Exactly Rao-Blackwellized Unscented Particle Filters for SLAM

Chanki Kim, Hyoungkyun Kim, and Wan Kyun Chung

***Abstract*— This paper addresses the limitation of the conventional Rao-Blackwellized unscented particle filters. The problem is on the usage of the overconfident optimal proposal distribution caused by perfect map assumption, so that predictive robot poses are sampled from the underestimated error covariance in the particle filtering process. The proposed solution computes more precise error covariance of the robot which contains uncertainties of the robot, map, and measurement noise. Experimental results using the benchmark dataset confirmed that the covariance of the proposed method is always larger than that of the conventional method while inducing slower increasing rate of the weight variance with less resamplings.**

I. INTRODUCTION

Simultaneous localization and mapping (SLAM) is the problem of determining the position and the heading of an autonomous vehicle moving through an unknown environment and, simultaneously, of estimating the features of the environment. This is a fundamental requirement for autonomous navigation of intelligent robots.

Among the solutions for the SLAM problem, performing nonlinear optimization to a vehicle pose graph constrained by observations can give us the most reliable answer. Since the graph contains all information collected during navigation, a problem lies on investigating realtime algorithms which compute the most likely configuration that maximizes the likelihood of the observations or the configuration that minimizes least square errors [1].

On the contrary, the solutions using probabilistic filters incrementally track the joint posterior of both vehicle and map states based on normally 1st order Markov assumption. According to the filter used and the type of the estimated map, a large variety of solutions have been proposed [2]–[7]. where the scalability problem with difficulty of data association is still an issue. Developing memory efficient algorithm with low complexity is also an issue for satisfying realtime operation ability.

This paper considers the recently developed Rao-Blackwellized unscented particle filter in which the vehicle state is tracked by using the unscented particle filter and the map is estimated by using the unscented Kalman filters [6], [8]. The full SLAM posterior in this framework is divided into the product of both the vehicle state posterior and the map state posterior, in which the map state conditionally depends on the vehicle state. The mean squared error and its variance of the map estimator are reduced based on Rao-Blackwell theorem. Thus the SLAM performance is dominantly influenced by the vehicle state estimator.

This paper points out an important problem of the conventional Rao-Blackwellized unscented particle filter. The problem is on the usage of the overconfident proposal distribution caused by the perfect map assumption without any prior basis, so that the particles are sampled from the underestimated covariance. This can curtail the filter durability.

The remainder of this paper is organized as follows. In Section II, we formally introduce the Rao-Blackwellized unscented particle filter and discuss about the problem. Section III describes a solution, called an exactly Rao-Blackwellized unscented particle filter, after understanding uncertainty components of an innovation covariance of the proposal distribution, and then discusses about a particle weight computation and the use of the unscented transformation in the data association process. In Section IV, we demonstrate the experimental results, and concluding remarks follows in Section V.

II. RELATED WORK

This section presents the conventional Rao-Blackwellized unscented particle filter algorithm [6], [8] for SLAM purpose then discusses about the problem issued in this paper.

A. Rao-Blackwellized Unscented Particle Filter for SLAM

1) Predicting vehicle state: We consider the measurement noise as an additive term so that the augmented state at the prediction has smaller dimension ($L = N_{\text{xv}} + N_Q$, where N_{xv} is the robot state dimension and N_Q is the process noise dimension) than that of the full augmentation, used in [6], while getting computational benefit.

$$\mathbf{x}_{t-1}^{a[m]} = \left(\begin{array}{c}\mathbf{x}_{t-1}^{[m]} \\ \hline \mathbf{0}\end{array}\right), \quad \mathbf{P}_{t-1}^{a[m]} = \left(\begin{array}{c|c}\mathbf{P}_{t-1}^{[m]} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{Q}\end{array}\right)$$

where $\mathbf{x}_{t-1}^{[m]}$ and $\mathbf{P}_{t-1}^{[m]}$ are the previously estimated state and covariance, respectively. m is particle index and $\mathbf{Q}$ is the white process noise variance.

