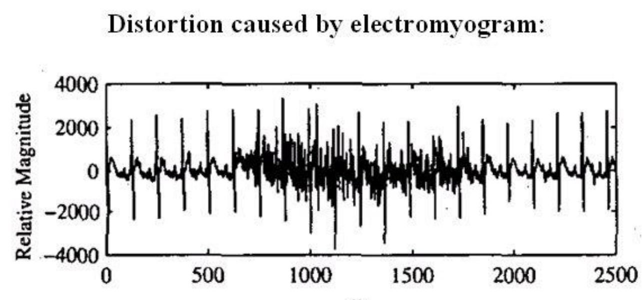
Signal segmentation and waveform characterization

Biosignal processing I, 521273S

Autumn 2019

Short-time analysis of signals

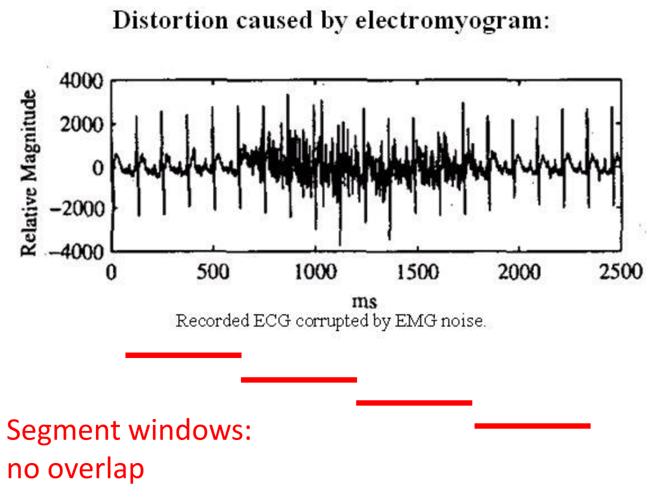
- Signal statistics may vary in time: nonstationary
  - how to compute signal characterizations?
- Signal must be partitioned into homogeneous segments
  - Operation is called segmentation
  - Statistics remain approximately the same in each segment: quasi-stationary segments
  - Characteristics is then computed for each segment separately
- Fixed and adaptive segmentation

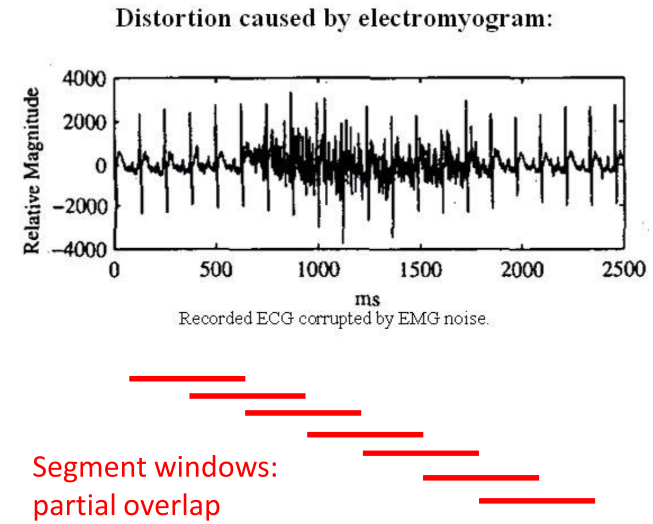


Recorded ECG corrupted by EMG noise.

