FRACTIONAL FACTORIAL DESIGN AND PLACKETT-BURMAN DESIGNS

ORTHOGONALITY OF DESIGN

- A design is called orthogonal if for any two design factors (in design matrix), each factor-level combination has the same number of runs.
- That is, for any two factors (columns) the number of runs at each of the four factor-level combinations (-1,-1), (-1,+1), (+1,-1), (+1,+1) is the same.
- In fact, both full factorial and fractional factorial designs have this property.

NOTE: ORTHOGONAL DESIGN

- **This property can be checked by *multiplying any two columns* and summing the products → if it's zero for any pairs, then the design is orthogonal. (similar to vector inner product = 0→ perpendicular)**
- Orthogonal design is very nice since it implies effects are estimated independently.

→ In calculating the main effect of A, the main effect of B has no influence; whenever A is at +1 (or -1), B is equally likely to be at +1 or -1. → Any change in one effect is canceled out in the estimate of any other effect.

PLACKETT-BURMAN (PB) DESIGNS

- Plackett-Burman designs are an alternative to fractional-factorial screening designs.
- We looked at 2^{k-p} designs, which give us designs that have 8, 16, 32, 64, 128, etc. number of runs.
- Note: There is a pretty big gap between 16 and 32, 32 to 64, etc.
- We sometimes need other alternative designs besides these with a different number of observations, due to budget/time restrictions.

FRACTIONAL FACTORIAL TO PB DESIGN

- **What is a price to pay going from FFD to PB design?**
- **PB design maintains orthogonal property among the main factors only.**
  - PB design should be used with caution in that interaction effects (if they are 'active') are hard to detect by PB design.
- **FFD has orthogonal property among main factors and interactions.**

PLACKETT-BURMAN (PB) DESIGNS

- PB designs exist for

  N = 12, [16], 20, 24, 28, [32], 36, 40, 44, 48, ...

  → <u>any number which is divisible by four.</u>

- PB designs are Resolution III designs, meaning you can estimate main effects clear of other main effects. Main effects are confounded with other two-way interactions.

PLACKETT-BURMAN (PB) DESIGNS VS. FFD

- **However, the actual confounding patter is not simple**

E.g.,

In FFD, Resolution III design, each 2-factor interaction is confounded with a single main effect.

- In PB, each 2-factor interaction is confounded with more than one main effects.
- → This is a price to pay moving from FFD to PB.

PLACKETT-BURMAN (PB) DESIGNS

- **If the run size is a power of 2 (N=8, 16, 32, …), then PB designs are identical to the fractional factorial designs w/ Resolution III.**
- **In a resolution III design, main effects are aliased with 2-way interactions. Therefore, you should only use these designs when you are willing to assume that 2-way interactions are negligible.**
- **Use Plackett-Burman designs to identify the most important factors early in the experimentation phase.**

PLACKETT-BURMAN (PB) DESIGNS

- NOTE: PB designs are useful only when you are trying to screen out main effects.
- For example, where there are 8 factors, you can use 2^{8-4} FFD with resolution IV (16 runs), OR you can use the PB design with 12 runs, if economy of run size is important.
- After screening for important factors (say, 8 factors to 3 factors), then perform a further experiment based on Full Factorial design.

PLACKETT-BURMAN (PB) DESIGNS

- JMP Demo
- DOE > Screening Design
- Load Responses, Load Factors (if you have them already in files, e.g. Reactor Response.jmp, Reactor Factors.jmp)
- Fill in responses. (Plackett Burman.jmp)
- Run Model Fit: showing only a single significant effect.

PLACKETT-BURMAN (PB) DESIGNS: CASE STUDY

- **Experimental design on the front lines of marketing: Testing new ideas to increase direct mail sales by Gordon H. Bell, Johannes Ledolter, Arthur J. Swersey (2006)**
  International Journal of Research in Marketing
- **Paper uploaded onto Canvas**
- ***Effective use of direct mails to reach new customers?***
- ***Planning and execution of a large-scale mailing of a credit card offer***

THE MARKETING TEAM IDENTIFIED 19 FACTORS TO TEST: 20 RUNS

Factor	(−) Control	(+) New idea
A: Envelope teaser	General offer	Product-specific offer
B: Return address	Blind	Add company name
C: "Official" ink-stamp on envelope	Yes	No
D: Postage	Pre-printed	Stamp
E: Additional graphic on envelope	Yes	No
F: Price graphic on letter	Small	Large
G: Sticker	Yes	No
H: Personalize letter copy	No	Yes
I: Copy message	Targeted	Generic
J: Letter headline	Headline 1	Headline 2
K: List of benefits	Standard layout	Creative layout
L: Postscript on letter	Control version	New P.S.
M: Signature	Manager	Senior executive
N: Product selection	Many	Few
O: Value of free gift	High	Low
P: Reply envelope	Control	New style
Q: Information on buckslip	Product info	Free gift info
R: 2nd buckslip	No	Yes
S: Interest rate	Low	High

NOTES

- The firm's marketing team used OFAT in the past to identify main effects and an optimal combination.
- Factors A-E were approaches aimed at getting more people to look inside the envelope, while the remaining factors related to the offer inside.
- Factor G (sticker) refers to the peeling-off sticker at the top of the letter to be applied by the customer to the order form.
- Factor N (product selection) refers to the number of different credit card images that a customer could choose from.
- The term "buckslip" (factors Q and R) describes a small separate sheet of paper that highlights product information.

