

Boundary Reduction of Helmholtz Scattering Problem for exterior domains

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Let D be an open Lipschitz bounded domain, $D^c := \mathbb{R}^d \setminus \overline{D}$ and $\Gamma := \partial D$, $d = 2, 3$ with exterior normal field \mathbf{n} pointing by convention toward the exterior domain. We use the superscript $i = 1$ (resp. $i = 2$) when referring to D^c (resp. D). For a given scalar field U , we introduce the Dirichlet (resp. Neumann) traces on Γ as:

$$\gamma_0 U^i := U|_\Gamma \text{ and } \gamma_1 U := \langle \mathbf{n}, \nabla U|_\Gamma \rangle. \quad (1)$$

For a given κ , we define the Helmholtz equation operator $L_\kappa : U \mapsto \Delta U + \kappa^2 U$. For an index $\beta \in \{0, 1\}$ us consider the following exterior boundary value problems (\mathbf{EP}_β) , which consist in finding the scattered field induced in response to a given incident field:

Exterior Problems

Given $\kappa > 0$ and $U^{inc} \in H_{loc}^1(D^c)$ with $L_\kappa U^{inc} = 0$ in D^c , find $U := (U^{scat} + U^{inc}) \in H_{loc}^1(D^c)$ such that

$$\begin{cases} \Delta U + \kappa^2 U = 0 & \text{in } D^c, \\ \gamma_\beta U = 0 & \text{on } \Gamma, \\ \left| \frac{\partial}{\partial r} U^{scat} - i\kappa U^{scat} \right| = o(r^{\frac{1-d}{2}}) & \text{for } r \rightarrow \infty. \end{cases} \quad (2)$$

Let us introduce the boundary integral operators $\mathcal{V}_\kappa, \mathcal{K}_\kappa, \mathcal{W}_\kappa, \mathcal{K}'_\kappa, \mathcal{I}$ defined on Γ . Then, we have the following propositions:

Proposition 1. *The sound-soft problem (\mathbf{EP}_0) is equivalent to each of the following BIEs given on Γ :*

$$\begin{cases} \mathcal{V}_\kappa \gamma_1 U &= \gamma_0 U^{inc} \text{ and} \\ \mathcal{V}_\kappa \gamma_1 U^{scat} &= -(\mathcal{K}_\kappa - \frac{1}{2} \mathcal{I}) \gamma_0 U^{inc}. \end{cases} \quad (3)$$

Proposition 2. *The sound-hard problem (\mathbf{EP}_1) is equivalent to each of the following BIEs given on Γ :*

$$\begin{cases} \mathcal{W}_\kappa \gamma_0 U &= \gamma_1 U^{inc} \text{ and} \\ \mathcal{W}_\kappa \gamma_0 U^{scat} &= (\mathcal{K}'_\kappa + \frac{1}{2} \mathcal{I}) \gamma_1 U^{inc}. \end{cases} \quad (4)$$