Fixed segmentation

- Partition the signal into fixed length segments of length M samples
  - M must be selected manually (through trial and error, or via domain knowledge)
- For example: Short-Time Fourier Transform (STFT)
  - STFT is computed for each segment (FFT, windowing, zero-padding)
  - Segment overlap can be used: e.g. 50%





STFT example: phonocardiogram

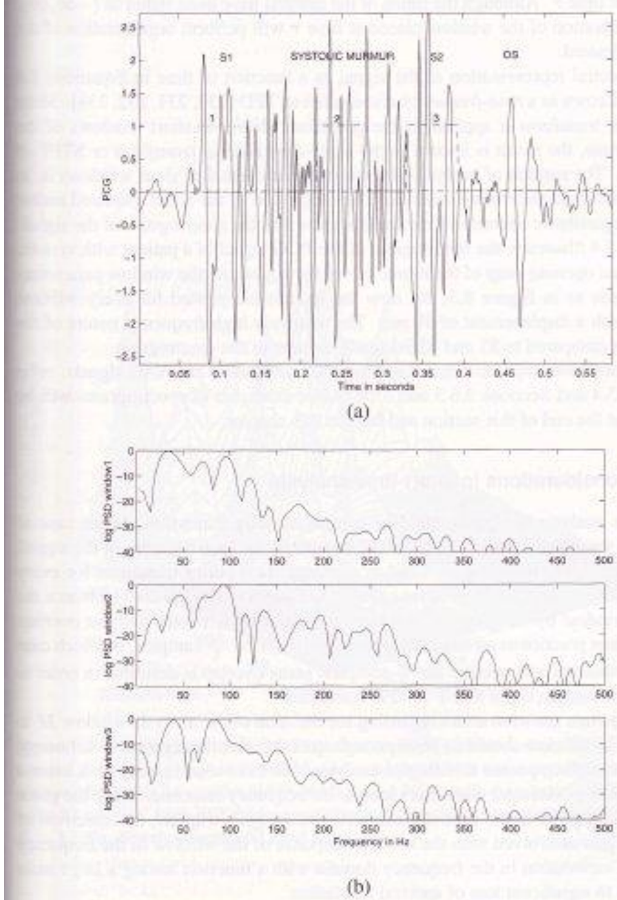


Figure 8.3 (a) PCG signal of a patient (female, 14 months) with systolic murmur and opening snap (OS) of the mitral valve. Three short-time analysis windows are superimposed, each one being a rectangular window of duration 64 *ms*. (b) Log PSDs of the three windowed signal segments. Each FFT was computed with zero-padding to a total length 256 samples. $f_s = 1\ kHz$. See also Figure 6.12.

$P_{dB}(f) = 10 log_{10}(P(f)/P_{max})$

Logarithmic PSD (power spectral density) in three consecutive windows, normalized power axis

- 64 sample windows with 32 sample overlap
- 256 sample FFT, zero-padding

STFT example: phonocardiogram

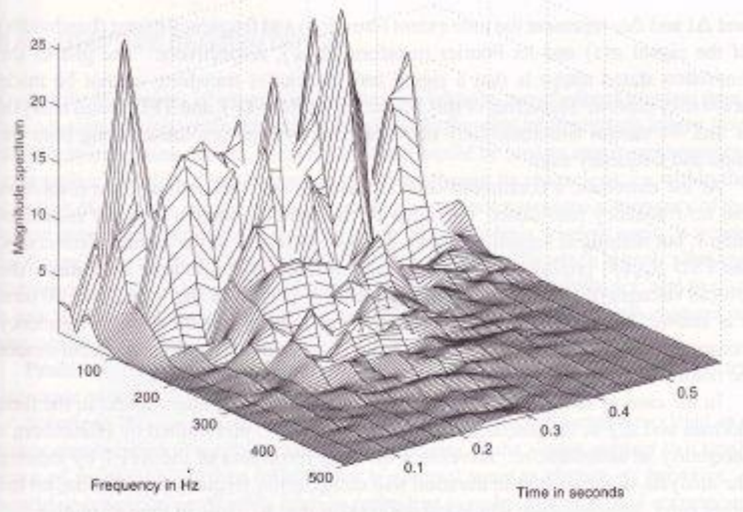


Figure 8.4 Spectrogram of the PCG signal of a patient (female, 14 months) with systolic murmur and opening snap of the mitral valve, computed with a moving short-time analysis window of duration 64 samples (64 ms with $f_s = 1\ kHz$), with the window advance interval being 32 samples. Each FFT was computed with zero-padding to a total length 256 samples. $f_s = 1\ kHz$. See also Figures 6.12 and 8.3.