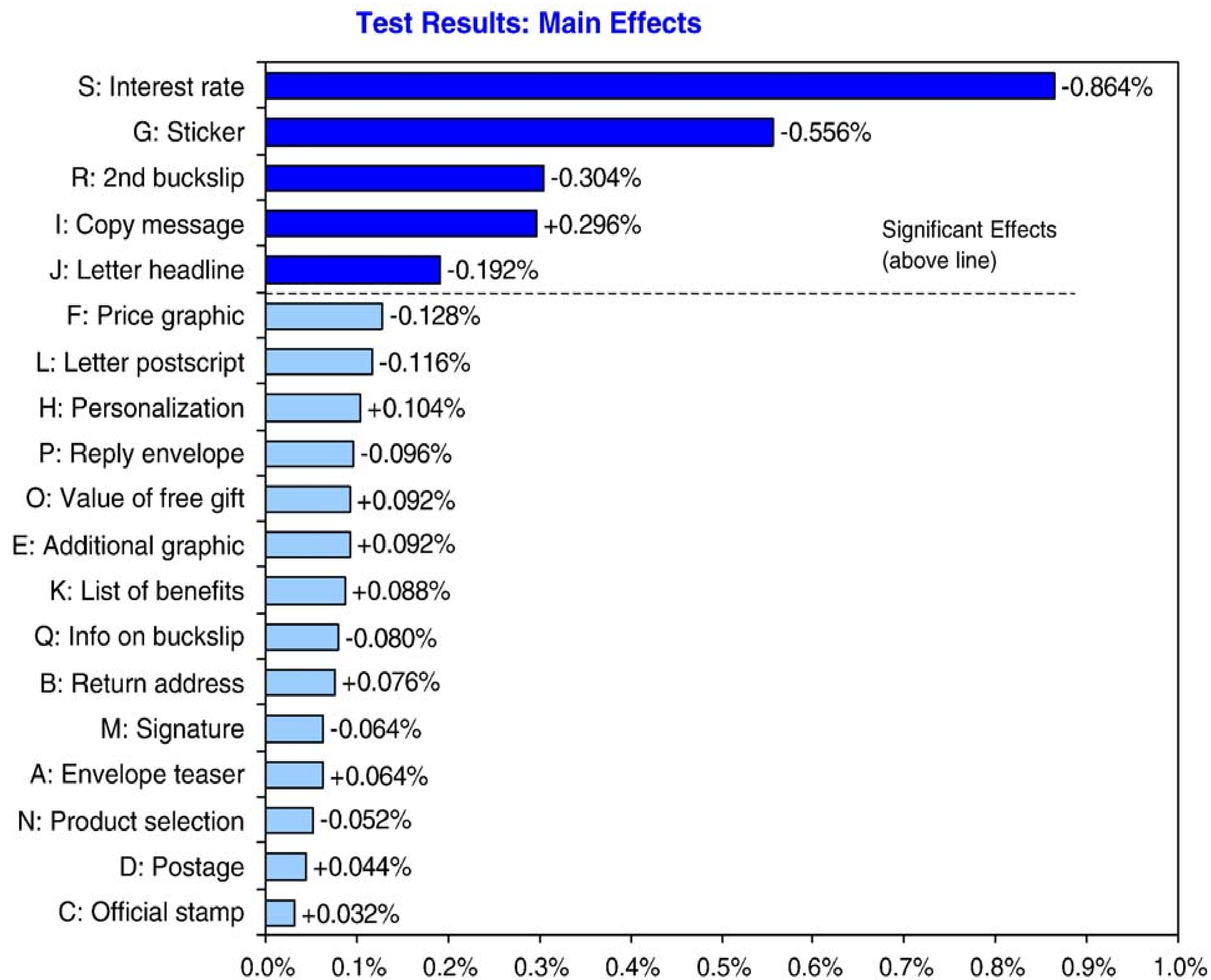
A total of 100,000 (100,000/20runs = 5,000 in each run) people, randomly chosen from a list of potential customers, participated in the experiment.

Response rates in the 20-run Plackett–Burman design

Test cell	Envelope teaser A	Return address B	“Official” ink-stamp on envelope C	Postage D	Additional graphic on envelope E	Price graphic on letter F	Sticker G	Personalize letter copy H	Copy message I	Letter headline J
1	+	+	−	−	+	+	+	+	−	+
2	−	+	+	−	−	+	+	+	+	−
3	+	−	+	+	−	−	+	+	+	+
4	+	+	−	+	+	−	−	+	+	+
5	−	+	+	−	+	+	−	−	+	+
6	−	−	+	+	−	+	+	−	−	+
7	−	−	−	+	+	−	+	+	−	−
8	−	−	−	−	+	+	−	+	+	−
9	+	−	−	−	−	+	+	−	+	+
10	−	+	−	−	−	−	+	+	−	+
11	+	−	+	−	−	−	−	+	+	−
12	−	+	−	+	−	−	−	−	+	+
13	+	−	+	−	+	−	−	−	−	+
14	+	+	−	+	−	+	−	−	−	−
15	+	+	+	−	+	−	+	−	−	−
16	+	+	+	+	−	+	−	+	−	−
17	−	+	+	+	+	−	+	−	+	−
18	−	−	+	+	+	+	−	+	−	+
19	+	−	−	+	+	+	+	−	+	−
20	−	−	−	−	−	−	−	−	−	−

