Review of Basic Inferential Statistics

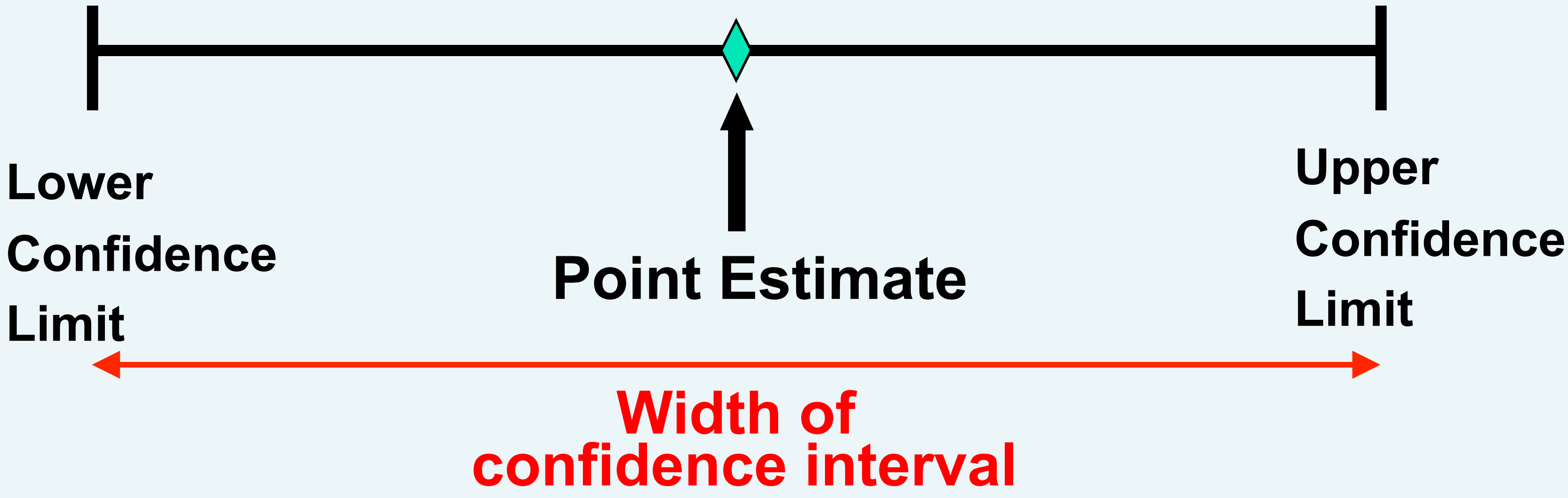
Confidence Interval Estimation,
Sample Size

Point Estimates: How much uncertainty?

We can estimate a Population Parameter …		with a Sample Statistic
Mean	μ	$\bar{X}$
Proportion	π	p

Point and Interval Estimates

- A point estimate is a single number,
- a confidence interval provides additional information about the variability of the estimate

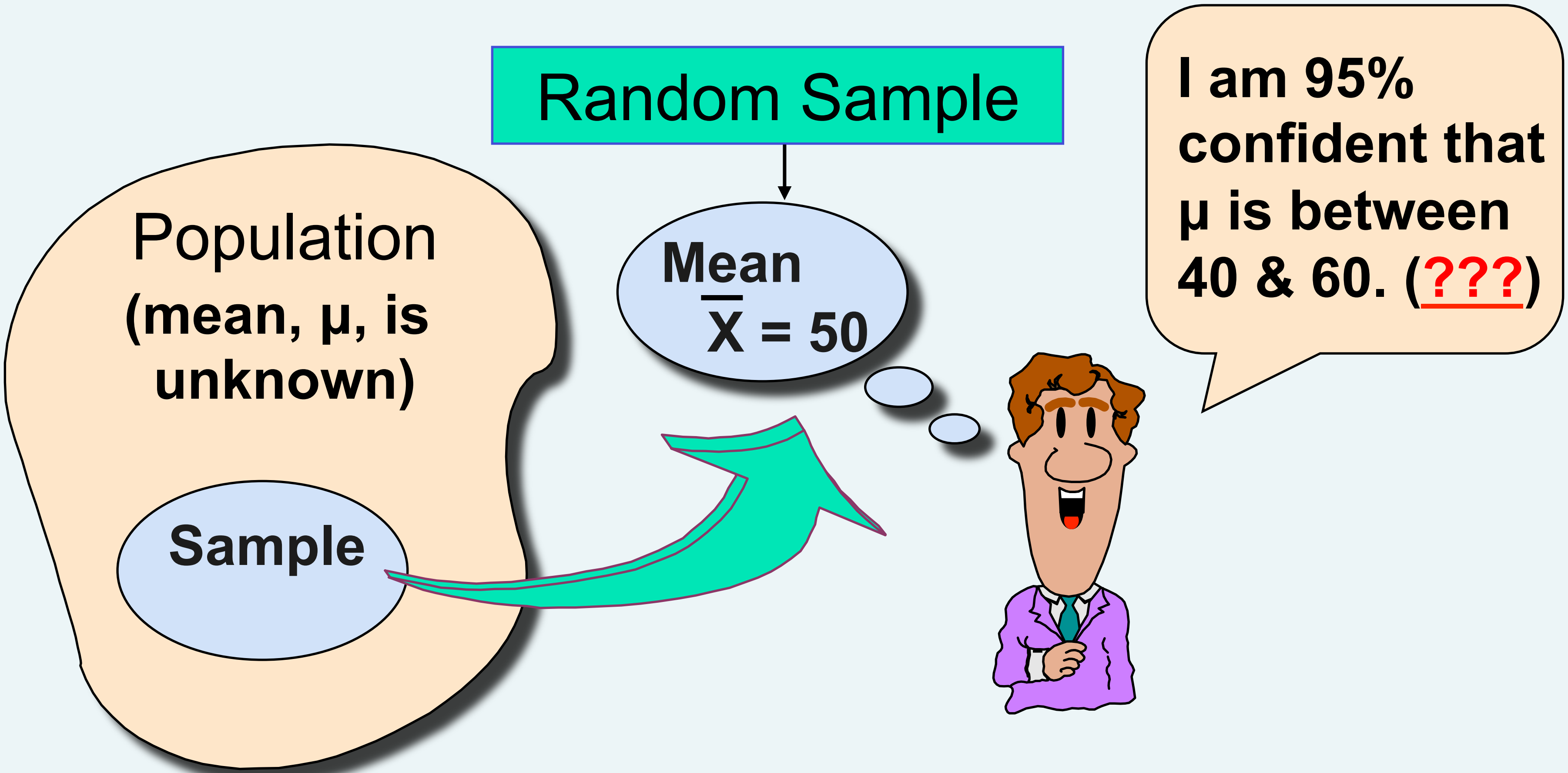


Terminology

- Estimator: a random variable used to estimate an unknown parameter. It has a distribution, a mean, a variance.
  - Sample mean

- Estimate: a number as a realization of an estimator (when we actually run the experiment and observe the data), e.g., realization of sample mea
- Statistic: any function of random samples, e.g., estimator, test statistic

Estimation Process



What is the meaning of C.I.?

Symbol Check

$\mu_{\bar{x}}$ The mean of the sample means.

$\sigma_{\bar{x}}$ The standard deviation of the sample means.
Also called "the standard error of the mean."

Central Limit Theorem

Compare Averages NOT Individuals

- Jacob Bernoulli (1654-1705)
- The ‘Father of Uncertainty’

- “Even the most stupid of men, by some instinct of nature, by himself and without any instruction, is convinced that the more observations have been made, the less danger there is of wandering from one’s goal.”

Central Limit Theorem

Compare Averages NOT Individuals

- As the sample size (n) becomes large, the distribution of averages becomes approximately normal.
- The variance of the averages is smaller than the variance of the individuals by a factor of n.

$$\sigma_{\bar{y}}^2 = \frac{\sigma^2}{n}$$

σ (sigma) symbolizes true standard deviation

- The mean of the distribution of averages is the same as the mean of distribution of individuals.

$$\mu_{\bar{y}} = \mu_{y_i}$$

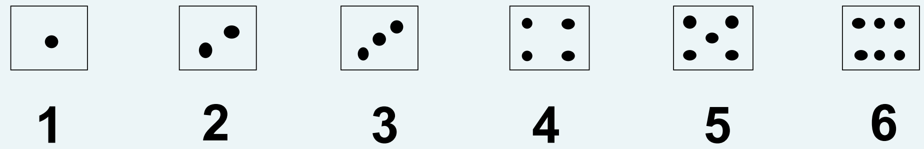
μ (mu) symbolizes true population mean

The CLT applies regardless of the distribution of the individuals.

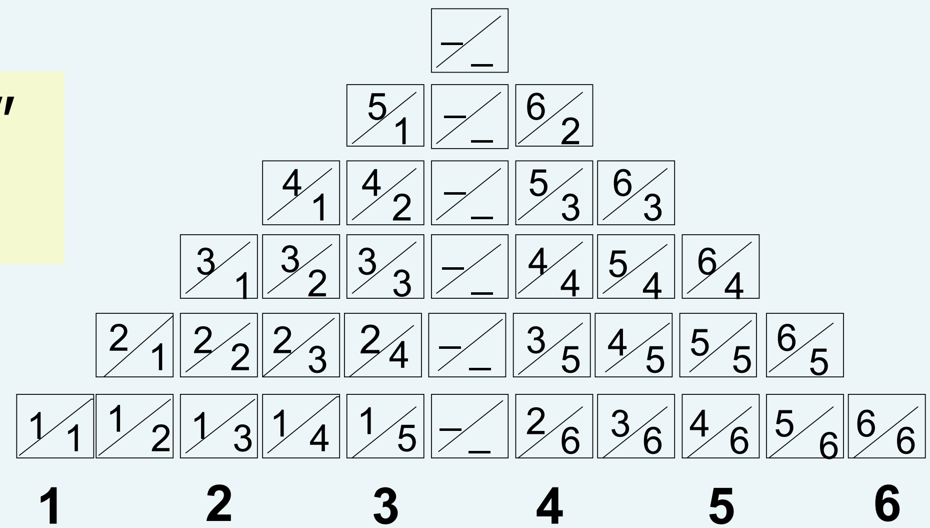
Central Limit Theorem

Illustration using Dice

- Individuals are uniform; averages tending toward normal!



Averages of Two Dice

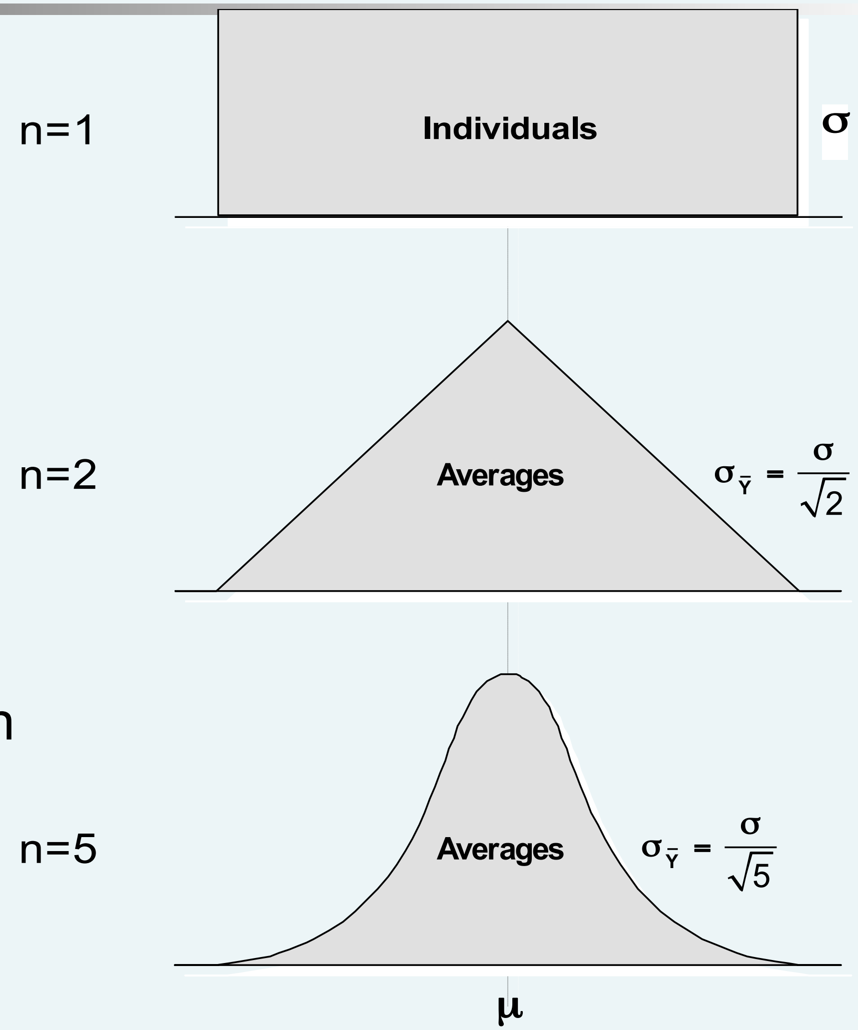


Example: "snakeyes" [1/1] is the only way to get an average of one.

Central Limit Theorem

Uniform Distribution

- As the sample size (n) becomes large, the distribution of averages becomes approximately normal.
- The variance of the averages is smaller than the variance of the individuals by a factor of n.
- The mean of the distribution of averages is the same as the mean of distribution of individuals.



Mathematical Theory… The Central Limit Theorem!

