ONE MORE EXAMPLE BEFORE BLOCK DESIGN

Airline Delays.jmp

Question: Is there significant difference in airline delays between airline companies?

RANDOMIZED COMPLETE BLOCK DESIGN: RECALL

Example 2 (modified): Suppose the firm believes the 15 stores are not homogeneous, namely, there possibly are store effects (e.g, one particular store makes its sales particularly high under all three promotions).

Then, it would be difficult to recognize differences among the treatments, due to the additional noise from store effects.

It may be better to observe each store under all three in-store promotions (e.g., assign each promotion randomly to three one-week periods).

In this design, each of the 15 stores acts as a *block*.

Within each block, treatments are assigned at random.

It is a 'Complete' Block Design, because each of the 3 promotions is assigned to every block.

EXAMPLE

- **A fast food franchise is test-marketing 3 new menu items. To find out if they have the same popularity, 5 franchise restaurants are randomly chosen for participation in the study.**
- **In accordance with the randomized block design, each restaurant will be test marketing all 3 new menu items. (This is a generalization of the pair-wise t-test setting!!)**
- **Furthermore, a restaurant will test market only one menu item per week, and it takes 3 weeks to test market all menu items. *The testing order of the menu items for each restaurant is randomly assigned as well.***

THE RANDOMIZED BLOCK DESIGN

Like One-Way ANOVA, we test for equal population means (for different factor levels, for example)...

...but we want to control for possible variation from a second factor (with two or more levels)

A block design is used when you want to account for KNOWN sources of variation in your experiment (e.g. people, restaurant, store effect).

Levels of the secondary factor are called blocks

TEST FOR BLOCK EFFECT

$$H_0 : \mu_{1.} = \mu_{2.} = \mu_{3.} = \ldots = \mu_{r.}$$

H_1 : Not all block means are equal

$$F_{STAT} = \frac{MSBL}{MSE}$$

Blocking test: $df_1 = r - 1$

$df_2 = (r - 1)(c - 1)$

$\boxed{\text{Reject } H_0 \text{ if } F_{STAT} > F_\alpha}$

PARTITIONING THE VARIATION

Total variation can now be split into three parts:

$$SST = SSB + SSBL + SSE$$

SST = Total variation
SSB = Between-Group variation
SSBL = Between-Block variation
SSE = Random variation

PARTITIONING THE VARIATION

Total variation can now be split into three parts:

$$\boxed{SST} = \boxed{SSA} + SSBL + \boxed{SSE}$$

SST and SSA are computed as they were in One-Way ANOVA

SSE = SST – (SSA + SSBL)

SUM OF SQUARES FOR BLOCKS

$$SST = SSB + SSBL + SSE$$

$$SSBL = c\sum_{i=1}^{r}(\bar{X}_{i.} - \bar{\bar{X}})^2$$

Where:

c = number of groups (e.g. 3 menus)

r = number of blocks (e.g. 5 restaurants)

$\bar{X}_{i.}$ = mean of all values in block i

$\bar{\bar{X}}$ = grand mean (mean of all data values)

MEAN SQUARES

$$\text{MSBL} = \text{Mean square blocking} = \frac{\text{SSBL}}{\text{r} - 1}$$

$$\text{MSB} = \text{Mean square among groups} = \frac{\text{SSB}}{\text{c} - 1}$$

$$MSE = \text{Mean square error} = \frac{SSE}{(r-1)(c-1)}$$

Randomized Block ANOVA Table

Source of Variation	SS	df	MS	F
Among Blocks	SSBL	r - 1	MSBL	$\frac{\text{MSBL}}{\text{MSE}}$
Among Groups	SSB	c - 1	MSB	$\frac{\text{MSB}}{\text{MSE}}$
Error	SSE	(r–1)(c-1)	MSE	
Total	SST	rc - 1		

c = number of groups
r = number of blocks
rc = total number of observations
df = degrees of freedom

