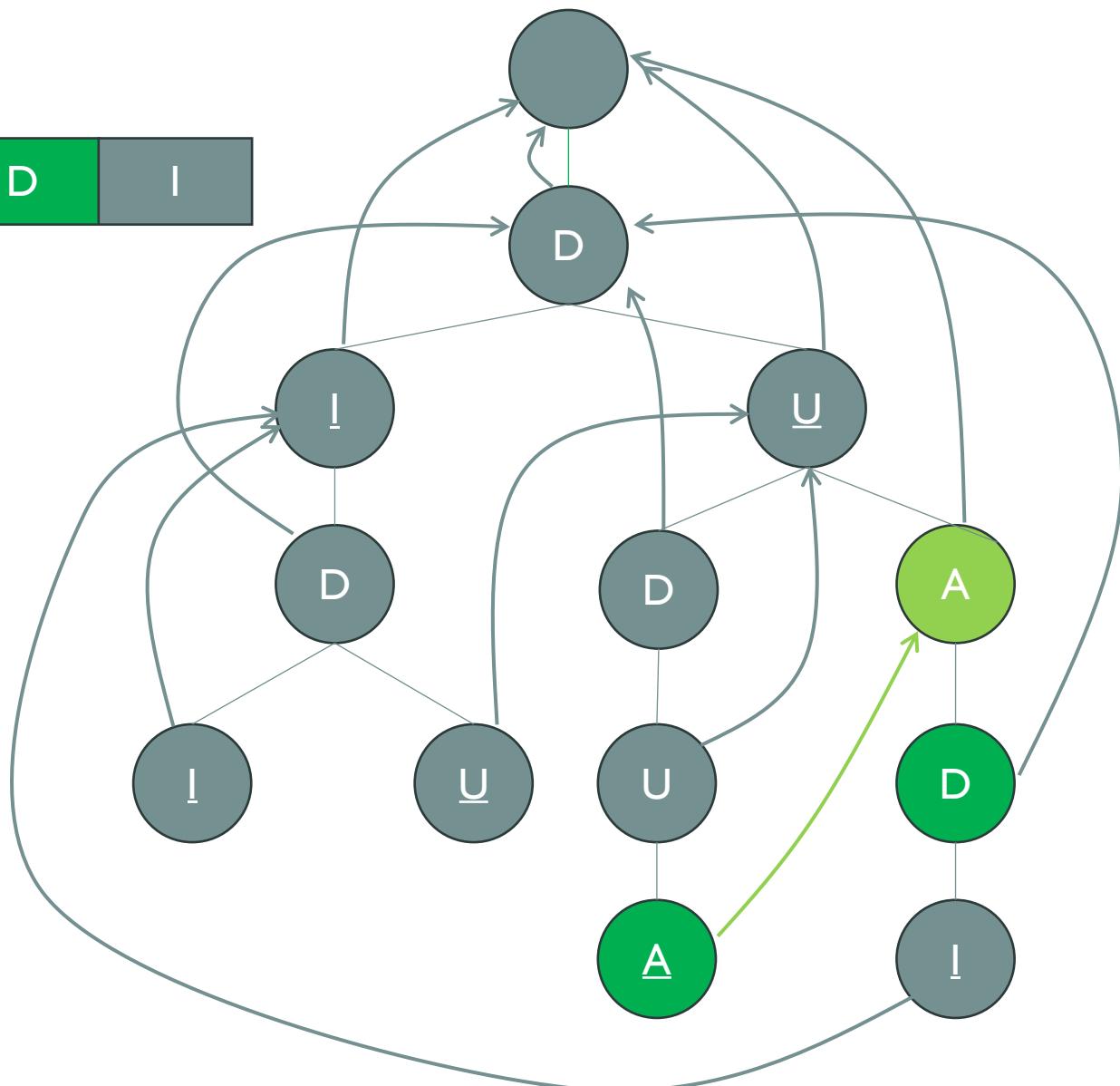
# The pattern strings occur in the text:

DI

DIDU

DU

DUDUA



The text: “DIDUDUADI”



