```
> L1 := [
        [0, 0, 4], [4, 0, 0], [0, 4, 0],
        [1, 3, 0], [3, 1, 0], [1, 0, 3], [3, 0, 1], [0, 1, 3], [0,
  3, 1],
        [2, 2, 0], [2, 0, 2], [0, 2, 2],
        [1, 1, 2], [1, 2, 1], [2, 1, 1]
      ]:
> L2 := [
         [0, 0, 0],
         [0, 0, 1], [1, 0, 0], [0, 1, 0],
         [0, 0, 2], [2, 0, 0], [0, 2, 0], [1, 1, 0], [0, 1, 1],
  [1, 0, 1],
        [0, 0, 3], [3, 0, 0], [0, 3, 0],
        [1, 2, 0], [2, 1, 0], [1, 0, 2], [2, 0, 1], [0, 1, 2],
  [0, 2, 1], [1, 1, 1]
      ]:
> L3 := [op(L1), op(L2)]:
> ## any [a11, a21, a31, a12, a22, a32] in S1 satisfies
  ## a11 + a21 + a31 = a12 + a22 + a32 = 4

  p := []:
  S1 := []:
  count := 0:
  for i from 1 to nops(L1) do
    for j from 1 to nops(L1) do
       p := [op(L1[i]), op(L1[j])]:
       if min(p[1],p[4])+min(p[2], p[5])+min(p[3],p[6])=1 and
          (p[1]-p[4])*(p[2]-p[5])*(p[3]-p[6])<>0
       then
              S1 := [op(S1), p]:
       fi:
    od:
  od:
> nops(S1);
```


(1)

```
> sym := {}:
  for j from 1 to nops(S1) do
      t := {j}:
      for i from 1 to nops(S1) do
          p1 := S1[j]:  p2 := S1[i]:
          if p2 = [p1[2], p1[1], p1[3], p1[5], p1[4], p1[6]] or
             p2 = [p1[3], p1[2], p1[1], p1[6], p1[5], p1[4]] or
             p2 = [p1[1], p1[3], p1[2], p1[4], p1[6], p1[5]] or
             p2 = [p1[3], p1[1], p1[2], p1[6], p1[4], p1[5]] or
             p2 = [p1[2], p1[3], p1[1], p1[5], p1[6], p1[4]]
          then
               t := t union {i}:
          fi:
      od:
      sym := sym union {t}:
  od:
  sym;
```

(2)

{{1, 2, 3, 4, 5, 6}, {7, 11, 13, 17, 19, 23}, {8, 10, 14, 16, 20, 22}, {9, 12, 15, 18, 21, 24}, {25, 26, 27, 28, 29, 30}, {31, 32, 33, 34, 35, 36}} **(2)**

```
> sym2 := {}:
  for j from 1 to nops(S1) do
      for i from 1 to nops(S1) do
           p1 := S1[j]:  p2 := S1[i]:
           if p1[1]=p2[4] and p1[2]=p2[5] and
              p1[3]=p2[6] and p1[4]=p2[1] and
              p1[5]=p2[2] and p1[6]=p2[3] then
              sym2 := sym2 union {{j, i}}:
           fi:
      od:
  od:
  sym2;
```

{{1, 33}, {2, 35}, {3, 31}, {4, 34}, {5, 32}, {6, 36}, {7, 16}, {8, 19}, {9, 27}, {10, 13}, {11, 22}, {12, 29}, {14, 23}, {15, 25}, {17, 20}, {18, 30}, {21, 26}, {24, 28}} **(3)**

```
> # There are 3 equivalence classes in S1
  [seq(S1[k], k in sym[1])]:
  [seq(S1[k], k in sym[6])]: #(10)
```

[[0, 0, 4, 1, 2, 1], [0, 0, 4, 2, 1, 1], [4, 0, 0, 1, 1, 2], [4, 0, 0, 1, 2, 1], [0, 4, 0, 1, 1, 2], [0, 4, 0, 2, 1, 1]]

[[1, 1, 2, 4, 0, 0], [1, 1, 2, 0, 4, 0], [1, 2, 1, 0, 0, 4], [1, 2, 1, 4, 0, 0], [2, 1, 1, 0, 0, 4], [2, 1, 1, 0, 4, 0]] **(4)**

```
> [seq(S1[k], k in sym[2])]:
  [seq(S1[k], k in sym[3])]: #(9)
```

[[1, 3, 0, 3, 0, 1], [3, 1, 0, 0, 3, 1], [1, 0, 3, 3, 1, 0], [3, 0, 1, 0, 1, 3], [0, 1, 3, 1, 3, 0], [0, 3, 1, 1, 0, 3]]

[[1, 3, 0, 0, 1, 3], [3, 1, 0, 1, 0, 3], [1, 0, 3, 0, 3, 1], [3, 0, 1, 1, 3, 0], [0, 1, 3, 3, 0, 1], [0, 3, 1, 3, 1, 0]] **(5)**

```
> [seq(S1[k], k in sym[4])]:
  [seq(S1[k], k in sym[5])]: #(8)
```

[[1, 3, 0, 2, 0, 2], [3, 1, 0, 0, 2, 2], [1, 0, 3, 2, 2, 0], [3, 0, 1, 0, 2, 2], [0, 1, 3, 2, 2, 0], [0, 3, 1, 2, 0, 2]]