Now we compute $2L+1$ sigma points by decomposing the scaled covariance using the unscented transformation parameters, λ_L, then transform the points through the process model, $f(\cdot)$:

$$\chi_{t-1}^{a[m]} = \left(\mathbf{x}_{t-1}^{a[m]}, \mathbf{x}_{t-1}^{a[m]} \pm \left(\sqrt{(L+\lambda_L)\mathbf{P}_{t-1}^{a[m]}}\right)\right)$$
$$\bar{\chi}^{[i][m]} = f(\chi_{x,t-1}^{a[i][m]}, \mathbf{u}_t + \chi_{u,t-1}^{a[i][m]})$$

Chanki Kim, Hyoungkyun Kim, and Wan Kyun Chung are with the Department of Mechanical Engineering at the Pohang University of Science and Technology(POSTECH), Korea. Email:{minekiki,wkchung,angramai}@postech.ac.kr

where $\chi_{x,t-1}^{a[i][m]}$ and $\chi_{u,t-1}^{a[i][m]}$ are the state and process noise terms in $\chi_{t-1}^{a[i][m]}$. $\mathbf{u}_t$ is the control input at t

The predictive state and covariance are given by a linear weighted regression using the weights $w_{g,L}^{[i]}$ and $w_{c,L}^{[i]}$ (see [6] and [7] for determining appropriate weight values):

$$\hat{\mathbf{x}}_t^{[m]} = \sum_{i=0}^{2L} w_{g,L}^{[i]} \bar{\chi}_t^{[i][m]}$$
$$\bar{\mathbf{P}}_t^{[m]} = \sum_{i=0}^{2L} w_{c,L}^{[i]} (\bar{\chi}_t^{[i][m]} - \hat{\mathbf{x}}_t^{[m]})(\bar{\chi}_t^{[i][m]} - \hat{\mathbf{x}}_t^{[m]})^T$$

2) Correction:

1) Computing proposal density: To generate the proposal density, firstly a predictive measurement $\hat{\mathbf{y}}_t^{[m]}$ is computed using a landmark mean location $\mu_{m_t,t-1}^{[m]}$:

$$\mathcal{Y}_t^{[i][m]} = h(\bar{\chi}_t^{[i][m]}, \mu_{m_t,t-1}^{[m]}), \quad i = 0 \sim 2L \tag{1}$$
$$\hat{\mathbf{y}}_t^{[m]} = \sum_{i=0}^{2L} w_{g,L}^{[i]} \mathcal{Y}_t^{[i][m]}$$

where $h(\cdot)$ is the observation model.
After computing an innovation and cross covariances which are given by:

$$\mathbf{V}_t^{[m]} = \sum_{i=0}^{2L} w_{c,L}^{[i]} (\mathcal{Y}_t^{[i][m]} - \hat{\mathbf{y}}_t^{[m]})(\mathcal{Y}_t^{[i][m]} - \hat{\mathbf{y}}_t^{[m]})^T$$
$$\mathbf{V}_t^{[m]} = \mathbf{V}_t^{[m]} + \mathbf{R}, \tag{2}$$

$$\mathbf{\Sigma}_t^{xy[m]} = \sum_{i=0}^{2L} w_{c,L}^{[i]} (\bar{\chi}_t^{[i][m]} - \hat{\mathbf{x}}_t^{[m]})(\mathcal{Y}_t^{[i][m]} - \hat{\mathbf{y}}_t^{[m]})^T, \tag{3}$$

we obtain the updated density that presents the optimal proposal distribution of the unscented particle filter:

$$\mathbf{x}_t^{[m]} = \hat{\mathbf{x}}_t^{[m]} + \mathbf{\Sigma}_t^{xy[m]} (\mathbf{V}_t^{[m]})^{-1} (\mathbf{z}_t - \hat{\mathbf{y}}_t^{[m]})$$
$$\mathbf{P}_t^{[m]} = \bar{\mathbf{P}}_t^{[m]} - \mathbf{\Sigma}_t^{xy[m]} (\mathbf{V}_t^{[m]})^{-1} (\mathbf{\Sigma}_t^{xy[m]})^T,$$

where, $\mathbf{R}$ is the measurement noise variance and $\mathbf{z}_t$ is current observation.

2) Computing particle weighs: Using the innovation covariance $\mathbf{V}_t^{[m]}$, we compute the importance weight of each particle $w_t^{[m]}$.
3) Sampling: Then, next generation particle is sampled from the proposal density and the vehicle state error covariance is reset to initial matrix for unbiased estimation
4) Estimating map: After finishing the vehicle state estimation, the multiple hypothetical maps are updated using the unscented filter based on the provided vehicle poses.

3) Resampling: The resampling process is operated to converge the particles, and its location in the algorithm sequence is flexible.

