FACTORIAL DESIGN AND FRACTIONAL FACTORIAL DESIGN

NO REPLICATIONS? USE NORMAL PROBABILITY PLOT

- **With no replication (each combination of conditions is only tested once), the traditional p-values cannot be calculated.**
- **This is because you would have 0 degrees of freedom for estimating variance.**
- **Under the null hypothesis that all effects (main and interactions) are 0 the estimated effect sizes will be linear combinations of normal errors (recall assumption on the normality).**
- **Therefore, effects on the normal plot that deviate from normality are suggestive of significant effects.**

NO REPLICATIONS?
USE NORMAL PROBABILITY PLOT

- **Simply pass the vector of estimated effects to the *qqnorm* in R or similar functions in SAS or JMP.**
- **The sorted data are plotted vs. values selected to make the resulting image look close to a straight line if the data are approximately normally distributed. Deviations from a straight line suggest departures from normality.**
- **The normal probability plot shows the *raw* estimates plotted against their normal quantiles.**
- **The *half* normal plot shows the *absolute* values of effects against their normal quantiles.**

RECALL THIS REAL EXAMPLE: TWO LEVEL FACTORIAL DESIGN

Kitchen scientists* conducted a 2^3 factorial experiment on microwave popcorn. The factors are:

A. Brand of popcorn (Costly, Cheap)
B. Time in microwave (4 min., 6 min
C. Power setting (75%, 100%)

A panel of neighborhood kids rated taste from one to ten scale and weighed the un-popped kernels (UPKs).

* For full report, see Mark and Hank Andersons' *Applying DOE to Microwave Popcorn*, PI Quality 7/93, p30. (Uploaded at CANVAS)

TWO LEVEL FACTORIAL DESIGN
AS EASY AS POPPING CORN!

Run Ord	A Brand expense	B Time minutes	C Power percent	R_1 Taste rating*	R_2 UPKs oz.	Std Ord
1	Costly	4	75	75	3.5	2
2	Cheap	6	75	71	1.6	3
3	Cheap	4	100	81	0.7	5
4	Costly	6	75	80	1.2	4
5	Costly	4	100	77	0.7	6
6	Costly	6	100	32	0.3	8
7	Cheap	6	100	42	0.5	7
8	Cheap	4	75	74	3.1	1

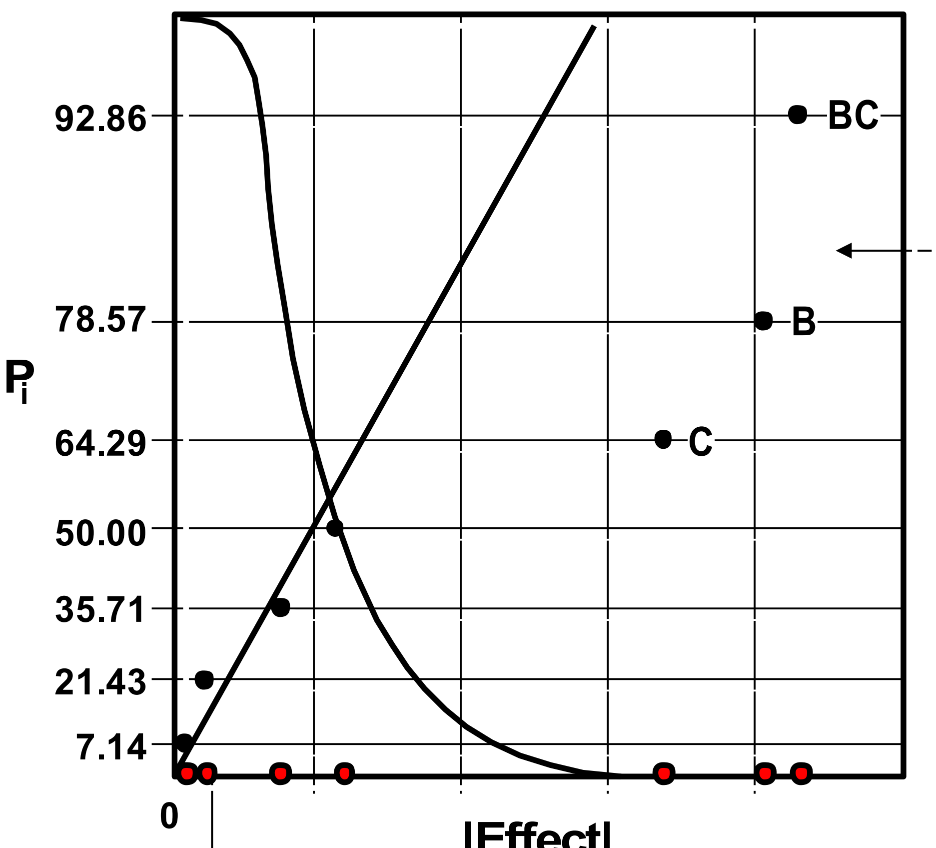
- Average scores multiplied by 10 to make the calculations easier.
  - *It is also important to run experiments in random order*

TWO LEVEL FACTORIAL DESIGN

Run Ord	A Brand expense	B Time minutes	C Power percent	R_1 Taste rating	R_2 UPKs oz.	Std Ord
1	+	–	–	75	3.5	2
2	–	+	–	71	1.6	3
3	–	–	+	81	0.7	5
4	+	+	–	80	1.2	4
5	+	–	+	77	0.7	6
6	+	+	+	32	0.3	8
7	–	+	+	42	0.5	7
8	–	–	–	74	3.1	1

Factors shown in coded values.
What is the effect of factor B? (-20.5)

HALF NORMAL PROBABILITY PLOT
THE POPCORN EXAMPLE—TASTE RESPONSE
SORTING THE VITAL FEW FROM THE TRIVIAL MANY.