List of benefits K	Postscript on letter L	Signature M	Product selection N	Value of free gift O	Reply envelope P	Information on buckslip Q	2nd buckslip R	Interest rate S	Orders	Response rate
−	+	−	−	−	−	+	+	−	52	1.04%
+	−	+	−	−	−	−	+	+	38	0.76%
−	+	−	+	−	−	−	−	+	42	0.84%
+	−	+	−	+	−	−	−	−	134	2.68%
+	+	−	+	−	+	−	−	−	104	2.08%
+	+	+	−	+	−	+	−	−	60	1.20%
+	+	+	+	−	+	−	+	−	61	1.22%
−	+	+	+	+	−	+	−	+	68	1.36%
−	−	+	+	+	+	−	+	−	57	1.14%
+	−	−	+	+	+	+	−	+	30	0.60%
+	+	−	−	+	+	+	+	−	108	2.16%
−	+	+	−	−	+	+	+	+	39	0.78%
+	−	+	+	−	−	+	+	+	40	0.80%
+	+	−	+	+	−	−	+	+	49	0.98%
−	+	+	−	+	+	−	−	+	37	0.74%
−	−	+	+	−	+	+	−	−	99	1.98%
−	−	−	+	+	−	+	+	−	86	1.72%
−	−	−	−	+	+	−	+	+	43	0.86%
+	−	−	−	−	+	+	−	+	47	0.94%
−	−	−	−	−	−	−	−	−	104	2.08%

SCREENING (INITIAL) ANALYSIS



FIVE SIGNIFICANT FACTORS

- **S− or Low interest rate: Increasing the credit card interest rate reduces the response by 0.864 percentage points. In addition, it was very clear based on the firm's financial models that the gain from the higher rate would be much less than the loss due to the decrease in the number of customers.**
- **G− or Sticker: The sticker (G−) increases the response by 0.556 percentage points, resulting in a gain much greater than the cost of the sticker.**
- **R− or No 2nd buckslip: A main effect interpretation shows that adding another buckslip reduces the number of buyers by 0.304 percentage points. One explanation offered for this surprising result was that the buckslip added unnecessary information and obscured the simple "buy now" offer. A more compelling explanation is that the significant effect is not the result of the main effect of factor R, but is due to an interaction between two other factors.**
- **I+ or Generic copy message: The targeted message (I−) emphasized that a person could choose a credit card design that reflected his or her interests, while the generic message (I+) focused on the value of the offer. The creative team was certain that appealing to a person's interests would increase the response, but they were wrong. The generic message increased the response by 0.296 percentage points.**
- **J− or Letter headline #1: The result showed that all "good" headlines were not equal. The best wording increased the response by 0.192 percentage points.**

FURTHER ANALYSIS OF THE RESULTS

- **The confounding of main effects and interactions introduces some uncertainty into our interpretation of the results.**
- **We cannot be certain which combinations of main effects and interactions are responsible for the significant estimates. Use, e.g., interaction plots to screen out significant ones, and then run regression models.**

Regression of response rate on S, G, SG, I, and J

Predictor	Coefficients	StdError	*t*-ratio	*P*-value
Constant	1.298	0.044	29.46	0.000
S	−0.432	0.044	−9.80	0.000
G	−0.278	0.044	−6.31	0.000
SG	0.151	0.046	3.29	0.005
I	0.118	0.045	2.62	0.020
J	−0.066	0.045	−1.46	0.166

$$\text{Rate} = 1.298 - (0.432)\text{S} - (0.278)\text{G} + (0.151)\text{SG} + (0.118)\text{I} - (0.066)\text{J};\ R^2 = 0.921.$$

PLACKETT AND BURMAN DESIGNS

- **Their 1964 paper in the journal *Biometrika***
- **For *k* factors, it requires *X* number of runs**
  - *X* = next multiple of 4 ≥ *k (e.g., k=11 -> X= 12)*

PB design matrix construction:

- It starts from the first *row (not column)*
- First row = from P&B paper
- Subsequent rows = circular right shift of preceding row
- Last row = all (-1)

PB DESIGN MATRIX (7 FACTORS)

Runs	Factors							Response
	A	B	C	D	E	F	G	
1	+1	+1	+1	-1	+1	-1	-1	9
2	-1	+1	+1	+1	-1	+1	-1	
3	-1	-1	+1	+1	+1	-1	+1	
4	+1	-1	-1	+1	+1	+1	-1	
5	-1	+1	-1	-1	+1	+1	+1	
6	+1	-1	+1	-1	-1	+1	+1	
7	+1	+1	-1	+1	-1	-1	+1	
8	-1	-1	-1	-1	-1	-1	-1	
Effect								

PB DESIGN MATRIX

Runs	Factors							Response
	A	B	C	D	E	F	G	
1	+1	+1	+1	-1	+1	-1	-1	9
2	-1	+1	+1	+1	-1	+1	-1	11
3	-1	-1	+1	+1	+1	-1	+1	
4	+1	-1	-1	+1	+1	+1	-1	
5	-1	+1	-1	-1	+1	+1	+1	
6	+1	-1	+1	-1	-1	+1	+1	
7	+1	+1	-1	+1	-1	-1	+1	
8	-1	-1	-1	-1	-1	-1	-1	
Effect								