B. Discussion

SLAM problem is fundamentally different from the problem of sequential localization and mapping from the fact that the map state is not perfectly known:

$$p(\mathbf{x}_t, \mathbf{m}|\mathbf{z}^t, \mathbf{u}^t) = \underbrace{p(\mathbf{x}_t|\mathbf{z}^t, \mathbf{u}^t)}_{\star} p(\mathbf{m}|\mathbf{x}_t, \mathbf{z}^t, \mathbf{u}^t) \tag{4}$$

where, $\mathbf{m}$ means the entire map state. The superscript t is history of random variable until t.

The limitation of the conventional algorithm is on the usage of the overconfident proposal density in which the map uncertainty is not considered through the innovation covariance (2). The map $\mathbf{m}$ is assumed as known in $\star$ without any prior basis. So the particles are sampled from the underestimated covariance every update time. This can curtail the filter durability.

III. EXACTLY RAO-BLACKWELLIZED UNSCENTED PARTICLE FILTER

In this section, the analytical solution of the innovation covariance is firstly derived to understand the uncertainty components, and then the exactly Rao-Blackwellized proposal posterior using the unscented transformation is proposed.

Using Bayes rule and Markov assumption, we expand the proposal density:

$$\begin{aligned} p(\mathbf{x}_t|\mathbf{z}^t, \mathbf{u}^t) &= \eta p(\mathbf{z}^t|\mathbf{x}^t, \mathbf{u}^t) p(\mathbf{x}_t|\mathbf{u}^t) \\ &\overset{\text{Markov}}{=} \eta p(\mathbf{z}_t|\mathbf{x}_t) \underbrace{p(\mathbf{x}_t|\mathbf{u}_t)}_{\mathcal{N}(f(\mathbf{x}_{t-1}, \mathbf{u}_t), \bar{\mathbf{P}}_t)} \end{aligned}$$

After taking the map state into account in the likelihood using total probability theorem, analytical linearization around the estimated mean states of the robot $\hat{\mathbf{x}}_t$ and feature $\mu_{m_t,t-1}$ is considered. Then, the likelihood can be obtained by a convolution of two densities, which represent uncertainties of both the measurement and the map:

$$p(\mathbf{z}_t|\mathbf{x}_t) = \underbrace{\int \underbrace{p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{m}_t)}_{\mathcal{N}(h(\mathbf{x}_t, \mathbf{m}_t), \mathbf{R})} \underbrace{p(\mathbf{m}_t|\mathbf{x}_t)}_{\mathcal{N}(\mu_{m_t,t-1}, \mathbf{\Sigma}_{m_t,t-1})} d\mathbf{m}_t}_{\mathcal{N}(h(\hat{\mathbf{x}}_t, \mu_{m_t,t-1}) + \mathbf{G}_x(\mathbf{x}_t - \hat{\mathbf{x}}_t), \mathbf{G}_f \mathbf{\Sigma}_{m_t,t-1} \mathbf{G}_f^T + \mathbf{R})} \tag{5}$$

where $\mathbf{m}_t$ is the landmark state at time step t. $\mathbf{\Sigma}_{m_t,t-1}$ is the covariance of the landmark observed at t, but registered previously. $\mathbf{G}_x$ and $\mathbf{G}_f$ are Jacobians.

The proposal distribution is given by

$$p(\mathbf{x}_t|\mathbf{z}^t, \mathbf{u}^t) = \eta' \exp[-\frac{1}{2}(\mathbf{A}^T \mathbf{Z}_t^{-1} \mathbf{A} + \tilde{\mathbf{x}}_t^T \bar{\mathbf{P}}_t^{-1} \tilde{\mathbf{x}}_t)],$$

where, $\mathbf{A} = \mathbf{z}_t - \hat{\mathbf{z}}_t - \mathbf{G}_x(\mathbf{x}_t - \hat{\mathbf{x}}_t)$, $\mathbf{Z}_t = \mathbf{G}_f \mathbf{\Sigma}_{m_t,t-1} \mathbf{G}_f^T + \mathbf{R}$, $\hat{\mathbf{z}}_t = h(\hat{\mathbf{x}}_t, \mu_{m_t})$, and $\tilde{\mathbf{x}}_t = \mathbf{x}_t - \hat{\mathbf{x}}_t$.

