Time Series Analysis on Quarterly U.S. GDP from 1947(1) to 2018(3)

Abstract:

In this project, I focused on the analysis of quarterly U.S.GDP from 1947(1) to 2018(3). The data contains seasonally adjusted real U.S. gross dominant product in billions, posted on trading economics.

The quarterly U.S. GDP can be treated as a time-series and after log transformation, it becomes the quarterly U.S. GDP growth rate and can be treated as a stationary process. By observing the sample ACF and PACF, I fitted several ARIMA models for the data. Then I used several methods to select the most proper model ARIMA (3,1,2). Next, I did the prediction for ten quarters ahead by the model, which was in a reasonable range by looking at the prediction interval. After that, I did some spectral analysis and found that there is no obvious seasonality performance of the data. After all the analysis, I noticed some limitations of my model that may cause potential problems afterwards and needed to do further adjustments for deeper analysis.

Introduction:

The project aims at analyzing the Quarterly U.S. GDP's performance by time and tries to generate a proper model to do the prediction and have deeper sights on the moving trend and seasonal features of GDP in the U.S.

The Quarterly U.S. GDP (Gross domestic product) is a monetary measure of the market value of all the final goods and services produced in the U.S. in a quarter[1]. GDP is often

[1] GDP, Wikipedia, https://en.wikipedia.org/wiki/Gross_domestic_product

considered to be the "world's most powerful statistical indicator of national development and progress". Therefore, by looking at the U.S. GDP from 1947 to 2018, we can have direct and overall idea of how the U.S economics had developed by time.

The dataset I used is seasonally adjusted real quarterly U.S. GDP. Using quarterly data can help us analysis the economic periods within a year. Also, having more observations can make our fitted models more precise. Besides, the dataset is adjusted for price changes and is, therefore, net of inflation[2], which is equivalent to control the variable of inflation and thus can make our analysis more accurate.

Now, the report will start with a general statistical methods introduction with a follow-up results representation, at last will have a further discussion.

Statistical methods:

I used a series of statistical methods regarding to time-series during the process of analyzing: building a stationary process, fitting the ARIMA models, selecting the models, doing further prediction and spectral analysis.

Firstly, apply some transformations to convert the data as a stationary process. Since the economic indicator is usually be considered in growth rate (in percent change) rather than only the actual or adjusted value, I did the log transformation for data detrending. Therefore, my data was assigned by a new meaning: Quarterly U.S growth rate.

Then, I looked at the cut-off and tail-off performance in the sample ACF and PACF of the

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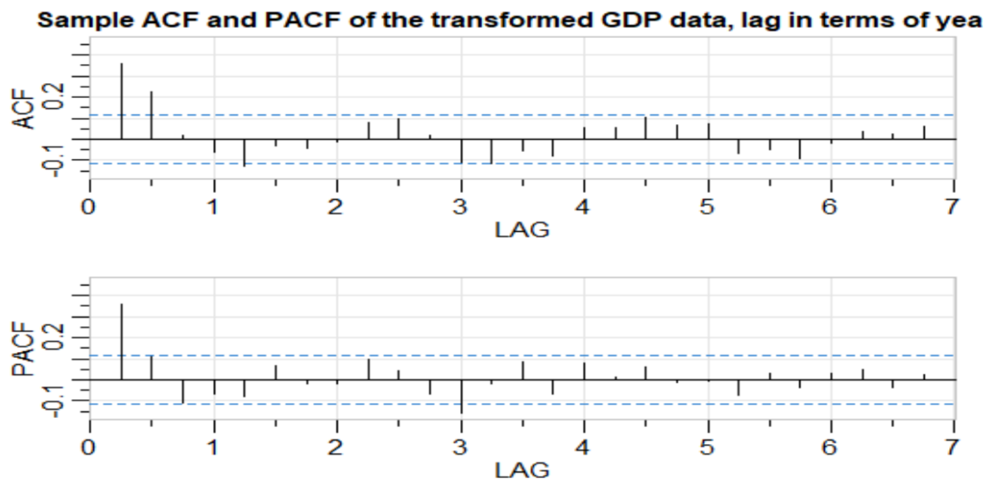
[2] GDP, Investopedia, https://www.investopedia.com/terms/g/gdp.asp

GDP growth rate and fitted the proper model.