PB DESIGN MATRIX

Runs	Factors							Response
	A	B	C	D	E	F	G	
1	+1	+1	+1	-1	+1	-1	-1	9
2	-1	+1	+1	+1	-1	+1	-1	11
3	-1	-1	+1	+1	+1	-1	+1	2
4	+1	-1	-1	+1	+1	+1	-1	1
5	-1	+1	-1	-1	+1	+1	+1	9
6	+1	-1	+1	-1	-1	+1	+1	74
7	+1	+1	-1	+1	-1	-1	+1	7
8	-1	-1	-1	-1	-1	-1	-1	4
Effect								

PB DESIGN MATRIX

Runs	Factors							Response
	A	B	C	D	E	F	G	
1	+1	+1	+1	-1	+1	-1	-1	9
2	-1	+1	+1	+1	-1	+1	-1	11
3	-1	-1	+1	+1	+1	-1	+1	2
4	+1	-1	-1	+1	+1	+1	-1	1
5	-1	+1	-1	-1	+1	+1	+1	9
6	+1	-1	+1	-1	-1	+1	+1	74
7	+1	+1	-1	+1	-1	-1	+1	7
8	-1	-1	-1	-1	-1	-1	-1	4
Effect	65							

PB DESIGN MATRIX

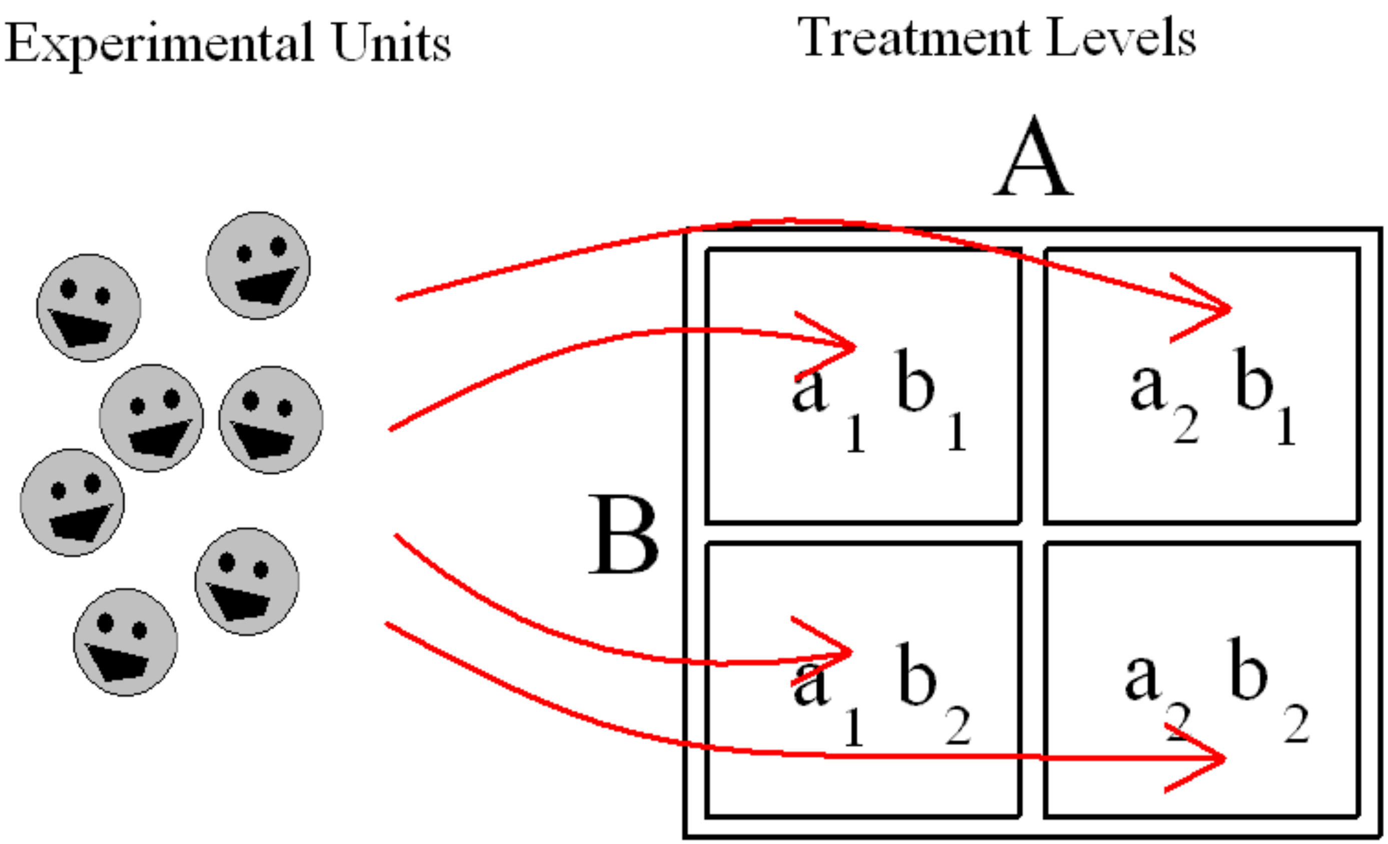
Runs	Factors							Response
	A	B	C	D	E	F	G	
1	+1	+1	+1	-1	+1	-1	-1	9
2	-1	+1	+1	+1	-1	+1	-1	11
3	-1	-1	+1	+1	+1	-1	+1	2
4	+1	-1	-1	+1	+1	+1	-1	1
5	-1	+1	-1	-1	+1	+1	+1	9
6	+1	-1	+1	-1	-1	+1	+1	74
7	+1	+1	-1	+1	-1	-1	+1	7
8	-1	-1	-1	-1	-1	-1	-1	4
Effect	65	-45						