The first and second moments of the proposal are expressed by following equations:

$$\begin{aligned} \mathbf{x}_t &= \hat{\mathbf{x}}_t + (\mathbf{G}_x^T \mathbf{Z}_t^{-1} \mathbf{G}_x + \bar{\mathbf{P}}_t^{-1})^{-1} \mathbf{G}_x^T \mathbf{Z}_t^{-1} (\mathbf{z}_t - \hat{\mathbf{z}}_t) \\ &= \hat{\mathbf{x}}_t + \bar{\mathbf{P}}_t \mathbf{G}_x^T (\mathbf{G}_x \bar{\mathbf{P}}_t \mathbf{G}_x^T + \mathbf{Z}_t)^{-1} (\mathbf{z}_t - \hat{\mathbf{z}}_t) \end{aligned} \tag{6}$$

$$\mathbf{P}_t = \bar{\mathbf{P}}_t - \underbrace{\bar{\mathbf{P}}_t \mathbf{G}_x^T}_{\text{cross cov.}} (\underbrace{\mathbf{G}_x \bar{\mathbf{P}}_t \mathbf{G}_x^T + \mathbf{Z}_t}_{\text{innovation cov.}})^{-1} \mathbf{G}_x \bar{\mathbf{P}}_t \tag{7}$$

TABLE I

LOCALIZATION POSTERIOR VS. PROPOSAL DENSITY OF RBPF

	$p(\mathbf{x}_t \mid \mathbf{z}^t, \mathbf{u}^t, \mathbf{m})$	$p(\mathbf{x}_t \mid \mathbf{z}^t, \mathbf{u}^t)$
$\mathbf{S}_t$	$\mathbf{G}_x \bar{\mathbf{P}}_t \mathbf{G}_x^T + \mathbf{R}$	$\mathbf{G}_x \bar{\mathbf{P}}_t \mathbf{G}_x^T + \mathbf{Z}_t$
$\mathbf{P}_t$[1]	$(\bar{\mathbf{P}}_t^{-1} + \mathbf{G}_x^T \mathbf{R}^{-1} \mathbf{G}_x)^{-1}$	$(\bar{\mathbf{P}}_t^{-1} + \mathbf{G}_x^T \mathbf{Z}_t^{-1} \mathbf{G}_x)^{-1}$
$\mathbf{S}_t$	$\mathbf{V}_t$	$\mathcal{S}_t$
$\mathbf{P}_t$	$\bar{\mathbf{P}}_t - \mathbf{\Sigma}_t^{xy} \mathbf{S}_t^{-1} (\mathbf{\Sigma}_t^{xy})^T$	

We note that the innovation covariance of Kalman filter consists of uncertainties of the robot, map, and measurement:

$$\mathbf{S}_t = \mathbf{G}_x \bar{\mathbf{P}}_t \mathbf{G}_x^T + \mathbf{Z}_t, \tag{8}$$

A difference between localization posterior and proposal distribution for particle filtering SLAM is whether or not the error covariance contains the map uncertainty as summarized in Tab. I.

A. Exactly Rao-Blackwellized Unscented Particle Filter

As the proposal distribution of Rao-Blackwellized particle filter is computed by means of the unscented transformation, the map uncertainty should be precisely considered through the innovation covariance.

1) The augmented state and its covariance are determined by considering uncertainties of the robot and landmark states:

$$\mathbf{x}_{\mathrm{a}} = \left(\begin{array}{c} \hat{\mathbf{x}}_t^{[m]} \\ \mu_{m_t}^{[m]} \end{array} \right), \ \mathbf{P}_{\mathrm{a}} = \left(\begin{array}{c|c} \bar{\mathbf{P}}_t^{[m]} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{\Sigma}_{m_t}^{[m]} \end{array} \right)$$

2) Using the augmented state dimension $N_{\mathrm{a}} = N_{\mathrm{xv}} + N_F$ and the related scaling parameter λ_{a}, sigma points $\zeta^{[m]}$ which contains the uncertainties are calculated. N_F is the landmark dimension. Then, we obtain the predictive measurement $\hat{\mathbf{y}}_t^{\mathrm{a}[m]}$ and the innovation covariance $\mathcal{S}_t$ after operating the nonlinear transformation of the $2N_{\mathrm{a}}+1$ sigma points through the observation model:

