Spectral analysis of biosignals

Biosignal processing I, 521273S

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Introduction

- Oscillations of biosignals often have intuitive physiological interpretation (breathing rate, heart rate, alpha rhythm,…)
- Frequencies of interest are low, typically: fMRI <3Hz, EEG <30 Hz, EMG <200Hz, ECG <300Hz
- Careful signal preprocessing needed due to artifacts and noise (movement, blinks, muscle activity,…)
- Wide-sense stationarity assumption (WSS)
  - First- and second order statics remain the same over the signal: the mean and the autocorrelation function (and spectrum)
- Spectral analysis is challenged by non-stationarity
  - Changing conditions due to body movements, reactions of body to various external/internal impulses, medication, injury etc.

Non-parametric methods for power-spectral density (PSD) estimation

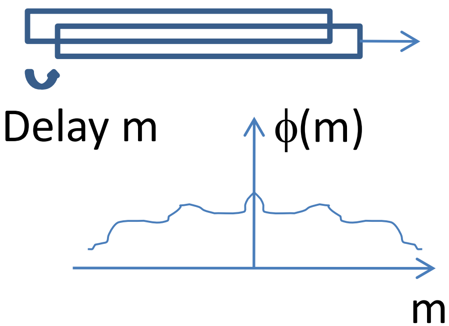
- no assumption is made of the signal generating process (such as being a stochastic process: AR, MA, ARMA)
- Discrete Fourier Transform, DFT, is the most common method
  - Can be computed effectively with Fast Fourier Transform, FFT
- often, signal is divided into segments, spectrum is estimated for each, and finally averaging of the segment-wise spectrums

Periodogram

- Periodogram estimate of PSD can be achieved through Fourier transforming the autocorrelation function of the signal

$$S(\omega) = \sum_{m=-(N-1)}^{N-1} \phi(m) e^{-j\omega m}$$

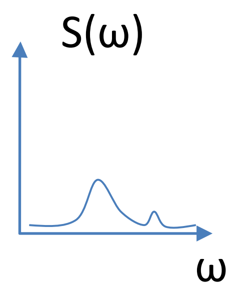
$$\phi(m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} x(n)x(n+m)$$



- PSD can also be computed from DFT of the signal:

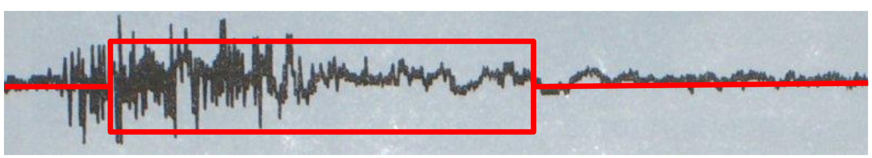
$$S(\omega) = \frac{1}{N} \left| X(\omega) \right|^2$$

$$X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$



Signal windowing

- Multiplication of signal samples by window function samples
- Often recommended for power spectrum estimation
- Several alternatives; small differences in practice
- Hamming and Hanning often used