PB DESIGN MATRIX

Runs	Factors							Response
	A	B	C	D	E	F	G	
1	+1	+1	+1	-1	+1	-1	-1	9
2	-1	+1	+1	+1	-1	+1	-1	11
3	-1	-1	+1	+1	+1	-1	+1	2
4	+1	-1	-1	+1	+1	+1	-1	1
5	-1	+1	-1	-1	+1	+1	+1	9
6	+1	-1	+1	-1	-1	+1	+1	74
7	+1	+1	-1	+1	-1	-1	+1	7
8	-1	-1	-1	-1	-1	-1	-1	4
Effect	65	-45	75	-75	-75	73	67	

BLOCKS IN FACTORIAL DESIGNS

COMPLETELY RANDOMIZED TWO-FACTOR DESIGN



Completely Randomized Design:

All treatments are randomized in the same way

BLOCKINGA

✿In most experiments, causal factors for y (be they known or unknown) are subdivided into three groups:

- 1) Factors whose effects are under study, by systematically varying their levels according to an experimental design's specifications ("experimental factors"); e.g., Effect of Tires on Stopping Distance
- 2) Factors not under study, whose levels are held fixed for the entire experiment ("background effects"); e.g., Driver
- 3) Factors whose levels and effects on y are uncontrolled ("random error"); e.g., unknown effects on Stop Distance.

✿Note that for any experiment to be really useful, the "background effect" factors must not interact with the factors under study.

✿Also, the factors with really BIG effects should not be left to vary out of control as random error.

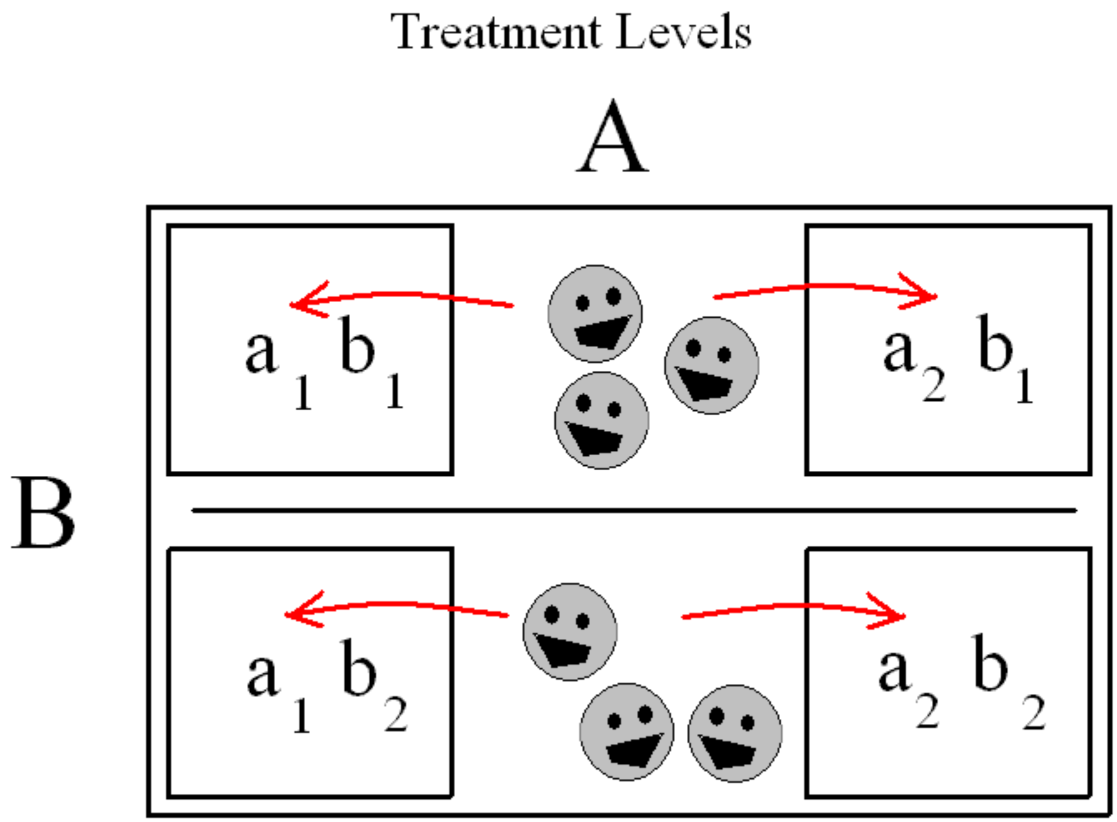
BLOCKING

- ✿In many instances factors which are not of interest cannot be held fixed for the entire experiment; yet, we do not want to let them vary out of control. These are the instances in which we can often use *blocking*.
- ✿Examples: Effects due to different
  - batches of raw material
  - operators,
  - machines, locations,
  - time periods (day 1, day 2, …),
  - Age, gender, etc.