$$\zeta^{[m]} = \left(\mathbf{x}_{\mathrm{a}},\ \mathbf{x}_{\mathrm{a}} \pm \left(\sqrt{(N_{\mathrm{a}}+\lambda_{\mathrm{a}})\mathbf{P}_{\mathrm{a}}}\right)\right)$$
$$\mathcal{A}^{[i][m]} = h(\zeta^{\mathrm{xv},[i][m]}, \zeta^{\mu,[i][m]})$$
$$\hat{\mathbf{y}}_t^{\mathrm{a}[m]} = \sum_{i=0}^{2N_{\mathrm{a}}} w_{g,N_{\mathrm{a}}}^{[i]} \mathcal{A}^{[i][m]}$$
$$\mathcal{S}_t^{[m]} = \sum_{i=0}^{2N_{\mathrm{a}}} w_{c,N_{\mathrm{a}}}^{[i]} (\mathcal{A}^{[i][m]} - \hat{\mathbf{y}}_t^{\mathrm{a}[m]})(\mathcal{A}^{[i][m]} - \hat{\mathbf{y}}_t^{\mathrm{a}[m]})^T + \mathbf{R}$$

where $\zeta^{\mathrm{xv},[i][m]}$ and $\zeta^{\mu,[i][m]}$ are robot and landmark terms in the i-th sigma points of the m-th particle respectively.

Cross covariance is computed by considering robot terms of the sigma points $\zeta^{\mathrm{xv},[m]}$:

$$\mathbf{\Sigma}_t^{xy[m]} = \sum_{i=0}^{2N_{\mathrm{a}}} w_{c,N_{\mathrm{a}}}^{[i]} (\zeta^{\mathrm{xv},[i][m]} - \hat{\mathbf{x}}_t^{[m]})(\mathcal{A}_t^{[i][m]} - \hat{\mathbf{y}}_t^{\mathrm{a}[m]})^T$$

[1]The covariance is represented by a block matrix identity.

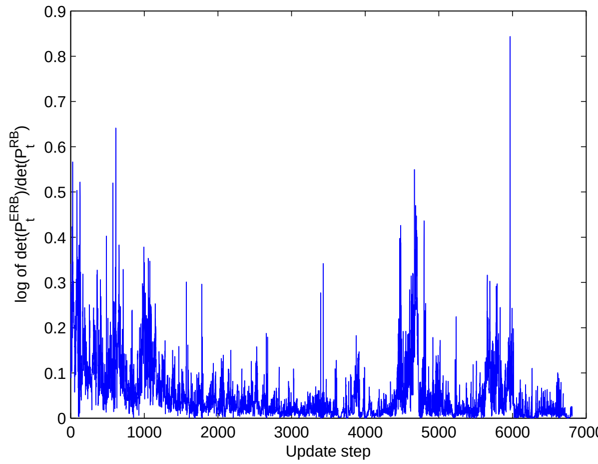


Fig. 1. Log of the ratio comparing the ERB proposal distribution to that of the RB. The minimum value is $9.5e-5$.

Then, the first and second moments of the proposal density can be computed by using Kalman update equation:

$$\mathbf{x}_t^{[m]} = \hat{\mathbf{x}}_t^{[m]} + \mathbf{\Sigma}_t^{xy[m]} (\mathcal{S}_t^{[m]})^{-1} (\mathbf{z}_t - \hat{\mathbf{y}}_t^{\mathrm{a}[m]}) \tag{9}$$
$$\mathbf{P}_t^{[m]} = \bar{\mathbf{P}}_t^{[m]} - \underbrace{\mathbf{\Sigma}_t^{xy[m]}}_{\equiv \bar{\mathbf{P}}_t \mathbf{G}_x^T} \underbrace{(\mathcal{S}_t^{[m]})^{-1}}_{\equiv \mathbf{S}_t^{-1}} (\mathbf{\Sigma}_t^{xy[m]})^T \tag{10}$$

On the one hand, $\hat{\mathbf{x}}_t^{[m]}$ is the predictive robot state in the case of batch update when dealing multiple observations (we use the batch process in the experiments of this paper). On the other hand, $\hat{\mathbf{x}}_t^{[m]}$ becomes a sequentially updated robot state in the case of serial update scheme (we refer to [6] for more discussion about the sequential measurement update).