If all possible random samples, each of size *n*, are taken from any population with a mean μ and a standard deviation σ, the sampling distribution of the sample means (averages) will:

1. have mean: $\boxed{\mu_{\bar{x}} = \mu}$

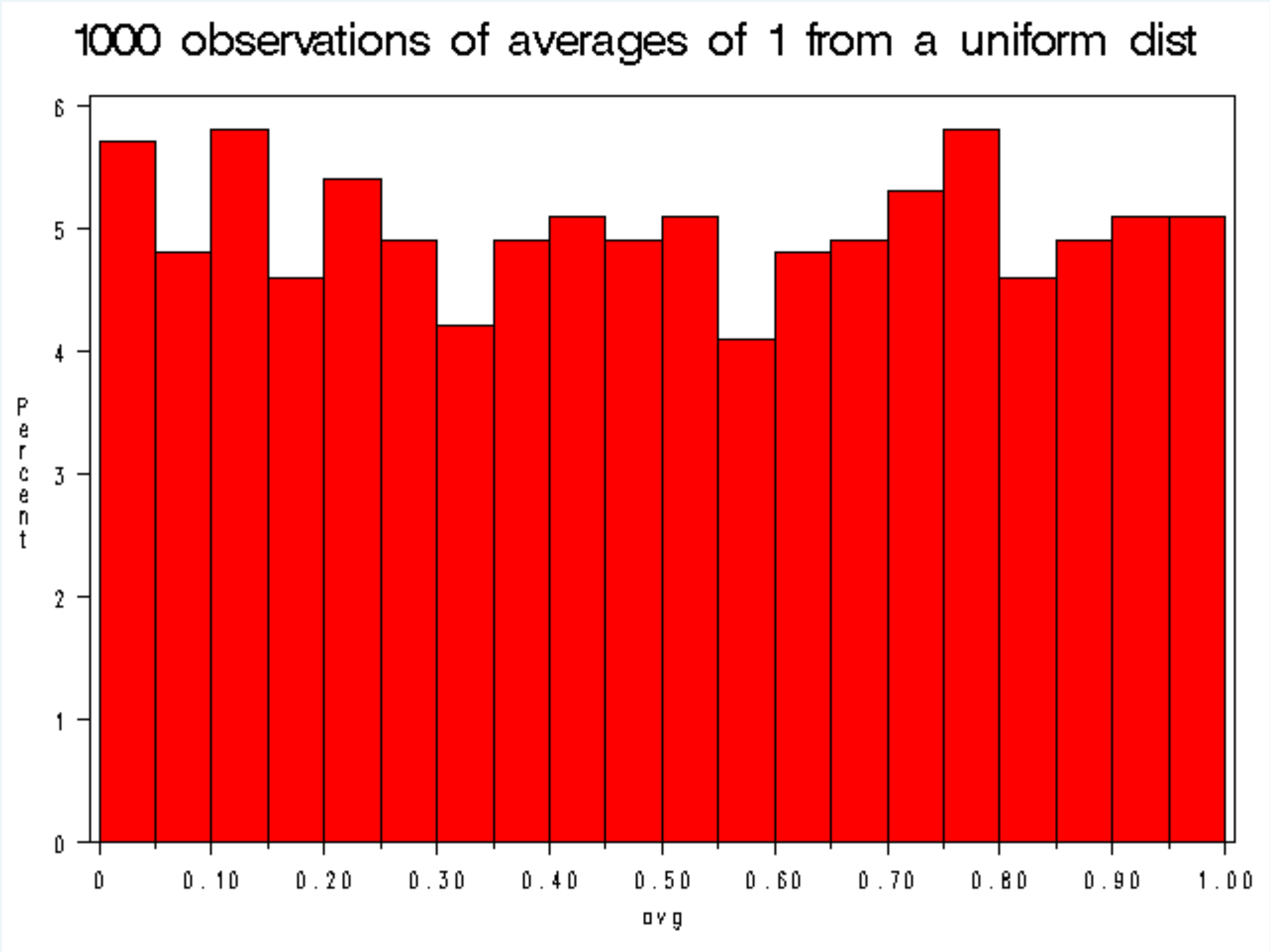
2. have standard deviation: $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

3. be approximately normally distributed regardless of the shape of the parent population (normality improves with larger *n*). It all comes back to Z!

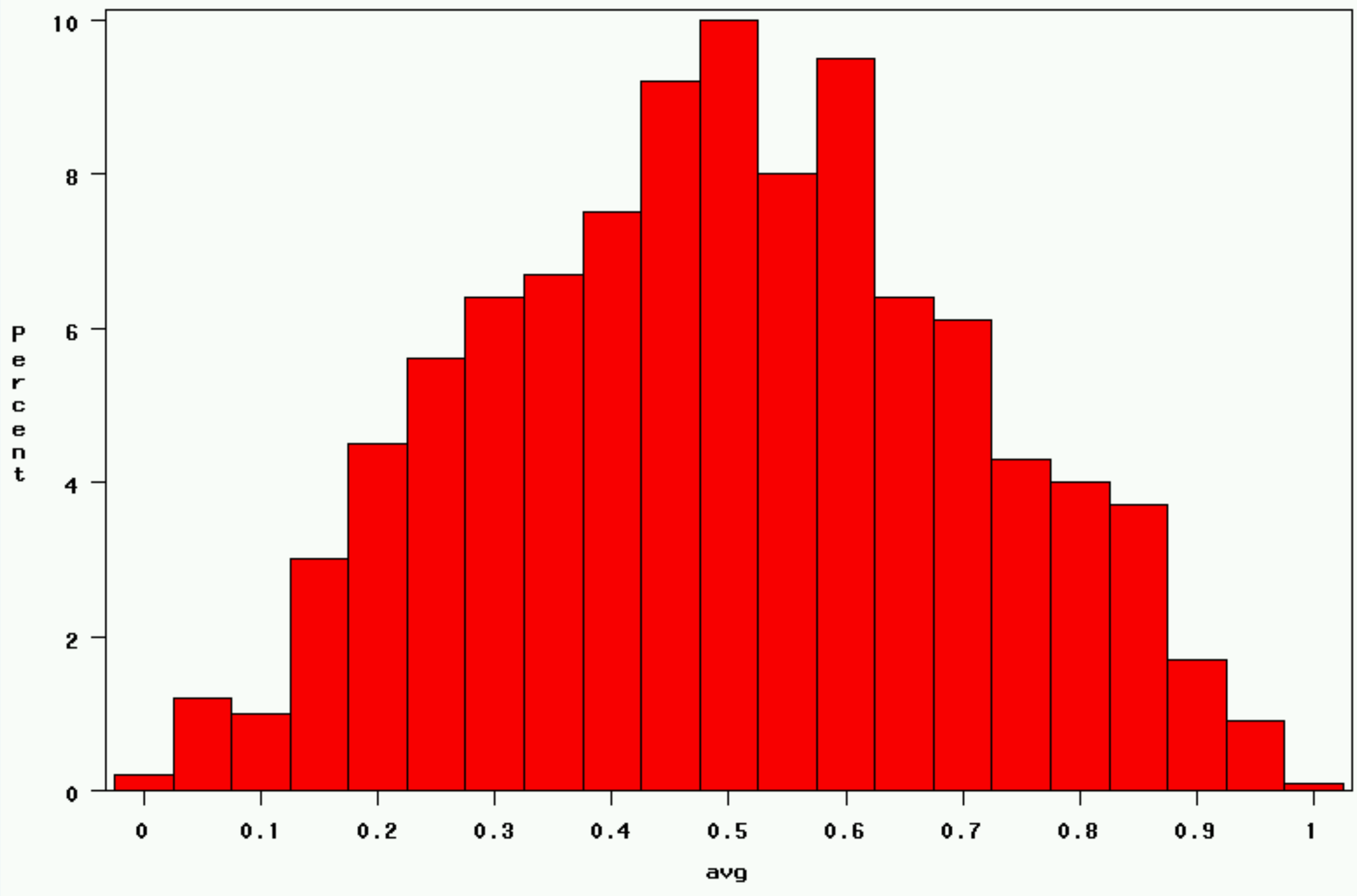
Computer simulation: CLT

1. Pick any probability distribution with finite mean and variance.
2. Tell the computer to randomly generate 1000 observations from that probability distributions.
3. Plot the “observed” values in a histogram.
4. Next, tell the computer to randomly generate 1000 averages-of-2 (randomly pick 2 and take their average) from that probability distribution. Plot “observed” averages in histograms.
5. Repeat for averages-of-10, and averages-of-100.

Uniform on [0,1]: average of 1 (original distribution)

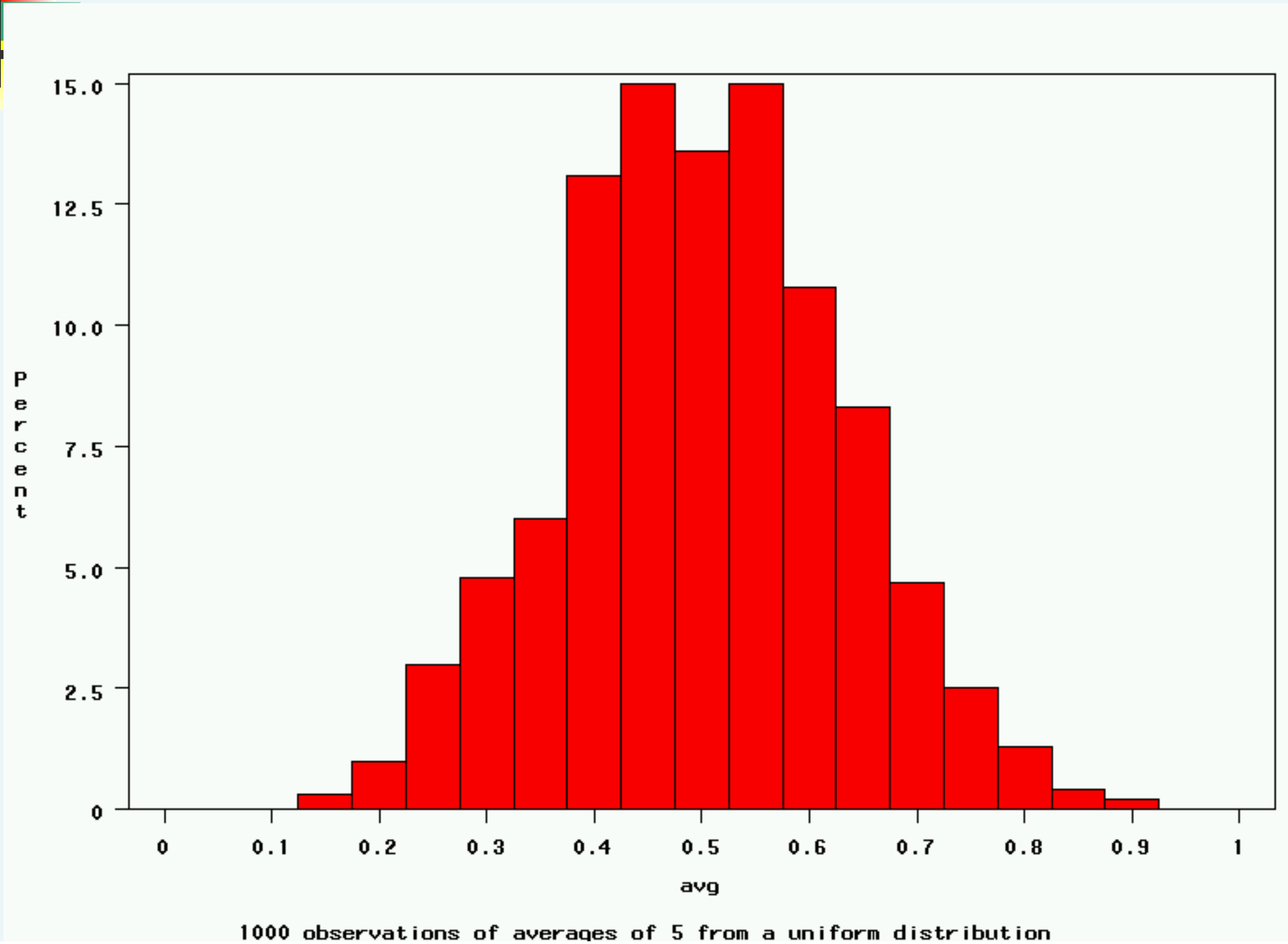


Uniform: 1000 averages of 2 (why)

