IEEE

IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Laser Gyros

IEEE Aerospace and Electronic Systems Society

Sponsored by the
Gyro and Accelerometer Panel

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Abstract: Specification and test requirements for a single-axis ring laser gyro (RLG) for use as a sensor in attitude control systems, angular displacement measuring systems, and angular rate measuring systems are defined. A standard specification format guide for the preparation of a single-axis RLG is provided. A compilation of recommended procedures for testing an RLG, derived from those presently used in the industry, is also provided.

Keywords: dithered gyro, gyro, gyroscope, inertial instrument, inertial sensor, optical gyro, resonant cavity, ring laser gyro, RLG, Sagnac effect, Sagnac gyro.

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Introduction

> This introduction is not part of IEEE Std 647-2006, IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Laser Gyros.

This standard is a minor revision of IEEE Std 647-1995 that corrects errors made in the publication process and adds minor technical improvements. It consists of two parts.

Part I is a specification format guide for the preparation of a laser gyro specification. It provides a common meeting ground of terminology and practice for manufacturers and users. The user is cautioned not to overspecify; only those parameters that are required to guarantee proper instrument performance in the specific application should be controlled. In general, the specification should contain only those requirements that can be verified by test or inspection. Parameters in addition to those given in this standard are not precluded.

Part II is a compilation of recommended procedures for testing a laser gyro. These procedures, including test conditions to be considered, are derived from those currently in use. For a specific application, the test procedure should reflect the requirements of the specification; therefore, not all tests outlined in this standard need be included, nor are additional tests precluded. In some cases, alternative methods for measuring performance characteristics have been included or indicated.

The intent is for the specification writer to extract the applicable test conditions and equipment requirements from Clause 11 for inclusion in the appropriate clauses listed under 6.5. Similarly, it is intended that the writer extract the applicable test procedures from Clause 12 for inclusion in the appropriate subclauses listed under 6.6. Part II can also be used as a guide in the preparation of a separate laser gyro test specification with appropriate clause numbering. In general, the intent is for the specification writer to ensure consistency and traceability between Part II test procedures and Part I requirements for performance, mechanical, electrical, environmental, reliability, and quality assurance. To that end, a test procedure should not be listed in Part II unless a related requirement exists in Part I.

Blank spaces in the text of this document permit the specification writer to insert specific information such as parameter values and their tolerances, clause numbers, etc. Brackets are used to enclose alternative choices of dimensional units, signs, axes, etc. Boxed statements are included for information only and are not part of the specification or test procedures. The following standards were used in the development of this standard.

ANSI/IEEE Std 260.1, IEEE Standard Letter Symbols for Units of Measurement (SI Units, Customary Inch-Pound Units, and Certain Other Units).

ANSI/IEEE Std 268, American National Standard for Metric Practice.

ANSI/IEEE Std 280, IEEE Standard Letter Symbols for Quantities Used in Electrical Science and Electrical Engineering.

ANSI/IEEE Std 315, IEEE Graphic Symbols for Electrical and Electronics Diagrams.

IEEE Std 528, IEEE Standard for Inertial Sensor Terminology.

This standard defines the requirements and test procedures for a single-axis laser gyro in terms unique to the laser gyro. The requirements contained herein cover applications where the laser gyro is used as an angular motion sensor in navigation and control systems.

The term *laser gyro* is accepted to include the electronics necessary to operate the gyro and to condition the output signal. The laser gyro provides an output frequency proportional to inertial angular rate about its input axis.

Annex A lists various laser gyro design features for which this format is applicable. The list is not intended to make any suggestion regarding the selection of particular design features that might restrict the free choice of manufacturers.

Annex B is an overview of dynamic and stochastic modeling.

Annex C is an overview of noise process variance analysis as a method for determination of the drift rate coefficients and the quantization coefficient.

Annex D is a bibliography.

Notice to users

Errata

Errata, if any, for this and all other standards can be accessed at the following URL: http://standards.ieee.org/reading/ieee/updates/errata/index.html. Users are encouraged to check this URL for errata periodically.

Interpretations

Current interpretations can be accessed at the following URL: http://standards.ieee.org/reading/ieee/interp/index.html.