Significant effects (the vital few) are outliers. They are too big to be explained by noise.

* The fitted line indicates where you would expect the points to fall if the effects were zero.

Negligible effects (the trivial many) will be N(0, σ), so they fall near zero on straight line.

BI&A[C.Lee]

PERFORM TWO-WAY ANOVA WITH B, C, B*C EFFECTS ONLY – THE REST EFFECTS (A, A*B, A*C) ARE CONSIDERED AS ERRORS (RESIDUAL)

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	df	Mean Square	F Value	Prob > F
Model	2343.00	3	781.00	31.56	0.0030
B-Time	*840.50*	*1*	*840.50*	*33.96*	*0.0043*
C-Power	*578.00*	*1*	*578.00*	*23.35*	*0.0084*
BC	*924.50*	*1*	*924.50*	*37.35*	*0.0036*
Residual	99.00	4	24.75		
Total	2442.00	7			

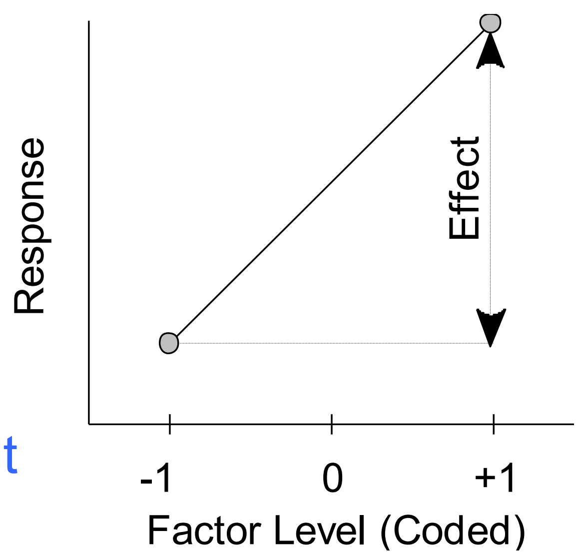
POPCORN ANALYSIS – TASTE

ANOVA COEFFICIENT ESTIMATES (LINEAR REGRESSION)

Factor	Coefficient Estimate	DF	Standard Error	95% CI Low	95% CI High
Intercept	66.50	1	1.76	61.62	71.38
B-Time	-10.25	1	1.76	-15.13	-5.37
C-Power	-8.50	1	1.76	-13.38	-3.62
BC	-10.75	1	1.76	-15.63	-5.87

Coefficient Estimate: One-half of the factorial effect

$$\text{Coefficient} = \Delta y / \Delta x = \Delta y / 2$$



Each estimated effect represents a 2-unit change (from -1 to +1). The slope is the change in the response per unit change of the factor.

POPCORN ANALYSIS – TASTE
PREDICTIVE REGRESSION EQUATION

Taste =

+66.50

-10.25*B

-8.50*C

-10.75*B*C

+ error

Regression coefficients tell us how the response would change.

The intercept in coded values is in the center (mean) of our design response.

Std	B	C	Pred y
1	−	−	74.50
2	−	−	74.50
3	+	−	75.50
4	+	−	75.50
5	−	+	79.00
6	−	+	79.00
7	+	+	37.00
8	+	+	37.00

JMP DEMO

1. Full Factorial Design → Make Table (Run Oder: Randomize)

2. Run the experiments, i.e., collect the data in each setting

3. Analyze the data (Fit Model)

Now, suppose your data are obtained:

1. Select **Help > Sample Data Library** and open Bicycle.jmp.
2. Select **Analyze > Fit Model**.
3. Select Y and click **Y**.
4. Select HBars through Brkfast and click **Add**.
5. Click **Run**.
6. From the red triangle menu next to Response Y, select **Effect Screening > Normal Plot**.

NOW, ‘FRACTIONAL’ FACTORIAL DESIGN – WHY, WHAT, HOW?

- FRACTIONAL FACTORIAL DESIGN (FFD) OFFERS REALLY POWERFUL APPROACH TO EXPERIMENTATION.

WHY FRACTIONAL FACTORIALS?

Full Factorials
No. of combinations
→
For
two-levels

In business/engineering, this is the sample size – number of experiments to carry out.

Full Factorials

Number of Factors	Number of Treatments
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512

WHY SO MANY TREATMENTS?

Full Factorials

Number Factors	Main Effects	Order of Interactions								
		2	3	4	5	6	7	8	9	10
2	2	1								
3	3	3	1							
4	4	6	4	1						
5	5	10	10	5	1					
6	6	15	20	15	6	1				
7	7	21	35	35	21	7	1			
8	8	28	56	70	56	28	8	1		
9	9	36	84	126	126	84	36	9	1	

"There tends to be a redundancy in full factorial designs"
– redundancy in terms of an excess number of interactions
Fractional factorial designs exploit this redundancy!