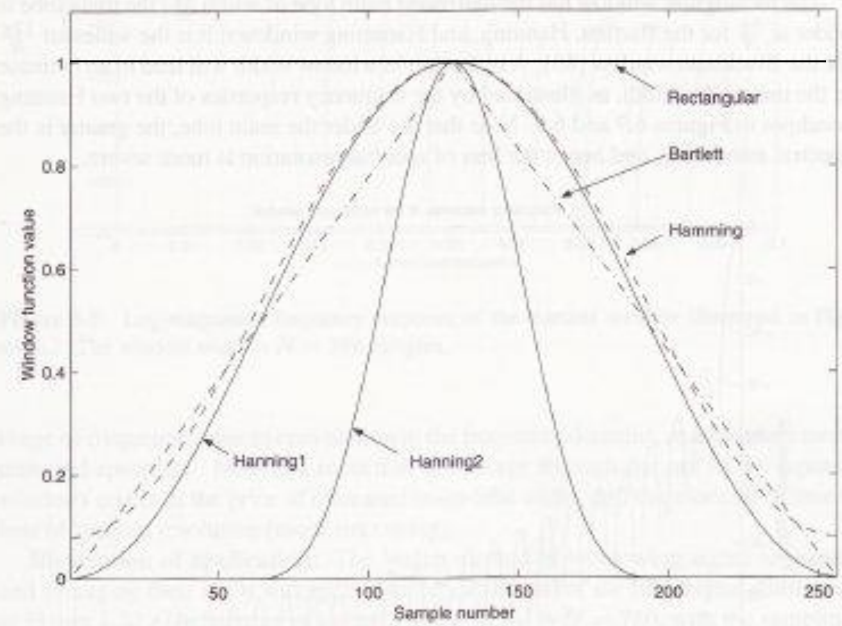
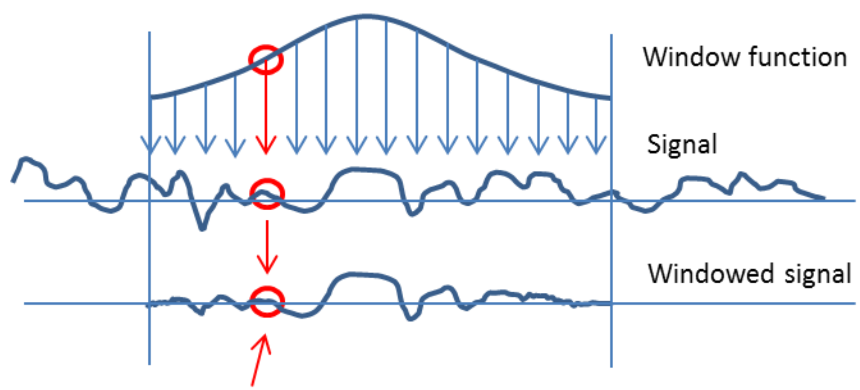


Figure 6.3 Commonly used window functions: rectangular, Bartlett, Hamming, and Hanning windows with $N = 256$ (Hanning1), and Hanning window with $N = 128$ samples (Hanning2). All windows are centered at the 128th sample.

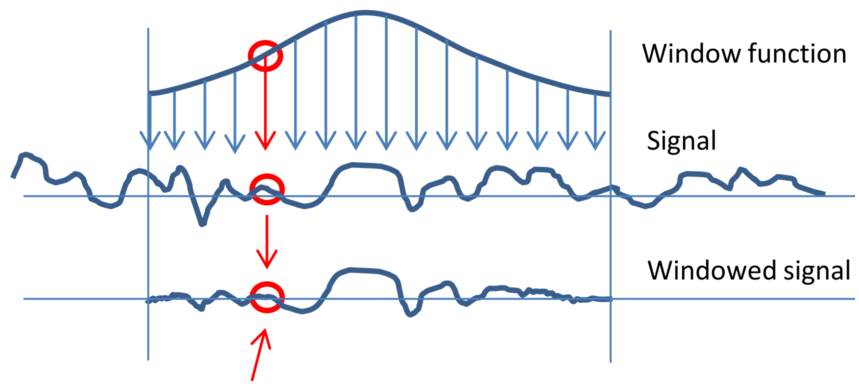


$$x_i(4)w(4)$$

Windowed power spectrum

- Signal *x* is multiplied sample-wise by the window function *w*
- Then, DFT is taken and squared

- Windowed periodogram



$x_i(4)w(4)$

$$S(\omega) = \frac{1}{ME_w}\left|\sum_{n=0}^{M-1} x_i(n)w(n)e^{-j\omega n}\right|^2$$

$$E_w = \frac{1}{M}\sum_{n=0}^{M-1} w^2(n)$$

Effect of windowing to PSD

- Convolution theorem: Multiplication in time domain corresponds to convolution in frequency domain (and vice versa)
- Spectral leaking effect: due to windowing, power leaks from actual frequencies to neighboring frequencies
  - Also smoothes the spectrum
  - Complicates interpretation of the spectrum shape