TESTING FOR FACTOR EFFECT

$$H_0 : \mu_{.1} = \mu_{.2} = \mu_{.3} = \cdots = \mu_{.c}$$

$$H_1 : \text{Not all population means are equal}$$

$$F_{STAT} = \frac{MSA}{MSE}$$

Main Factor test: $df_1 = c - 1$

$df_2 = (r - 1)(c - 1)$

$$\boxed{\text{Reject } H_0 \text{ if } F_{STAT} > F_\alpha}$$

EXAMPLE

- **A fast food franchise is test-marketing 3 new menu items. To find out if they have the same popularity, 5 franchisee restaurants are randomly chosen for participation in the study.**
- **In accordance with the randomized block design, each restaurant will be test marketing all 3 new menu items.**
- **Furthermore, a restaurant will test market only one menu item per week, and it takes 3 weeks to test market all menu items. *The testing order of the menu items for each restaurant is randomly assigned as well.***

MARKETING.JMP

- ***Are there differences in the mean popularity among the three menus?***
- ***<u>Does blocking make difference?</u>***

FACTORIAL DESIGN: TWO-WAY ANOVA

Examines the effect of

- Two factors of interest on the dependent variable
    - e.g., Percent carbonation and line speed on soft drink bottling process
- Interaction between the different levels of these two factors
    - e.g., Does the effect of one particular carbonation level depend on which level the line speed is set?

TWO-WAY ANOVA

(continued)

Assumptions

- Populations are normally distributed
- Populations have equal variances
- Independent random samples are drawn

TWO-WAY ANOVA
SOURCES OF VARIATION

Two Factors of interest: A and B

a = number of levels of factor A

b = number of levels of factor B

r = number of replications for each cell

n = total number of observations in all cells

n = abr

X_{ijk} = value of the k^{th} observation of level i of factor A and level j of factor B

TWO-WAY ANOVA SOURCES OF VARIATION

(continued)

SST = SSA + SSB + SSAB + SSE

Degrees of Freedom:

SST
Total Variation

n - 1

SSA
Factor A Variation

a – 1

SSB
Factor B Variation

b – 1

SSAB
Variation due to interaction between A and B

(a – 1)(b – 1)

SSE
Random variation (Error)

ab(r – 1)

TWO-WAY ANOVA DATA TABLE

Factor	Factor B			
A	1	2	...	b
1	X_{111}	X_{121}	...	X_{1b1}
	X_{112}	X_{122}	...	X_{1b2}
2	X_{211}	X_{221}	...	X_{2b1}
	X_{212}	X_{222}	...	X_{2b2}
:	:	:	:	:
a	X_{a11}	X_{a21}	...	X_{ab1}
	X_{a12}	X_{a22}	...	X_{ab2}