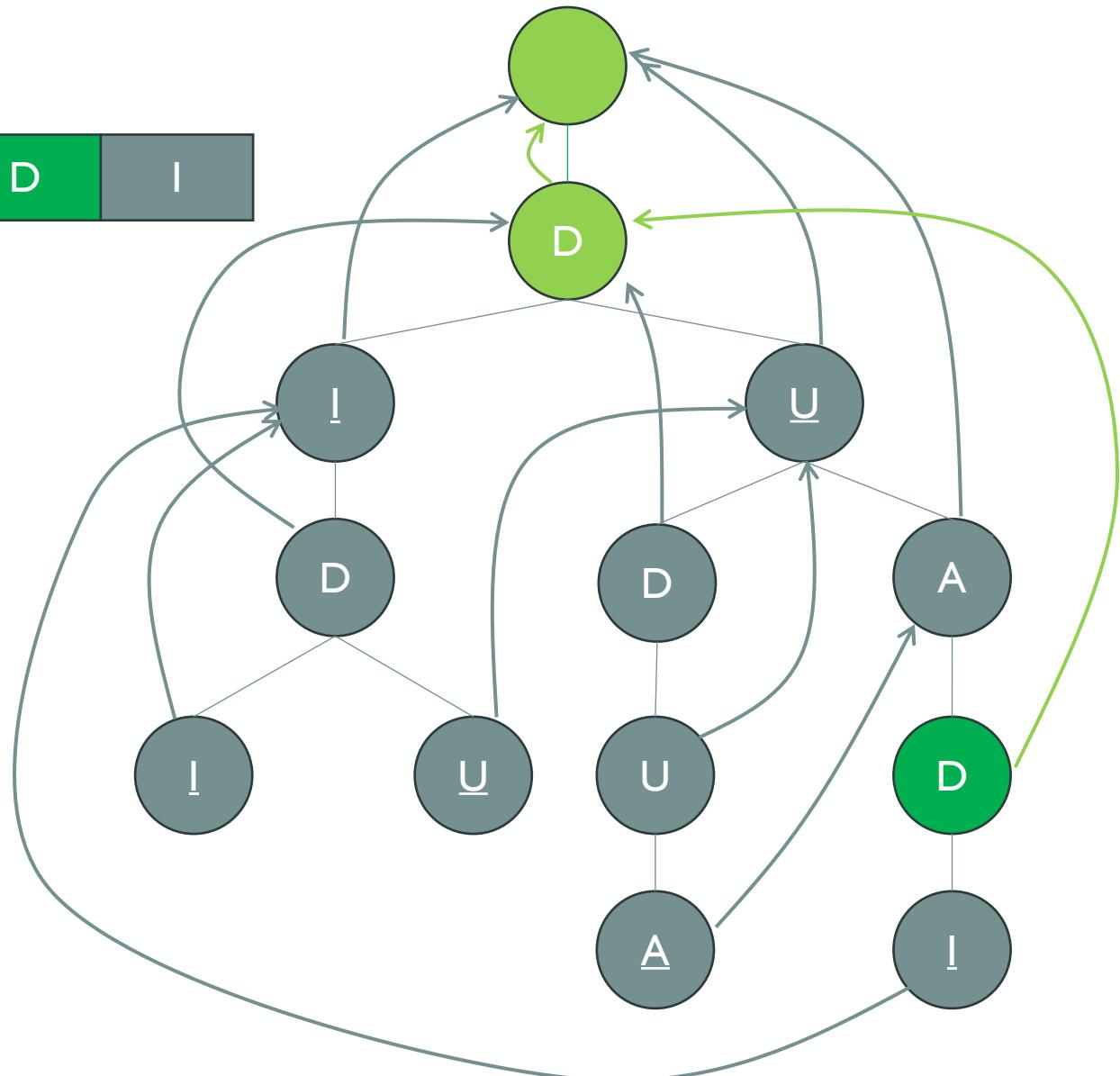
The pattern strings occur in the text:

DI

DIDU

DU

DUDUA



The text: “DIDUDUADI”