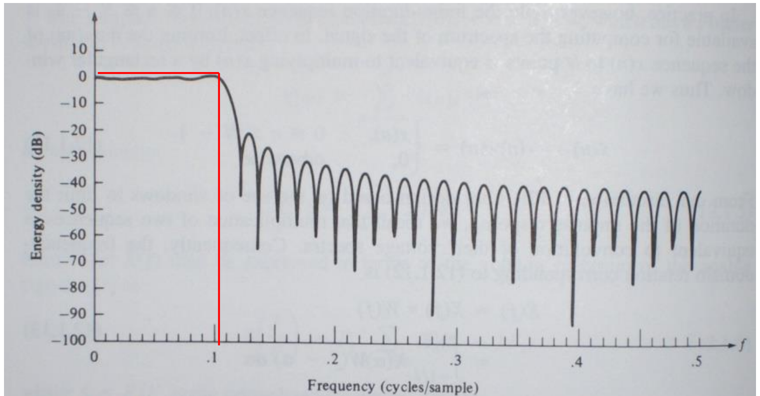


FIGURE 12.1 Spectrum obtained by convolving an $M = 61$ rectangular window with the ideal lowpass spectrum in Example 12.1.1.

Leaking with the rectangular window

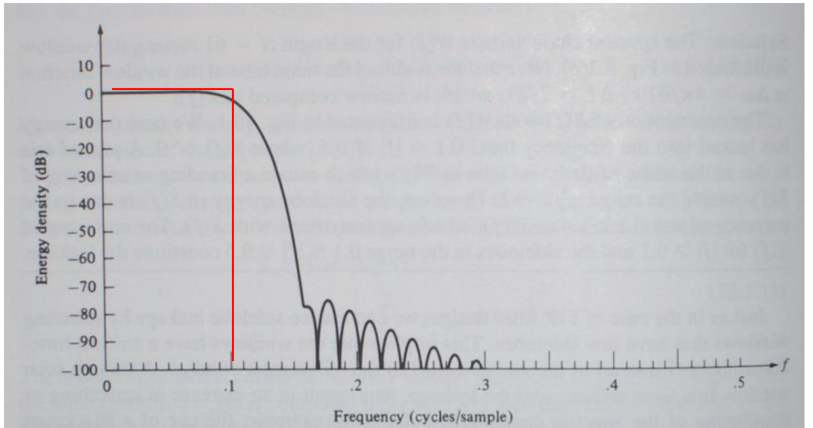
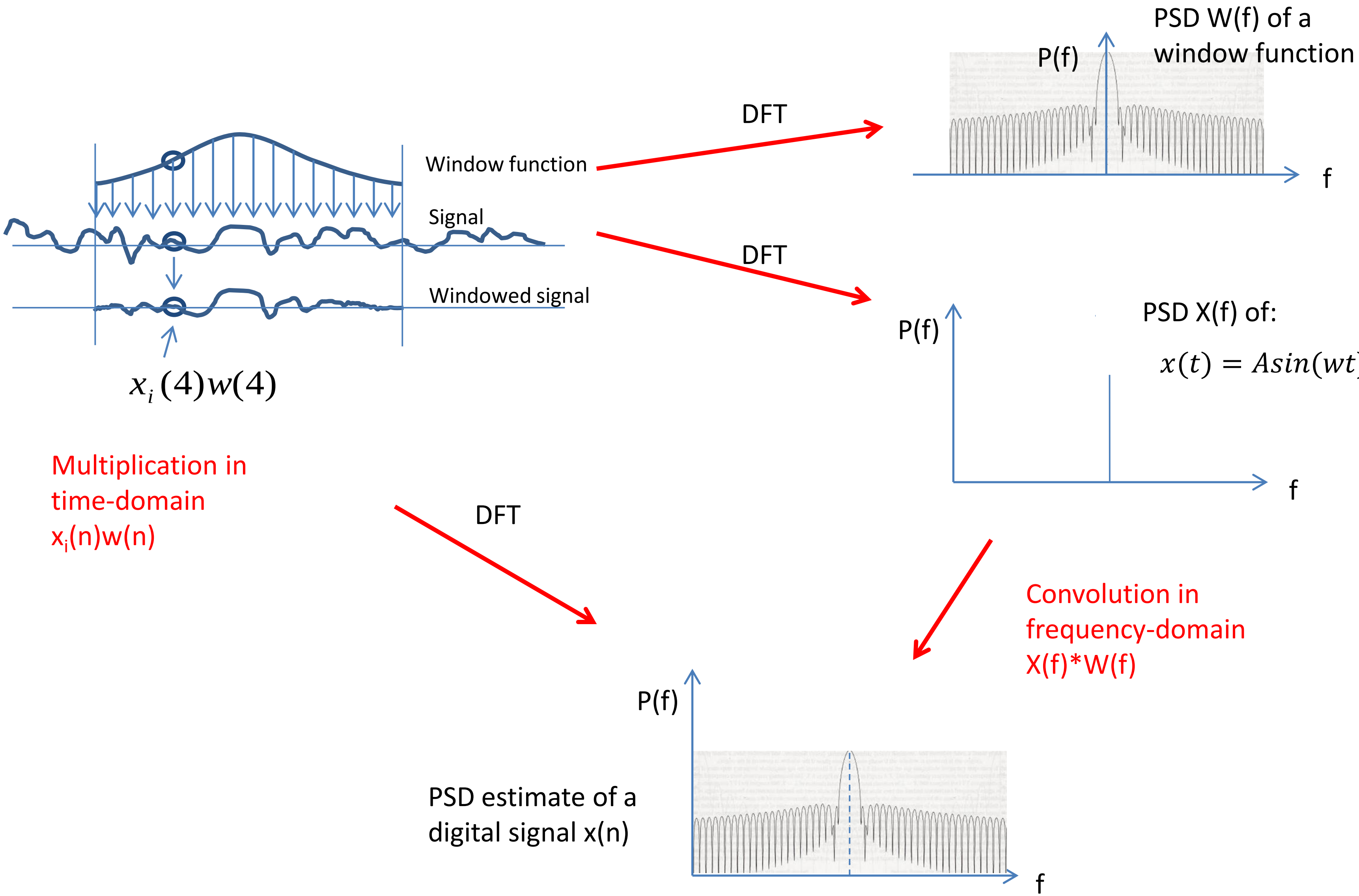