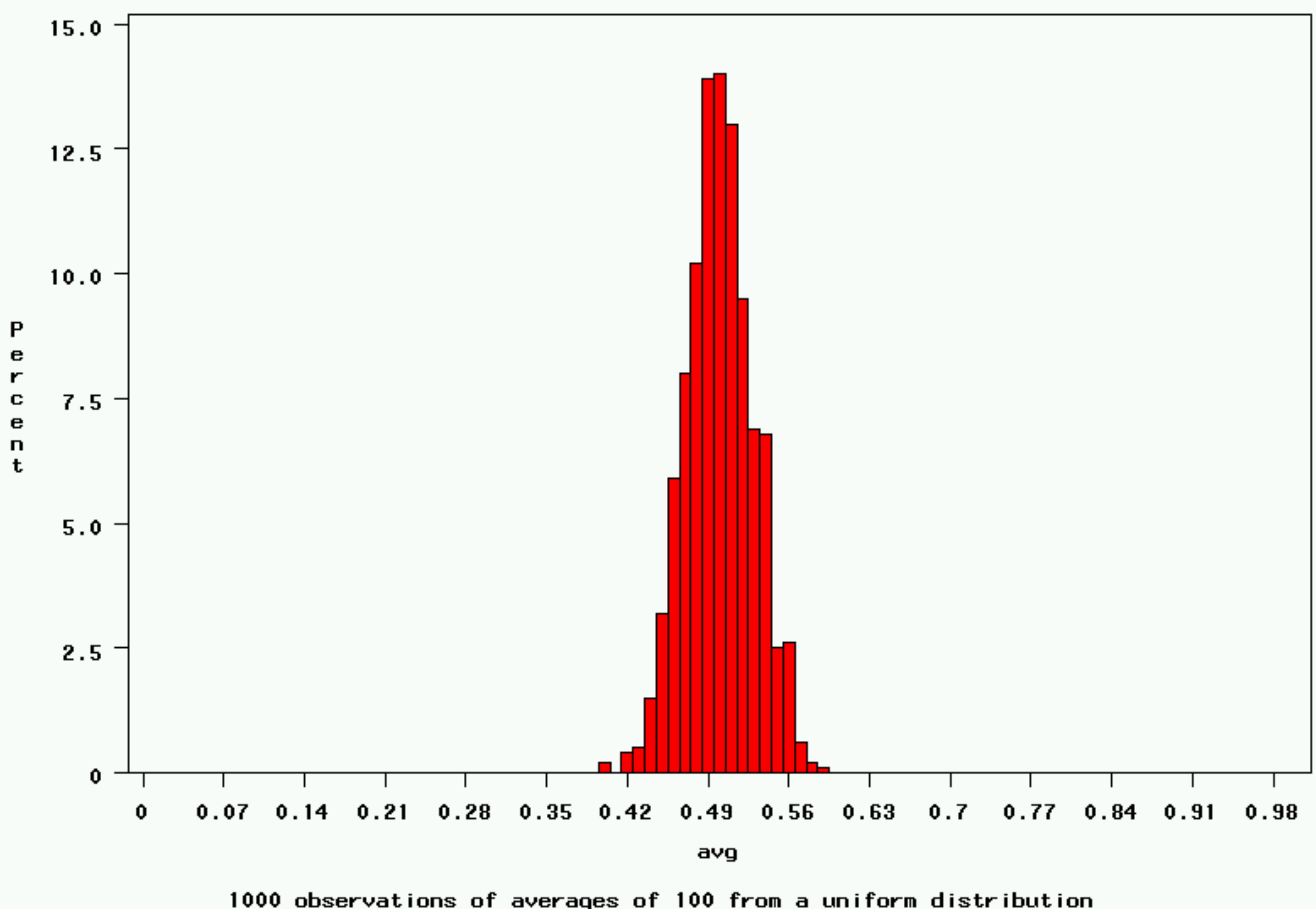


1000 observations of averages of 2 from a uniform distribution

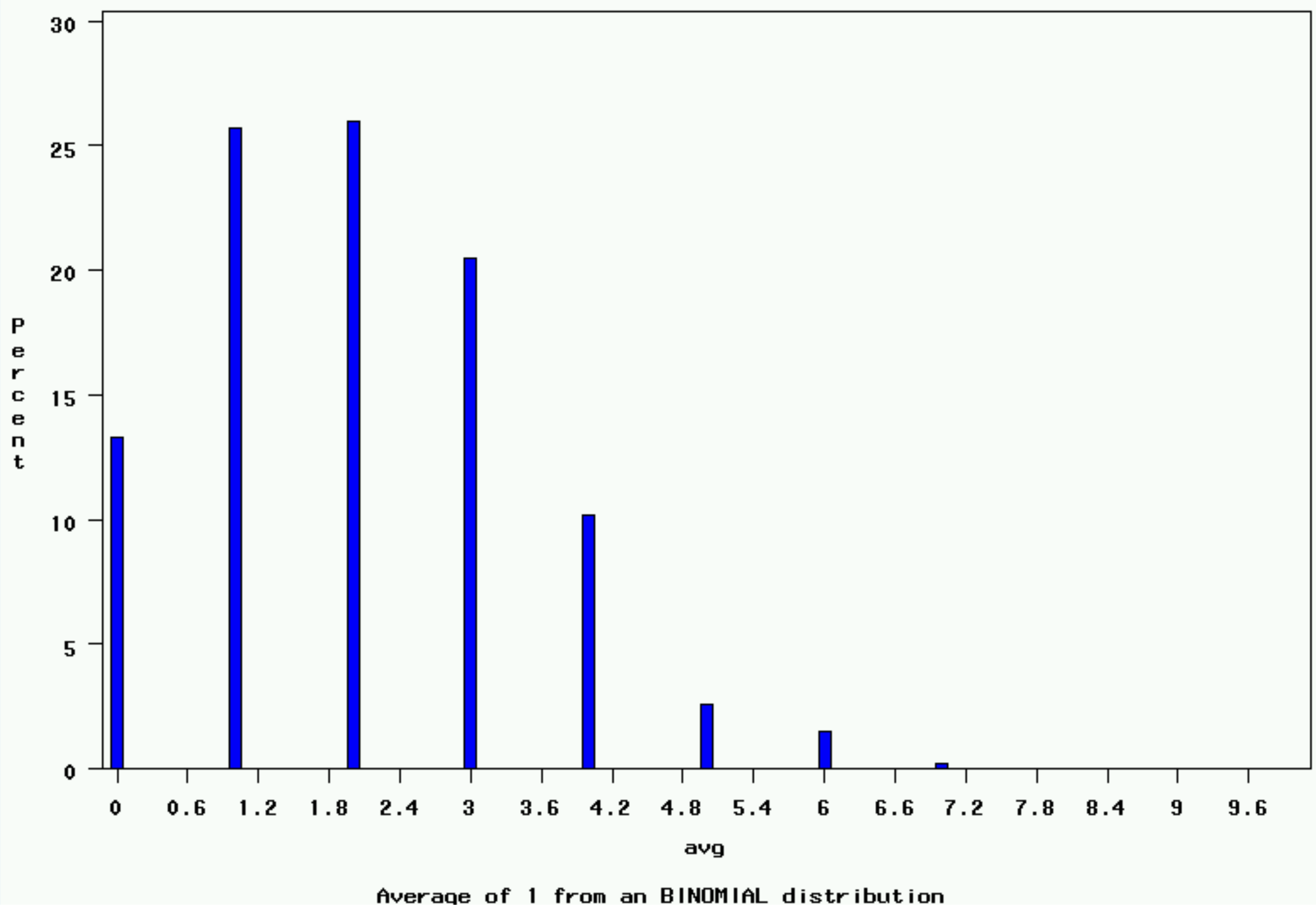
Uniform: 1000 averages of 5



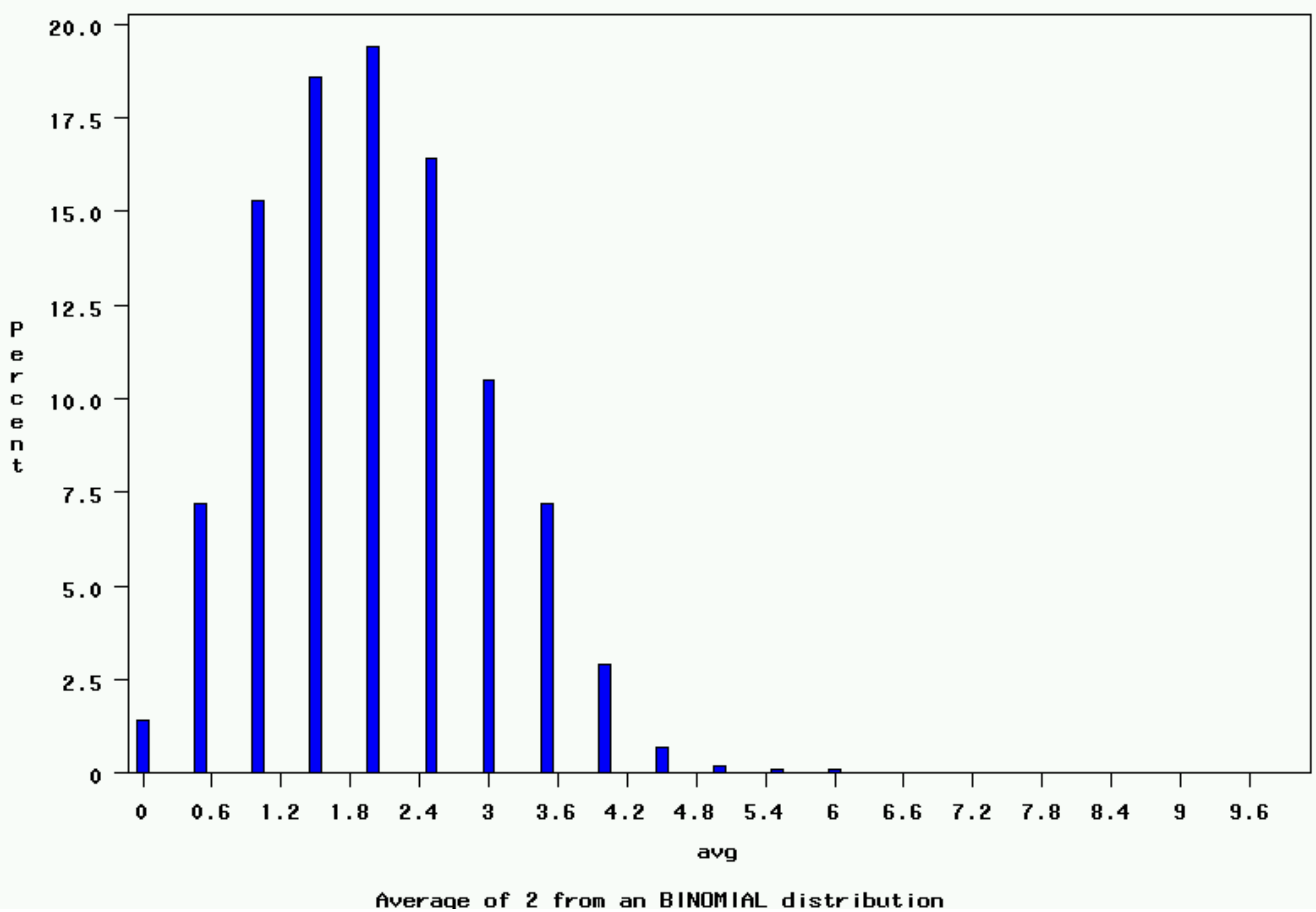
Uniform: 1000 averages of 100



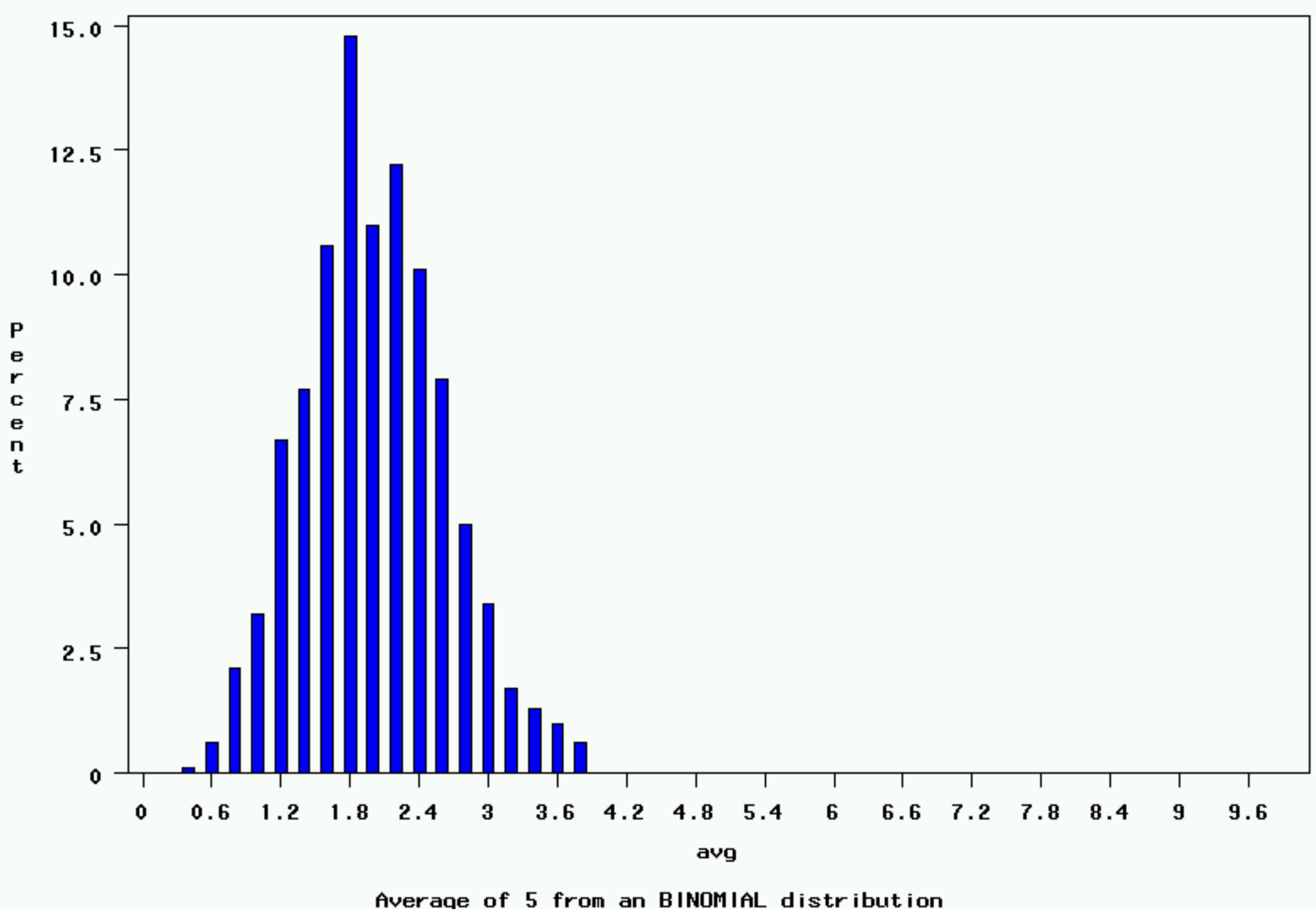
~Bin(40, .05): average of 1 (original distribution)



