CS211
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(1)

Fixed size ⇒ Amdahl's Law
inherently sequential

$$\psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\varphi(n)/p} = \frac{\sigma(n)\cdot\frac{1}{f}}{\sigma(n)+\sigma(n)\left(\frac{1}{f}-1\right)\cdot\frac{1}{p}} = \frac{1}{f+(1-f)\cdot\frac{1}{p}}$$

$$f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)} = 0.05$$

$$\psi(n,p) \le \frac{1}{0.05+\frac{0.95}{10}} \approx 6.9$$

2、

6% sequential operations
Fixed size ⇒ Amdahl's law

$$f = 0.06$$

$$\psi(n,p) \le \frac{1}{0.06+(1-0.06)\cdot\frac{1}{p}}$$

$$\Rightarrow p \le \frac{1}{0.06+0.94\cdot\frac{1}{p}} \Rightarrow p \ge 23.5$$

$\because p$ is integer $\therefore p \ge 24$

inherently sequential

3、

inherently sequential
Fixed size ⇒ Amdahl's law

$$\psi(n,p) = 50 \le \frac{1}{f+(1-f)\frac{1}{p}}$$

$$\Rightarrow f+(1-f)\frac{1}{p} \le 0.02$$

if p increase to ∞, the speedup achieve to the l

$$\psi(n,p) \le \frac{1}{f} \Rightarrow \frac{1}{f} \ge 50$$

$$\Rightarrow f \le 0.02$$

$\therefore$ the maximum fraction of computation that are inherently sequential is 0.02.

4、 fixed size ⇒ Amdahl's law
inherently sequential

$$\psi(n,p) = 9 \le \frac{1}{f+(1-f)\frac{1}{10}} \Rightarrow f+(1-f)\frac{1}{10} \le \frac{1}{9}$$

$$\frac{9}{10}f \le \frac{1}{90}$$

$$f \le 0.012$$

5、 scaled speed up ⇒ Gustafson-Barsis's law
~~real~~ parallel sequential.

$$s = \frac{\sigma(n)}{\sigma(n)+\varphi(n)\cdot\frac{1}{p}} = \frac{9}{242} = 0.037$$

$$\psi(n,p) = s + (1-s)\cdot p \approx 15.44$$

6、 parallel time
scaled speedup ⇒ Gustafson-Barsis's law

$$s = 0.01$$

$$\psi(n,p) = p + (1-p)s = 39.61$$

7、 ① use Amdahl's law.

for $\psi = 9$, $p = 10$ $\quad \psi(n,p) \le \frac{1}{f_1 + (1-f_1)/p} \Rightarrow 9 \le \frac{1}{f_1 + (1-f_1)\cdot\frac{1}{10}}$

$$10f_1 + (1-f_1) \le \frac{10}{9}$$

$$f_1 \le \frac{1}{81}$$

for $\psi = 90$ $p = 100$ $\quad 90 \le \frac{1}{f_2 + (1-f_2)\cdot\frac{1}{100}}$

$$f_2 \le \frac{1}{891}$$

$f_2 < f_1$ means
It's not perfectly true
to achieve 90 speedup when $p = 100$
only when inherently sequential
part $f \le \frac{1}{891}$, the statement
can be true for both.

7. ② use karp-flatt metric.

$$e_1 = \frac{\frac{1}{\psi} - \frac{1}{p}}{1 - \frac{1}{p}} = \frac{\frac{1}{9} - \frac{1}{10}}{1 - \frac{1}{10}} = \frac{1}{81}$$

$$e_2 = \frac{\frac{1}{90} - \frac{1}{100}}{1 - \frac{1}{100}} = \frac{1}{891}$$

because experimently determined serial fraction is decrease. which is imposible. for same problem

8. ① using Amdahl's law

$$\psi(n,p) = 2 \le \frac{1}{f + (1-f)/p}$$

$$2 \le \frac{1}{f + (1-f)\frac{1}{4}}$$

$$f \le \frac{1}{3}$$

the best condition is no overhead, means

$$\psi(n,16) \le \frac{1}{f + (1-f)\frac{1}{p}} = \frac{8}{3}$$

② using karp-flatt metric.

Assume e dont change to achive best speed up.

$$e = \frac{\frac{1}{\psi} - \frac{1}{p}}{1 - \frac{1}{p}} = \frac{1}{3}$$

$$\psi(n,16) = \frac{1}{e + (1-e)/p} = \frac{8}{3}$$

9. (a) $n \ge Cp$ $\quad M(f(p))/p = C^2p^2/p = C^2p$

(b) $M(f(p))/p = C^2 p\log^2 p / p = C^2 \log^2 p$

(c) $M(f(p))/p = C^2$

(d) $M(f(p))/p = C^2 p \log^2 p$

(e) $M(f(p))/p = C$

(f) $M(f(p))/p = p^{C-1} < p$

(g) $M(f(p))/p = p^{C-1} > p$

So the order is. Assume All $C>1$

$c \approx e > b > f > a > d > g$

$e = c$

∴ both bandwire consiat level, so they all have best salability

10、 $f(p) = 16n^2 \log_2 p$

(1) $24n^2 \le 1024 \times 1024^3$

$n \le 214039$

Assume all computation could be parallized. $\sigma(n) = 2n^3$

$\varphi(n) = 0$

$$\psi(5688, 1024) \le \frac{\sigma(n)+\varphi(n)}{\sigma(5688)+\varphi(5688)/1024+\kappa(n,p)}$$

$$\le \frac{2n^3}{\frac{2n^3}{1024}+16n^2\log_2 p}$$

$$\le \frac{n}{\frac{n}{1024}+8\log_2 p} = \frac{214039}{\frac{214039}{1024}+80} \approx 740$$

the maximum speed up is 740

(2)

$$\psi(n, 1024) \le \frac{2n^3}{\frac{2n^3}{1024}+\kappa(n,p)} = \frac{n}{\frac{n}{1024}+8\log_2 p}$$

$$256 \le \frac{n}{\frac{n}{1024}+80} \Rightarrow n \ge 27306.$$