FIGURE 12.2 Spectrum obtained by convolving an $M = 61$ Blackman window with the ideal lowpass spectrum in Example 12.1.1.

Leaking with the Blackman window

Explanation of spectral leakage in PSD



Spectrum of window function

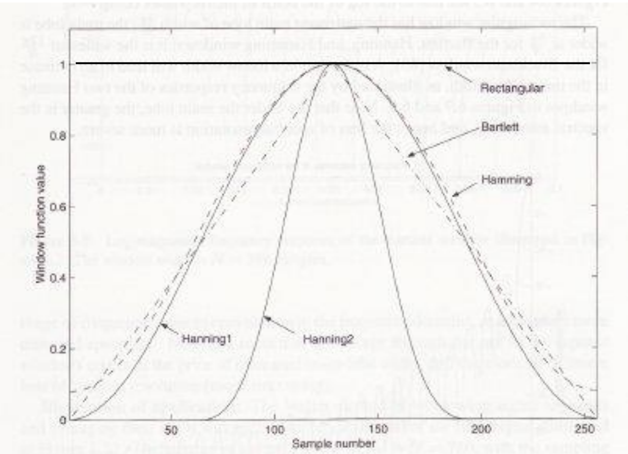


Figure 6.3 Commonly used window functions: rectangular, Bartlett, Hamming, and Hanning windows with $N = 256$ (Hanning1), and Hanning window with $N = 128$ samples (Hanning2). All windows are centered at the 128th sample.

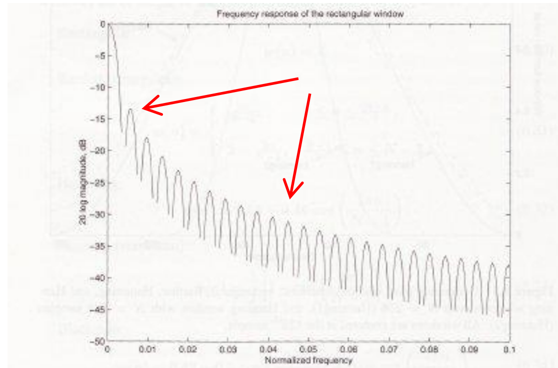


Figure 6.4 Log-magnitude frequency response of the rectangular window illustrated in Figure 6.3. The window width is $N = 256$ samples.