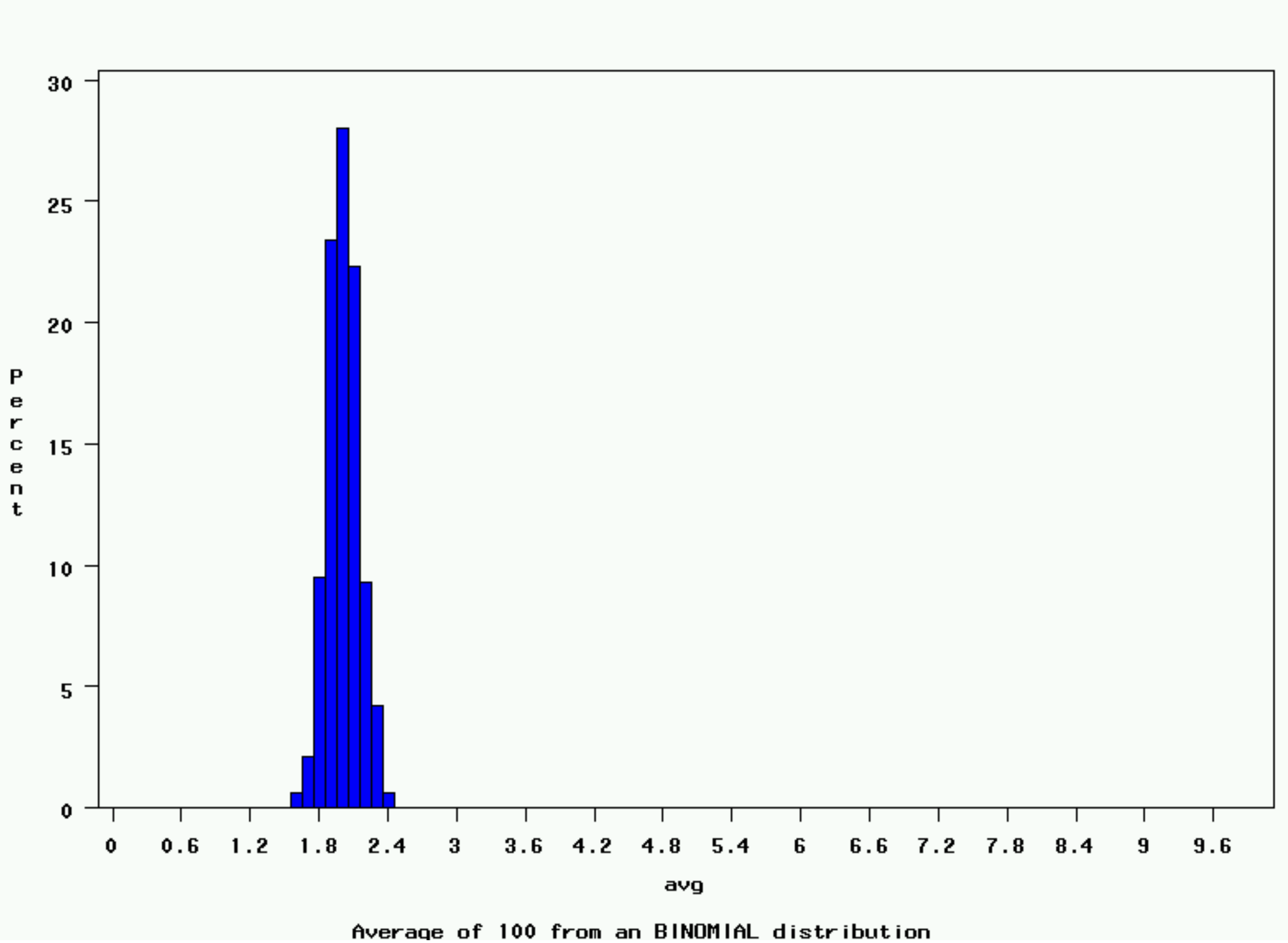
~Bin(40, .05): 1000 averages of 2



~Bin(40, .05): 1000 averages of 5



~Bin(40, .05): 1000 averages of 100



The Central Limit Theorem:

If all possible random samples, each of size *n*, are taken from any population with a mean μ and a standard deviation σ, the sampling distribution of the sample means (averages) will:

1. have mean: $\boxed{\mu_{\bar{x}} = \mu}$

2. have standard deviation: $\boxed{\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}}$

3. be approximately normally distributed regardless of the shape of the parent population (normality improves with larger *n*)

Central Limit Theorem caveats for small samples:

- For small samples:
  - The sample standard deviation is an imprecise estimate of the true standard deviation (σ); this imprecision changes the distribution to a T-distribution.
    - A t-distribution approaches a normal distribution for large n (≥ 50), but has fatter tails for small n (<50)
  - If the underlying distribution is non-normal, the distribution of the means would be non-normal.

How to construct C.I. for mean?

- From CLT, we have

Confidence Interval Recap.

- In practice you only take one sample of size n
- In practice you do not know μ so you do not know if the interval actually contains μ
- However you do know that 95% of the intervals formed in this manner will contain μ
- Thus, based on the one sample, you actually selected you can be 95% confident your interval will contain μ (this is a 95% confidence interval)

General Formula

- The general formula for all confidence intervals is:

Point Estimate ± (Critical Value)(Standard Error)

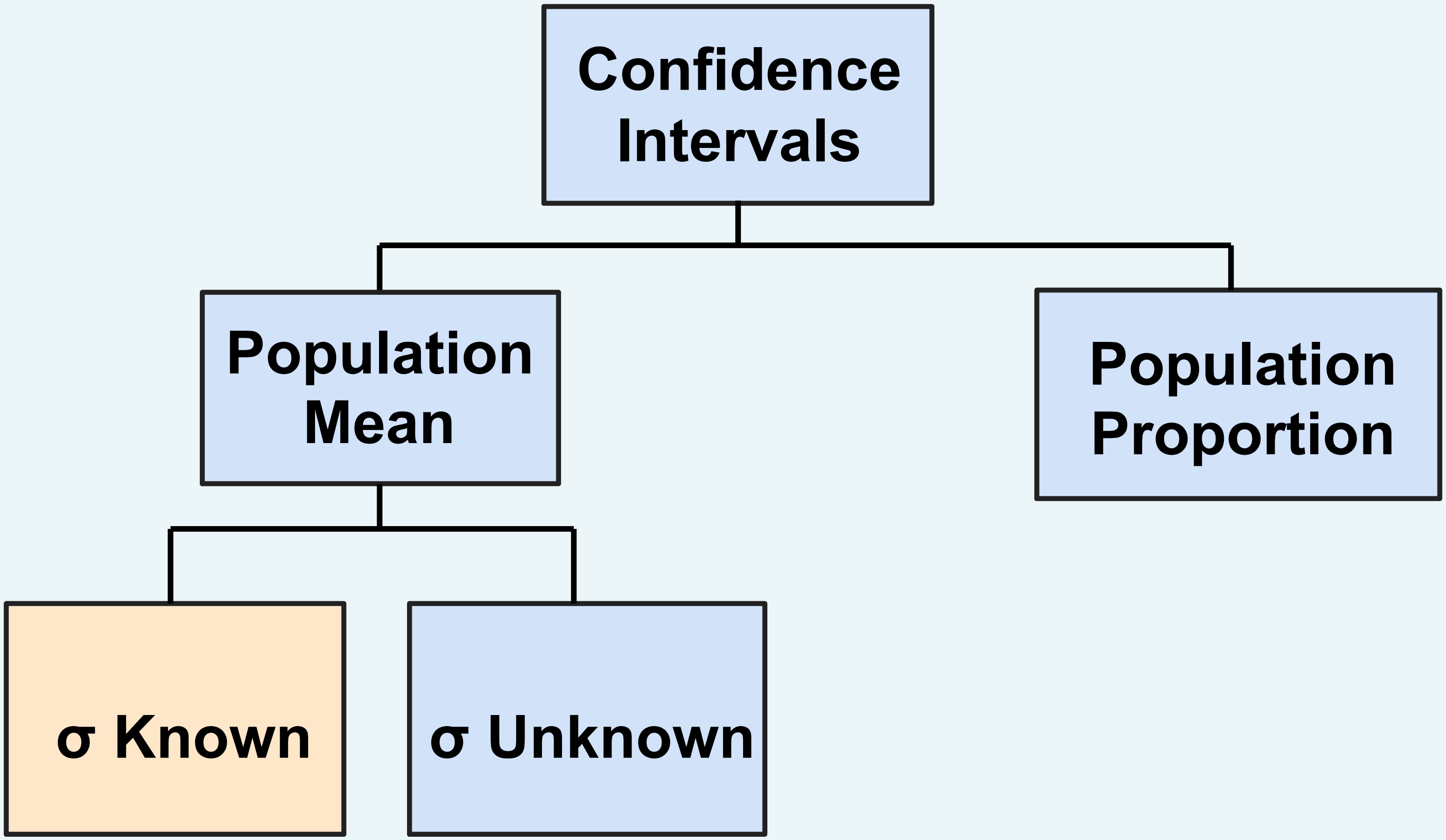
- Point Estimate is the sample statistic estimating the population parameter of interest
- Critical Value is a table value based on the sampling distribution of the point estimate and the desired confidence level
- Standard Error is the standard deviation of the point estimate

Confidence Level, $(1-\alpha)$

(continued)

- Suppose confidence level = 95%
- Also written $(1 - \alpha) = 0.95$, (so $\alpha = 0.05$)
- A relative frequency interpretation:
  - 95% of all the confidence intervals that can be constructed will contain the unknown true parameter
- A specific interval either will contain or will not contain the true parameter
  - No probability involved in a specific interval

Confidence Intervals



Confidence Interval for μ (σ Known)

- Assumptions
  - Population standard deviation σ is known
  - Population is normally distributed
  - If population is not normal, it should be a large sample
- Confidence interval estimate:

$$\boxed{\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}}$$

where $\bar{X}$ is the point estimate

$Z_{\alpha/2}$ is the normal distribution critical value for a probability of $\alpha/2$ in each tail

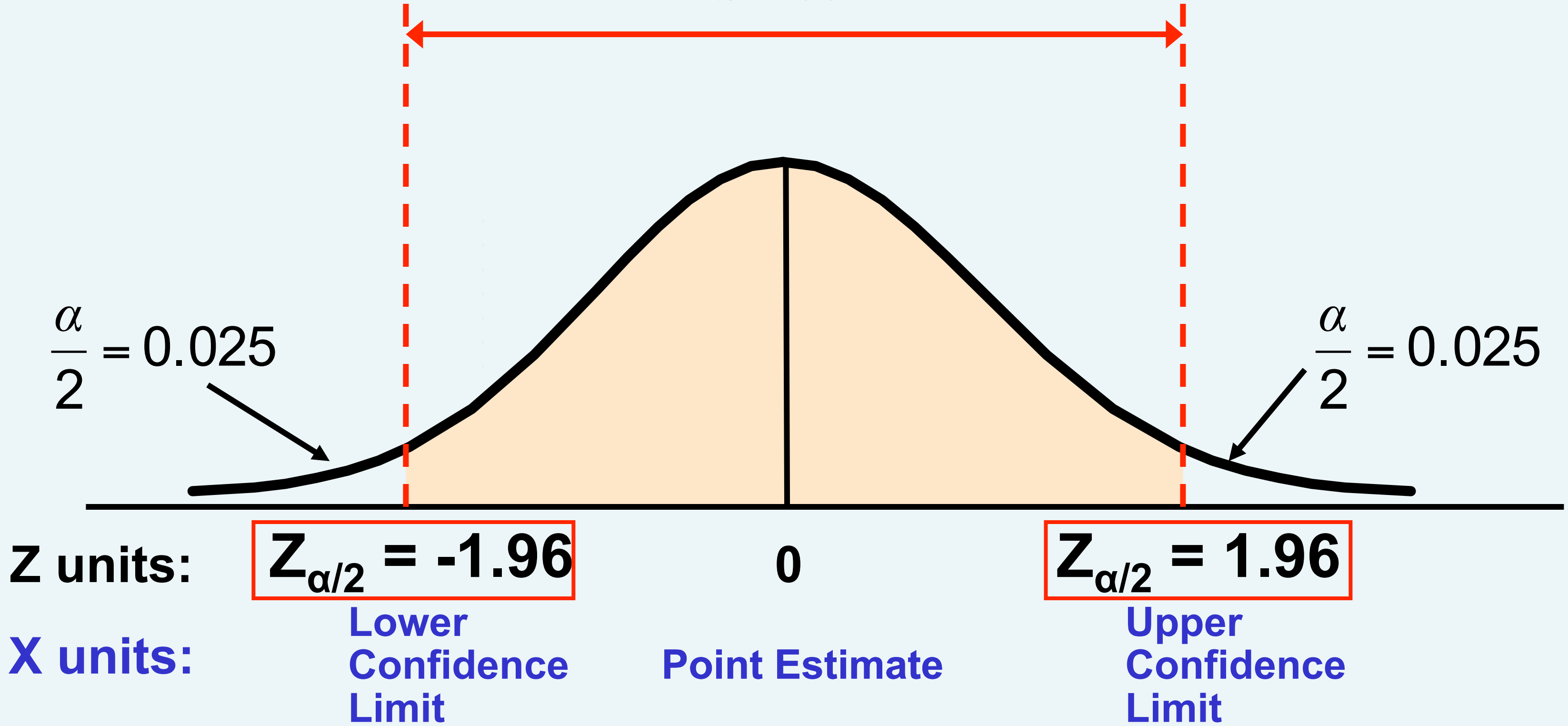
$\sigma/\sqrt{n}$ is the standard error

Finding the Critical Value, $Z_{\alpha/2}$

$$\boxed{Z_{\alpha/2} = \pm 1.96}$$

- Consider a 95% confidence interval:

$$1 - \alpha = 0.95 \text{ so } \alpha = 0.05$$



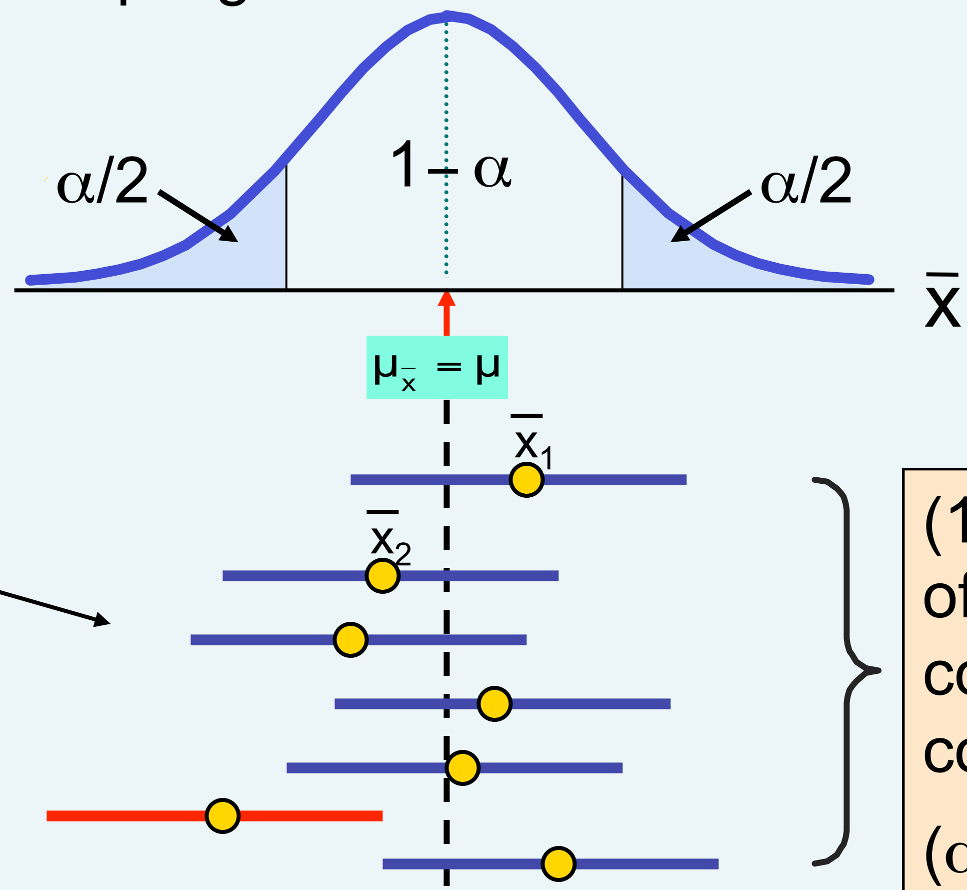
Common Levels of Confidence

- Commonly used confidence levels are 90%, 95%, and 99%

Confidence Level	***Confidence Coefficient,*** $1-\alpha$	$Z_{\alpha/2}$ ***value***
80%	**0.80**	**1.28**
90%	**0.90**	**1.645**
95%	**0.95**	**1.96**
98%	**0.98**	**2.33**
99%	**0.99**	**2.58**
99.8%	**0.998**	**3.08**
99.9%	**0.999**	**3.27**

Intervals and Level of Confidence

Sampling Distribution of the Mean



Confidence Intervals

Intervals extend from

$$\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

to

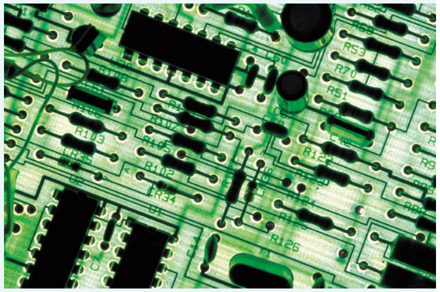
$$\bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$(1-\alpha)$x100% of intervals constructed contain μ;

(α)x100% do not.

Example

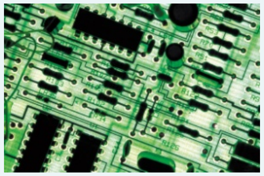
- A sample of 11 circuits from a large normal population has a mean resistance of 2.20 ohms. We know from past testing that the population standard deviation is 0.35 ohms.
- Determine a 95% confidence interval for the true mean resistance of the population.



Interpretation

- We are 95% confident that the true mean resistance is between 1.9932 and 2.4068 ohms

- Although the true mean may or may not be in this interval, 95% of intervals formed in this manner will contain the true mean

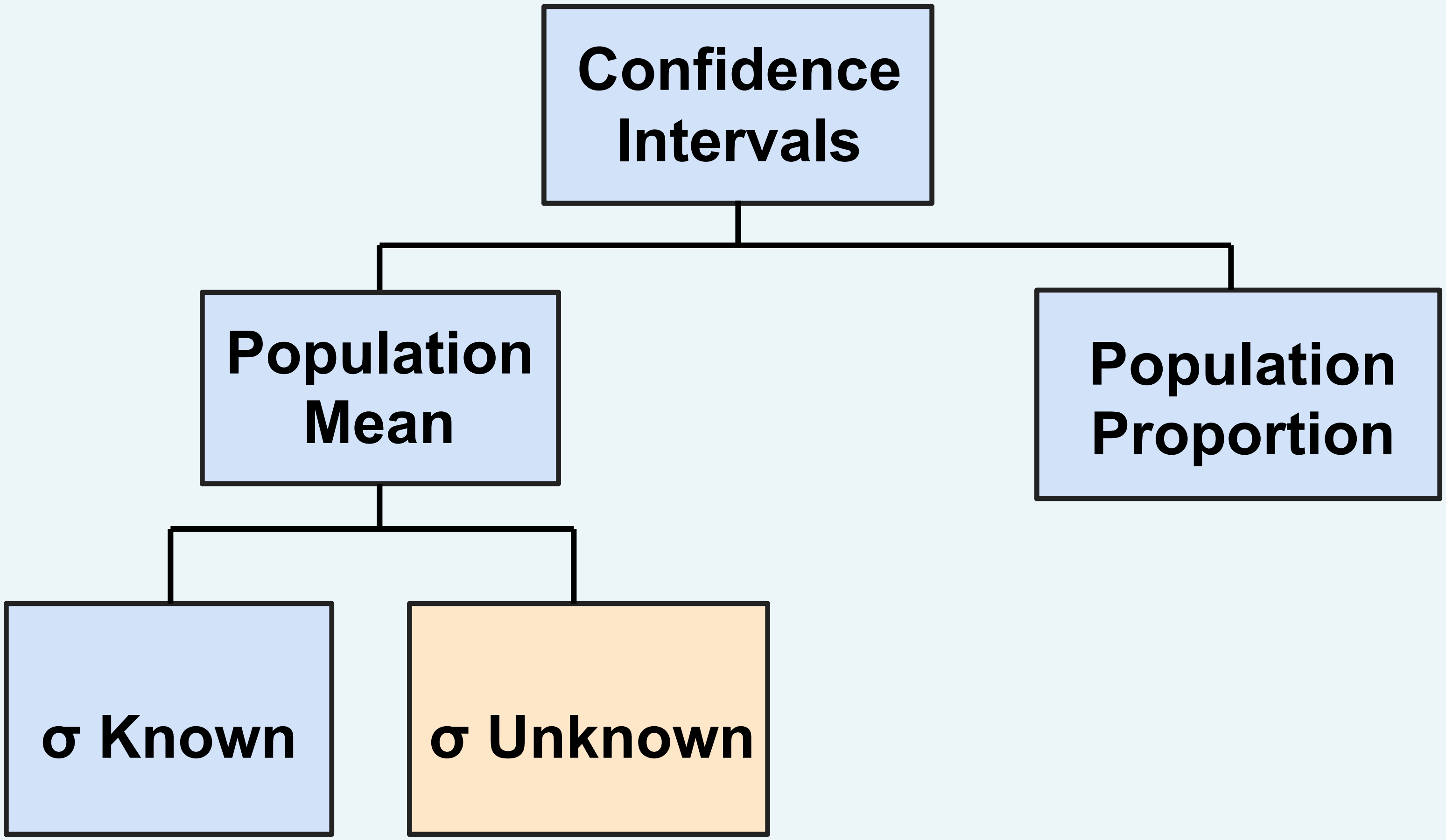


Example (mean)

- We look at data of ages 12-14 students' weights. The sample size is 40 with mean 105 pounds and a *sample* standard deviation of 22.2.

Form a 95% confidence interval for the heights of ages 12-14 students. (JMP Demo here)

Confidence Intervals



Do You Ever Truly Know σ?

- Probably not!

- In virtually all real world business situations, σ is not known.

- If there is a situation where σ is known then μ is also known (since to calculate σ you need to know μ.)

- If you truly know μ there would be no need to gather a sample to estimate it.

Confidence Interval for μ (σ Unknown)

- If the population standard deviation σ is unknown, we can substitute the sample standard deviation, *S*
- This introduces extra uncertainty, since *S* is variable from sample to sample
- So we use the *t* distribution instead of the normal distribution

Confidence Interval for μ (σ Unknown)

(continued)

- Assumptions
  - Population standard deviation is unknown
  - Population is normally distributed
  - If population is not normal, use large sample
- Use Student's t Distribution
- Confidence Interval Estimate:

$$\boxed{\overline{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}}$$

(where $t_{\alpha/2}$ is the critical value of the t distribution with n -1 degrees of freedom and an area of α/2 in each tail)

Student’s T story

- William Gosset's 1908 paper in Biometrika under the pseudonym “*Student*”
- Worked at the Guinness Brewery in Dublin, Ireland.
- Chemical properties of barley, when the sample size is small

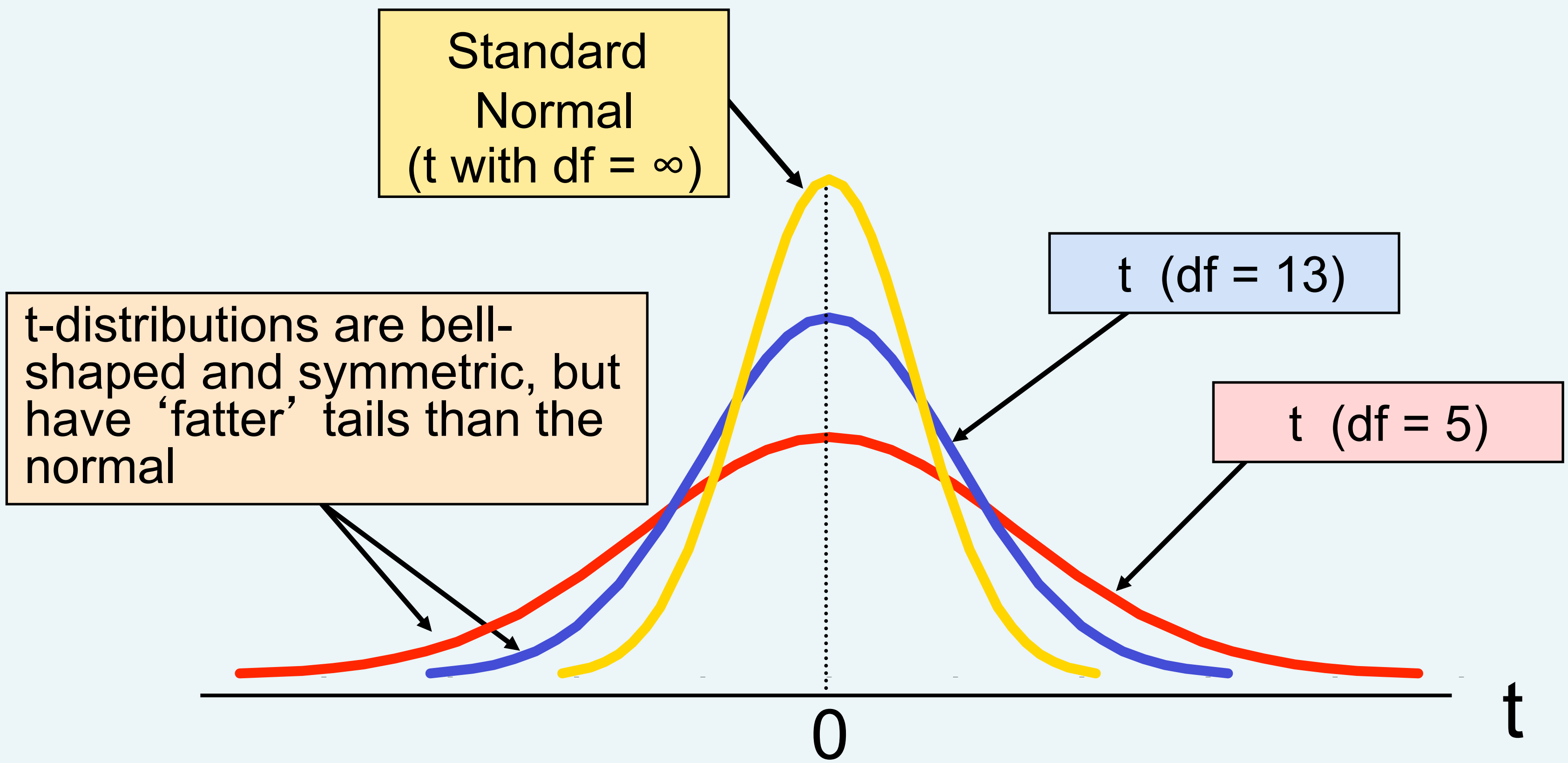
Student’s t Distribution has one parameter