Observation k

$$X_{ijk}$$

Level i Factor A

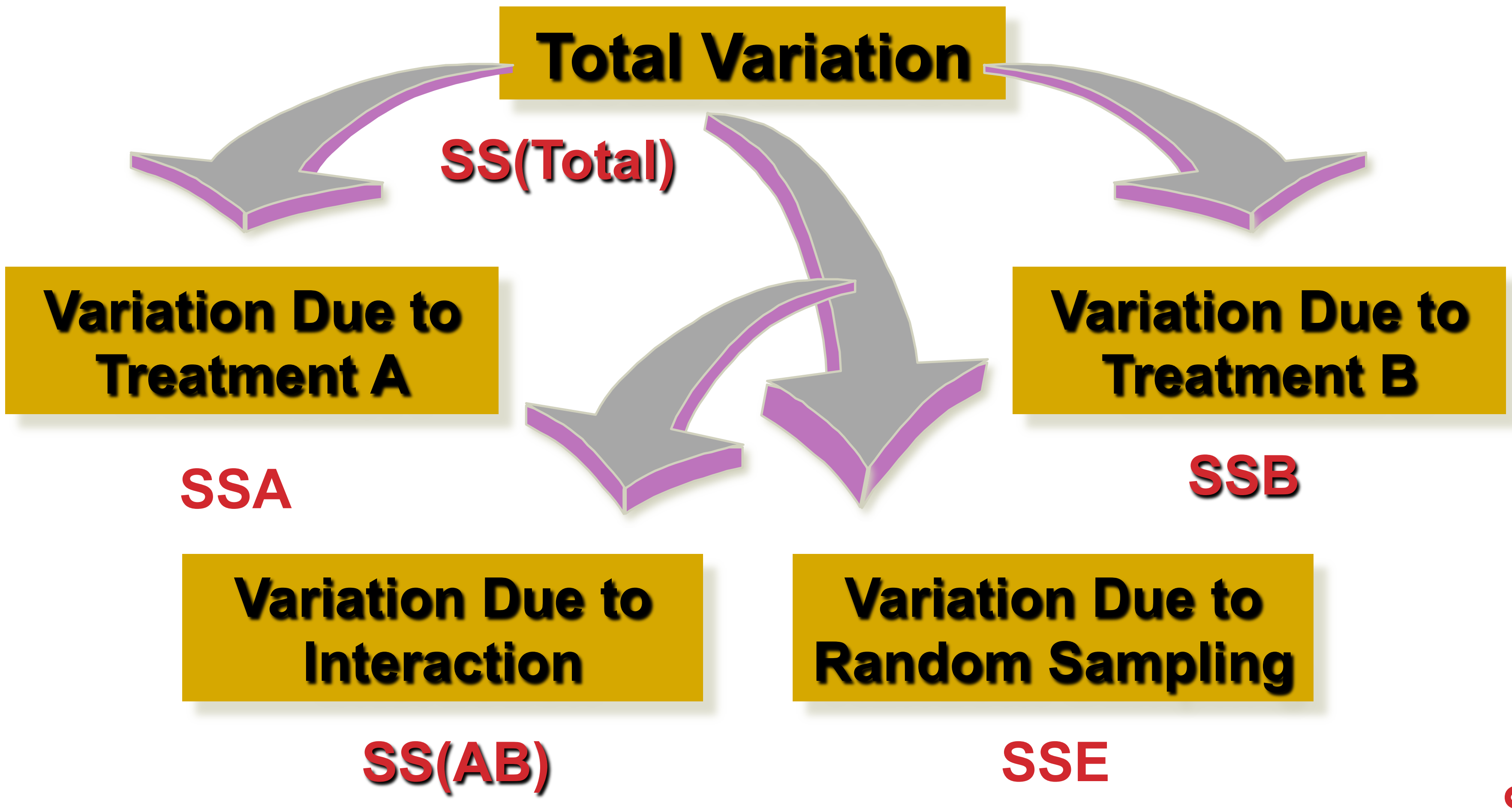
Level j Factor B

TWO-WAY ANOVA NULL HYPOTHESES

1. **No Difference in Means Due to Factor A**
   - H_0: $\mu_{1.} = \mu_{2.} = \ldots = \mu_{a.}$
2. **No Difference in Means Due to Factor B**
   - H_0: $\mu_{.1} = \mu_{.2} = \ldots = \mu_{.b}$
3. **No Interaction of Factors A & B**
   - H_0: $AB_{ij} = 0$

Main effect: Due to Factor itself (A, B)