From the sample ACF and PACF plot, we may conclude that the ACF is cutting off at 2 and the PACF is tailing off. This would suggest that the GDP growth rate follows an ARIMA (0,1,2) model. Also, we may conclude that the PACF is cutting off at 3 and the ACF is tailing off. This would suggest that we it follows an ARIMA (3,1,0) model. Moreover, we can say that the ACF is cutting off at 2 and the PACF is cutting off at 2, which leads to an ARIMA (3,1,2) model.

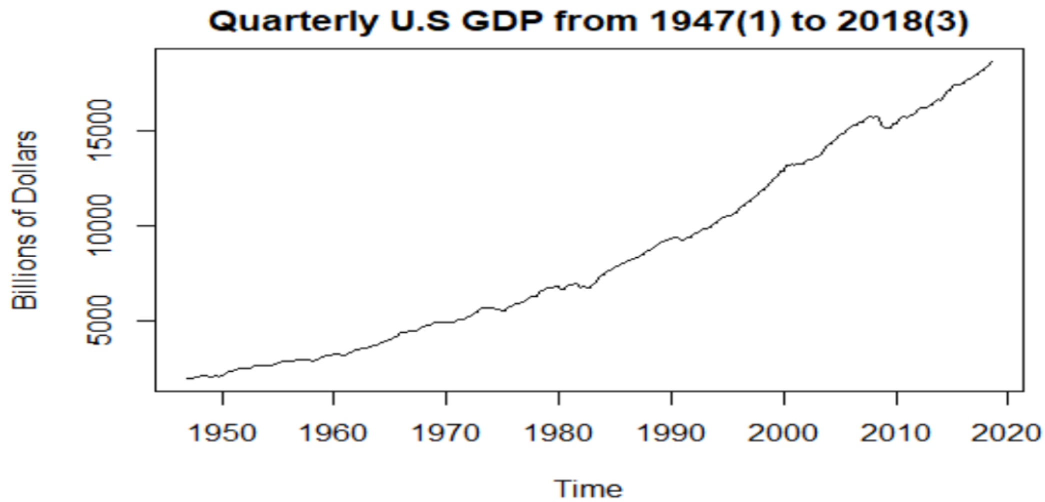
Next, do the model selection among the three models. I used the measurements in three dimensions: testing the significance of the parameters' estimations, analyzing the diagnostics, and comparing the values of AIC, BIC, AICc of the three models. After evaluating the performances in the three dimensions, pick the best model of the three.

After that, I did the forecast of 10 quarter for the data based on the model selected before and used 95% prediction interval to test. I also did some spectral analysis to look at the possible periods and seasonality of the data.

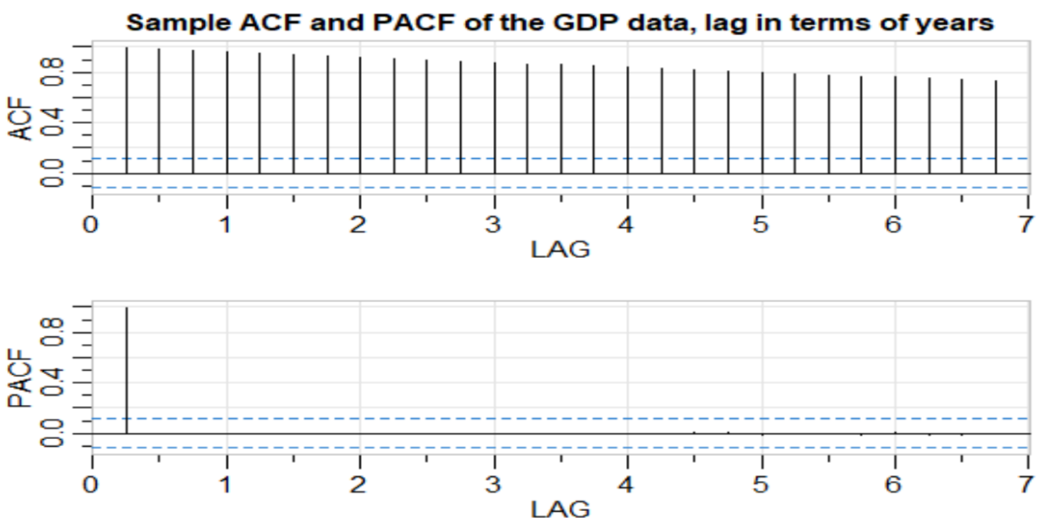
At last, I did an overview of my process and found some limitations of my model.