An alternative representation
of the spectrogram:
interpolated magnitude spectrum
as a landscape plot

- 64 sample windows with
  32 sample overlap
- 256 sample FFT, zero-padding

STFT example: speech signal

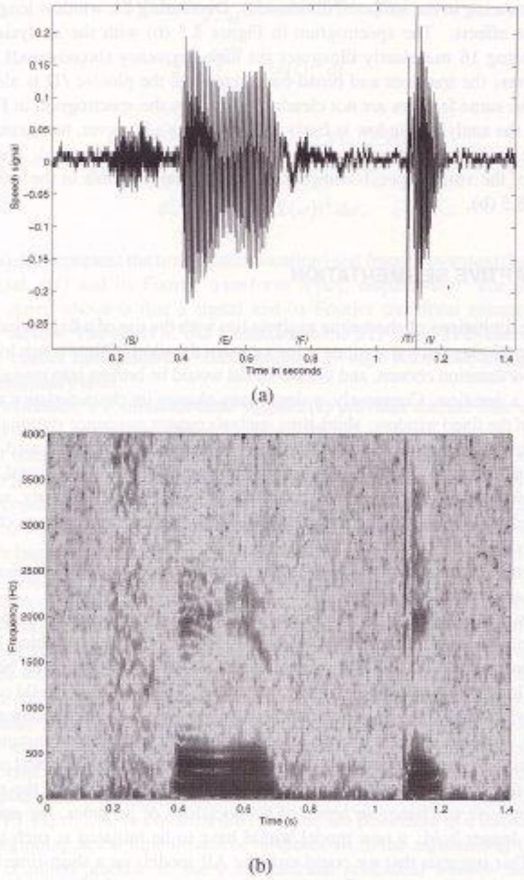


Figure 8.5 (a) Time-domain speech signal of the word "safety" uttered by a male speaker. (The signal is also illustrated in Figures 1.29, 3.1, and 3.3.) (b) Spectrogram (log PSD) of the signal computed with a moving short-time analysis window of duration 16 ms (128 samples with $f_s = 8\ kHz$), with the window advance interval being 8 ms.

An alternative representation of PSD in a sliding window: spectrogram - gray-level image of power spectrums

- 128 sample windows with
  64 sample overlap
- 128 sample FFT

Adaptive segmentation

- Partitioning of a nonstationary signal into quasi-stationary segments of variable duration
- Typically, a small window is anchored in the beginning of a segment, and another window slides forward
  - Difference in the window contents is measured from their spectrums
  - If large enough difference is found, the ending boundary of the segment has been found
  - A new anchoring point is set where the segment ended, and the process starts over again

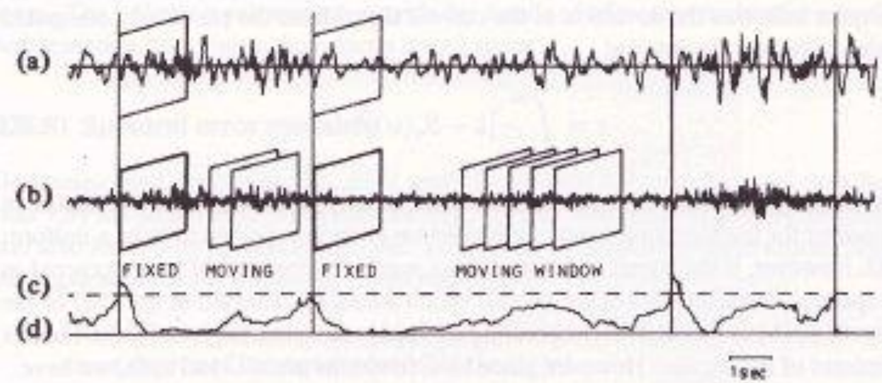
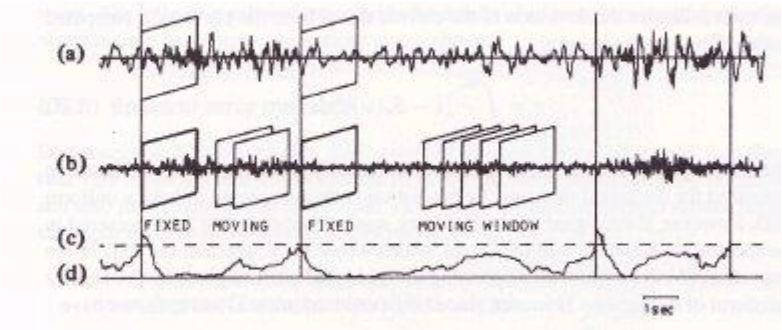


