Assignment 4 by Haobin Tang

1. Work out (in LaTeX) the equations for Absolute/Relative Risk Premia for CARA/CRRA respectively

CARA: Constant Absolute Risk-Aversion (CARA)

$$A(x) = \frac{-U''(x)}{U'(x)} = a$$

a is called Coefficient of Constant Absolute Risk-Aversion (CARA)

One example of U(x) is that:

$$U(x) = \frac{-e^{-ax}}{e}$$

Risk Primia:

For $a = 0$, $U(x) = x$ (note: $A(x) = \frac{-U''(x)}{U'(x)} = 0$) If the random outcome $x \sim N(\mu, \sigma^2)$

$$\mathbb{E} = \begin{cases} \dfrac{-e^{-a\mu+\frac{a^2\sigma^2}{2}}}{e} \\ \mu \end{cases}$$

$$x_{CE} = \mu - a\sigma^2$$

Absolute Risk Premium $\pi_A = \mu - x_{CE} = \frac{a\sigma^2}{2}$

CRRA:Constant Relative Risk-Aversion (CRRA)

$$R(x) = \frac{-U''(x) * x}{U'(x)} = \gamma$$

a is called Coefficient of Constant Absolute Risk-Aversion (CARA)

One example of U(x) is that:

$$U(x) = \frac{x^{1-\gamma}}{1-\gamma}$$

Risk Primia:

For $\gamma = 1$, $U(x) = log(x)$ (note: $R(x) = \frac{-U''(x)*x}{U'(x)} = 1$)

If the random outcome x is lognormal $log(x) \sim N(\mu, \sigma^2)$

$$\mathbb{E} = \begin{cases} \dfrac{-e^{\mu(1-\gamma)+\frac{\sigma^2}{2}(1-\gamma)^2}}{1-\gamma} \\ \mu \end{cases}$$

$$x_{CE} = e^{\mu+\frac{\sigma^2}{2}(1-\gamma)}$$

Relative Risk Premium $\pi = 1 - \frac{xCE}{\bar{x}} = 1 - e^{-\frac{\sigma^2\gamma}{2}}$

2. Write the solutions to Portfolio Applications covered in class with precise notation

We know from the problem that

$$logW \sim N(r + \pi(\mu - r) - \frac{\pi^2\sigma^2}{2}, \pi^2\sigma^2)$$

From the section on CRRA Utility, we know we need to maximize:

$$\begin{aligned} & r + \pi(\mu - r) - \frac{\pi^2\sigma^2}{2} + \frac{\pi^2\sigma^2(1-\gamma)}{2} \\ & = r + \pi(\mu - r) - \frac{\pi^2\sigma^2\gamma}{2} \end{aligned}$$

To maximize this objective function we take a derivative of π:

$$\begin{aligned} (r + \pi(\mu - r) - \frac{\pi^2\sigma^2\gamma}{2})d\pi &= 0 \\ (\mu - r) - \pi\sigma^2\gamma &= 0 \\ \pi^* &= \frac{\mu - r}{\gamma\sigma^2} \end{aligned}$$