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Attention is called to the possibility that implementation of this standard may require use of subject matter covered by patent rights. By publication of this standard, no position is taken with respect to the existence or validity of any patent rights in connection therewith. The IEEE shall not be responsible for identifying patents or patent applications for which a license may be required to implement an IEEE standard or for conducting inquiries into the legal validity or scope of those patents that are brought to its attention.

Participants

This standard represents a large-scale group effort. A total of 143 individuals attended 46 meetings of the Gyro and Accelerometer Panel during preparation of this standard.

The following individuals on the Gyro and Accelerometer Panel were major contributors to IEEE Std 647-2006:

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A total of 50 individuals attended 12 meetings of the Gyro and Accelerometer Panel during the preparation of this revision. The following individuals on the Gyro and Accelerometer Panel contributed to this standard:

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The following members of the individual balloting committee voted on this standard. Balloters may have voted for approval, disapproval, or abstention.

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Robert Dahlgren
George Erickson
Jean-François Kieffer
Vladimir Skvortzov
Daniel Tazartes
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Angela Ortiz
IEEE Standards Program Manager, Technical Program Development

Contents

IEEE Standard Specification Format Guide and Test Procedure for Single-Axis Laser Gyros

1. Overview

1.1 Scope

This standard defines the specification and test requirements for a single-axis laser gyro for use as a sensor in attitude control systems, angular displacement measuring systems, or angular rate measuring systems, including the electronics necessary to operate the gyro and to condition the output signals.

1.2 Purpose

This standard provided a common meeting ground of terminology and practice for manufacturers and users. The user is cautioned not to overspecify—only those parameters that are required to guarantee proper instrument performance in the specific application should be controlled. In general, the specification should contain only those requirements that can be verified by test or inspection. Part II of this standard can also be used as a guide in the preparation of a separate laser gyro test specification. In general, the intent is for the specification writer to ensure consistency and traceability between Part II test procedures and Part I requirements for performance, mechanical, electrical, environmental, and quality assurance.

1.3 Document structure

Part I (Clause 3 through Clause 8) of this standard is a specification format guide for the preparation of a laser gyro specification. Part II (Clause 9 through Clause 12) is a compilation of recommended procedures for testing a laser gyro.

2. Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

None

Part I—Specification format[1]

3. Specification format overview

This specification defines the requirements for a single-axis laser gyro to be used as a sensor in [an inertial measure unit, _____]. This includes the electronics necessary to operate the gyro and to condition the output signal.

4. Applicable documents

The following documents form a part of the specification to the extent specified in this specification. In the event of any conflict between the requirements of this specification and the listed documents, the requirements of this specification shall govern.[2]

NOTE—Give identification number, title, date of issue, and revision letter of each listed document.

4.1 Standards

4.1.1 Government

NOTE—MIL-P-116H, Methods of Preservation

4.1.2 Industry/technical

4.1.3 Company

4.2 Standards

4.2.1 Government

NOTE—

MIL-STD-105	Sampling Procedure and Tables for Inspection by Attributes.
MIL-STD-461	Electromagnetic Interference Characteristics, Emission, and Susceptibility Requirements for the Control of Electromagnetic Interference.
MIL-STD-462	Electromagnetic Interference Characteristics, Measurement of.
MIL-STD-704C	Aircraft Electrical Power Characteristics.
MIL-STD-781C	Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution.
MIL-STD-785B	Reliability Program for Systems and Equipment Development and Production.
MIL-STD-810	Environmental Test Methods and Engineering Guidelines.

[1] The sample forms in Part I of this guide are included for information and example only. Users of this guide may freely reproduce and modify the forms in Part I so that they can be used for their intended purpose, so long as each page is reproduced in its entirety, including any and all copyright language.
[2] Notes in text, tables, and figures of a standard are given for information only and do not contain requirements needed to implement this standard.

4.2.2 Industry/technical

4.2.3 Company

4.3 Drawings

4.3.1 Government

4.3.2 Industry/technical

4.3.3 Company

4.4 Bulletins

4.4.1 Government

4.4.2 Industry/technical

4.4.3 Company

4.5 Other publications

NOTE—Other applicable documents should be listed under the appropriate paragraph.

5. Requirements

5.1 Description

The major components of the laser gyro herein specified are: a _____ gain medium, a closed path optical resonator utilizing _____, a _____ path length control, and a combining optics readout device (including pulse forming electronics) consisting of _____.