Figure 8.7 Adaptive segmentation of EEG signals via use of SEM. (a) Original EEG signal. The rectangular window at the beginning of each adaptive segment indicates the signal window to which the AR model has been optimized. (b) Prediction error. The initial ACF of the error is computed over the fixed window; the running ACF of the error is computed over the moving window. (c) Segmentation threshold. (d) SEM. The vertical lines represent the segmentation boundaries. Reproduced with permission from G. Bodenstein and H.M. Praetorius, Feature extraction from the electroencephalogram by adaptive segmentation, *Proceedings of the IEEE*, 65(5):642–652, 1977. ©IEEE.

Spectral Error Measure method (SEM)

- SEM uses autoregressive modeling (AR) of the signal spectrum
  - All-pole signal modeling in the stochastic process theory: useful when signal contains oscillating components in one or more frequencies



- AR(p) model is computed for the fixed window (at time index value n=0): the anchored window
  - Linear prediction of signal samples from p previous samples

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + e_t$$

- The AR(p) model based linear prediction of signal x_t is applied in the sliding window (n) to generate prediction error signal e_t for the sliding window of size N

$$e_t = x_t - \sum_{k=1}^{p} a_k(t) x_{t-k}$$

- Autocorrelation function $\phi_e(n,m)$ of e_t is computed for the sliding window (at time index n)

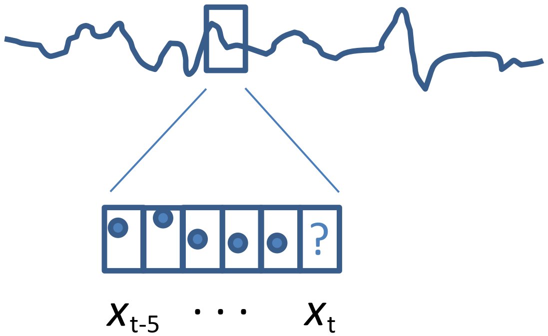
$$\phi_e(n,m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} e_t(n) e_t(n+m)$$

- SEM is computed for sliding window (n)
  - First term compares error signal powers in the windows 0 and n
  - Second term considers prediction error whiteness in window n
  - 1 < M ≤ N, to be specified by application design

$$SEM(n) = \left[\frac{\phi_e(0,0)}{\phi_e(n,0)} - 1\right]^2 + 2\sum_{k=1}^{M}\left[\frac{\phi_e(n,k)}{\phi_e(n,0)}\right]^2$$

SEM cont'd: explanations

- **Linear prediction** with AR(p) model
  - Establish an AR(p) model for the signal *x* -> model parameters p and a_k, k=1,…,p
  - Predict sample x_t by using p previous samples
  - Prediction is not perfect which results in a prediction error e_t



$$x_t = \sum_{k=1}^{5} a_k(t) x_{t-k} + e_t$$

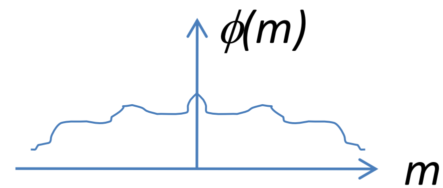
SEM cont'd: explanations

- **Autocorrelation function**:
  the correlation of a signal with itself at different points in time
- Informally, it is the similarity between observations as a function of the time lag *m* between them
- It is a mathematical tool for finding repeating patterns, such as the presence of a periodic signal obscured by noise
- Signal power spectrum can be computed by Fourier transforming the autocorrelation function