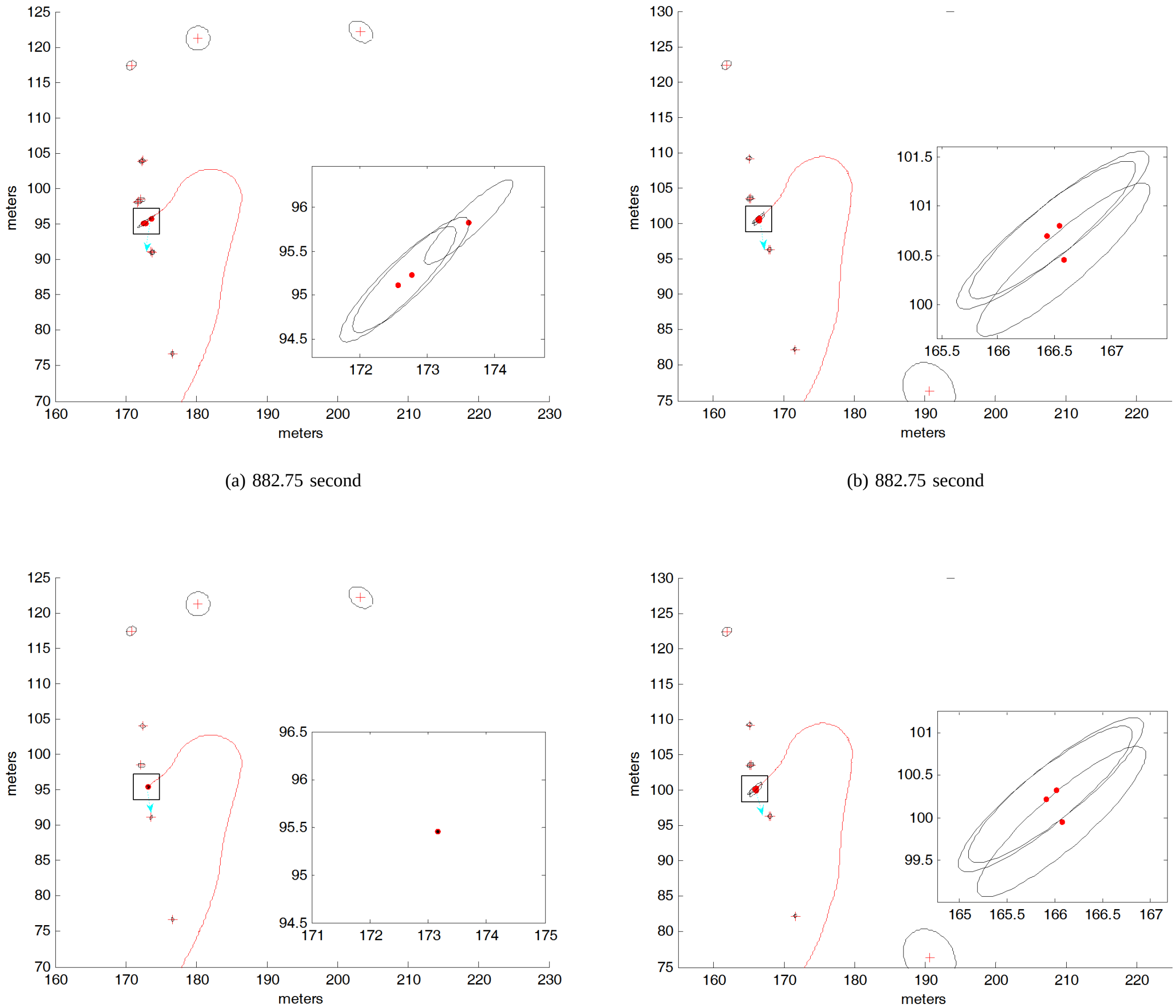
The particle weights that represent a confidence level of each hypothesis can be computed by using the innovation covariance as follows:

$$w_t^{[m]} = |2\pi \mathcal{S}_t^{[m]}|^{-0.5} \exp[-\frac{1}{2}((\nu_t^{[m]})^T (\mathcal{S}_t^{[m]})^{-1} \nu_t^{[m]})], \tag{11}$$

where $\nu_t^{[m]} = \mathbf{z}_t - \hat{\mathbf{y}}_t^{\mathrm{a}[m]}$.

Different from unimodal Gaussian filters, there is no mathematical basis in data association process to enforce the unscented transformation to a filter, which uses sampling based robot state estimator likes Rao-Blackwellized particle filter, even though states are estimated by using the unscented filter.

However, skewed uncertainty propagation caused by analytical linearization can still be a problem at computing the predictive measurement in the data association process (highly nonlinear observation models such as vision sensors intensify this). Thus, it is advantageous to use sigma points which are calculated by augmenting the landmark covariance $\mathbf{\Sigma}_{m_t}^{[i][m]}$ and the measurement noise variance $\mathbf{R}$ for more precise Mahalanobis distances [10].