The particular laser gyro specified may have additional major components as follows: a _____ anti-lock means, temperature control or temperature compensation, power supplies for _____, and an excitation control.

NOTE—To fill in the blanks, refer to Annex A for examples. For gyros that do not include built-in anti-lock mechanisms, the test equipment and test procedures must provide the necessary anti-lock motion as a part of the test environment. Failure to provide anti-lock motion may result in damage to the gyro.

5.2 General requirements

5.2.1 Precedence

In the event of conflict among the purchase agreement, this specification, and other documents referred to in this standard, the order of precedence shall be as follows:

a) Purchase agreement

b) This standard and its applicable drawings (see 5.4.2 and 5.5.1)

c) Other applicable documents (see Clause 4)

5.3 Performance

Performance characteristics shall be as specified in this subclause.

The tolerances include a summation of manufacturing and test errors, and environmental effects. Any exception due to environmental effects will be listed in 5.6.3.1.

NOTE—When required by the application, the sensitivity of a gyro characteristic to variations in voltage, frequency, temperature, or other variables should be included in the subclause specifying that characteristic.

5.3.1 Input rates

5.3.1.1 Maximum input rate

The maximum input rate about the gyro input axis (IA) shall be ± _____ [°/s, _____].

NOTE—The positive and negative input rate limits need not be equal. Multiple rate limits may be specified.

5.3.1.2 Minimum input rate

The minimum rate of rotation about the IA for which the performance is of the specified quality shall be ± _____ [°/s, _____].

NOTE—This characteristic is important for laser gyros without built-in anti-lock devices. The minimum input rate at which the gyro can be operated without damage should be specified.

5.3.2 Gyro scale factor, S

The gyro scale factor shall be between _____"/p and _____"/p.

NOTE 1—The scale factor is the angular input in arc seconds (") equivalent to a digital pulse (p) output. The scale factor is directly proportional to the total pathlength and operating wavelength, and inversely proportional to the enclosed ring area.

NOTE 2—This scale factor range includes the manufacturing tolerance. The test procedure will result in a nominal scale factor that is defined in the model equation of 8.3.

5.3.2.1 Gyro scale factor errors

a) Asymmetry: _____ ppm.

> NOTE—The specific input rotational rate(s) at which the scale factor asymmetry is to be determined may be specified. The asymmetry is the ratio of the difference in magnitudes of scale factor measured for clockwise and counterclockwise inputs to one-half the sum of the same magnitudes.

b) Nonlinearity: _____ ppm deviation [maximum spread, 1σ _____] from the nominal scale factor.

> NOTE—The rates over which the specification is valid should be specified. For some gyros it may be appropriate to include a specification and test for linearity error.

c) Repeatability: _____ ppm [maximum spread, 1σ _____].

> NOTE—Thermal cycles and other environmental exposures, shutdowns, time between runs, remounting, and additional factors pertinent to the particular application should be specified.

d) Stability: _____ ppm, [maximum spread, 1σ, _____] for _____ [h, _____].

5.3.2.2 Gyro scale factor sensitivities

a) Temperature: the gyro scale factor temperature sensitivity resulting from a change in steady-state operating temperature shall not exceed _____ ppm/°C from the nominal scale factor.
b) Temperature gradient: the change in gyro scale factor resulting from a change in steady-state temperature difference measured across the gyro case in the _____ [IA, other] direction shall be _____ ± _____ ppm/Δ°C.

> NOTE—The specific points on the case where the temperatures are measured, the temperature difference, and the specific temperature should be specified.

c) Other sensitivities

> NOTE—Additional sensitivities may be specified such as those due to variations in supply voltage, rate of temperature change, orientation, acceleration, vibration, magnetic field, radiation, and other environments pertinent to the particular application.

5.3.3 Drift rate, *D*, *E*

5.3.3.1 Bias and random, *D*

5.3.3.1.1 Bias, D_F

_____ ± _____ °/h.

a) Repeatability: _____ °/h [maximum spread, 1σ, _____].

NOTE—Thermal cycles and other environmental exposures, shutdowns, time between runs, remounting, and additional factors pertinent to the particular application should be specified.

b) Warm-up, $D_F(t)$: _____ ± _____ °/h from _____ [s, _____] to _____ [s, _____] after turn-on.