Signal segment and it's copy:

Time lag *m*

$$\phi(m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} x(n)x(n+m)$$



$$S(\omega) = \sum_{m=-(N-1)}^{N-1} \phi(m) e^{-j\omega m}$$

SEM cont'd: explanations

Autocorrelation function in EEG:

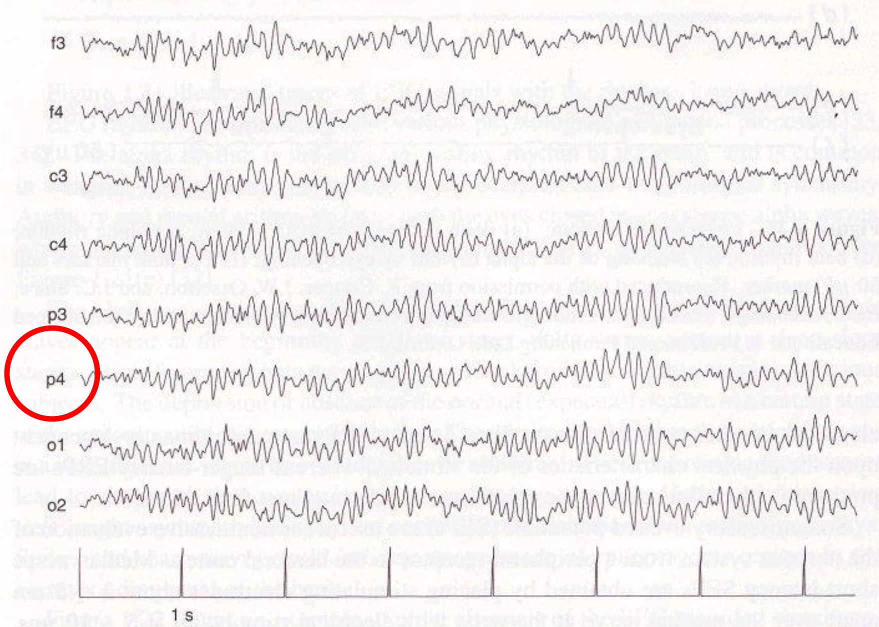


Figure 1.22 Eight channels of the EEG of a subject displaying alpha rhythm. See Figure 1.20 for details regarding channel labels. Data courtesy of Y. Mizuno-Matsumoto, Osaka University Medical School, Osaka, Japan.