(a) 882.75 second

(b) 882.75 second

(c) 883.05 second

(d) 883.05 second

Fig. 2. The left column is the RB and the right column is the ERB. The robot and landmark states are illustrated with the three sigma confidence bounds. The red line is the estimated robot trajectory and the cyan arrows indicate the observations detected through the laser range finder. The red cross and red dot mean the mean location of the landmark and robot, respectively. The black ellipses are the covariance estimates.

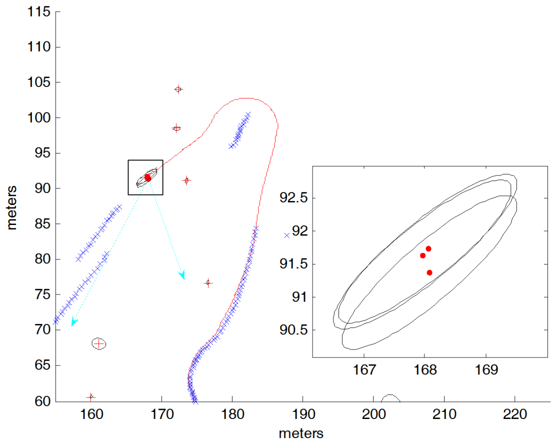
IV. EXPERIMENTAL VALIDATION

To validate the proposed method experimentally, we consider the benchmark Victoria Park dataset courtesy of E. Nebot [9]. The dataset is widely used in the SLAM research field for validating a difference of algorithms including the well-known scalability problem. In the experiment, a truck equipped with the SICK laser range finder with a 180 degree frontal field-of-view was driven around the park covering a path of over $3.5km$. Trees were used as point landmarks. Although there were many spurious observations including dynamic objects, we do not additionally process these observations for the estimation. We reduce the perception range of the laser from $80m$ to $30m$ for challenging SLAM. The process and measurement noises for the filters are (2m/s, 6rad/s) and (1m, $3°$), respectively.

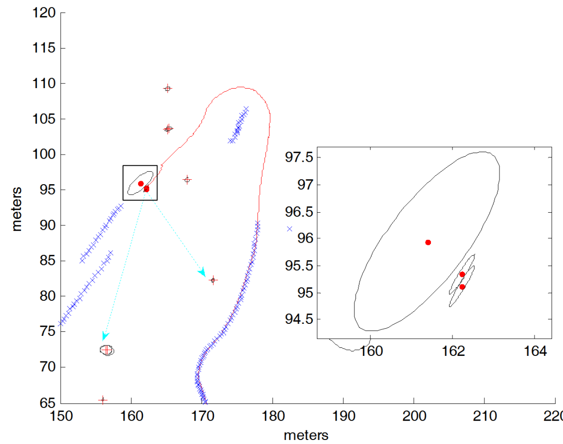
To validate the overconfidence of the proposal distribution, we firstly measure and compare magnitudes of the robot state error covariance just after the measurement update. Experimental conditions including a random seed are exactly the same.

Different proposal densities cause differences in estimating both the robot trajectory and map state, as well as opportunities of measurement updates. The total number of estimated landmarks also becomes different. So we need to square time step of one algorithm with another for fair comparison. For this, we compute covariance determinants of the two proposal distributions: the exactly Rao-Blackwellized unscented particle filter (ERB) and the conventional Rao-Blackwellized unscented particle filter (RB) [6]. These computations are done at every update step while performing the estimation based on the ERB, so the determinant of the RB is calculated regardless of the estimation result.

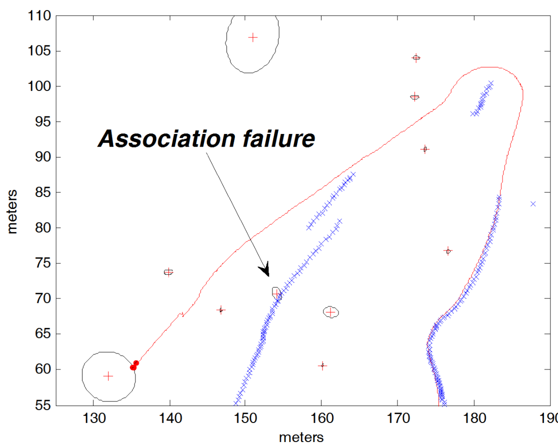
Figure 1 is the log of the determinant ratio of the proposal distributions at each measurement update step. The determinant is computed with considering the particle weights as