NOTE—The drift rate limits of the gyro, during a certain time interval after turn-on, should be specified for certain applications. The limits may be specified as a function of time after turn-on. More than one function may be specified for different temperatures at the time of turn-on.

5.3.3.1.2 Random, D_R

NOTE—Random drift rate is usually defined in terms of the Allan variance components. These components should be specified. See D_R terms in 8.3 and Annex C.

a) Angle random walk (rate white noise) coefficient, N. _____ °/h^{½} maximum.
b) Bias instability coefficient, B. _____ °/h maximum.
c) Rate random walk coefficient, K. _____ (°/h)/h^{½} maximum.
d) Ramp coefficient, R. _____ (°/h)/h maximum.

NOTE—For some applications it may be sufficient to define random drift as the standard deviation of the output. The time interval and integration time should be specified.

5.3.3.1.3 Measurement noise, D_Q

Combined quantization and anti-lock residual noise coefficient, Q _____" maximum.

NOTE—Measurement noise is usually defined as the Allan variance component Q. See 8.3 and Annex C.

5.3.3.2 Environmentally sensitive, E

5.3.3.2.1 Temperature, D_T

The change in bias drift due to a change in the steady-state operating temperature shall not exceed _____ (°/h)/°C.

NOTE—The specific point or points where the temperature is measured and the temperature range should be specified.

5.3.3.2.2 Temperature gradient, $D_{\nabla T}$

The change in the bias drift resulting from a change in steady-state temperature difference measured across the gyro case in the _____ [IA, other} direction shall not exceed ____ (°/h)/∇°C.

NOTE—The specific points on the case where the temperature is measured, the temperature gradient(s), and the test temperature(s) should be specified.

5.3.3.2.3 Magnetic

The change in gyro bias resulting from a change in steady-state flux density shall not exceed ± _____ °/h over the range _____ to _____ [mT, G].

NOTE—Gyro axis and characteristics of the field should be defined. If exposure to a varying field is required, the nature of the variation should be described. Demagnetization may be necessary following exposure.

5.3.3.2.4 Other sensitivities

5.3.4 IA alignment characteristics

5.3.4.1 IA misalignment

_____ [mrad, ", _____], maximum with respect to the input reference axis (IRA).

NOTE—The specific direction of IA misalignment may be important in some applications and should be specified with respect to L reference axis (LRA) and N reference axis (NRA) defined in 5.4.3.

5.3.4.2 IA misalignment repeatability

_____ [mrad, ", _____], [1σ, peak-to-peak, _____].

NOTE—Thermal cycles and other environmental exposures, shutdowns, time between runs, remounting, and additional factors pertinent to the particular application should be specified.

5.3.4.3 IA misalignment sensitivities

NOTE—Sensitivities, such as those due to temperature, thermal gradients, rate of temperature change, acceleration, vibration, and additional environments pertinent to the particular application may be specified.

5.3.5 Operating temperature

_____ ± _____ °C.

NOTE—The preceding operating temperature applies only to temperature controlled gyros. Typically, laser gyros are not designed with internal temperature controls. Temperature sensors may be built into the gyro for use in characterizing its temperature sensitivity. For gyros intended to operate without temperature control, see 5.6.4.

5.3.6 Turn-on time

The gyro output rate shall be within _____ [°/h, _____] of the input rate within _____ [s, _____] after the application of power.

5.3.7 Warm-up time

_____ [s, _____], maximum.

5.3.8 Polarities

5.3.8.1 Input axis

The positive IA shall conform to [drawing number _____, Figure _____].

5.3.8.2 Output signals

The output signals representing gyro rotation shall conform to [drawing number _____, Figure _____].

5.3.9 Reference constants

NOTE—These constants are for reference only. They are not specified independently, because they may vary within the framework of the specification, or because they are difficult or impossible to measure independently in a complete gyro. Nominal values are listed.

5.3.9.1 Laser beam wavelength

_____ [μm, _____].

5.3.9.2 Beam path length

_____ cm.

5.3.9.3 Dithered element IA moment of inertia

_____gm-cm^2.

5.3.10 Anti-lock residual

> NOTE 1—Anti-lock means introduce extraneous output, which should be considered in the data processing.
>
> NOTE 2—Signal processing techniques to mitigate the effect and the residual error after compensation should be specified.
>
> NOTE 3—Note that oscillatory motion of the gyro base caused by mechanical interaction of the dithered gyro with the sensor mounting assembly in a real system is actual rate and not anti-lock residual.