- The t is a family of distributions
- The $t_{\alpha/2}$ value depends on degrees of freedom (d.f.)
  - Number of observations that are free to vary after sample mean has been calculated

$$\boxed{d.f. = n - 1}$$

Student' s t Distribution

Note: $t \longrightarrow Z$ as n increases

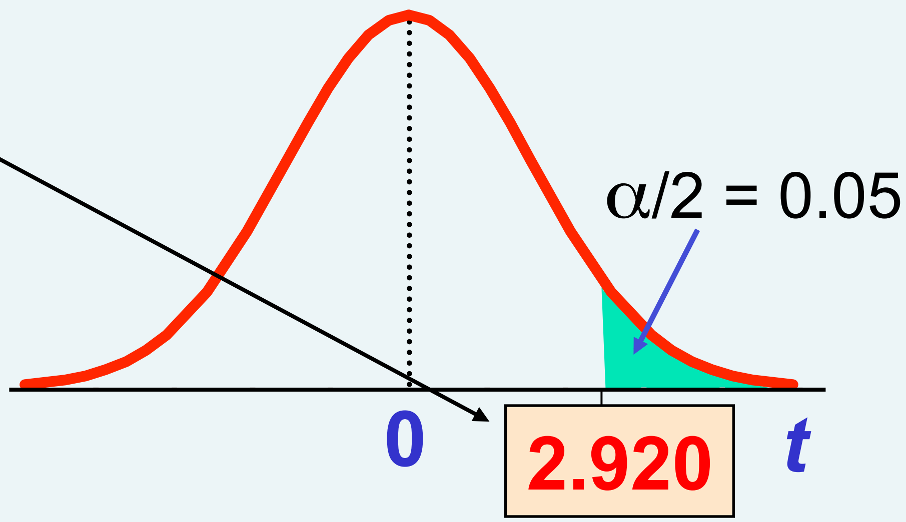


Student's t Table: e.g., t(39) at upper tail 0.025 = 2.023

	Upper Tail Area		
df	.25	.10	**.05**
1	1.000	3.078	6.314
2	0.817	1.886	**2.920**
3	0.765	1.638	2.353

The body of the table contains t values, not probabilities

Let: n = 3
df = n - 1 = 2
α = 0.10
$\alpha/2$ = 0.05



Selected t distribution values

With comparison to the Z value

Confidence Level	t (10 d.f.)	t (20 d.f.)	t (30 d.f.)	Z (∞ d.f.)
0.80	1.372	1.325	1.310	1.28
0.90	1.812	1.725	1.697	1.645
0.95	2.228	2.086	2.042	1.96
0.99	3.169	2.845	2.750	2.58

Note: $t \longrightarrow Z$ as n increases

Example of t distribution confidence interval

A random sample of n = 25 has $\bar{X}$ = 50 and S = 8. Form a 95% confidence interval for μ

- d.f. = n – 1 = 24, so $t_{\alpha/2} = t_{0.025} = 2.0639$

The confidence interval is

$$\bar{X} \pm t_{\alpha/2} \frac{S}{\sqrt{n}} = 50 \pm (2.0639) \frac{8}{\sqrt{25}}$$

Example of t distribution confidence interval

(continued)

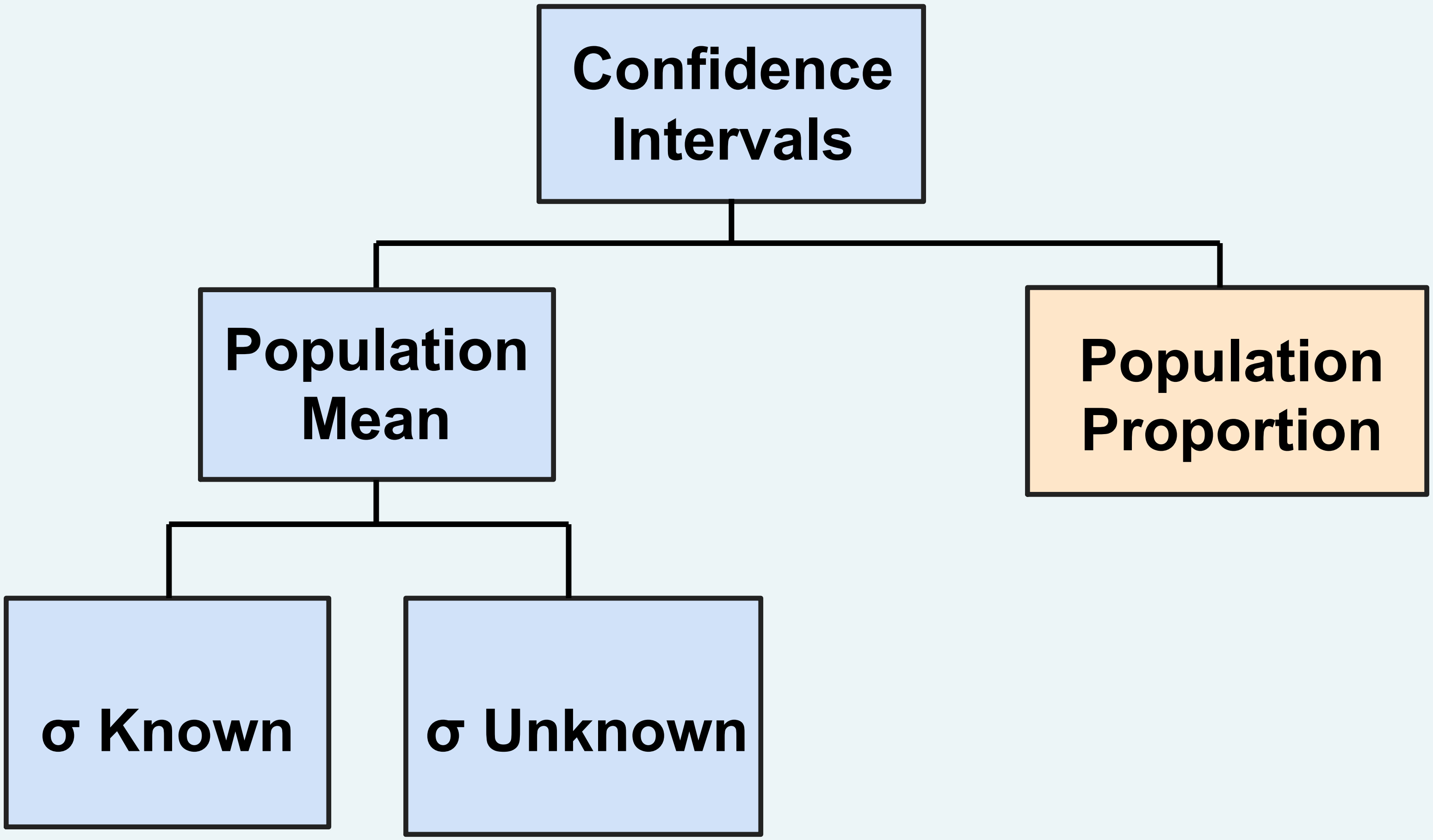
- **Interpreting this interval requires the assumption that the population you are sampling from is approximately a normal distribution (*especially since n is only 25*).**
- This condition can be checked by creating a:
  - Normal probability plot (JMP Demo here)

Single population mean (**small n, normally distributed trait**)

$$\frac{\text{observed mean} - \text{true mean}}{\frac{s}{\sqrt{n}}} ---> T(n-1)$$

$$\text{confidence interval} = \text{observed mean} \pm \mathrm{T}_{n-1,\alpha/2} * \left(\frac{s}{\sqrt{n}}\right)$$

Confidence Intervals



Confidence Intervals for the Population Proportion, π

- Point estimate of π?
- For instance, we are interested in the customer population proportion of preferring Uber service.
- Sample of size n=500 customers and count those who like it. So, *the sample proportion p* is given by:

Confidence Intervals for the Population Proportion, π

(continued)

- Recall that the distribution of the sample proportion is approximately normal if the sample size is large (why?), with standard deviation

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

- We will estimate this with sample data:

$$\sqrt{\frac{p(1-p)}{n}}$$

Confidence Interval Endpoints

- Upper and lower confidence limits for the population proportion are calculated with the formula

$$p \pm Z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

- where
  - $Z_{\alpha/2}$ is the standard normal value for the level of confidence desired
  - p is the sample proportion
  - n is the sample size
- General rule of thumb for Normal approximation to Binomial: must have **np > 5 and n(1-p) > 5**

Example

- A random sample of 100 people shows that 25 are left-handed.
- Form a 95% confidence interval for the true proportion of left-handers

Interpretation

- We are 95% confident that the true percentage of left-handers in the population is between

  16.51% and 33.49%.

- Although the interval from 0.1651 to 0.3349 may or may not contain the true proportion, 95% of intervals formed from samples of size 100 in this manner will contain the true proportion.

Confidence Intervals

The value of the statistic in my sample (eg., mean, proportion, etc.)

Point estimate ± (measure of how confident we want to be) × (standard error)

From a Z table or a T table, depending on the sampling distribution of the statistic.

Standard error of the statistic.

Now, Determining Sample Size

- **Determining Sample Size**
  - **For the Mean**
  - **For the Proportion**

Sample Size Determinatioin

- The required sample size can be found to reach a desired margin of error (e) with a specified level of confidence (1 - α)

- The margin of error is also called sampling error
  - the amount added and subtracted to the point estimate to form the confidence interval

Determining Sample Size

Determining Sample Size

For the Mean

$$\bar{X} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Sampling error (margin of error)

$$e = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Determining Sample Size

(continued)

Determining Sample Size

For the Mean

$$e = Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Now solve for n to get

$$n = \frac{Z_{\alpha/2}^{2}\, \sigma^{2}}{e^{2}}$$

Determining Sample Size

(continued)

- To determine the required sample size for the mean, you must know:
  - The desired level of confidence $(1 - \alpha)$, which determines the critical value, $Z_{\alpha/2}$
  - The acceptable sampling error, e
  - The standard deviation, σ

Required Sample Size Example

If $\sigma = 45$, what sample size is needed to estimate the mean within ± 5 with 90% confidence?

So the required sample size is **n =**

If σ is unknown

- If unknown, σ can be estimated when using the required sample size formula
  - Select a pilot sample and estimate σ with the sample standard deviation, S

Determining Sample Size

(continued)

Determining Sample Size

For the Proportion

$$e = Z\sqrt{\frac{\pi(1-\pi)}{n}}$$

Now solve for n to get

$$n = \frac{Z^2\,\pi(1-\pi)}{e^2}$$

Determining Sample Size

(continued)

- To determine the required sample size for the proportion, you must know:
  - The desired level of confidence $(1 - \alpha)$, which determines the critical value, $Z_{\alpha/2}$
  - The acceptable sampling error, e
  - The true proportion of events of interest, π
    - π can be estimated with a pilot sample if necessary (or conservatively use 0.5 as an estimate of π)

Required Sample Size Example

How large a sample would be necessary to estimate the true proportion defective (say, of new i-phone) in a large population within ±3%, with 95% confidence?

(Assume a pilot sample yields $p = 0.12$)

Required Sample Size Example

(continued)

Solution:

For 95% confidence, use $Z_{\alpha/2} = 1.96$

$e = 0.03$

$p = 0.12$, so use this to estimate π

$$n = \frac{Z_{\alpha/2}^{\ 2}\ \pi(1-\pi)}{e^2} = \frac{(1.96)^2(0.12)(1-0.12)}{(0.03)^2} = 450.74$$

So use n = 451

Fundamentals of Hypothesis Testing

Pop Quiz

- As part of opinion survey, a simple random sample of 400 persons age 25 and over is taken in Hoboken. The total years of schooling completed by the sample person is 4,635. So their average educational level is 4,635/400, about 11.6 years. The Std. dev. of the sample is 4.1 years.
  A) 95% CI? B) 50 surveys' CI's

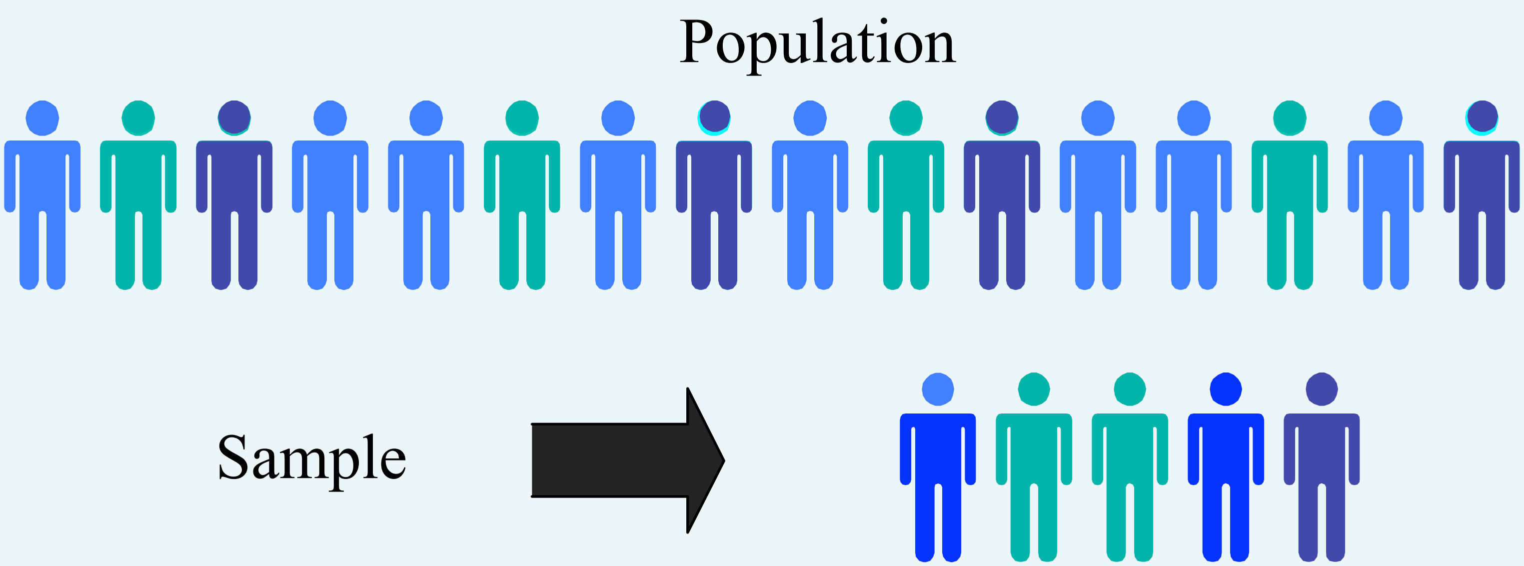
H_0 vs. H_1

- H_0 The Null Hypothesis: Begin with the assumption that the null hypothesis is true
  - Similar to the notion of innocent until proven guilty

- Refers to the status quo or historical value
- H_1 The Alternative Hypothesis
- Challenges the status quo
- Is generally the hypothesis that the researcher is trying to prove

The Hypothesis Testing Process

- Claim: The population mean age is 50.
  - H_0: $\mu = 50$, H_1: $\mu \neq 50$
- Sample the population and find sample mean.