BLOCKING

When we block an experiment, we subdivide its runs into groups, usually of equal size, called *blocks*. Ideally, within each block the background effects are fixed. It's o.k. if they vary block to block, without interacting with the experimental factors. The order of the runs within each block is randomized. An additional Factor(s), Blocks, is used in the analysis.

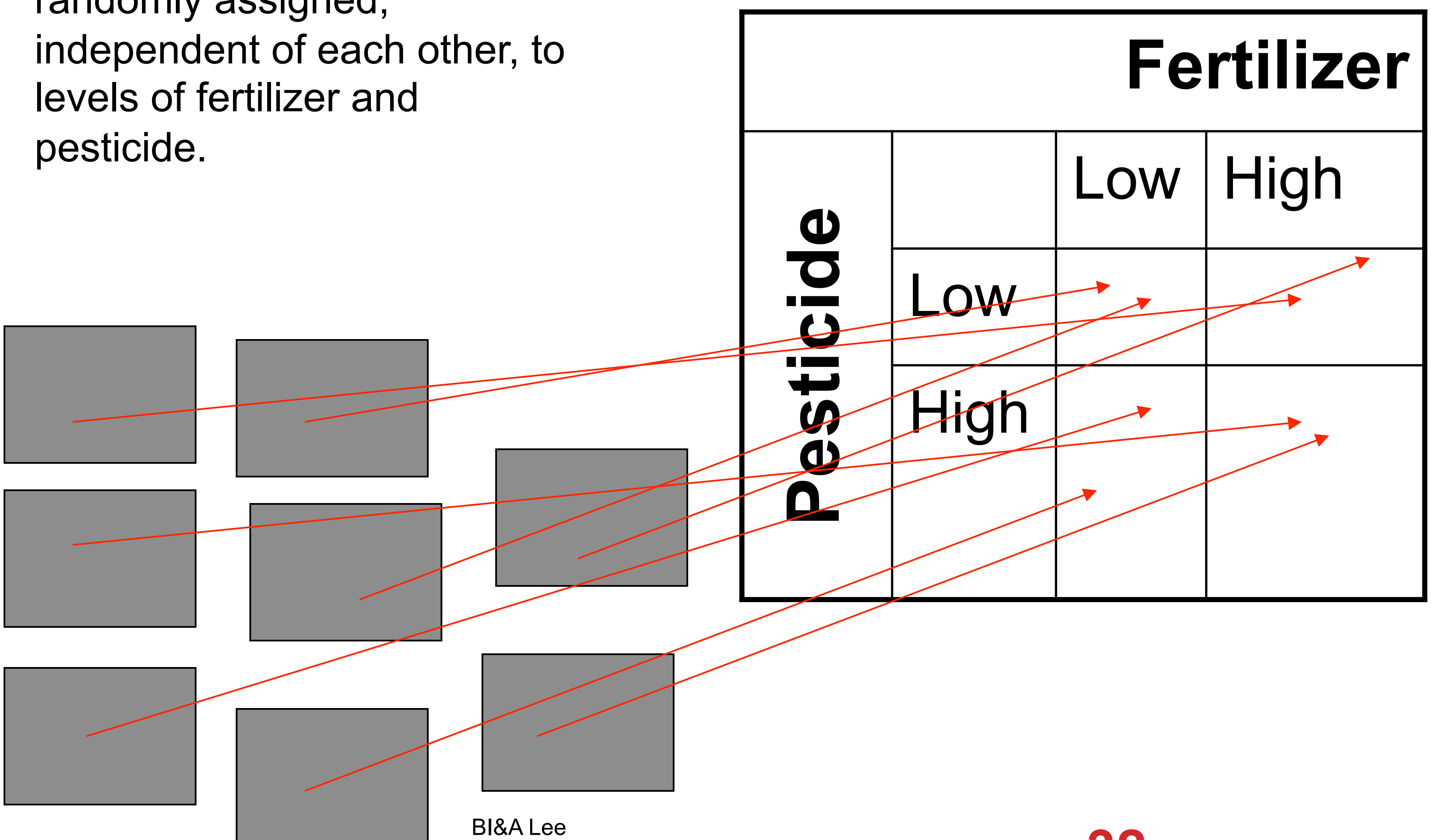
RANDOMIZED COMPLETE BLOCK DESIGN



Each block is randomized separately

COMPLETELY RANDOMIZED DESIGN

Plots (in the field) are randomly assigned, independent of each other, to levels of fertilizer and pesticide.