D I D U D U A D I

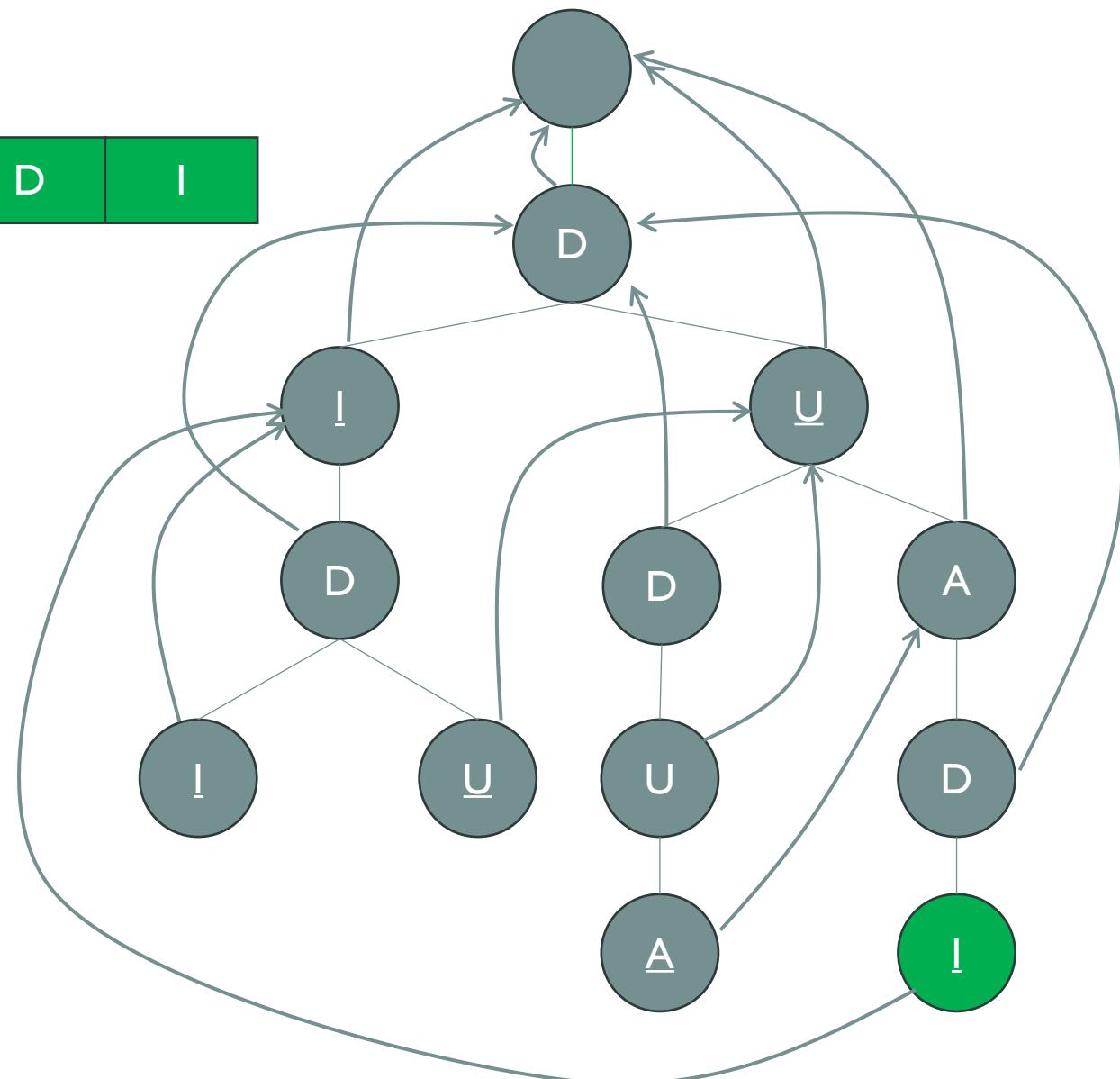
The pattern strings occur in the text:

DI

DIDU

DU

DUDUA



The text: “DIDUDUADI”

D I D U D U A D I

The pattern strings occur in the text:

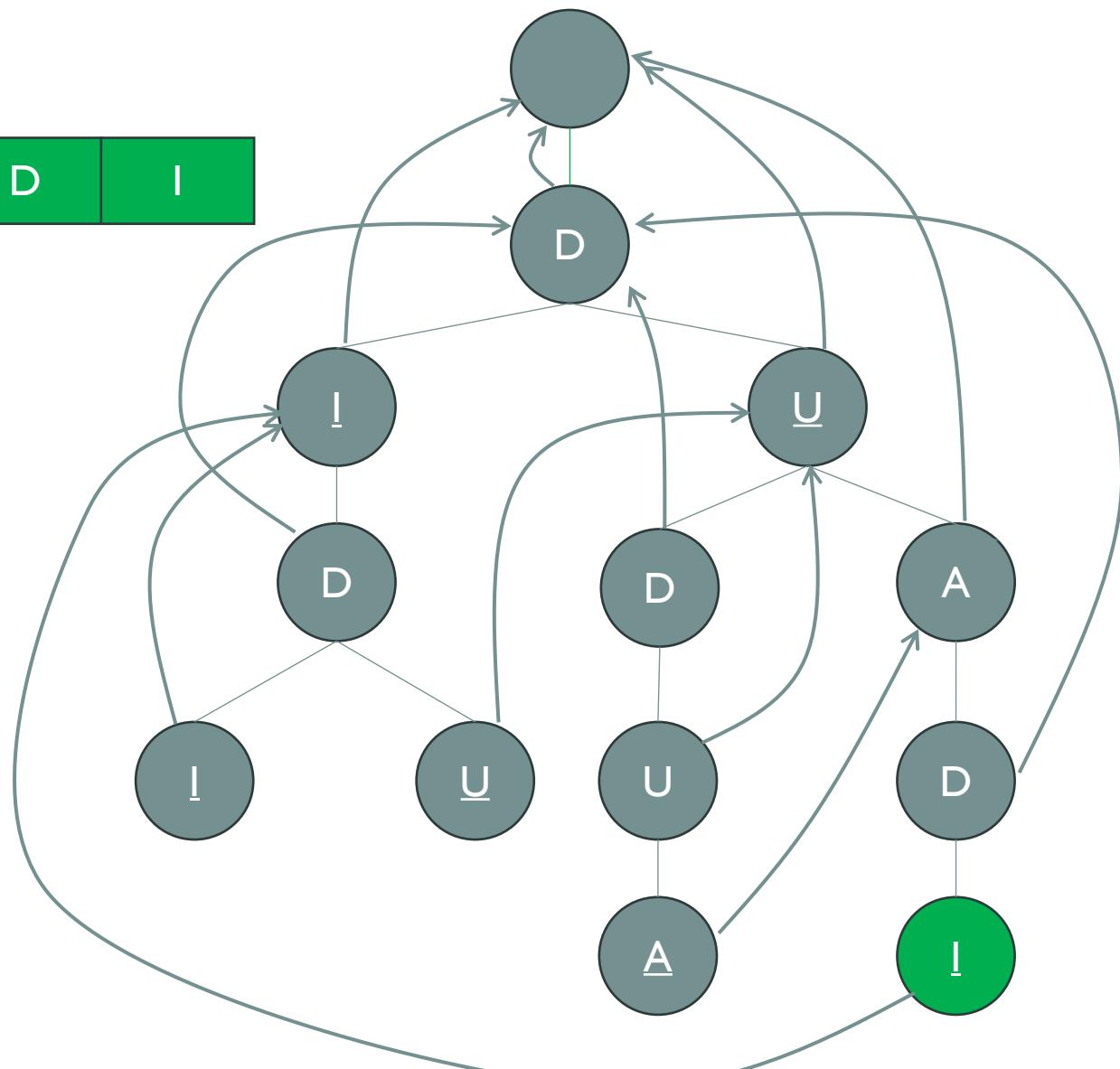
DI

DIDU

DU

DUDUA

DUADI



The text: “DIDUDUADI”

