

```
> L1 := [[0, 3],[3, 0],[1, 2],[2, 1]];
      LI := [[0,3],[3,0],[1,2],[2,1]]
```

(1)

```
> L2 := [[0, 0], [1, 0], [0, 1], [1, 1], [0, 2], [2, 0]];
      L2 := [[0,0],[1,0],[0,1],[1,1],[0,2],[2,0]]
```

(2)

```
> #we study the case that a_11+a_21 = 3, b_11+b_21 < 3, and a_12+
a_22 = 3
#note it is symmetric to a_11+a_21 < 3, b_11+b_21 = 3, and a_12+
a_22 = 3
```

```
> p := [];
S1 := [];
S2 := [];
count := 0;
for i from 1 to nops(L1) do
  for j from 1 to nops(L2) do
    for k from 1 to nops(L1) do
      t := [];
      p := [op(L1[i]), op(L2[j]), op(L1[k])];
      if p[1]=p[3] or p[2]=p[4] then
        next;
      fi:
      #t record the condition type
      if max(p[1], p[3])<p[5] then
        t := [op(t), 1]:
      fi:
      if max(p[2], p[4])<p[6] then
        t := [op(t), 2]:
      fi:
      if min(p[1], p[3])>p[5] and p[5]>0 then
        t := [op(t), 3]:
      fi:
      if min(p[2], p[4])>p[6] and p[6]>0 then
        t := [op(t), 4]:
      fi:
      if ((has(t, 1) or has(t, 3)) and p[1]<>p[3])
        or
        ((has(t, 2) or has(t, 4)) and p[2]<>p[4]))
        and p[1]*p[3]*p[5]=0
        and p[2]*p[4]*p[6]=0
      then
        count := count + 1:
        S1 := [op(S1), p];
        S2 := [op(S2), t];
        # print(p); print(t);
      fi:
    od:
  od:
od:
count;
```

```

> sym := {}:
for j from 1 to nops(S1) do
  for i from 1 to nops(S1) do
    p1 := S1[j]: p2 := S1[i]:
    if p1[1]=p2[2] and p1[2]=p2[1] and
      p1[3]=p2[4] and p1[4]=p2[3] and
      p1[5]=p2[6] and p1[6]=p2[5] then
      sym := sym union {{j, i}}:
    fi:
  od:
od:
sym;

{{1, 5}, {2, 6}, {3, 7}, {4, 8}, {9, 15}, {10, 14}, {11, 16}, {12, 17}, {13, 18}}
> S3 := []:
S4 := []:
for i from 1 to nops(sym) do
  if S2[sym[i][1]]=1 then
    S3 := [op(S3), S1[sym[i][1]]]:
    S4 := [op(S4), S2[sym[i][1]]]:
  else
    S3 := [op(S3), S1[sym[i][2]]]:
    S4 := [op(S4), S2[sym[i][2]]]:
  fi:
od:
for i from 1 to nops(S3) do
  print(S3[i], S4[i]);
od:

sym2 := {}:
for j from 1 to nops(S3) do
  for i from 1 to nops(S3) do
    p1 := S3[j]: p2 := S3[i]:
    if p1[1]=p2[3] and p1[2]=p2[4] and
      p1[3]=p2[1] and p1[4]=p2[2]
    then
      sym2 := sym2 union {{j, i}}:
    fi:
  od:
od:
nops(S3);
#The 9 elements are listed in the first column of Table 3
#For instance, the first element means [a11, a21, b11, b21, a12,
a22] = [0, 3, 1, 0, 3, 0]
[0, 3, 1, 0, 3, 0], [1]
[0, 3, 1, 0, 2, 1], [1]
[0, 3, 1, 1, 3, 0], [1]
[0, 3, 2, 0, 3, 0], [1]
[2, 1, 0, 0, 3, 0], [1]
[1, 2, 0, 0, 3, 0], [1]
[1, 2, 0, 0, 2, 1], [1]

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(4)

[1, 2, 0, 1, 3, 0], [1]
 [2, 1, 0, 2, 3, 0], [1]

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(5)

```
> flag := [1$nops(S3)]:
for i from 1 to nops(S3) do
  if has(S4[i], 1) and S3[i][6]=0
    and (S3[i][3]-S3[i][1])*(S3[i][4]-S3[i][2])<0 then
    flag[i] := 0:
  fi:
  if has(S4[i], 2) and S3[i][5]=0
    and (S3[i][3]-S3[i][1])*(S3[i][4]-S3[i][2])<0 then
    flag[i] := 0:
  fi:
  if flag[i]=1 then
    print(S3[i], S4[i]);
  fi:
od:
#This is a program to check if  $\pi_{\sigma}^{-1}(\sigma) \cap C = \emptyset$  in a simple way.
#For these 5 elements from the above 9 might,  $\pi_{\sigma}^{-1}(\sigma) \cap C$  might not be empty
#We proved in the paper (see Page 19, Lemma 5.5) that for [0, 3, 1, 0, 2, 1],  $\pi_{\sigma}^{-1}(\sigma) \cap C$  is still empty
#The last 4 are recored in the bold/colored cells. For these elements,  $\pi_{\sigma}^{-1}(\sigma) \cap C$  are non-empty.
```

[0, 3, 1, 0, 2, 1], [1]
 [2, 1, 0, 0, 3, 0], [1]
 [1, 2, 0, 0, 3, 0], [1]
 [1, 2, 0, 0, 2, 1], [1]
 [1, 2, 0, 1, 3, 0], [1]

(6)