The Karp-Flatt metric; revisited

From Chapter 7.5 of the textbook, we recall that the Karp-Flatt metric can be used to calculate an experimentally determined serial fraction e of a parallel computation as

$$e = \frac{1/\Psi - 1/p}{1 - 1/p},$$

where Ψ is the measured speedup using p processors. Studying the growth of e as a function of p can give us valuable information about the quality of the parallelization.

Unfortunately, the definition of e was incorrectly given on slide 23 of the lecture slides set "*Performance analysis*". Namely, $e(n,p)$ should not be defined as $\frac{\sigma(n)+\kappa(n,p)}{\sigma(n)+\varphi(n)}$, but instead should be defined as

$$e(n,p) = \frac{(p-1)\sigma(n) + p\,\kappa(n,p)}{(p-1)T(n,1)} = \frac{\sigma(n) + \frac{p}{p-1}\kappa(n,p)}{T(n,1)}\,. \tag{1}$$

The above correct definition of $e(n,p)$ can be found on page 167 of the textbook.

This is because, as we know as the starting point,

$$\begin{aligned} T(n,1) &= \sigma(n) + \varphi(n), \\ T(n,p) &= \sigma(n) + \kappa(n,p) + \frac{\varphi(n)}{p}, \end{aligned}$$

where $\kappa(n,p)$ denotes the parallelization overhead. Therefore, following the definition of $e(n,p)$ as given in (1), we will have

$$\begin{aligned} e(n,p)T(n,1) &= \frac{\sigma(n) + \frac{p}{p-1}\kappa(n,p)}{T(n,1)}T(n,1) \\ &= \sigma(n) + \frac{p}{p-1}\kappa(n,p), \end{aligned}$$

and

$$\begin{aligned}
\frac{1-e(n,p)}{p}T(n,1) &= \frac{1}{p}\left(T(n,1) - e(n,p)T(n,1)\right) \\
&= \frac{1}{p}\left(\sigma(n) + \varphi(n) - \sigma(n) - \frac{p}{p-1}\kappa(n,p)\right) \\
&= \frac{\varphi(n)}{p} - \frac{1}{p-1}\kappa(n,p)\,.
\end{aligned}$$

Consequently, combining the above two results, we will get

$$\begin{aligned}
e(n,p)T(n,1) + \frac{1-e(n,p)}{p}T(n,1) &= \sigma(n) + \frac{p}{p-1}\kappa(n,p) + \frac{\varphi(n)}{p} - \frac{1}{p-1}\kappa(n,p) \\
&= \sigma(n) + \kappa(n,p) + \frac{\varphi(n)}{p} \\
&= T(n,p)\,.
\end{aligned}$$

At the same time, the definition of speedup Ψ can be used to give the following relation between Ψ and $e(n,p)$:

$$\begin{aligned}
\Psi(n,p) &= \frac{T(n,1)}{T(n,p)} \\
&= \frac{T(n,1)}{\left(e(n,p) + \frac{1-e(n,p)}{p}\right)T(n,1)} \\
&= \frac{1}{e(n,p) + \frac{1-e(n,p)}{p}}.
\end{aligned}$$

When the value of Ψ is known and the above equation is solved with respect to e, we will recover exactly the Karp-Flatt metric. Note also that the formula for e given on the top of page 168 of the textbook is incorrect.