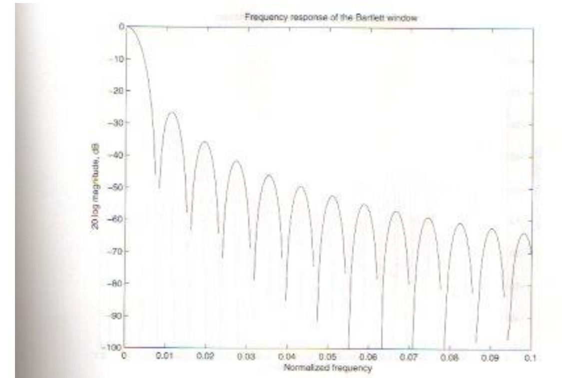


Figure 6.5 Log-magnitude frequency response of the Bartlett window illustrated in Figure 6.3. The window width is $N = 256$ samples.

rectangular $W_R(\omega) = \dfrac{\sin[\omega(2N-1)/2]}{\sin(\omega/2)}$

Bartlet $W_B(\omega) = \dfrac{1}{N}\left[\dfrac{\sin(\omega N/2)}{\sin(\omega/2)}\right]^2$

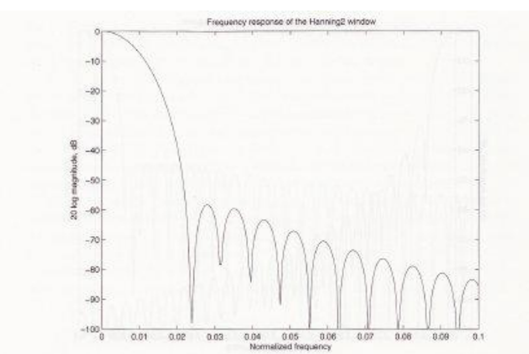


Figure 6.8 Log-magnitude frequency response of the Hanning2 window illustrated in Figure 6.3. The window width is $N = 128$ samples.

Hanning 2

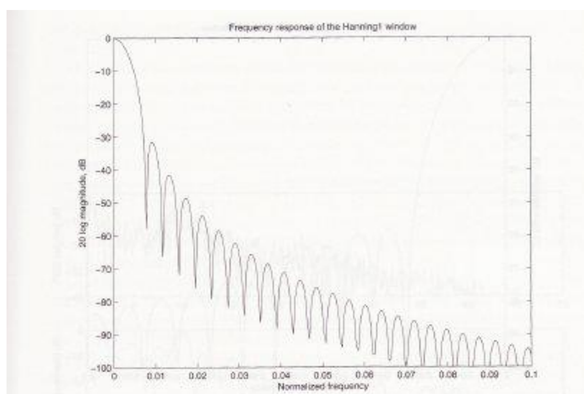


Figure 6.7 Log-magnitude frequency response of the Hanning1 window illustrated in Figure 6.3. The window width is $N = 256$ samples.

Hanning 1

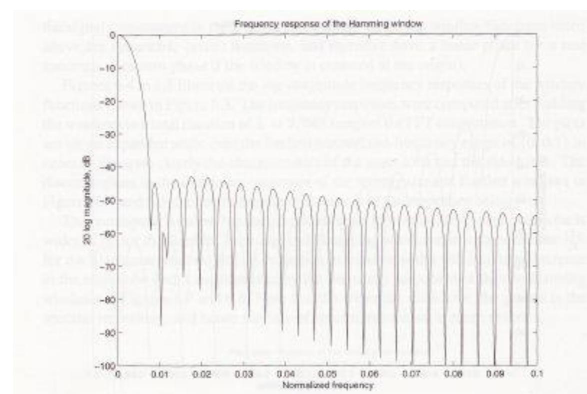
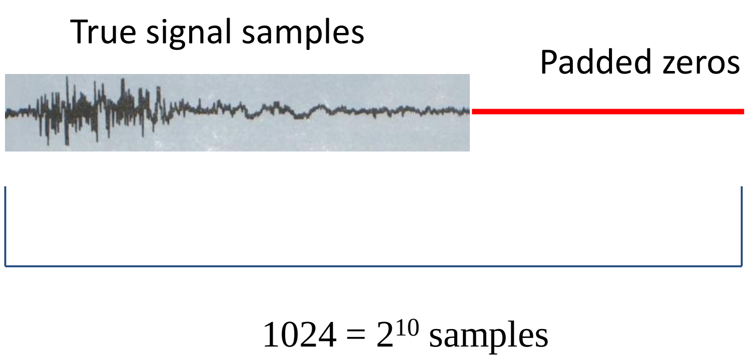


Figure 6.6 Log-magnitude frequency response of the Hamming window illustrated in Figure 6.3. The window width is $N = 256$ samples.