TWO-WAY ANOVA
TOTAL VARIATION PARTITIONING



INTERACTION

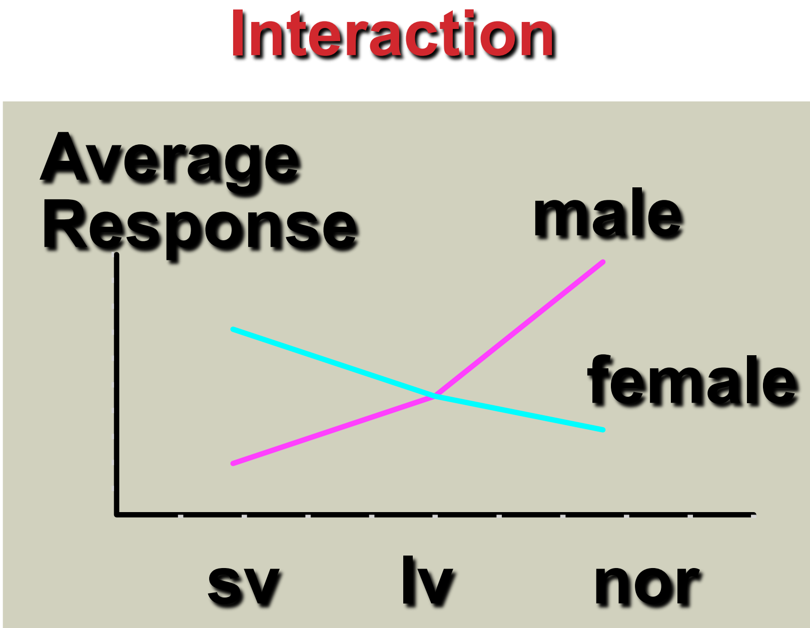
- **Occurs When Effects of One Factor Vary According to Levels of Other Factor**

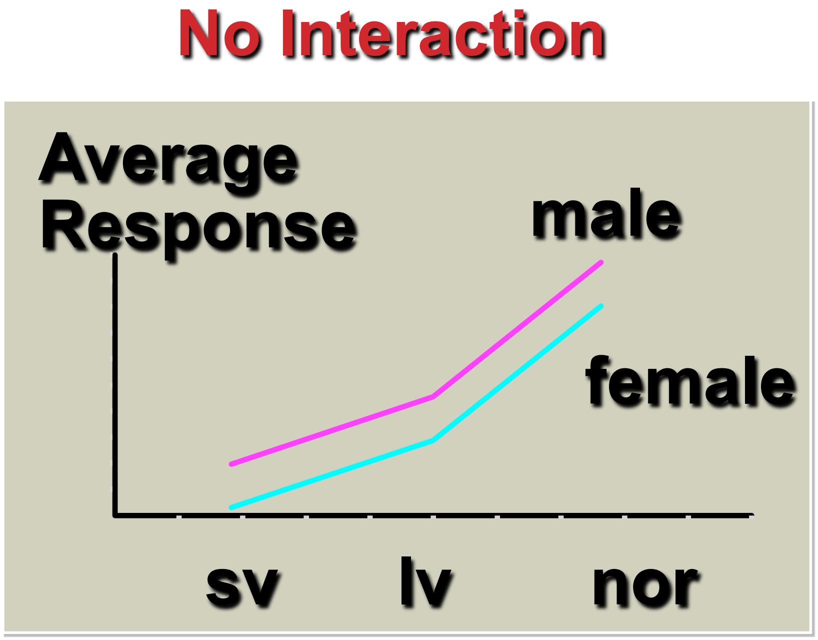
  - Example: What is the interaction of gender and ethnicity with respect to income? **Is the effect of gender different for different categories of ethnicity?**

  - Example: What is the combined effect of gender and dietary method on systolic blood pressure? **Is the effect of gender different of different categories of dietary groups?**