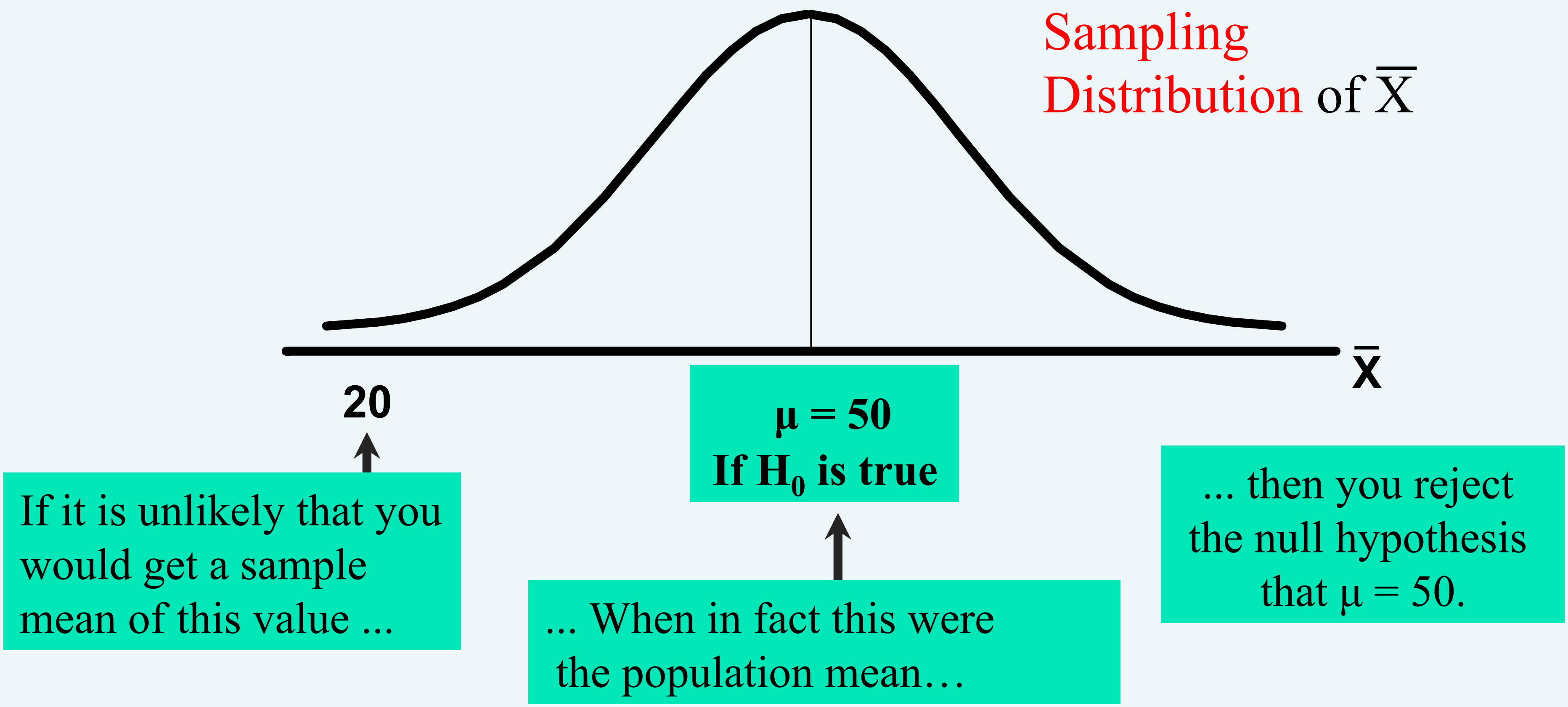
The Hypothesis Testing Process

(continued)

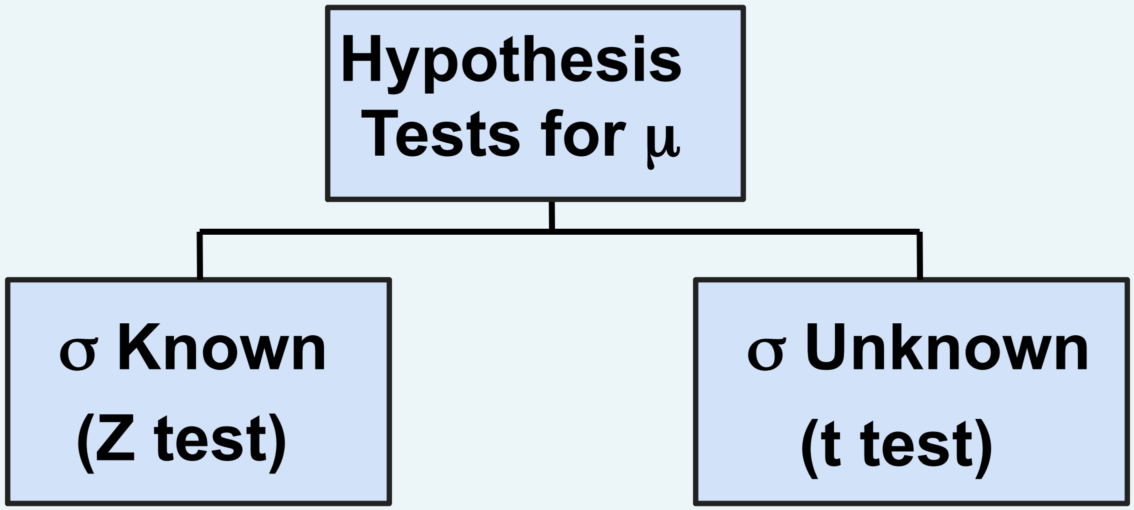
- Suppose the sample mean age was $\overline{X} = 20$.
- This is significantly lower than the claimed mean population age of 50.
- If the null hypothesis were true, the probability of getting such a different sample mean would be very small, so you reject the null hypothesis.
- In other words, getting a sample mean of 20 (or smaller) is so unlikely if the population mean was 50, you conclude that the population mean must not be 50.

The Hypothesis Testing Process

(continued)



Hypothesis Tests for the Mean



Z Test of Hypothesis for the Mean (σ Known)

- Convert sample statistic ($\bar{X}$) to a Z_{STAT} test statistic

Hypothesis Tests for μ

σ Known (Z test)

σ Unknown (t test)

The test statistic is:

$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

→ Can be thought of as "Signal to Noise Ratio"

p-Value Approach to Testing

- p-value: simply a numerical 'measure' (in (0,1)) of consistency of the observed data with the null hypothesis in support of the alternative (closer to 0 → stronger support of the alternative)

- p-value: Probability of obtaining a test statistic equal to or more extreme than the observed sample value, given H_0 is true

p-Value Approach to Testing: Interpreting the p-value

- Compare the p-value with α
  - If p-value $< \alpha$, reject H_0
  - If p-value $\geq \alpha$, do not reject H_0
- Remember
  - If the p-value is low then H_0 must go

Possible Errors in Hypothesis Test Decision Making

- **Type I Error**
  - Reject a true null hypothesis
  - Considered a serious type of error
  - The probability of a Type I Error is α
    - Called level of significance of the test
    - Set by researcher in advance
- **Type II Error**
  - Failure to reject false null hypothesis
  - The probability of a Type II Error is $\boldsymbol{\beta}$

Possible Errors in Hypothesis Test Decision Making

(continued)

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No Error Probability 1 - α	Type II Error Probability β
Reject H_0	Type I Error Probability α	No Error Probability 1 - β

Quiz 1: The level of significance increases if sample size increases (True, False)

Quiz 2: The type II error will decrease if sample size increases (True, False)

The 5 Step p-value approach to Hypothesis Testing

1. State the null hypothesis, H_0 and the alternative hypothesis, H_1
2. Choose the level of significance, α, and the sample size, n
3. Determine the appropriate test statistic and sampling distribution
4. Collect data and compute the value of the test statistic and the p-value
5. Make the statistical decision and state the managerial conclusion. If the p-value is < α then reject H_0, otherwise do not reject H_0. Report p-value. State the managerial conclusion in the context of the problem

p-value Hypothesis Testing Example

Test the claim that the true mean # of TV sets in US homes is equal to 3.

(Assume $\sigma = 0.8$)

1. State the appropriate null and alternative hypotheses
   - H_0: $\mu = 3$ H_1: $\mu \neq 3$ (This is a two-tail test)
2. Specify the desired level of significance and the sample size
   - Suppose that $\alpha = 0.05$ and $n = 100$ are chosen for this test

p-value Hypothesis Testing Example

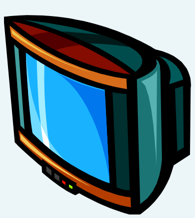
(continued)

3. Determine the appropriate technique
   - σ is assumed known so this is a Z test.
4. Collect the data, compute the test statistic and the p-value
   - Suppose the sample results are

     n = 100, $\bar{X}$ = 2.84 (σ = 0.8 is assumed known)

So the test statistic is:

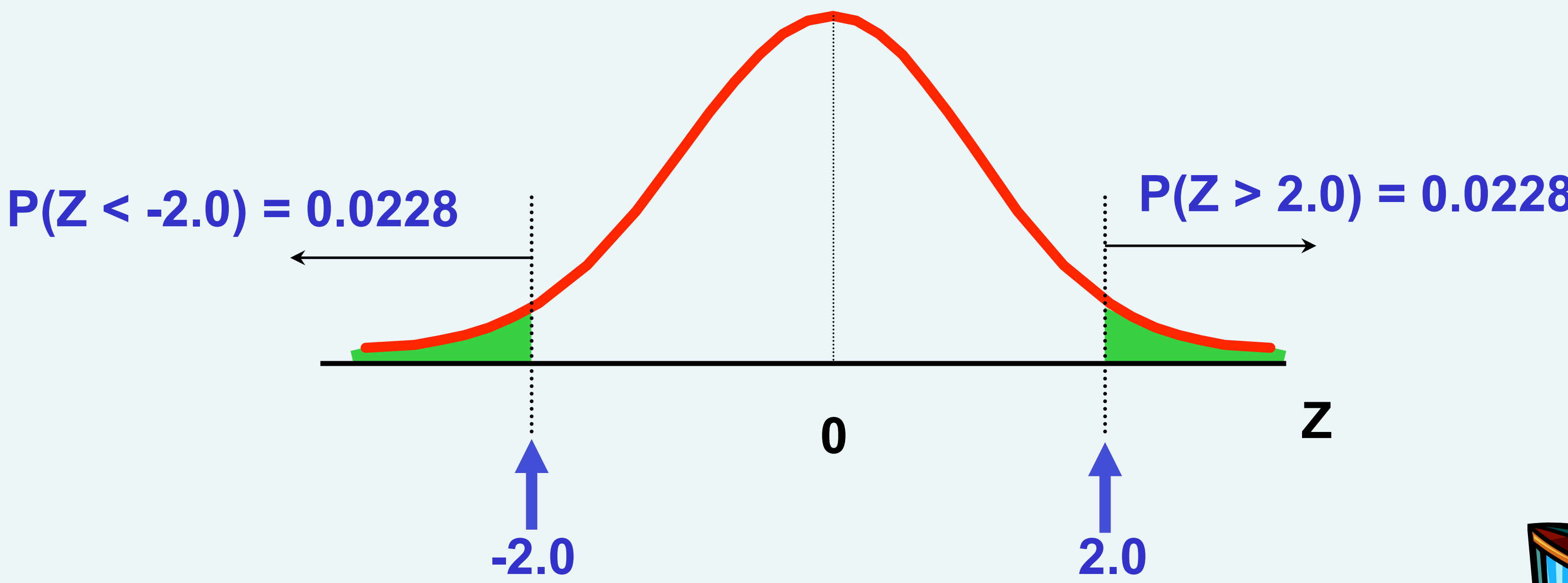
$$Z_{STAT} = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{2.84 - 3}{\frac{0.8}{\sqrt{100}}} = \frac{-.16}{.08} = -2.0$$



p-Value Hypothesis Testing Example: Calculating the p-value

4. (continued) Calculate the p-value.

- How likely is it to get a Z_{STAT} of -2 (or something further from the mean (0), in either direction) if H_0 is true?

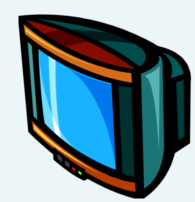


p-value = 0.0228 + 0.0228 = 0.0456

p-value Hypothesis Testing Example

(continued)

- 5. Is the p-value < α?
  - Since p-value = 0.0456 < α = 0.05 Reject H_0
- 5. (continued) State the managerial conclusion in the context of the situation.
  - There is sufficient evidence to conclude the average number of TVs in US homes is not equal to 3.



Hypothesis Testing: σ Unknown

- If the population standard deviation is unknown, you instead use the sample standard deviation S.

- Because of this change, you use the t distribution instead of the Z distribution to test the null hypothesis about the mean.

- When using the t distribution you must assume the population you are sampling from follows a normal distribution.