PRINCIPLES OF FRACTIONAL FACTORIAL DESIGNS

1. The Pareto principle states that there might be a lot of factors, but very few are important.

2. The Sparsity of Effects principle states that usually the more important effects are main effects and low-order interactions.

3. These designs can be used in sequential experimentation; that is, additional design points can be added to these designs at later time, after we first "screen out" the important factors.

FFD: WHAT

- ❑ **Sparsity of effect principle:**

  Most systems are dominated by some of

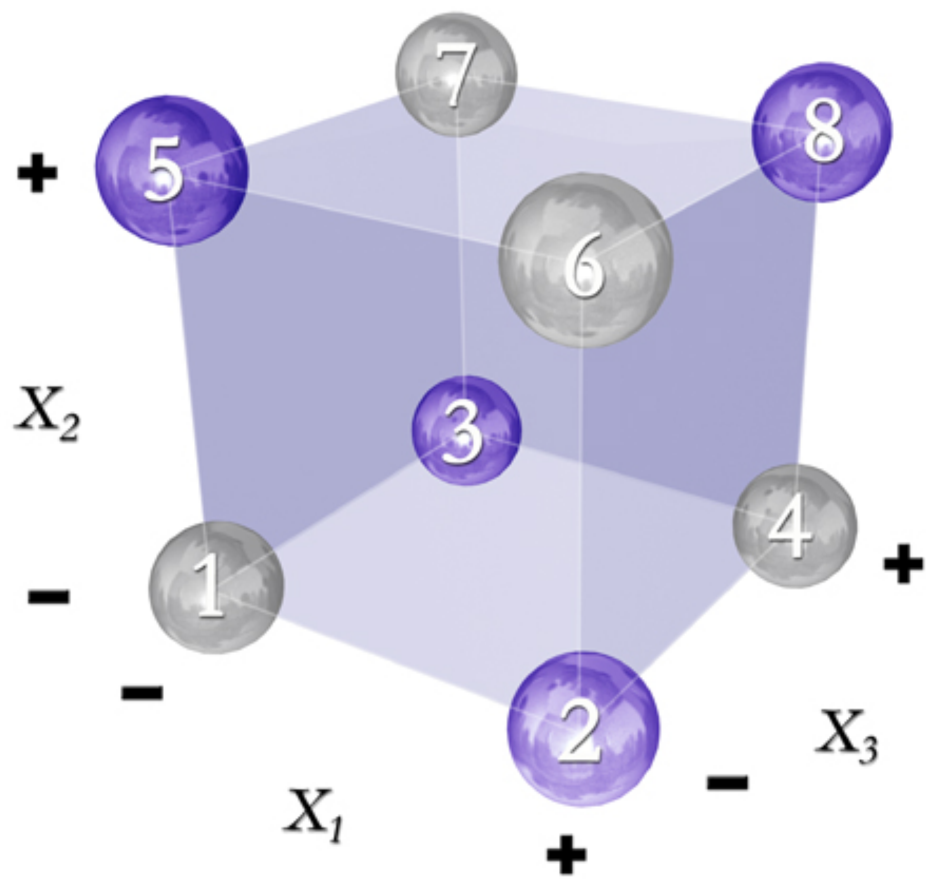
  the main effects and low–order interactions, and

  most high–order interactions are negligible.

- **In Fractional Factorial Design:**
  we intentionally (& wisely) *confound* the effects of interest with those that are negligible (higher-order) ones.

- **Two effects are said to be cofounded if it is not impossible to separate them out.**

What is the principled approach?



X_1	X_2	X_3
+	−	−
−	−	+
−	+	−
+	+	+

Notion of exploiting redundancy in interactions
→ Set X3 column equal to
the X1X2 interaction column