GRAPHS OF INTERACTION

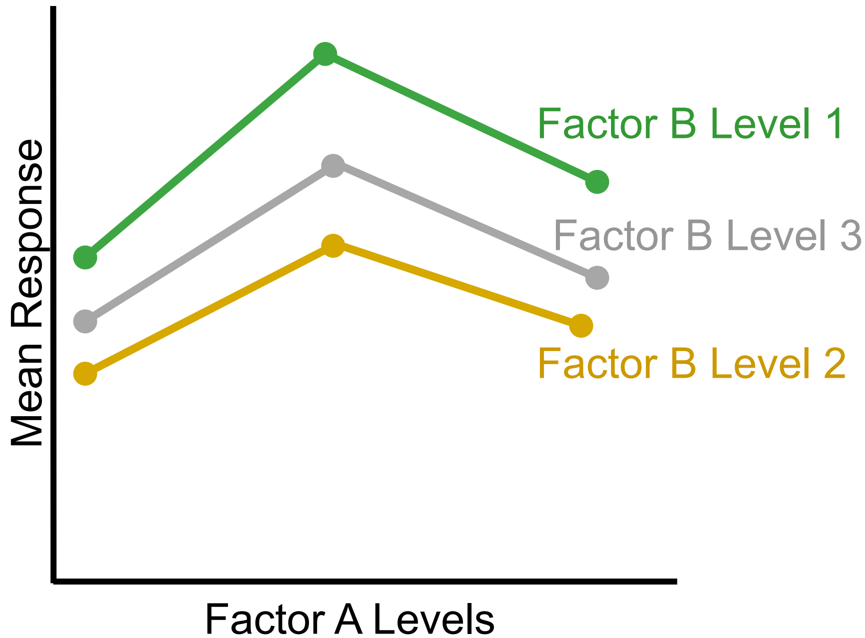
Effects of Gender (male or female) & dietary group (sv, lv, nor) on systolic blood pressure



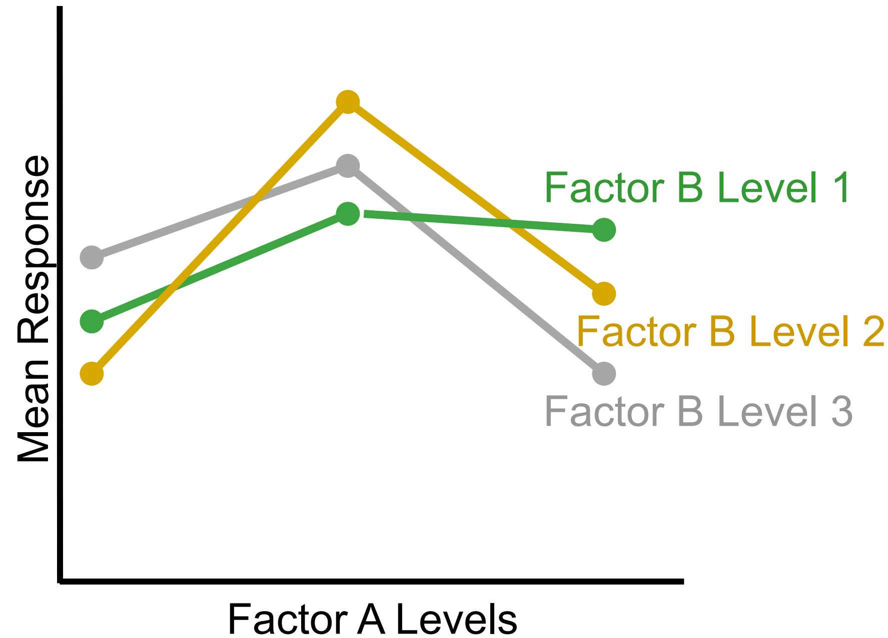


EXAMPLES: INTERACTION VS. NO INTERACTION

No interaction: line segments are parallel



- **Interaction is present: some line segments not parallel**



TWO-WAY ANOVA F-TEST EXAMPLE

Effect of diet (sv-strict vegetarians, lv-lactovegetarians, nor-normal) and gender (female, male) on systolic blood pressure.

Question: Test for interaction and main effects at the .05 level.

TWO-WAY ANOVA: NOTATION

(continued)

$$\bar{\bar{X}} = \frac{\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r} X_{ijk}}{abr} = \text{Grand Mean}$$

$$\bar{X}_{i..} = \frac{\sum_{j=1}^{b}\sum_{k=1}^{r} X_{ijk}}{br} = \text{Mean of } i^{\text{th}} \text{ level of factor A } (i = 1, 2, \ldots, a)$$

$$\bar{X}_{.j.} = \frac{\sum_{i=1}^{a}\sum_{k=1}^{r} X_{ijk}}{ar} = \text{Mean of } j^{\text{th}} \text{ level of factor B } (j = 1, 2, \ldots, b)$$

$$\bar{X}_{ij.} = \sum_{k=1}^{r} \frac{X_{ijk}}{r} = \text{Mean of cell ij}$$

a = number of levels of factor A
b = number of levels of factor B
r = number of replications in each cell