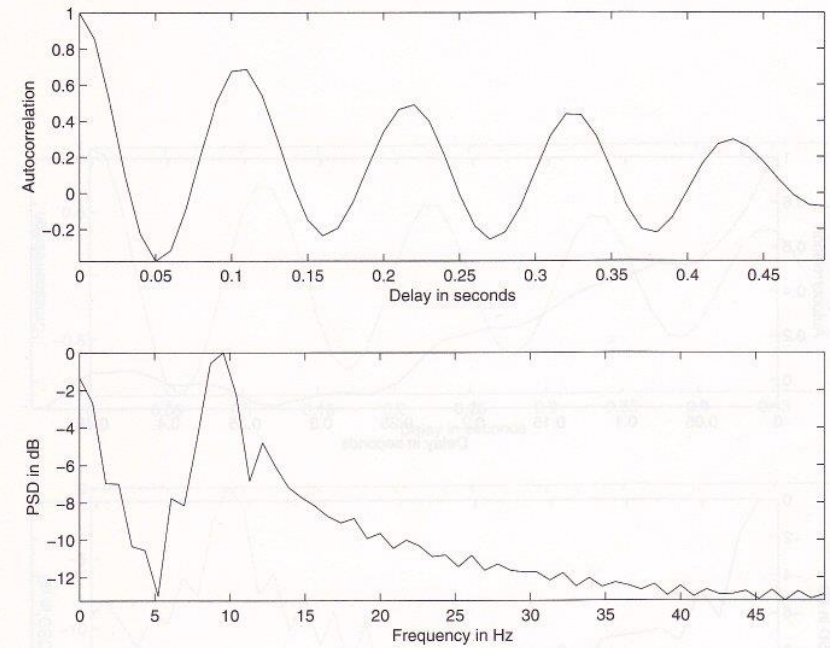


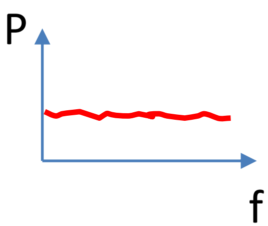
Figure 4.8 Upper trace: ACF of the $4.67 - 5.81\ s$ portion of the p4 channel of the EEG signal shown in Figure 1.22. Lower trace: The PSD of the signal segment in dB, given by the Fourier transform of the ACF.

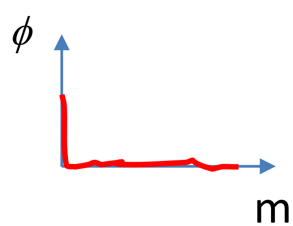
SEM cont'd: explanations

$$x_t = \sum_{k=1}^{5} a_k(t) x_{t-k} + e_t$$

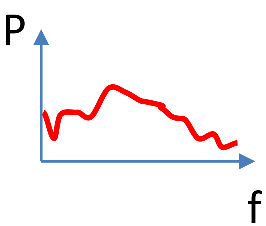
- If the signal follows the assumed model, the prediction error e_t is white noise
  - The model thus explains all structure in the signal
  - Flat power spectrum
  - Autocorrelation function: Dirac delta function

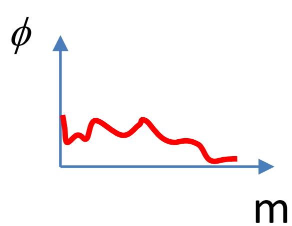
  $$\phi_e(n,m) \approx 0, m \geq 1$$





- If the signal does not follow the assumed model, the error is not white noise
  - Spectrum is not flat
  - Autocorrelation function is more complex containing many non-zero values



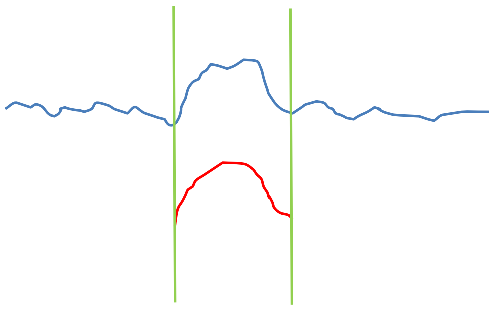


Waveform analysis

Correlation coefficient

- A detected waveform is compared to a model waveform
- Cross-correlation is computed and normalized between [-1, 1]
- An absolute value close to 1 indicates a close match
  - In practice, a value above 0.8 can be considered good (depending on the application)
- (Compare with the matched filter!)

$$\gamma_{xy} = \frac{\sum_{n=0}^{N-1} x(n)y(n)}{\left[\sum_{n=0}^{N-1} x^2(n) \sum_{n=0}^{N-1} y^2(n)\right]^{1/2}}$$



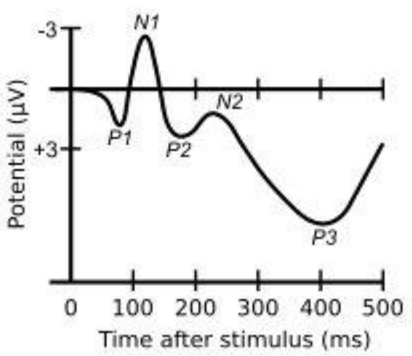
Envelope extraction

- Estimation of trends in signal activity or energy

- A simple method:
  - Full-wave rectify the signal (absolute value)
  - Smooth the result (moving average, Butterworth low-pass filter,...)