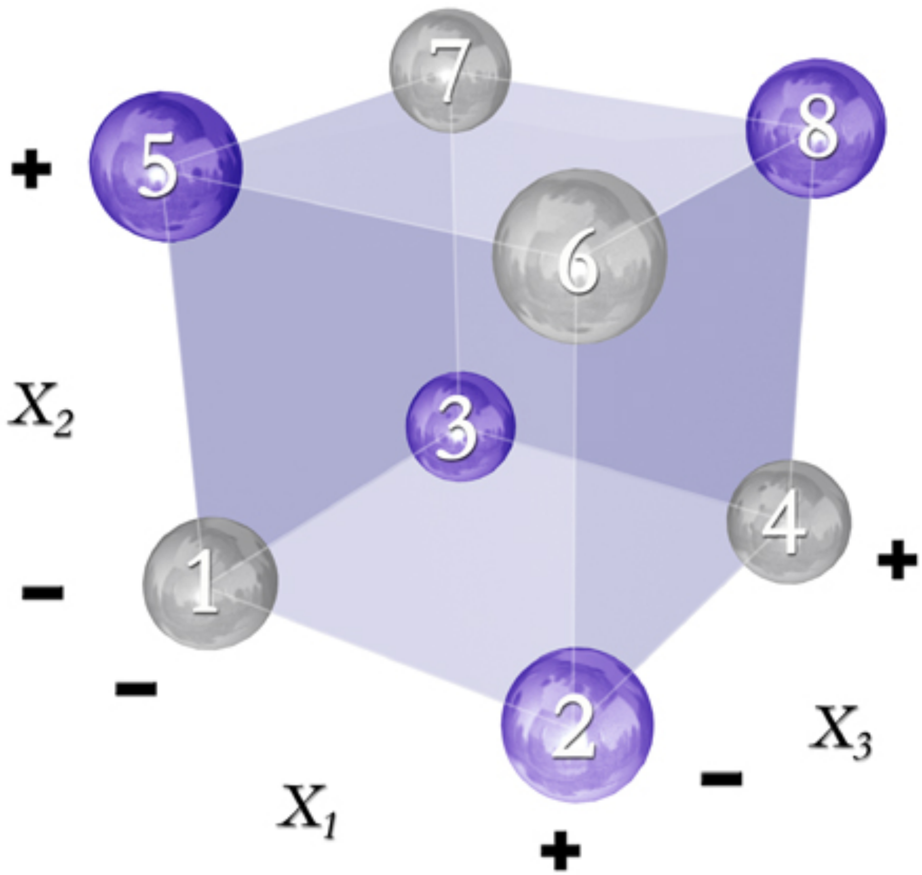
Fractional Factorial Notation

$$2_R^{k-p}$$

- "2" indicates each factor has two levels
- "k" indicates the number of factors included
- "p" indicates the fraction to be run
  "p" also indicates the number of "extra" factors that need to be placed into the base design
- "R" indicates the resolution of the design

Notion of "resolution" → Later

Regular Fractional Factorial Designs



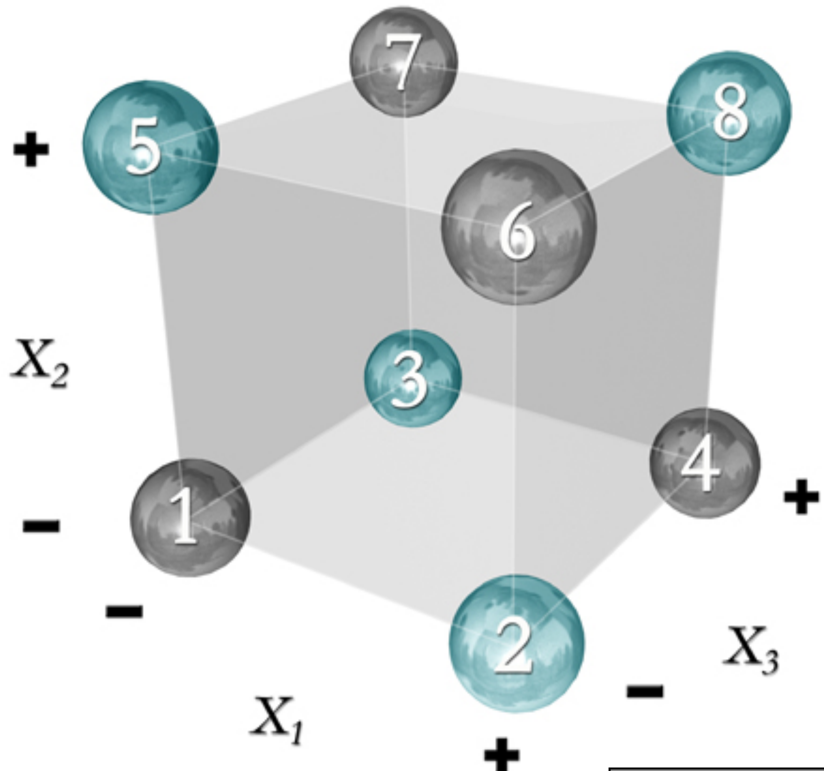
X_1	X_2	X_3
+	−	−
−	−	+
−	+	−
+	+	+

Half fraction of a 2^3 design = 2^{3-1} design
3 factors studied -- 1-half fraction
→ 8/2 = 4 runs

Resolution III (later)

Confounding

→ NO FREE LUNCH!!!



X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3
+	−	−	−	−	+
−	−	+	+	−	−
−	+	−	−	+	−
+	+	+	+	+	+

X3=X1X2 → ??

Each main effect is aliased *with a two factor interaction!!*

X3 = X1X2 → X1X3 = X2 and X2X3 = X1

(main effects aliased with two-factor interactions) – *Resolution III design*

Want to study 5 factors (1,2,3,4,5) using a 2^4 = 16-run design
i.e., construct half-fraction of a 2^5 design
= 2^{5-1} design

2^4 Unreplicated Experiment

Place x5 Here? (x4)

Place x5 Here? (x3x4)

Place x5 Here? (x1x2x3x4)

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
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+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
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-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Want to study 5 factors (1,2,3,4,5) using a 2^4 = 16-run design i.e., construct half-fraction of a 2^5 design = 2^{5-1} design

Place x5 Here?   Place x5 Here?   Place x5 Here?

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
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+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
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-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
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-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

For half-fractions, always best to alias the new (additional) factor with the highest-order interaction term

Half Fractions of Highest Resolution

Step 1: Write out the full factorial for the first *k-1* factors.

Step 2: Associate the last (k^{th}) factor into the column labeled: $X_1 X_2 \cdots X_{k-1}$ (that is, the highest order interaction column).

