Normalized innovation error squared (NIS) based Kalman Filter Auto-tuning

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June 22, 2022

1 Preliminaries

1.1 Kalman Filter

A Kalman filter mainly consists of two steps:

Predict

Predicted (*a priori*) state estimate

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k$$

Predicted (*a priori*) estimate covariance

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$$

Update

Innovation pre-fit residual

$$\tilde{y}_k = z_k - H_k \hat{x}_{k|k-1}$$

Innovation covariance

$$S_k = H_k P_{k|k-1} H_k^T + R_k$$

Optimal Kalman gain

$$K_k = P_{k|k-1} H_k^T S_k^{-1}$$

Updated (*a posteriori*) state estimate

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{y}_k$$

Updated (*a posteriori*) estimate covariance

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

The performance of Kalman filter highly depends on the two parameters:

- State noise covariance matrix: Q
- Observation noise covariance matrix: R

Our task is to establish an auto-tuning process to find the optimal value of these two parameters under a given situation.

More details about Kalman filter see also [Faragher, 2012].

1.2 Objective Function

Here we consider the Normalized Innovation Error Squared (NIS), which is computed from

$$\epsilon_{z,k} = \tilde{y}_k^T S_{k|k-1}^{-1} \tilde{y}_k$$

It is often assumed that the prediction and observation errors are Gaussian. So when the dynamical consistency conditions are met under the chosen parameters (Q and R), $\epsilon_{z,k}$ will be χ^2-distributed random variables with n_z degrees of freedom, i.e.:

$$E[\epsilon_{z,k}] \approx n_z$$

where n_z refers to the measurement(observation)-vector dimension. Then an objective function for parameters tuning based on NIS can be build:

$$J_{NIS}(Q, R) = \left| \log \left(\frac{\sum_{k=1}^{T} \epsilon_{z,k}/T}{n_z} \right) \right|$$

A close to zero value of this objective function will reveal a dynamical consistent estimation of the observations by the Kalman filter using the chosen parameters.

More information see also [Chen et al., 2018], [Chen et al., 2019].

1.3 Optimization

Here we use the Tree-structured Parzen Estimators (TPE) algorithm for optimization. TPE is a kind of Sequential Model-Based Optimization (SMBO) algorithm (Algorithm 1) which iterates between fitting models and using the calculated objective function value to make choices about which parameters to investigate next. When the searching iteration ends, SMBO will return the parameters which generate the optimal objective function value. The main idea of TPE is similar to Bayesian optimization but the algorithm is different. This approach can find the optimal objective function value with out stuck in the local minimum (or maximum) as well but behaves better than Bayesian optimization with Gaussian process regression.

Algorithm 1 Sequential Model-Based Optimization (SMBO)
$\boldsymbol{R}$ keeps track of all target algorithm runs performed so far and their performances (objective function value), $\mathcal{M}$ is SMBO's model, $\vec{\boldsymbol{\Theta}}_{new}$ is a list of promising configurations.
(Modified from [Hutter et al., 2011])

Input: Target algorithm A with parameter configuration space $\boldsymbol{\Theta}$; instance set Π; objective function J
Output: Optimized (incumbent) parameter configuration, $\boldsymbol{\theta}_{inc}$

1: $[\boldsymbol{R}, \boldsymbol{\theta}_{inc}] \leftarrow Initialize(\boldsymbol{\Theta}, \Pi)$
2: **repeat**
3: $\quad \mathcal{M} \leftarrow FitModel(R)$
4: $\quad \vec{\boldsymbol{\Theta}}_{new} \leftarrow SelectConfigurations(\mathcal{M}, \boldsymbol{\theta}_{inc}, \boldsymbol{\Theta})$
5: $\quad [\boldsymbol{R}, \boldsymbol{\theta}_{inc}] \leftarrow Intensify(\vec{\boldsymbol{\Theta}}_{new}, \boldsymbol{\theta}_{inc}, \mathcal{M}, \boldsymbol{R}, \Pi, J)$
6: **until** *termination criteria met*
7: ***return*** $\boldsymbol{\theta}_{inc}$

More information about SMBO: [Hutter et al., 2011], and TPE: [Bergstra et al., 2011].

2 Example: Finding the optimal Kalman filter parameters for fruit fly *Drosophila melanogaster* trajectory smoothing

Suppose that we need to find the optimal Kalman filter parameters for smoothing the recorded *Drosophila* trajectory in an arena. We already have several recorded trajectories (observations) which are consists of the x and y position (in pixel unit) of the fly in the arena. The basic idea is that we can first find the optimal filter parameters based on some recorded trajectories through the auto-tuning procedure described above, and then test the filter performance on a test trajectory. Then the optimal parameters found can be used in the smoothing of other *Drosophila* trajectories.

2.1 Build the Kalman filter

We consider a 4-dimensional state

References

James Bergstra, Rémi Bardenet, Yoshua Bengio, and Balázs Kégl. Algorithms for hyper-parameter optimization. *Advances in neural information processing systems*, 24, 2011.

Zhaozhong Chen, Christoffer Heckman, Simon Julier, and Nisar Ahmed. Weak in the nees?: Auto-tuning kalman filters with bayesian optimization. In *2018 21st International Conference on Information Fusion (FUSION)*, pages 1072–1079. IEEE, 2018.

Zhaozhong Chen, Nisar Ahmed, Simon Julier, and Christoffer Heckman. Kalman filter tuning with bayesian optimization. *arXiv preprint arXiv:1912.08601*, 2019.

Ramsey Faragher. Understanding the basis of the kalman filter via a simple and intuitive derivation [lecture notes]. *IEEE Signal processing magazine*, 29(5):128–132, 2012.

Frank Hutter, Holger H Hoos, and Kevin Leyton-Brown. Sequential model-based optimization for general algorithm configuration. In *International conference on learning and intelligent optimization*, pages 507–523. Springer, 2011.