Hamming

Effect of zero padding in spectral resolution

- Zero padding: appending zero samples at the end of signal segment in order to make the length 2^N for FFT
- Only the number of true signal samples determines the spectral resolution
- Zero-padding only introduces interpolation of frequency samples



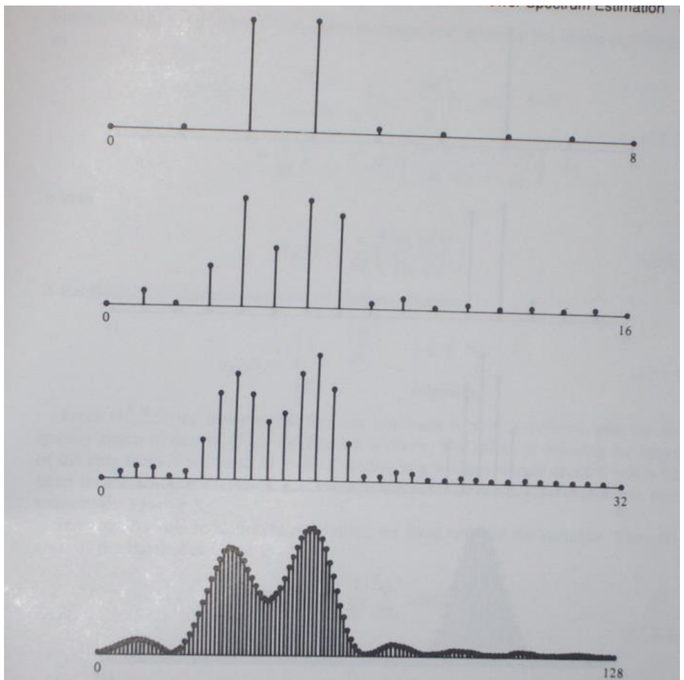


FIGURE 12.4 Spectra of two sinusoids with frequency separation $\Delta f = 0.06$.

Only two spectral peaks are present when different numbers of zeros have been padded at the end of window

Disadvantages of non-parametric methods

- require long signals for good frequency resolution
- spectral leakage due to windowing
  - side-lobes cause energy migration and spectral smoothing
  - may mask weak signal components

- parametric (model-based) methods have none of these disadvantages
  - but assume a parametric model of a signal
  - such as stochastic processes: AR, MA, ARMA