$$y(t) = |x(t)|$$

- For pulse waveform extraction synchronized (time-aligned) averaging may be needed if strong noise is present
  - For example, Event-Related Potential (ERP) in EEG
  - Measurement is repeated hundreds of times, pulses are averaged after synchronization



Envelogram

- Envelogram estimate: magnitude of the analytic signal *y*(t) formed using *x*(t) and its Hilbert transform x_H(t)

$$y(t) = x(t) + jx_H(t)$$

$$x_H(t) = \int_{-\infty}^{\infty} \frac{x(t)}{\pi(t-\tau)} d\tau$$

- Practical algorithm:
  1. Compute DFT of x(t)
  2. Set the negative-frequency terms to zero
  3. Multiply positive-frequency terms by 2
  4. Compute inverse DFT of the result
  5. The magnitude of the result gives the envelogram estimate

(DFT = Discrete Fourier Transform)

Envelope examples (phonocardiogram)

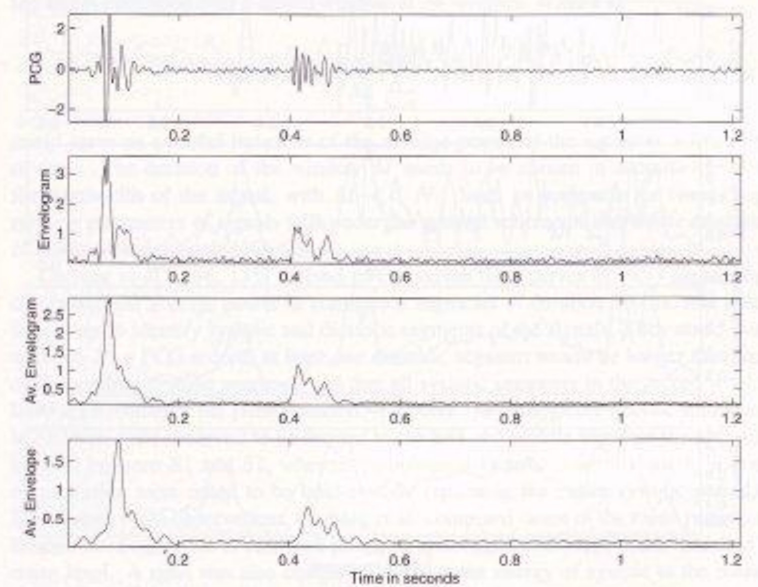


Figure 5.6 Top to bottom: PCG signal of a normal subject (male, 23 years); envelogram estimate of the signal shown; averaged envelogram over 16 cardiac cycles; averaged envelope over 16 cardiac cycles. The PCG signal starts with S1. See Figure 4.27 for an illustration of segmentation of the same signal.

- Signal
- Envelogram of the signal
- Synchronously averaged envelogram (N=16)
- Synchronously averaged envelope (N=16)