D I D U D U A D I

The pattern strings occur in the text:

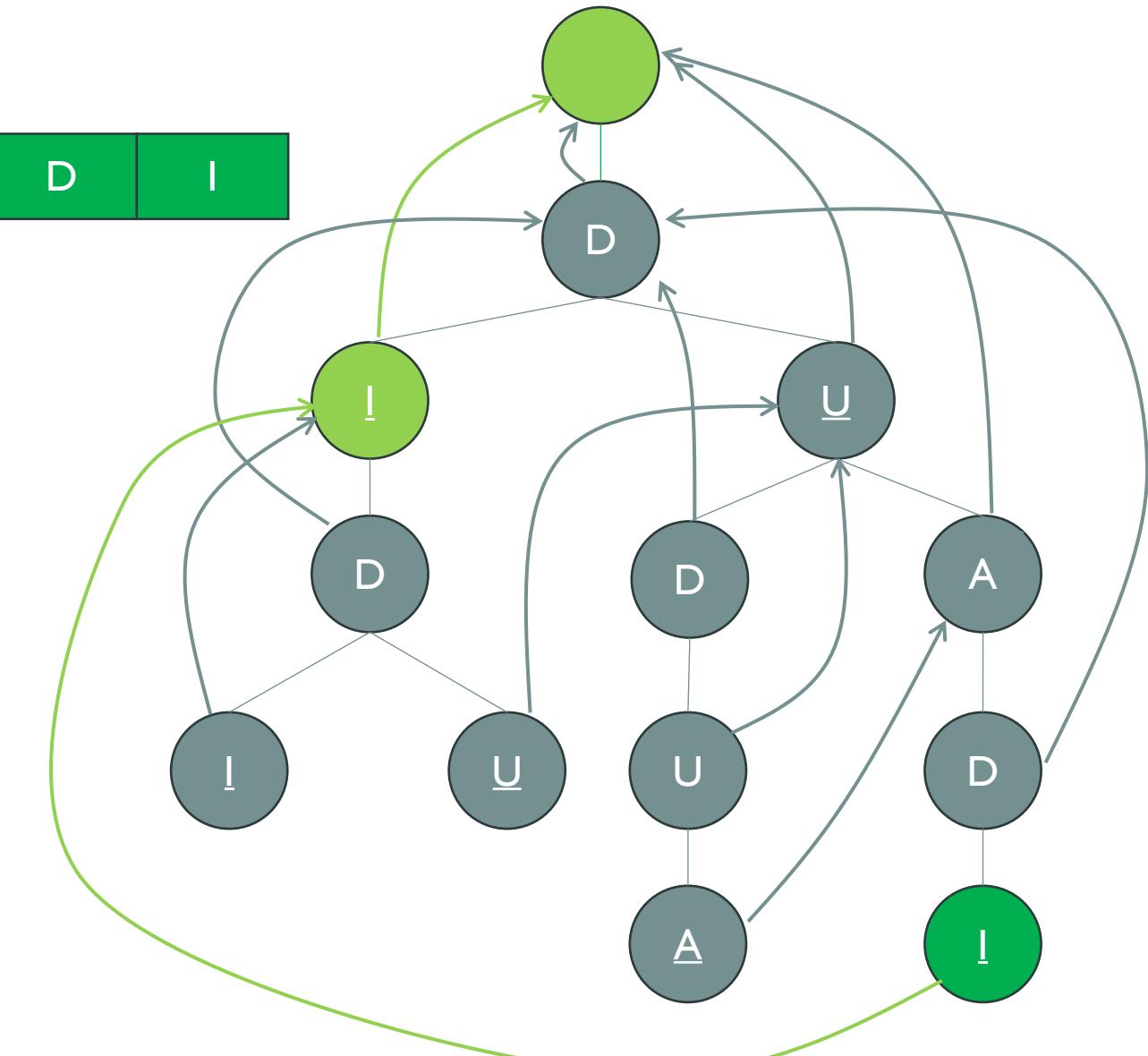
DI

DIDU

DU

DUDUA

DUADI