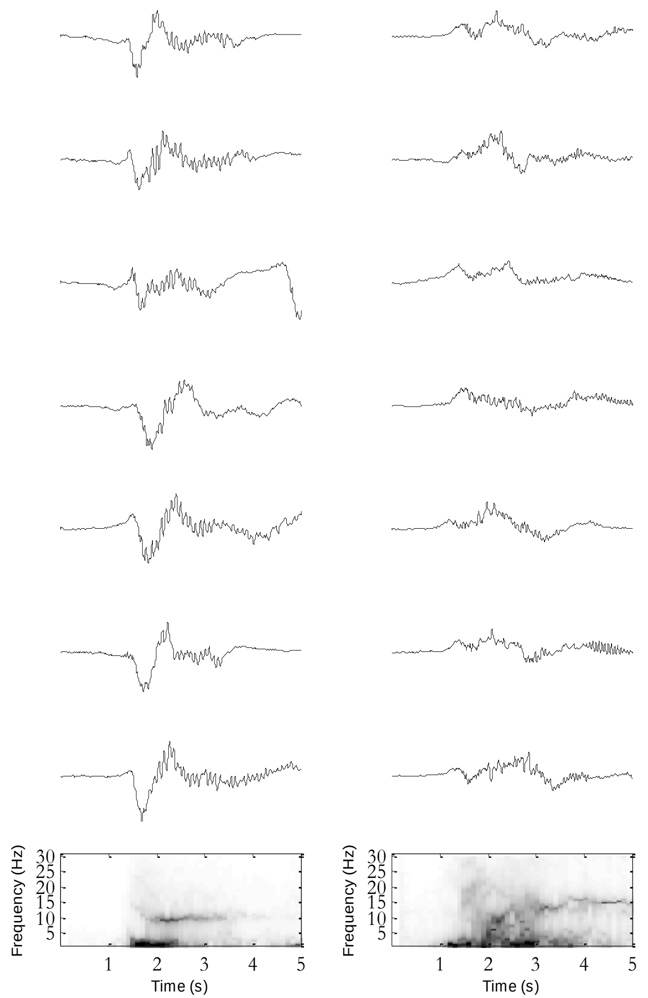
Additional issues

- Spectrum estimation with unevenly sampled signals: e.g., Lomb algorithm
- Non-stationary dynamics: baseline fluctuation
  - Linear / nonlinear detrending
    - baseline estimation and subtraction
    - decide first what is signal and what is noise
- Filter banks, wavelets, empirical mode decomposition, Wigner-Ville distributions and some other time-domain decomposition techniques can be used instead if fast variation of the signal needs to be analyzed

Spectrogram estimation

- Compute windowed periodogram for each (possibly partly overlapping) signal segment
- Display them side-by-side in vertical orientation
- Shows the how the spectrum of the signal changes over time

- Short-Time Fourier Transform (STFT)



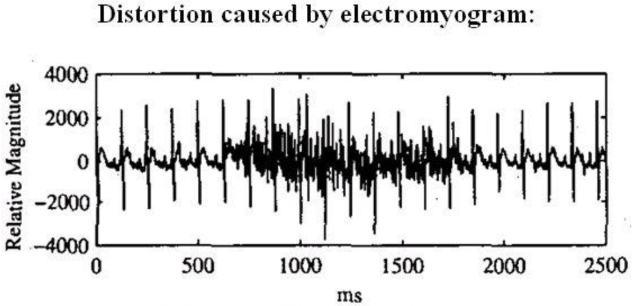
Welch procedure for averaging periodograms

- The signal is segmented into K consecutive (partially overlapping) parts of lenght M samples, spectrum is computed for each, and the average is taken
- Results in a smoothed PSD estimate

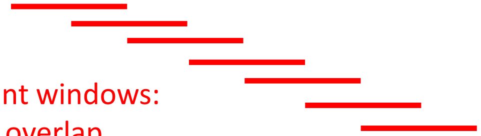
$$S_{W_i}(\omega) = \frac{1}{ME_w} \left| \sum_{n=0}^{M-1} x_i(n) w(n) e^{-j\omega n} \right|^2$$

$$E_w = \frac{1}{M} \sum_{n=0}^{M-1} w^2(n)$$

$$S_W(\omega) = \frac{1}{K} \sum_{i=1}^{K} S_{W_i}(\omega)$$



Recorded ECG corrupted by EMG noise.



Segment windows:
partial overlap

AR Spectrum Estimate

- Autoregressive model (AR) from stochastic process theory
- A parametric model with order *p*, all-poles model
- Suitable for signals with spectrum with sharp peaks and wide valleys
- Model order *p* should be at least the total number of spectral peaks in order to model them all (including peaks in both positive and negative frequencies)
- Often recommended: $N/3 \leq p \leq N/2$, N is signal length in samples
  - However, this depends on the application: prior knowledge of the spectral content is important
  - For example: p=20 is enough for ECG tachogram analysis even for 1-hour signal segments
- It is the most often used stochastic model of signals

$$x_t = \sum_{k=1}^{p} a_k(t) x_{t-k} + e_t$$

$$\hat{S}(\omega) = \frac{E_p}{\left| 1 + \sum_{k=1}^{p} \hat{a}_p(k) e^{-j\omega k} \right|^2}$$