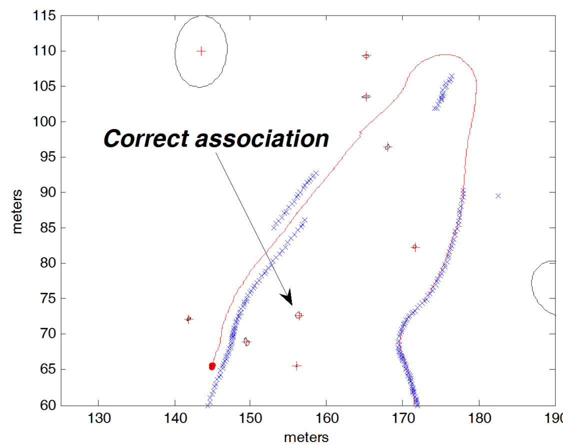
(e) 886.25 second



(f) 886.25 second



(g) 900 second



(h) 900 second

Fig. 3. The left column is the RB and the right column is the ERB. The robot and landmark states are illustrated with the three sigma confidence bounds. The blue cross path is the relative gps data. The red line is the estimated robot trajectory and the cyan arrows indicate the observations detected through the laser range finder. The red cross and red dot mean the mean location of the landmark and robot, respectively. The black ellipses are the covariance estimates.

expressed by the following equation:

$$\det(\mathbf{P}_t^{(\text{ERB,RB})}) = \sum_{m=1}^{M} w_t^{[m]} \det(\mathbf{P}_t^{(\text{ERB,RB}),[m]}),$$

where, $\det(\mathbf{P}_t^{(\cdot)})$ is the covariance determinant of the proposal density. M is the total number of particles and 100 particles are used.

From the fact that the log value over the entire update step does not decrease under zero, we know that $\mathbf{P}_t^{\text{ERB}} > \mathbf{P}_t^{\text{RB}}$. This means the covariance $\mathbf{P}_t^{\text{RB}}$ is always underestimated.

The log of the ratio momentarily increases when the robot moves around unknown area, because an uncertainty of newly observed map component is large. It decreases again whenever the uncertainty of the map component is reduced by repeated updates.

Figure 2 and 3 show a dangerousness of this underestimated proposal distribution of the RB. In this second experiment, we allocate the small number of particles (3 particles) for minimizing the robustness originated from the multiple hypotheses, to confirm the effectiveness of the proposed method. Experimental conditions including a random seed are exactly the same.

In Fig. 2 (c), the resampling occurred due to large variance of the particle weights in the RB while the ERB still maintains the particle diversity with proper amount of uncertainties for the robot state estimate as shown in Fig. 2 (d). This means that the proposal density of the ERB is more similar to the true posterior, $p(\mathbf{x}_t|\mathbf{z}^t, \mathbf{u}^t)$.

In Fig. 3 (f), the two particles could find the correct identities of the landmarks, then it could succeed to update the proposal distribution. The RB diverged, because uncertainty of the proposal distribution is too small to sample the next generation robot state for providing the Mahalanobis distances which satisfy the χ^2 distribution.

This result tells us that the underestimated proposal distribution can eventually degenerate the filter performance. One can circumvent this limitation by using some expedients like

1) using more particles: this is an expedient that uses the inherent robustness from multiple hypotheses of the particle filtering algorithm.
2) using tolerant process and measurement noises: this can make the conventional Rao-Blackwellized unscented particle filter perform even with single particle. In this case, the estimation accuracy can be degraded.

However, these can not be the solution.

V. CONCLUDING REMARKS

In this paper, we pointed out the important problem of the conventional Rao-Blackwellized unscented particle filtering SLAM: the usage of the overconfident proposal distribution. We say that researchers in this field should perceive this problem and use the correct method proposed in this paper for their applications.

We experimentally validated the solution using the benchmark dataset under the fair conditions. We firstly compared the magnitude of the covariance of the proposal densities at every update step. The result reveals that covariance of the proposal distribution of the proposed method is always larger than that of the conventional method. As demonstrated in the second experiment, this larger covariance caused the slower increasing rate of the weight variance with less resamplings. From this fact we conclude that the proposal distribution provided in this paper is more similar to the true posterior.

VI. ACKNOWLEDGMENTS

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