Results:

In this project, we focus on quarterly U.S.GDP from 1947(1) to 2018(3), n=287 observations. The data contains seasonally adjusted real U.S. gross dominant product.



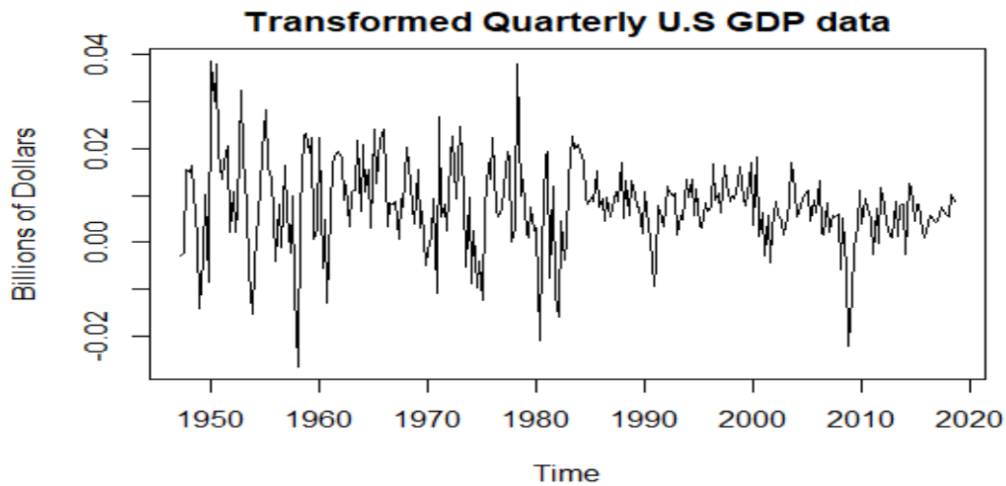
From the plot of the data, we see there is a strong upward trend.



The sample ACF of the data does not decay to zero very fast as h increases.

Therefore, a slow decay in the sample ACF indicates that doing the differencing may be necessary.

Since GDP is an economics value, it is better to consider in growth rate rather than only the actual value. So, I used the log transformation as data detrending.



After transformation, the data represents the U.S. quarterly GDP growth rate. Now from the plot of the GDP growth rate, we can see there is no obvious pattern.

And as been shown in the previous section, the sample ACF is no longer has slow decay. Therefore, we can conclude that the growth rate appears to be a stable process.

From the explanation of the ARIMA model fitting in statistical methods section, there are three models on the list: ARIMA (0,1,2), ARIMA (3,1,0) and ARIMA (3,1,2) for our quarterly GDP growth rate data.

For ARIMA (0,1,2) model and ARIMA (3,1,0) model:

```
$ttable
    Estimate     SE  t.value p.value
ma1  -0.7209 0.0478 -15.0664       0
ma2  -0.2593 0.0472  -5.4913       0

$AIC
[1] -6.572409

$AICc
[1] -6.57226

$BIC
[1] -6.533962
```

```
$ttable
    Estimate     SE t.value p.value
ar1  -0.4564 0.0586 -7.7841  0.0000
ar2  -0.1752 0.0641 -2.7335  0.0067
ar3  -0.1430 0.0587 -2.4350  0.0155

$AIC
[1] -6.436686

$AICc
[1] -6.436387

$BIC
[1] -6.385423
```

$X_t = W_t - 0.7209W_{t-1} - 0.2593W_{t-2}$ is our estimated ARIMA(0,1,2) model.

The estimated value for θ_1 is -0.7209, with p-value of 0 (smaller than 0.05), which is significant. The estimated value for θ_2 is -0.2593, with p-value of 0 (smaller than 0.05), which is significant. The constant term is not siginificant due to the p-value larger than 0.05, however, there's no meaning of the constant term in this situation: the quarterly U.S GDP growth rate does not need a staring point or a required certain level to be stays with. So we set no.constant=TRUE here and ignored the constant term.