Example: Event related changes in spectrum by time-varying AR

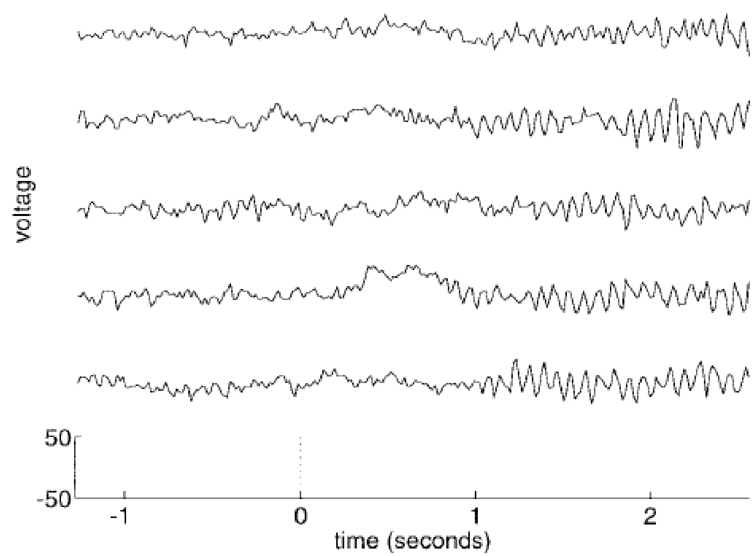
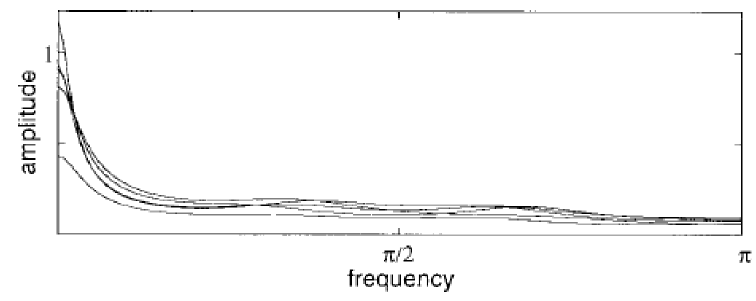
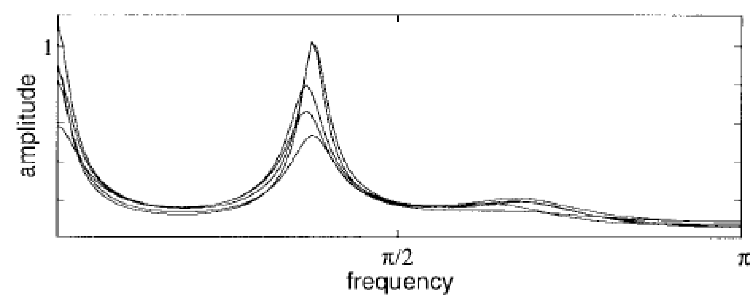


Fig. 1. Five EEG samples measured during event-related synchronization test. Vertical scale in microvolts and time in seconds. The trigger time of the auditory stimulus (vertical dotted line) is set at time $t = 0$.



(a)



(b)

Characteristics of PSD (1/2)

- Mean frequency $\bar{f} = f_s \frac{2}{NE_x} \sum_{k=0}^{N/2} k \left| X(\omega_k) \right|^2$ $E_x = \frac{1}{N} \sum_{k=0}^{N-1} \left| X(\omega_k) \right|^2$
  - f_s: sampling frequency

- Median frequency $f_{med} = \frac{m}{N} f_s$
  with the largest m such that $\frac{2}{NE_x} \sum_{k=0}^{m} \left| X(\omega_k) \right|^2 < \frac{1}{2}$ $0 \le m \le \frac{N}{2}$

- Variance $f_{m2} = f_s \frac{2}{NE_x} \sum_{k=0}^{N/2} \left(k - \bar{k}\right)^2 \left| X(\omega_k) \right|^2$

  where $\bar{k}$ is the frequency sample index corresponding to $\bar{f}$

Characteristics of PSD (2/2)

- Skewness $$f_{m3} = f_s \frac{2}{NE_x} \sum_{k=0}^{N/2} \left(k - \bar{k}\right)^3 \left|X(\omega_k)\right|^2$$

- Kurtosis $$f_{m4} = f_s \frac{2}{NE_x} \sum_{k=0}^{N/2} \left(k - \bar{k}\right)^4 \left|X(\omega_k)\right|^2$$

- Fraction of power in frequency band (f_1 : f_2)
$$E(f_1, f_2) = \frac{2}{NE_x} \sum_{k=k_1}^{k_2} \left|X(\omega_k)\right|^2$$
  - k_1 and k_2 are indexes corresponding to frequencies f_1 and f_2, correspondingly
- Spectral power ratio $$E(f_1, f_2) \big/ E(f_3, f_4)$$