TWO-WAY ANOVA VARIATIONS

Total Variation:

$$SST = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(X_{ijk} - \bar{\bar{X}})^2$$

Factor A Variation:

$$SSA = br\sum_{i=1}^{a}(\bar{X}_{i..} - \bar{\bar{X}})^2$$

Factor B Variation:

$$SSB = ar\sum_{j=1}^{b}(\bar{X}_{.j.} - \bar{\bar{X}})^2$$

TWO-WAY ANOVA VARIATIONS

(continued)

Interaction Variation:

$$\mathrm{SSAB} = r\sum_{i=1}^{a}\sum_{j=1}^{b}(\overline{\mathrm{X}}_{\mathrm{ij.}} - \overline{\mathrm{X}}_{\mathrm{i..}} - \overline{\mathrm{X}}_{\mathrm{.j.}} + \overline{\overline{\mathrm{X}}})^2$$

Sum of Squares Error:

$$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{r}(X_{ijk} - \overline{X}_{ij.})^2$$

MEAN SQUARE CALCULATIONS

$$MSA = \text{Mean square factor A} = \frac{SSA}{a-1}$$

$$MSB = \text{Mean square factor B} = \frac{SSB}{b-1}$$

$$MSAB = \text{Mean square interaction} = \frac{SSAB}{(a-1)(b-1)}$$

$$MSE = \text{Mean square error} = \frac{SSE}{ab(r-1)}$$

TWO-WAY ANOVA: THE F TEST STATISTICS

F Test for Factor A Effect

H_0: $\mu_{1..} = \mu_{2..} = \mu_{3..} = \bullet \bullet = \mu_{r..}$

H_1: Not all $\mu_{i..}$ are equal

$$F_{STAT} = \frac{MSA}{MSE}$$

Reject H_0 if $F_{STAT} > F_\alpha$

F Test for Factor B Effect

H_0: $\mu_{.1.} = \mu_{.2.} = \mu_{.3.} = \bullet \bullet = \mu_{.c.}$

H_1: Not all $\mu_{.j.}$ are equal

$$F_{STAT} = \frac{MSB}{MSE}$$

Reject H_0 if $F_{STAT} > F_\alpha$

F Test for Interaction Effect

H_0: the interaction of A and B is equal to zero

H_1: interaction of A and B is not zero

$$F_{STAT} = \frac{MSAB}{MSE}$$

Reject H_0 if $F_{STAT} > F_\alpha$

TWO-WAY ANOVA SUMMARY TABLE

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F
Factor A	SSA	$a - 1$	MSA $= SSA/(a - 1)$	$\frac{MSA}{MSE}$
Factor B	SSB	$b - 1$	MSB $= SSB/(b - 1)$	$\frac{MSB}{MSE}$
AB (Interaction)	SSAB	$(a - 1)(b - 1)$	MSAB $= SSAB / (a - 1)(b - 1)$	$\frac{MSAB}{MSE}$
Error	SSE	$ab(r - 1)$	MSE $= SSE/ab(r - 1)$	
Total	SST	$n - 1$		

FEATURES OF TWO-WAY ANOVA F TEST

Degrees of freedom always add up

- n-1 = (a-1) + (b-1) + (a-1)(b-1) +ab(r-1)
- Total = factor A + factor B + interaction + error

The denominators of the F Test are always the same but the numerators are different

The sums of squares always add up

- SST = SSE + SSA + SSB + SSAB
- Total = error + factor A + factor B + interaction

EXAMPLES

Analgesics.jmp (painkiller, drugs, male/female)

Is Male/Female factor significant?

Is there a Drug effect?

Is there an interaction effect? Is the effect of drugs different for different gender?