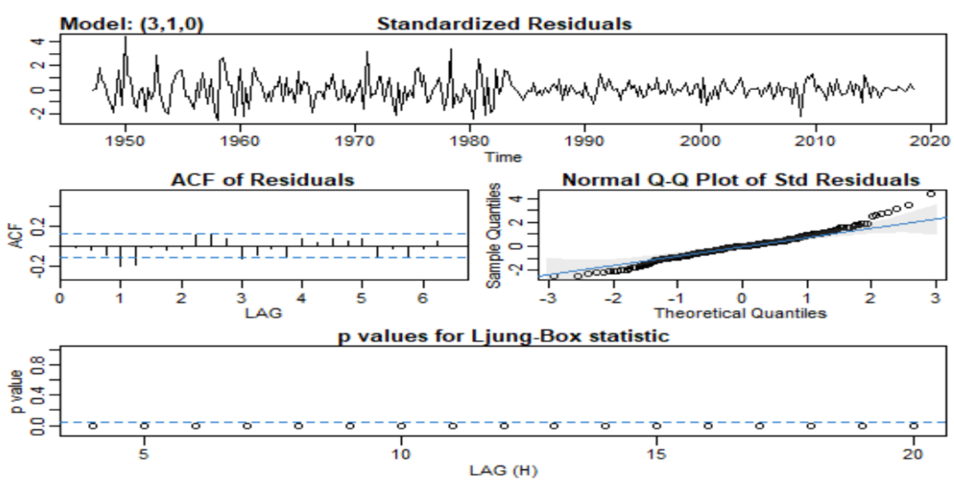
$X_t = -0.4564X_{t-1} - 0.1752X_{t-2} - 0.1430t - 3 + W_t$ is our estimated ARIMA(3,1,0) model. The estimated value for ϕ_1 is -0.4564, with p-value of 0 (smaller than 0.05), which is significant. The estimated value for ϕ_2 is -0.1752, with p-value of 0.0067 (smaller than 0.05), which is significant. The estimated value for ϕ_3 is -0.1430, with p-value of 0.015 (smaller than 0.05), which is significant. Similarly, the constant term is not siginificant due to the p-value larger than 0.05, And since there's no meaning of the constant term in this situation, we do not consider the constant term here.

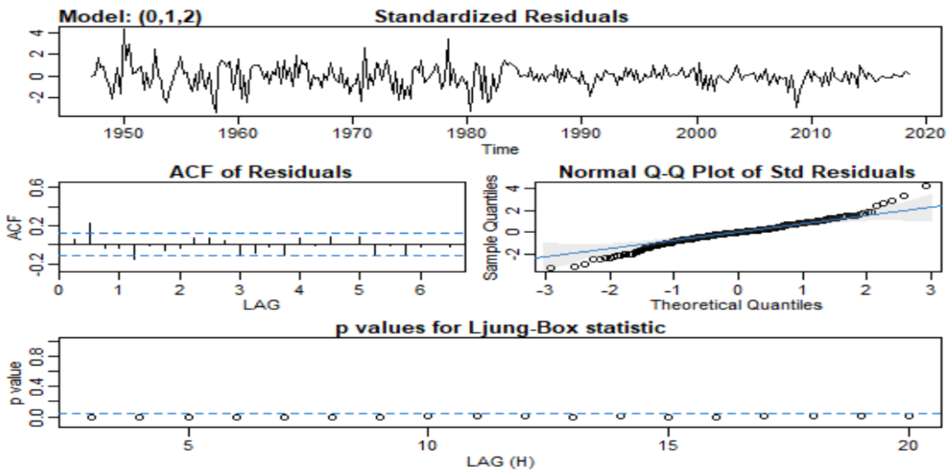
For ARIMA (3,1,2) model:

```
$ttable
    Estimate     SE t.value p.value
ar1   0.8409 0.2469  3.4064  0.0008
ar2  -0.0145 0.1136 -0.1275  0.8986
ar3  -0.1820 0.0591 -3.0793  0.0023
ma1  -1.5099 0.2478 -6.0942  0.0000
ma2   0.5168 0.2455  2.1057  0.0361

$AIC
[1] -6.619691

$AICc
[1] -6.618936

$BIC
[1] -6.542797
```

$X_t = 0.8409X_{t-1} - 0.0145X_{t-2} - 0.1820X_{t-3} + W_t - 1.5099W_{t-1} + 0.5168W_{t-2}$ is our estimated ARIMA(3,1,2) model. The estimated value for ϕ_1 is 0.8409, with p-value of 0.008 (smaller than 0.05), which is significant. The estimated value for ϕ_2 is -0.0145, with p-value of 0.8986 (larger than 0.05), which is not significant. The estimated value for ϕ_3 is -0.1820, with p-value of 0.0023 (smaller than 0.05), which is significant. The estimated value for θ_1 is -1.5099, with p-value of 0 (smaller than 0.05), which is significant. The estimated value for θ_2 is 0.5168, with p-value of 0.0361 (smaller than 0.05), which is significant. Similarly, the constant term is not siginificant due to the p-value larger than 0.05, And since there's no meaning of the constant term in this situation, we do not consider the constant term here.