What about bigger fractions?
Studying 6 factors with 16 runs?
¼ fraction of $2^6 = 2^{6-2}$ FFD

Place x5 Here

Place x6 Here

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
+	+	-	+	+	-	+	-	+	-	-	+	-	-	-
-	-	+	+	+	-	-	-	-	+	+	+	-	-	+
+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X2*X3*X4; X6 = X1*X2*X3*X4; → X5*X6 = X1 <u>Can we do better?</u>

2^{6-2} Experiment

Place x5 Here → x1x2x3

Place x6 Here → x2x3x4

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
-	+	+	-	-	-	+	+	-	-	-	+	+	-	+
+	+	+	-	+	+	-	+	-	-	+	-	-	-	-
-	-	-	+	+	+	-	+	-	-	-	+	+	+	-
+	-	-	+	-	-	+	+	-	-	+	-	-	+	+
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-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4 (yes, better!)

DESIGN 'GENERATORS' AND 'DEFINING RELATIONS'

Just saw that

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4

That is,

5 = 123; 6 = 234; 56 = 14

→ These relations are called the *generators* of the design.

The capital letter I denotes a column of all plus signs (think as +1's).

Multiplying both sides of the generators *by 5,* 6 and 56, we obtain

I = 1235 = 2346 = 1456

→ Call them the defining relations of the design.

DESIGN GENERATORS AND DEFINING RELATIONS

That is,

5 = 123; 6 = 234; 56 = 14
I = 1235 = 2346 = 1456

- → **Use the defining relations to find the confounding pattern among the effect estimates.**
- → **For example, to find what is confounded with 1 (the main effect of factor 1), we multiply both sides of the defining relation by column 1:**
  - → 1(I) = 11235 → 1 = 235
  - → Thus the sign in column 1 and the 235 interaction column are identical
  - → The main effect of factor 1 is confounded with the 235 interaction.

CONFOUNDING RELATIONSHIPS

I = 1235 = 2346 = 1456

Main-effects:

1=235=456; 2=135=346; 3=125=246; 4=…??

15-possible 2-factor interactions:

12=35

13=25

14=56

15=23=46

16=45

24=36

26=34

NOW, 'WORD' AND 'RESOLUTION'

Recall defining relations: I = 1235 = 2346 = 1456

→ Each term in the defining relations to the right of I is called a 'word.'

→ For example, there are 3 words in the above.

→ Resolution: The length of the shortest "word"

→ Resolution IV here; usually written as a roman numeral.

→ IV has to do with "(1+3 or 2+2)" confounding pattern.

Design Resolution

- Roman numeral subscript are usually used to denote design resolutions
- Designs of resolution III, IV and V are particularly important

RESOLUTION

Resolution III: (1+2)

Main effect aliased with 2-order interactions

e.g. 2^{3-1} design with I = ABC

Resolution IV: (1+3 or 2+2)

Main effect aliased with 3-order interactions and

2-factor interactions aliased with other 2-factor …

e.g. 2^{4-1} design with I = ABCD

Resolution V: (1+4 or 2+3)

Main effect aliased with 4-order interactions and

2-factor interactions aliased with 3-factor interactions

e.g. 2^{5-1} design with I = ABCDE

FFD OFFERS HOW …

- To distinguish between important and significant factors → one example of screening design
- For example, an experimenter would begin with a resolution III design, which is quite economical (e.g., 7 factors in 8 runs!)
- Sort out significant effects → At this point, we have an ambiguity due to confounding.
- Now, augment/modify initial experiment with additional runs → Clarify open questions.

SCREENING DESIGNS: E-MAIL ADVERTISING EXAMPLE

¼ fraction of $2^6 = 2^{6-2}$ FFD

Place x5 Here

Place x6 Here

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3	x1x2x4	x1x3x4	x2x3x4	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
-	+	-	-	-	+	+	-	-	+	+	+	-	+	-
+	+	-	-	+	-	-	-	-	+	-	-	+	+	+
-	-	+	-	+	-	+	-	+	-	+	-	+	+	-
+	-	+	-	-	+	-	-	+	-	-	+	-	+	+
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-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X2*X3*X4; X6 = X1*X2*X3*X4; → X5*X6 = X1

or I = 2345 = 12346 = 156 → Resolution III design

2^{6-2} Experiment

x1	x2	x3	x4	x1x2	x1x3	x1x4	x2x3	x2x4	x3x4	x1x2x3 (Place x5 Here)	x1x2x4	x1x3x4	x2x3x4 (Place x6 Here)	x1x2x3x4
-	-	-	-	+	+	+	+	+	+	-	-	-	-	+
+	-	-	-	-	-	-	+	+	+	+	+	+	-	-
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+	-	+	+	-	+	+	-	-	+	-	-	+	-	-
-	+	+	+	-	-	-	+	+	+	-	-	-	+	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

X5 = X1*X2*X3; X6 = X2*X3*X4 → X5*X6 = X1*X4

or I = 1235 = 2346 = 1456 → Resolution IV design

One half fraction of the 2^3 design

Run	Full 2^2 Factorial (Basic Design)		Resolution III, I = ABC		
	A	B	A	B	C = AB
1	-	-	-	-	+
2	+	-	+	-	-
3	-	+	-	+	-
4	+	+	+	+	+