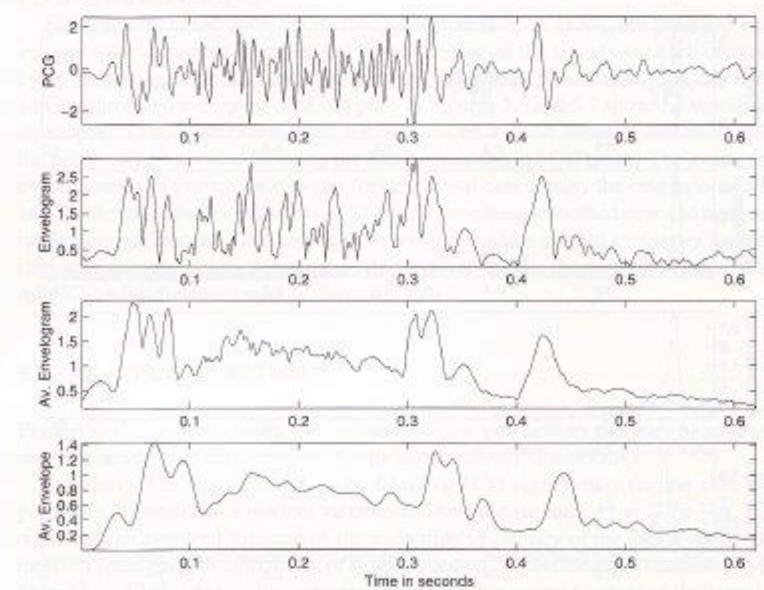


Figure 5.7 Top to bottom: PCG signal of a patient (female, 14 months) with systolic murmur (approximately $0.1 - 0.3\ s$), split S2 ($0.3 - 0.4\ s$), and opening snap of the mitral valve ($0.4 - 0.43\ s$); envelogram estimate of the signal shown; averaged envelogram over 26 cardiac cycles; averaged envelope over 26 cardiac cycles. The PCG signal starts with S1. See Figure 4.28 for an illustration of segmentation of the same signal.

- Signal
- Envelogram of the signal
- Synchronously averaged envelogram (N=26)
- Synchronously averaged envelope (N=26)

Other descriptors of activity level

- Root-mean-square (RMS) value of the signal or signal window
- Zero-crossing rate (ZCR): number of times the signal crosses zero-value per time unit
- Turns count: number of times that the signal amplitude changes direction

$$RMS = \left[\frac{1}{N} \sum_{n=0}^{N-1} x^2(n) \right]^{1/2}$$

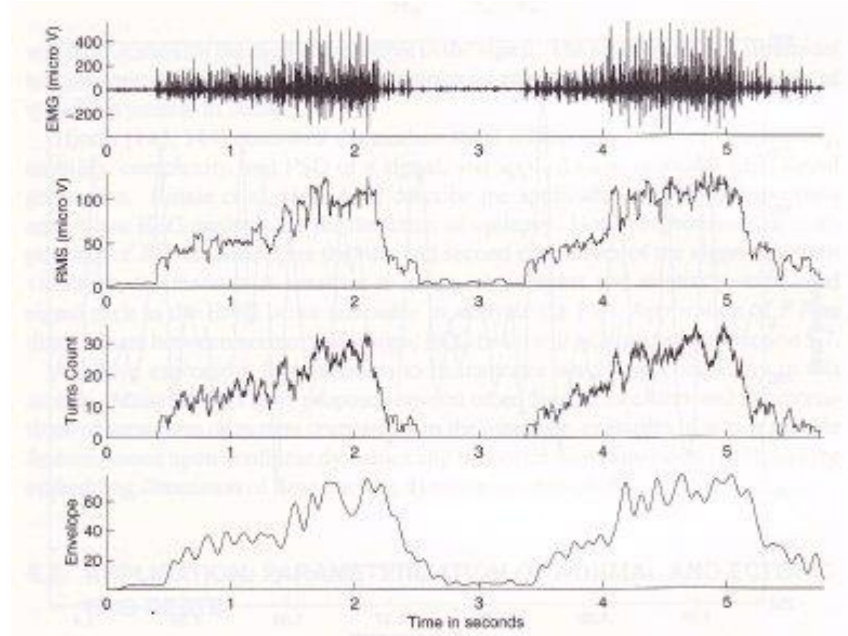


Figure 5.8 Top to bottom: EMG signal over two breath cycles from the crural diaphragm of a dog recorded via implanted fine-wire electrodes; short-time RMS values; turns count using Willison's procedure; and smoothed envelope of the signal. The RMS and turns count values were computed using a causal moving window of 70 *ms* duration. EMG signal courtesy of R.S. Platt and P.A. Easton, Department of Clinical Neurosciences, University of Calgary.