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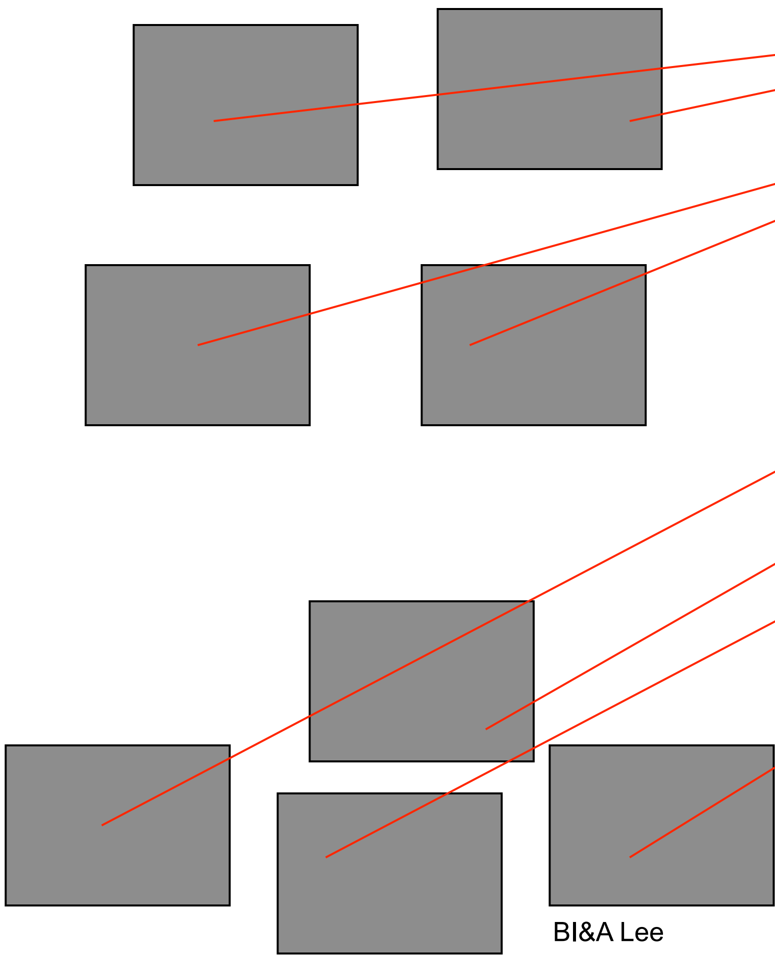
RANDOMIZED BLOCK DESIGN

		Fertilizer	
		Low	High
Field	North		
	South		



Plots in the north field are randomly assigned to low or high fertilizer. Plots in the south field are randomly assigned to low or high fertilizer. Field is a blocking factor.

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BLOCKS

- Group similar subjects into "blocks" and then randomize treatment applications into those.
- A blocking factor is one which accounts for some variability in the response (background effects)
- It is included in the model to reduce the error variance → If it's NOT included, then the random error term increases.
- Blocks are usually "random effects" factors:
- "Random effects" factors are those whose levels represent a sample from population, so that we are not interested in the means of the levels but only in what they tell us about the variability in response due to variability in that population.
- "Fixed effects" factors are those in which each level is considered to be important in its own right and we want to estimate the mean Y at that level.

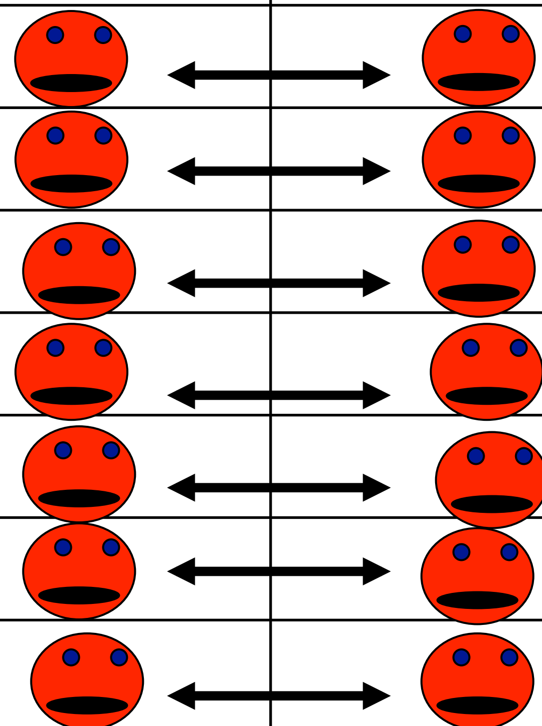
BLOCKING (CONTD.)

Another example of blocking

- Have pairs of subjects (chosen because they are twins, or are similar on some demographic variables, etc.)
- Within each pair randomly assign one treatment to one subject and the other treatment to the other.
- This works best if there are only two levels of the factor of interest. So here the blocks are of size 2.

Blocks of Size Two (Schizophrenia in twins study)

	Treatment: Low	Treatment: High
Pair 1		
Pair 2		
Pair 3		
Pair 4		
Pair 5		
Pair 6		
Pair 7		



RULE OF THUMB OF DESIGN OF EXPERIMENT

<u>Block what you can</u>, and randomize what you cannot – George Box

EXAMPLE: RECALL GOLF DRIVER YARDAGE

✿Suppose four factors (A,B,C,D each at two levels- Table on the next page) are to be studied with regard to their effects on

y = strength of a manufactured steel product.

✿Suppose we want to do eight runs, but the runs are so time consuming that only four can be done in a day.

✿Assign D to an effects column.

✿Assign Day to another effects column with its "Lo" level being Day 1, and its "Hi" level being Day 2.

✿*Questions:*

- (a) How would the four factors and a block be assigned to the design matrix?
- (b) Which runs are to be done each day?
- (c) What is the alias structure of the design?