Looking at the diagnostic plots for ARIMA (0,1,2) and ARIMA (3,1,0):





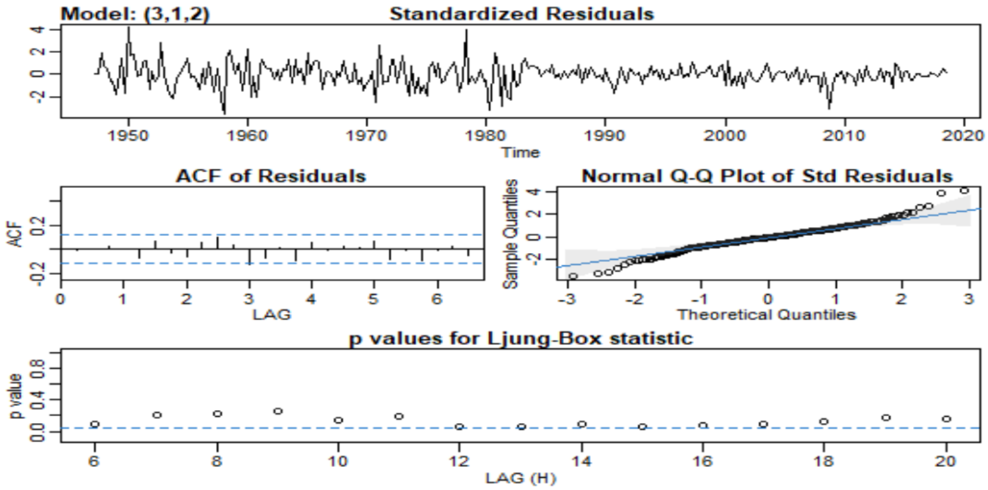
The standardized residuals show both have no obvious patterns. Few outliers are exceeding three standard deviations from the mean.

From the ACF Residuals plots, there are only a few spikes at some lags in both cases. This indicates that no apparent departure from the model the randomness assumption.

The residuals' normal Q-Q plots show that the assumption of normality is reasonable, except for some possible outliers. Some outliers are detected at the tails which indicate a deviation from normality.

The p-values for Ljung-Box statistics seems to be not above the reasonable significant level for all lags in both models. Although the p-values are very close to 0.05, the values are still below the benchmark level. So we need to reconsider whether to reject the null hypothesis that the residuals are independent.

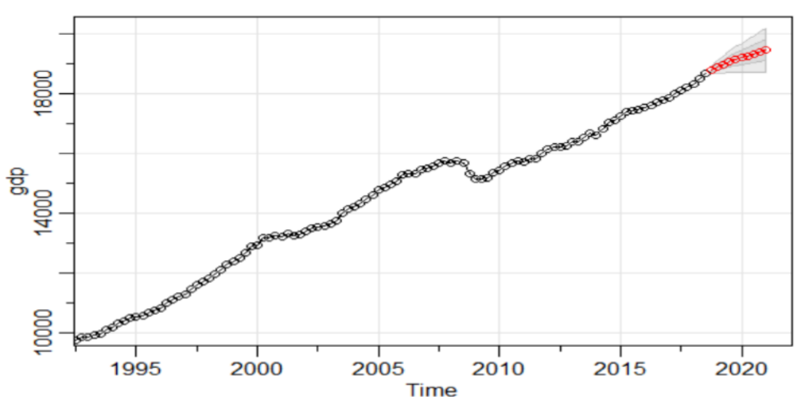
Overall, the ARIMA (0,1,2) and ARIMA (3,1,0) models' residuals seems to be iid and normal with mean zero and constant variance. But the p-values for Ljung-Box statistics indicate that the two models are both not the perfect ones.



For diagnostics for ARIMA (3,1,2), the standard residuals, ACF residual plots and the residual's normal Q-Q plots are quite similar to the previous two. However, the performance of the p-values in Ljung-Box statistics are much better: most of the p-values are above the significant level for all lags for both models. Although at some lags the p-values are very close to 0.05, it is in the reasonable range. So we can conclude that the residuals are independent.