[[2, 2, 0, 1, 0, 3], [2, 2, 0, 0, 1, 3], [2, 0, 2, 1, 3, 0], [2, 0, 2, 0, 3, 1], [0, 2, 2, 3, 1, 0], [0, 2, 2, 3, 0, 1]] **(6)**

```
> ## any [a11, a21, a31, a12, a22, a32] in S3 satisfies
  ## a11 + a21 + a31 = 4, a12 + a22 + a32 < 4
  ## This case is symmetric to the case that a11 + a21 + a31 < 4,
  a12 + a22 + a32 = 4

  p := []:
  S3 := []:
  count := 0:
  for i from 1 to nops(L1) do
    for j from 1 to nops(L2) do
       p := [op(L1[i]), op(L2[j])]:
       if min(p[1],p[4])+min(p[2], p[5])+min(p[3],p[6])=1 and
```

```
          (p[1]-p[4])*(p[2]-p[5])*(p[3]-p[6])<>0
       then
              S3 := [op(S3), p]:
       fi:
    od:
  od:
>
  sym2 := {}:
  for j from 1 to nops(S3) do
      t := {j}:
      for i from 1 to nops(S3) do
          p1 := S3[j]:  p2 := S3[i]:
          if p2 = [p1[2], p1[1], p1[3], p1[5], p1[4], p1[6]] or
             p2 = [p1[3], p1[2], p1[1], p1[6], p1[5], p1[4]] or
             p2 = [p1[1], p1[3], p1[2], p1[4], p1[6], p1[5]] or
             p2 = [p1[3], p1[1], p1[2], p1[6], p1[4], p1[5]] or
             p2 = [p1[2], p1[3], p1[1], p1[5], p1[6], p1[4]]
          then
               t := t union {i}:
          fi:
      od:
      sym2 := sym2 union {t}:
  od:
  sym2;
  nops(sym2);
  #There are 9 equivalence classes in S3
```

{{1, 2, 3}, {34, 39, 44}, {4, 7, 10, 13, 16, 19}, {5, 9, 11, 15, 17, 21}, {6, 8, 12, 14, 18, 20}, {22, 23, 26, 27, 30, 31}, {24, 25, 28, 29, 32, 33}, {35, 36, 40, 41, 45, 46}, {37, 38, 42, 43, 47, 48}}

9 **(7)**

```
> [seq(S3[k], k in sym2[1])]:
  [seq(S3[k], k in sym2[2])]:
  [seq(S3[k], k in sym2[3])]:
  [seq(S3[k], k in sym2[4])]:
  [seq(S3[k], k in sym2[5])]:
  [seq(S3[k], k in sym2[6])]:
  [seq(S3[k], k in sym2[7])]:
  [seq(S3[k], k in sym2[8])]:
  [seq(S3[k], k in sym2[9])]:
```

[[0, 0, 4, 1, 1, 1], [4, 0, 0, 1, 1, 1], [0, 4, 0, 1, 1, 1]]

[[1, 1, 2, 0, 0, 1], [1, 2, 1, 0, 1, 0], [2, 1, 1, 1, 0, 0]]

[[1, 3, 0, 0, 1, 1], [3, 1, 0, 1, 0, 1], [1, 0, 3, 0, 1, 1], [3, 0, 1, 1, 1, 0], [0, 1, 3, 1, 0, 1], [0, 3, 1, 1, 1, 0]]

[[1, 3, 0, 2, 0, 1], [3, 1, 0, 0, 2, 1], [1, 0, 3, 2, 1, 0], [3, 0, 1, 0, 1, 2], [0, 1, 3, 1, 2, 0], [0, 3, 1, 1, 0, 2]]

[[1, 3, 0, 0, 1, 2], [3, 1, 0, 1, 0, 2], [1, 0, 3, 0, 2, 1], [3, 0, 1, 1, 2, 0], [0, 1, 3, 2, 0, 1], [0, 3, 1, 2, 1, 0]]

[[2, 2, 0, 0, 1, 1], [2, 2, 0, 1, 0, 1], [2, 0, 2, 1, 1, 0], [2, 0, 2, 0, 1, 1], [0, 2, 2, 1, 1, 0], [0, 2, 2, 1, 0, 1]]

[[2, 2, 0, 1, 0, 2], [2, 2, 0, 0, 1, 2], [2, 0, 2, 1, 2, 0], [2, 0, 2, 0, 2, 1], [0, 2, 2, 2, 1, 0], [0, 2, 2,

```
    2, 0, 1 ]]
[ [1, 1, 2, 2, 0, 0 ], [ 1, 1, 2, 0, 2, 0 ], [ 1, 2, 1, 0, 0, 2 ], [ 1, 2, 1, 2, 0, 0 ], [2, 1, 1, 0, 0, 2 ], [2, 1, 1,
    0, 2, 0 ]]
[ [1, 1, 2, 3, 0, 0 ], [ 1, 1, 2, 0, 3, 0 ], [ 1, 2, 1, 0, 0, 3 ], [ 1, 2, 1, 3, 0, 0 ], [2, 1, 1, 0, 0, 3 ], [2, 1, 1,      (8)
    0, 3, 0 ]]
> # Finally we list the 12 representives recoreded in Table 1
  S3[44];
  S3[23];
  S3[4];
  S3[40];
  S3[24];
  S3[6];
  S3[42];
  S1[25];
  S1[8];
  S1[33];
  S3[2];
  S3[9];

                                [2, 1, 1, 1, 0, 0 ]
                                [2, 2, 0, 1, 0, 1 ]
                                [1, 3, 0, 0, 1, 1 ]
                                [1, 2, 1, 0, 0, 2 ]
                                [2, 2, 0, 1, 0, 2 ]
                                [1, 3, 0, 0, 1, 2 ]
                                [1, 2, 1, 0, 0, 3 ]
                                [2, 2, 0, 1, 0, 3 ]
                                [1, 3, 0, 0, 1, 3 ]
                                [1, 2, 1, 0, 0, 4 ]
                                [4, 0, 0, 1, 1, 1 ]
                                [3, 1, 0, 0, 2, 1 ]                 (9)
```