SIGNS TABLE

Standard Order	A	B	C	AB	AC	BC	ABC
1	-1	-1	-1	1	1	1	-1
2	1	-1	-1	-1	-1	1	1
3	-1	1	-1	-1	1	-1	1
4	1	1	-1	1	-1	-1	-1
5	-1	-1	1	1	-1	-1	1
6	1	-1	1	-1	1	-1	-1
7	-1	1	1	-1	-1	1	-1
8	1	1	1	1	1	1	1

EXAMPLE

a) Assign D and Block to any of columns AB, AC, BC or ABC. In this example, D was assigned to BC and Block was assigned to AC

b) Day 1: Runs 2, 4, 5, 7; Day 2: Runs 1, 3, 6, 8

c) Ignore the Block factor(s) when computing the design generator. Since D=BC, the design generator is I=BCD. The confounding structure is:

EXAMPLE: GOLF DRIVER

Factor	Level 1	Level 2
A. Club Head	Small	Large
B. Shaft	Steel	Graphite
C. Ball	Cheap	Expensive
D. Glove	No	Yes

EXAMPLE

Response	A+ ABCD	B+CD	C+BD	AB +ACD	Block +AC +ABD	D+BC	AD +ABC
38.4	-1	-1	-1	1	1	1	-1
49.2	1	-1	-1	-1	-1	1	1
30.5	-1	1	-1	-1	1	-1	1
40.4	1	1	-1	1	-1	-1	-1
43.7	-1	-1	1	1	-1	-1	1
53.1	1	-1	1	-1	1	-1	-1
34.3	-1	1	1	-1	-1	1	-1
42.5	1	1	1	1	1	1	1
8	4	4	4	4	4	4	4
41.51	9.57	-9.18	3.78	-0.53	-0.77	-0.83	-0.07

EXAMPLE

- ✿Based on the effects plot (normal probability plot) and assuming there are no interactions, factors A, B, and C appear to have strong effects on the distance.
- ✿To maximize the distance, A and C should be set at their Hi levels (large club head and expensive ball) and B should be set at its Lo level (steel shaft).
- ✿Factor D does not appear to have much effect on the distance, nor does there appear to be a block effect (which means that the mean distance did not vary much day to day).

BLOCKING

- This approach had lower resolution, but confounded only one two-way effect with Block.
- An alternate approach selects a high-resolution fractional factorial first (D=ABC), then assigns Block to an effects column.
- The alternative approach confounds 2 two-way effects with block.

FOUR BLOCKS

✿If an experiment can be run in not two days but four days (i.e., four blocks), no problem.

✿ *Two columns of the design matrix are allocated to "block" factors.*

✿For example, if we have five real factors A, B, C, D, and E, to be analyzed in sixteen runs (that is, four blocks of four runs each), we assign E=ABC, then assign blocks based on the signs of BCD and ACD.

- Block 1: runs having level -1 of both BCD and ACD.
- Block 2: runs having level -1 of BCD and level 1 of ACD.
- Block 3: runs having level 1 of BCD and level -1 of ACD.
- Block 4: runs having level 1 of both BCD and ACD.

FOUR BLOCKS
(FIVE-FACTOR EXPERIMENT IN FOUR BLOCKS OF FOUR RUNS EACH)

✿The table of column-allocations for the sixteen run fractional factorial design matrix assigns the block factors F and G to columns 14 and 13. The runs in each block are as shown below.

Block	BCD	ACD	Block Runs
1	-1	-1	1,8,12,13
2	-1	1	2,7,11,14
3	1	-1	3,6,10,15
4	1	1	4,5,9,16

FOUR BLOCKS

CONFOUNDING STRUCTURE

(FIVE-FACTOR EXPERIMENT IN FOUR BLOCKS OF FOUR RUNS EACH)

- The confounding structure for five factors in a sixteen run experiment is given:

A=BCE, B=ACE, C=ABE, D=ABCDE, E=ABC

AB=CE*, AD=BCDE, AE=BC, BD=ACDE, CD=ABDE, DE=ABCD, ABD=CDE, AC=BE, ACD=BDE

ADE=BCD

- Note that Block is confounded with BCD, ACD and BCDxACD=AB.

- Since AB=CE in the fractional factorial, CE is also confounded with Block and cannot be estimated in this design.

EXERCISE

TOOL LIFE

✿**An engineer is interested in the effects of four factors on y = life of a tool:**

A: Cutting speed
B: Cutting angle
C: Tool hardness
D: Cutting tool angle

✿**Two levels were selected for each factor; sixteen runs were planned. Because of the length of time involved in doing each test, four stations (=four blocks) were used.**

✿***Exercise: Allocate the sixteen runs to the four blocks. What is the confounding structure?***

SIGN TABLE

ABC	ABD	ACD	BCD	ABCD
-1	-1	-1	-1	1
1	1	1	-1	-1
1	1	-1	1	-1
-1	-1	1	1	1
1	-1	1	1	-1
-1	1	-1	1	1
-1	1	1	-1	1
1	-1	-1	-1	-1
-1	1	1	1	-1
1	-1	-1	1	1
1	-1	1	-1	1
-1	1	-1	-1	-1
1	1	-1	-1	1
-1	-1	1	-1	-1
-1	-1	-1	1	-1
1	1	1	1	1