Based on these diagnostics, we can see that the ARIMA (3,1,2) have the best performances: the models' residuals seem to be iid and normal with mean zero and constant variance.

Moreover, from looking at the AIC, AICc, and BIC, the ARIMA (3,1,2) have the smallest values among the three models. So, we select ARIMA (3,1,2) for our further analysis.

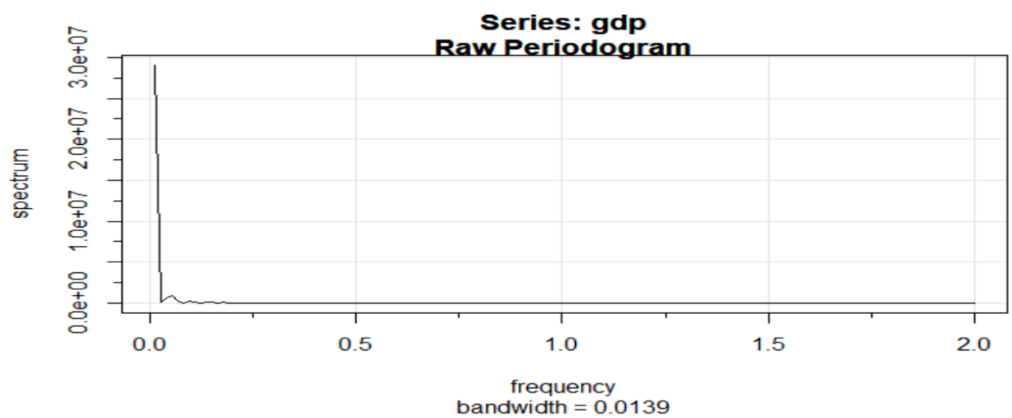


The above graph is the prediction plot of the Quarterly U.S.GDP 10 quarters ahead.

```
##    Prediction.for.Quarterly.U.S.GDP.. PI.95..Lower.Bound PI.95..Upper.Bound
## 1                            18788.95           18669.26           18908.65
## 2                            18888.59           18689.34           19087.84
## 3                            18976.88           18698.79           19254.96
## 4                            19055.97           18704.72           19407.23
## 5                            19129.37           18708.46           19550.28
## 6                            19198.18           18712.52           19683.83
## 7                            19264.12           18717.47           19810.77
## 8                            19327.77           18724.09           19931.45
## 9                            19389.98           18732.51           20047.45
## 10                           19451.04           18742.89           20159.19
```

The above data frame is the values of the ten forecasts and the 95% prediction intervals.

We can see there is an obvious upward trend of the quarterly U.S GDP from around 1994 to 2018. The 10 prediction values also follow an upward trend, which matches the general trend of the data. And the values of the 10 prediction intervals have no obvious outliers and jumping values, which means the prediction is smooth and proper.



The graph above is the periodogram for our Quarterly U.S GDP data. From the graph, we can see the peaks are mostly gathered close to frequency 0.

```
order(gdp.per$spec,decreasing = TRUE)[1:3]
```

```
## [1] 1 4 3
```

```
1/gdp.per$freq[c(1,4,3)]
```

```
## [1] 72 18 24
```

The first three predominant periods are 72, 18 and 24 with top three spectrum values.

An approximate 95% confidence interval for the periods 72, 18 and 24 are (19.51812,2843.848), (4.879531,710.962) and (6.506041,947.9494) respectively, which are too wide to be of much use. Moreover, the lower value 19.51812, 4.879531 and 6.506041 are lower than other periodogram ordinate, so we cannot say these values are significant.

Thus, all three confidence intervals are unable to establish significant of the peaks.

Discussion:

Overall, the quarterly U.S. GDP data varies from time. Its transformation: the quarterly U.S. GDP growth rate can be fitted by a certain statistical model to do the forecasting and seasonal analysis. After determining the proper model, the model predicts that the quarterly U.S. GDP will continue increasing for the next 10 quarters with big probability. And there is no obvious seasonality performance of our data.

Additionally, there are some limitations in the model selecting process. Firstly, one of the parameters of ARIMA (3,1,2) is not significant enough considering 0.05 significant level. Secondly, the standard residual plot of our model has a smaller variance after the year 1985. These limitations should be considered for further analysis.

Appendix: (references)

Data sources:

https://tradingeconomics.com/united-states/gdp

Further references:

https://en.wikipedia.org/wiki/Gross_domestic_product

https://www.investopedia.com/terms/g/gdp.asp