The resolution IV design with I = ABCD

- Suppose there are four main factors, A, B, C, D.
- We will use the 2^{4-1} with I = ABCD, as this will generate the highest resolution possible
    - We will first write down the basic design, which is 2^3 design
    - The basic design has eight runs but with three factors
    - To find the fourth factor levels, we solve I = ABCD for D

$$D * I = D * ABCD = ABCD^2 = ABC$$

The resolution IV design with I = ABCD

Run	A	B	C	D = ABC	Treatment Combination
	Basic Design				
1	-	-	-	-	(1)
2	+	-	-	+	ad
3	-	+	-	+	bd
4	+	+	-	-	ab
5	-	-	+	+	cd
6	+	-	+	-	ac
7	-	+	+	-	bc
8	+	+	+	+	abcd

SELECTING RESOLUTION IV DESIGNS

Consider an example with 6 factors in 16 runs (or 1/4 fraction)
Suppose 12, 13, and 14 are important and factors 5 and 6 have no interactions with any others

Set 12=35, 13=25, 14= 56 (for example) →

I = 1235 = 2346 = 1456 → Resolution IV design

<u>All possible 2-factor interactions are then given by:</u>
12=35
13=25
14=56
15=23=46
16=45
24=36
26=34

HOW TO CHOOSE APPROPRIATE DESIGN?

- **Software → for a given set of generators, will give design, resolution, and aliasing relationships**

  →SAS, JMP, Minitab, R (FrF2 package), …

- **Resolution III designs → easy to construct but main effects are aliased with 2-factor interactions**
- **Resolution V designs → also easy but not as economical (for example, 6 factors → need 32 runs)**
- **Resolution IV designs → balances in between**
- **But overall, it depends on case by case (budget/ time vs. accuracy issue)**

JMP DEMO

- **Screening Design offers Fractional Factorial Design.**

EXAMPLE--SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

- **An experimenter wanted to study the effect of 5 factors on corrosion rate of iron rebar (reinforcing bar, reinforcing steel) in only 8 runs by assigning D to column AB and E to column AC in the 3-factor 8-run signs table.**

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

- ✿**For this particular design, the experimenter used only 8 runs (1/4 fraction) of a 32 run (or 2^5) design (I.e., a 2^{5-2} design).**
- ✿**For each of these 8 runs, D=AB and E=AC. If we multiply both sides of the first equation by D, we obtain DxD=ABxD, or I=ABD.**
- ✿**Likewise, if we multiply both sides of E=AC by E, we obtain ExE=ACxE, or I=ACE.**

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

- **We can say the design is comprised of the 8 runs for which both ABD and ACE are equal to one (I=ABD=ACE).**

- **I=ABD=ACE is the design generator**

- **Their interaction is ABD x ACE = BCDE**

- **The first two rows of the confounding structure are provided below.**
  - Line 1: I = ABD = ACE = BCDE
  - Line 2:
    - AxI=AxABD=AxACE=AxBCDE
    - A=BD=CE=ABCDE

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

✿***U-Do-It Exercise.*** **Complete the remaining 6 non-redundant rows of the confounding structure for the corrosion experiment. Start with the main effects and then try any two-way effects that have not yet appeared in the alias structure.**

I=ABD=ACE=BCDE

A=BD=CE

B=

C=

D=

E=

BC=

BE=

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

✿The corrosion experiment generated the following data:

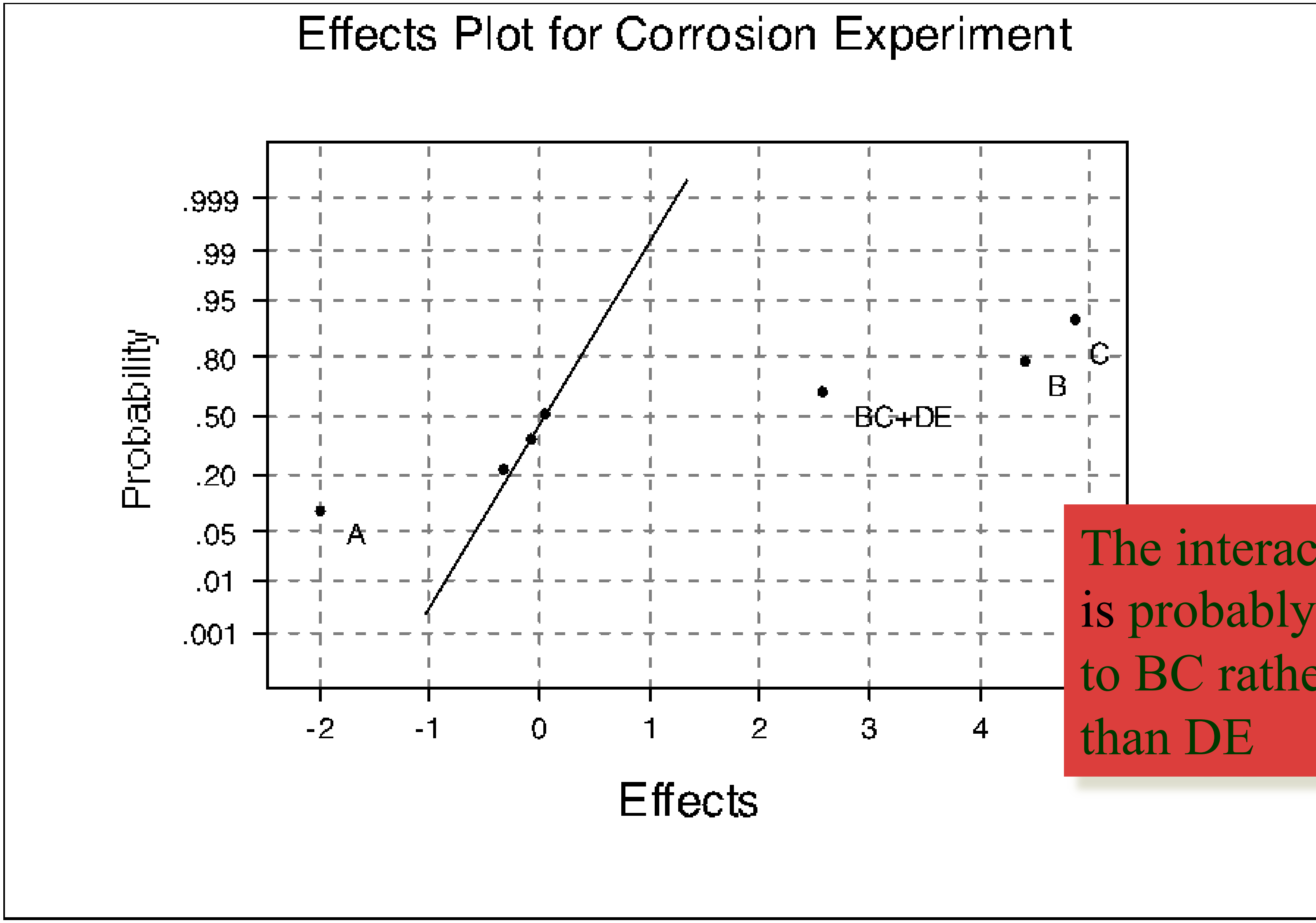
Standard Order	Corrosion Rate	A	B	C	D	E
1	2.71	-1	-1	-1	1	1
2	0.93	1	-1	-1	-1	-1
3	4.8	-1	1	-1	-1	1
4	2.53	1	1	-1	1	-1
5	4.89	-1	-1	1	1	-1
6	3.35	1	-1	1	-1	1
7	12.29	-1	1	1	-1	-1
8	9.92	1	1	1	1	1

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

Computation of Factor Effects

y	A+BD+CE	B+AD	C+AE	D+AB	E+AC	BC+DE	BE+CD
2.71	-1	-1	-1	1	1	1	-1
0.93	1	-1	-1	-1	-1	1	1
4.8	-1	1	-1	-1	1	-1	1
2.53	1	1	-1	1	-1	-1	-1
4.89	-1	-1	1	1	-1	-1	1
3.35	1	-1	1	-1	1	-1	-1
12.29	-1	1	1	-1	-1	1	-1
9.92	1	1	1	1	1	1	1
Sum: 41.42	-7.96	17.66	19.48	-1.32	0.14	10.28	-0.34
8	4	4	4	4	4	4	4
Avg: 5.178	Effect: -1.99	4.415	4.87	-0.33	0.035	2.57	-0.085

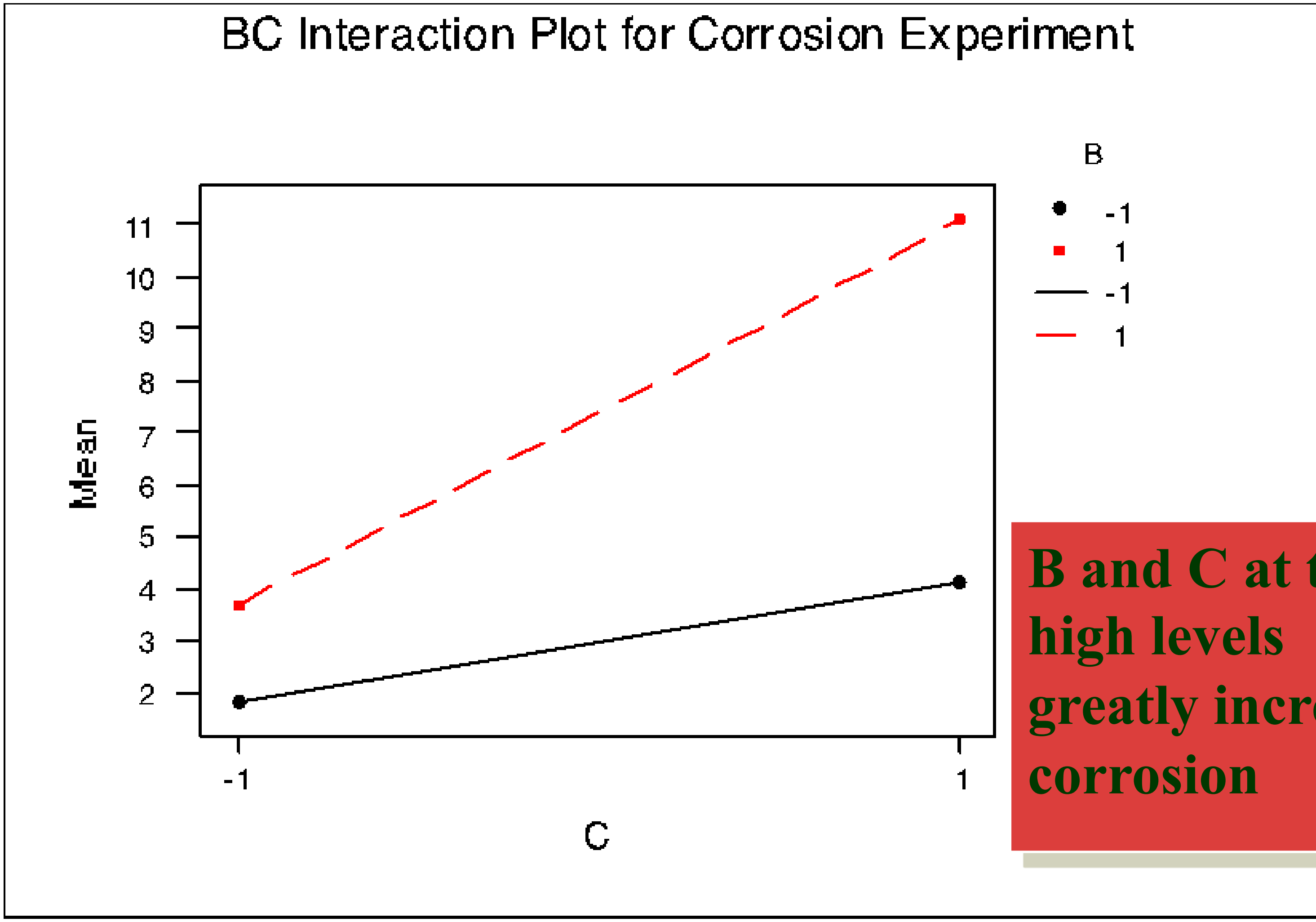
SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS



The interaction is probably due to BC rather than DE

BI&A Lee

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS



B and C at their high levels greatly increase corrosion

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

U-Do-It Exercise: If the experimenter <u>wishes to minimize</u> the corrosion rate, what is an optimal way, and what is its predicted corrosion rate?

SCREENING DESIGNS IN EIGHT RUNS: FIVE FACTORS IN 8 RUNS

U-Do-It Exercise Solution

The factor A should be set high, and the factors B and C should be low, and the interaction BC should be high.

So, the predicted corrosion rate is

5.178+(-1.99/2)-(4.415/2)-(4.87/2)+(2.57/2)

=.8255

E-MAIL ADVERTING EXAMPLE: 8 FACTORS

E-mail Advertising Experiment: Factors and Levels

Factor	Control (−)	New Idea (−)
A Link to online catalog	No	Yes
B Design of e-mail	Simple	Stronger brand image
C Partner promotions	None	Offers from two-partner companies
D Navigation bar on side	Current	Additional buttons
E Background color	White	Blue
F Discount offer	15% off	No discount
G Subject line	Exclusive e-mail offer	Special offer for our customers
H Free gift	None	Free pen-and-pencil set

THE DESIGN TABLE AND RESULTS: 2^{8-4}_IV

FACTOR								
A Link	*B* Design	*C* Partner	*D* Navigation Bar	*E* Color	*F* Discount	*G* Subject Line	*H* Gift	Response: Purchase Rate (%)
−	−	−	−	−	−	−	−	2.23
+	−	−	−	+	+	+	−	1.47
−	+	−	−	+	+	−	+	1.81
+	+	−	−	−	−	+	+	2.38
−	−	+	−	+	−	+	+	2.03
+	−	+	−	−	+	−	+	1.62
−	+	+	−	−	+	+	−	1.28
+	+	+	−	+	−	−	−	1.98
−	−	−	+	−	+	+	+	1.78
+	−	−	+	+	−	−	+	2.30
−	+	−	+	+	−	+	−	2.30
+	+	−	+	−	+	−	−	1.53
−	−	+	+	+	+	−	−	1.11
+	−	+	+	−	−	+	−	1.81
−	+	+	+	−	−	−	+	1.93
+	+	+	+	+	+	+	+	1.82

Each version of the e-mail was sent to 10,000 addresses randomly chosen from a list the firm had purchased.

EFFECT AND ESTIMATE: SIGNIFICANT EFFECTS ARE BOLDFACE

Average	1.8363
A	0.0550
B	0.0850
C	**−0.2775**
D	−0.0275
E	0.0325
F	**−0.5675**
G	0.0450
H	**0.2450**
$AB + CE + DF + GH$	0.0425
$AC + BE + DG + FH$	**0.1650**
$AD + BF + CG + EH$	0.0300
$AE + BC + DH + FG$	0.0250
$AF + BD + CH + EG$	0.0600
$AG + BH + CD + EF$	[illegible]

REMARKS

- The factors C (partner promotion), F(discount offer), and H (free gift) are significant.
- The fourth largest (in magnitude) is an estimate of AC+BE+DG+FH. These are 2-factor interactions confounded; we only know that their sum is 0.165.
- We do know F and H are significant.
- Effect heredity principle: significant interactions tend to involve significant factors.
- FH-interaction plot does reveal F (discount offer) at 15% discount + H at free gift yields highest response.