- All other steps, concepts, and conclusions are the same.

t Test of Hypothesis for the Mean
(σ Unknown)

- Convert sample statistic ($\bar{X}$) to a t_{STAT} test statistic

Hypothesis Tests for μ

σ Known (Z test)

σ Unknown (t test)

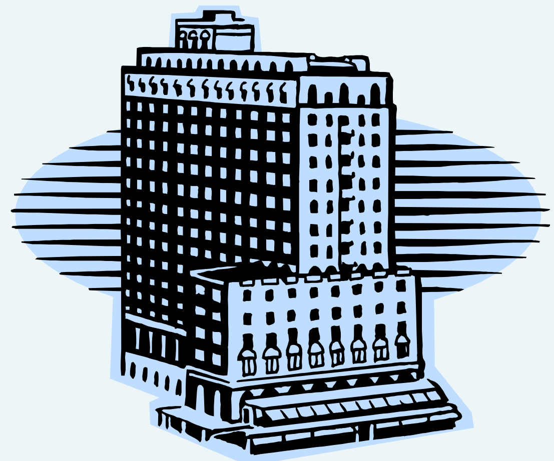
The test statistic is:

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}}$$

Example: Two-Tail Test (σ Unknown)

The average cost of a hotel room in New York is said to be \$168 per night. To determine if this is true, a random sample of 25 hotels is taken and resulted in an $\bar{X}$ of \$172.50 and an S of \$15.40. Test the appropriate hypotheses at $\alpha = 0.05$.

(Assume the population distribution is normal.)



H_0: $\mu = 168$
H_1: $\mu \neq 168$

Example Two-Tail t Test Using a p-value

One-Sample T

Test of mu = 168 vs not = 168

N	Mean	StDev	SE Mean	95% CI	T	P
25	172.50	15.40	3.08	(166.14, 178.86)	1.46	0.157

p-value > α
So do not reject H_0

One-Tail Tests

- In many cases, the alternative hypothesis focuses on a particular direction

H_0: $\mu \geq 3$
H_1: $\mu < 3$

This is a lower-tail test since the alternative hypothesis is focused on the lower tail below the mean of 3

H_0: $\mu \leq 3$
H_1: $\mu > 3$

This is an upper-tail test since the alternative hypothesis is focused on the upper tail above the mean of 3

Example: Upper-Tail t Test for Mean (σ unknown)

A phone industry manager thinks that customer monthly cell phone bills have increased, and now average over \$52 per month. The company wishes to test this claim. (Assume a normal population)

Form hypothesis test:

H_0: $\mu \le 52$ the average is not over \$52 per month

H_1: $\mu > 52$ the average **is** greater than \$52 per month (i.e., sufficient evidence exists to support the manager's claim)

Example: Test Statistic

(continued)

Obtain sample and compute the test statistic

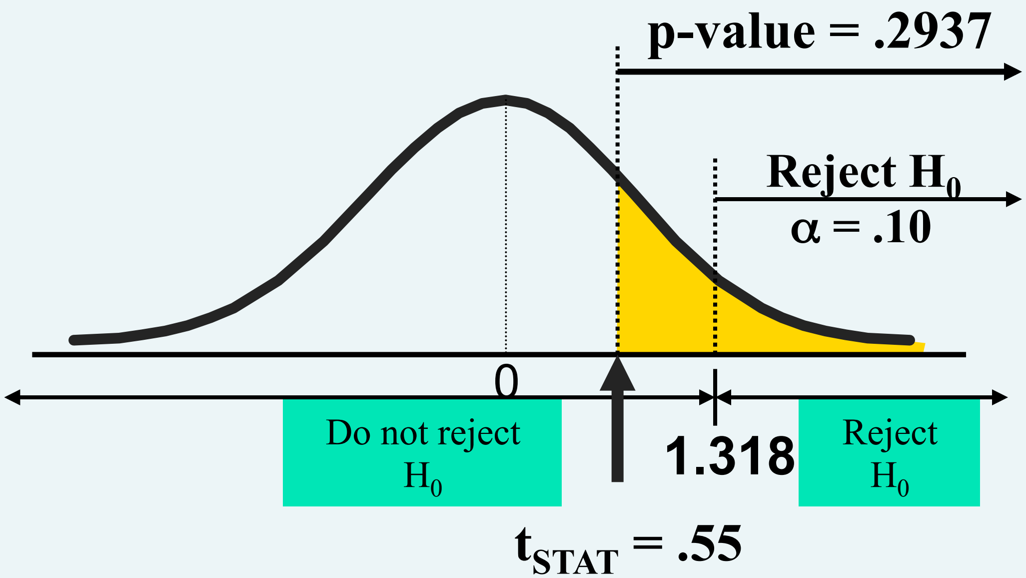
Suppose a sample is taken with the following results: $n = 25$, $\bar{X} = 53.1$, and $S = 10$

- Then the test statistic is:

$$t_{STAT} = \frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} = \frac{53.1 - 52}{\frac{10}{\sqrt{25}}} = 0.55$$

Example: Utilizing The p-value for The Test

- Calculate the p-value and compare to α (p-value below calculated using excel spreadsheet on next page)



Do not reject H_0 since p-value = .2937 > α = .10

Excel Spreadsheet Calculating The p-value for The Upper Tail t Test

t Test for the Hypothesis of the Mean

Data		
Null Hypothesis	**μ=**	**52.00**
Level of Significance		**0.1**
Sample Size		**25**
Sample Mean		**53.10**
Sample Standard Deviation		**10.00**

Intermediate Calculations	
Standard Error of the Mean	2.00
Degrees of Freedom	24
t test statistic	**0.55**
Upper Tail Test	
Upper Critical Value	1.318
p-value	0.2937
Do Not Reject Null Hypothesis	

JMP Demo here.

Hypothesis Tests for Proportions

- Involves categorical variables
- Two possible outcomes
  - Possesses characteristic of interest
  - Does not possess characteristic of interest
- Fraction or proportion of the population in the category of interest is denoted by π

Proportions

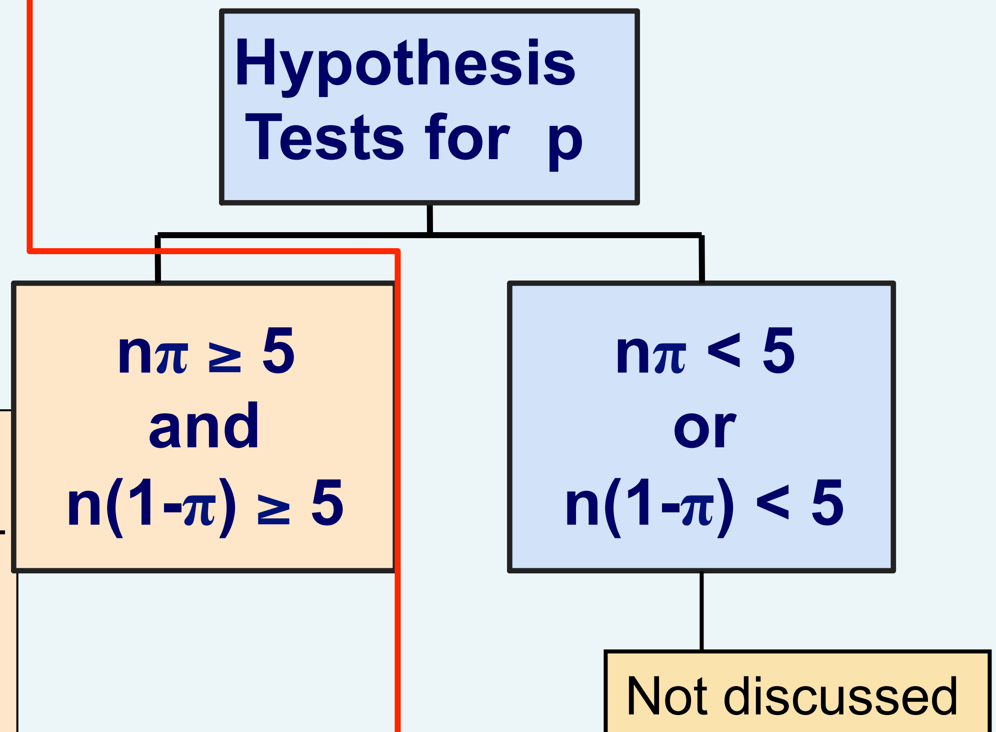
(continued)

- Sample proportion in the category of interest is denoted by p
  - $$p = \frac{X}{n} = \frac{\text{number in category of interest in sample}}{\text{sample size}}$$
- When both $n\pi$ and $n(1-\pi)$ are at least 5, p can be approximated by a normal distribution with mean and standard deviation
  - $$\mu_p = \pi \qquad \sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

Hypothesis Tests for Proportions

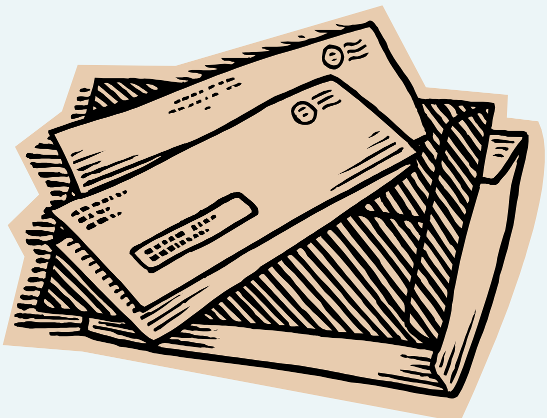
- The sampling distribution of p is approximately normal, so the test statistic is a Z_{STAT} value:

$$Z_{STAT} = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$$



Example: Z Test for Proportion

A marketing company claims that it receives 8% responses from its mailing. To test this claim, a random sample of 500 were surveyed with 25 responses. Test at the $\alpha = 0.05$ significance level.



Check:

$$n\pi = (500)(.08) = 40$$

$$n(1-\pi) = (500)(.92) = 460$$

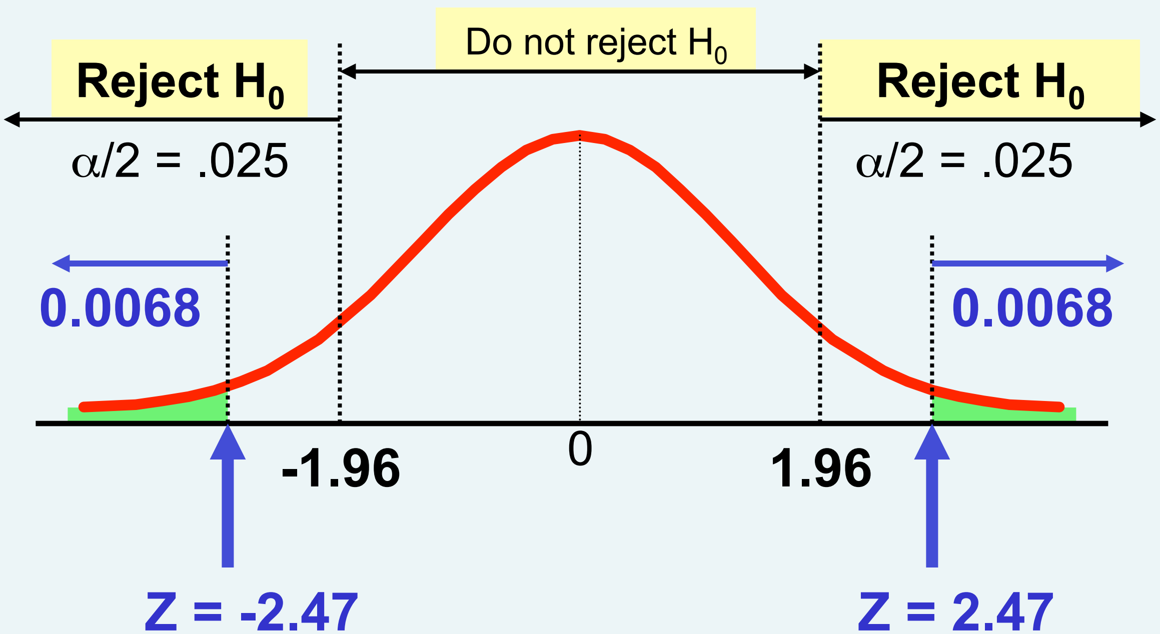
✓

p-Value Solution

(continued)

Calculate the p-value and compare to α

(For a two-tail test the p-value is always two-tail)



p-value = 0.0136:

$$P(Z \le -2.47) + P(Z \ge 2.47)$$

$$= 2(0.0068) = 0.0136$$

Reject H_0 since p-value = 0.0136 < α = 0.05

Possible Errors in Hypothesis Test Decision Making

(continued)

Possible Hypothesis Test Outcomes		
	Actual Situation	
Decision	H_0 True	H_0 False
Do Not Reject H_0	No Error Probability 1 - α	Type II Error Probability β
Reject H_0	Type I Error Probability α	No Error Probability 1 - β

Possible Errors in Hypothesis Test Decision Making

(continued)

- The confidence coefficient ($1-\alpha$) is the probability of not rejecting H_0 when it is true.

- The confidence level of a hypothesis test is ($1-\alpha$)*100%.

- The power of a statistical test ($1-\beta$) is the probability of rejecting H_0 when it is false.

Type I & II Error Relationship

- Type I and Type II errors cannot happen at the same time

Note: The stronger the evidence needed to reject the null hypothesis (that is, _________), the lower the chance that the null hypothesis will be rejected (that is, _________).

____

Factors Affecting Type II Error

- All else equal,
  - $\beta \uparrow$ when the difference between hypothesized parameter and its true value $\downarrow$
  - $\beta \uparrow$ when $\alpha \downarrow$
  - $\beta \uparrow$ when $\sigma \uparrow$
  - $\beta \uparrow$ when $n \downarrow$

Power of the Test

- Conclusions regarding the power of the test:

1. An increase in the level of significance (α) results in an increase in power
2. An increase in the sample size results in an increase in power
3. An increase in the difference between hypothesized parameter and its true value

Recommended reading:
http://www.intuitor.com/statistics/T1T2Errors.html
Judicial systems and statistical hypotheses