# The text: “DIDUDUADI”

# D | D U D U A D |

# The pattern strings occur in the text:

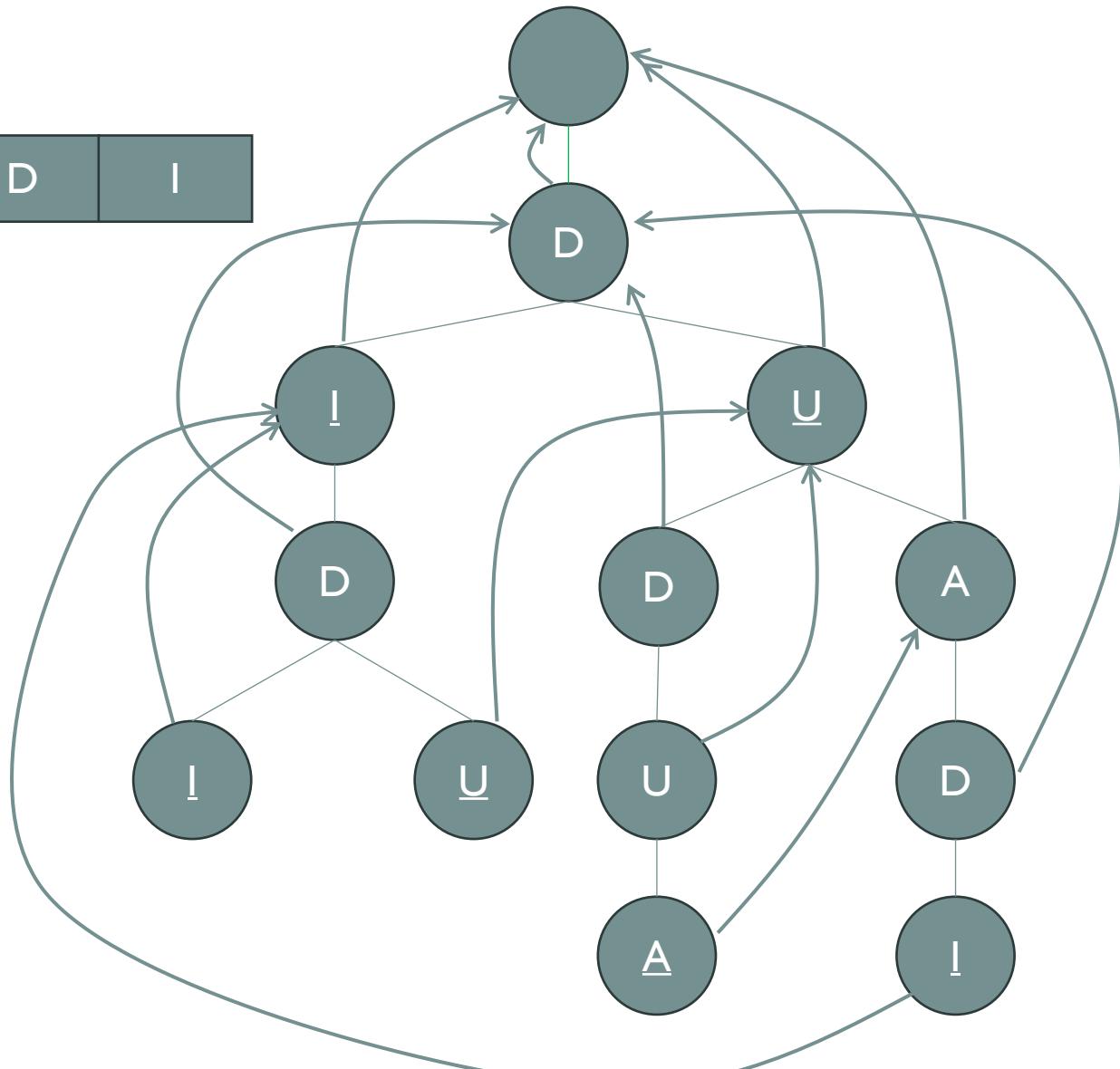
DI

DIDU

DU

DUDUA

DUADI





**THANK YOU**