5.4 Mechanical requirements

Mechanical characteristics shall be as specified in this subclause.

5.4.1 Exterior surface

All exterior surfaces shall withstand the environment specified herein and the handling expected in the normal course of operation, testing, and maintenance without deterioration that causes nonconformance to this specification.

> NOTE—Additional requirements controlling surface finish, workmanship, processing, etc., may be specified.

5.4.2 Outline and mounting dimensions

Shall conform to [drawing number _____, Figure _____].

> NOTE—Specify center of gravity, if required.

5.4.3 Gyro axes

The IRA, LRA, and NRA are mutually orthogonal right-handed reference axes defined with respect to the mounting provisions. The IRA is nominally parallel to the true gyro IA and shall conform to [drawing number _____, Figure _____] (see Figure 1).

> NOTE—The IRA is typically perpendicular to the mounting surface. LA and NA are nominally parallel to the LRA and NRA. Misalignment angles θ_L and θ_N represent rotations of IA about LRA and NRA, respectively.

5.4.4 Weight

_____ ± _____ [g, _____]

> NOTE—Specify those components such as cables, connectors, and electronics that are to be included in the weight.

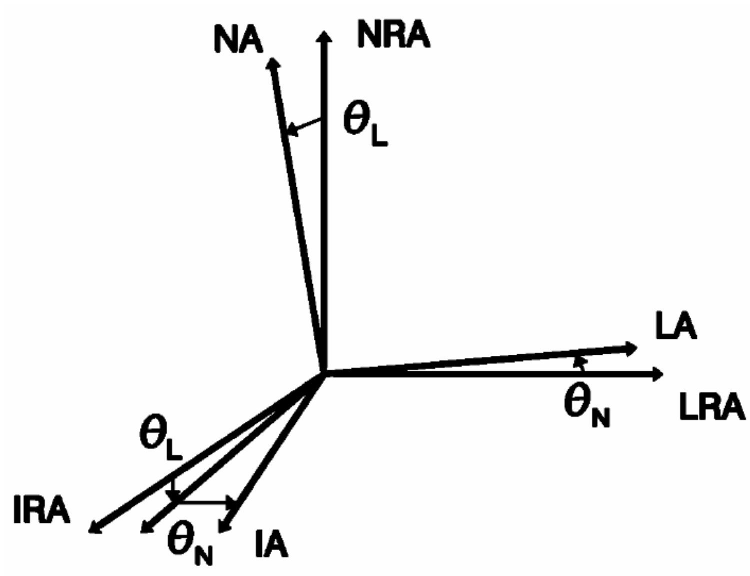


Figure 1—Gyro axes and misalignment angles

5.4.5 Seal

The gyro housing shall be sealed such that the leak rate is less than _____ [cm^3/s, _____] when subjected to pressure of _____ ± _____ Pa and gyro temperature of _____ ± _____ °C for a minimum of _____ min.

5.4.6 Mount reaction torque

NOTE—Mechanically dithered gyros generate oscillatory torques at the mounting interface that should be considered in testing and usage.

5.4.7 Identification of product

The gyro shall be identified on the surface and in the manner indicated in drawing number _____. Identification shall include the following:

a) Laser gyro
b) Specification number
c) Unit serial number
d) Axis identification marking as shown in [drawing number _____, Figure _____]
e) Manufacturer’s name or symbol
f) Safety warnings

NOTE—The purchase agreement may require additional identification, such as date of manufacture.

5.5 Electrical requirements

Electrical characteristics shall be as specified in this subclause.

5.5.1 Schematic

The electrical circuits shall be connected as shown in [drawing number _____, Figure _____].

> NOTE—The schematic may include circuits for monitoring laser discharge current, laser intensity, path length control, etc.

5.5.2 Impedances

The gyro impedances shall be _____.

> NOTE—Load impedances and impedances of excitation, monitoring, temperature sensing, and test circuits should be specified.

5.5.3 Input power and circuit excitations

The input power (current) shall not exceed _____W (A) with a circuit excitation of _____ ± _____V.

> NOTE—Requirements such as power factor, frequency, voltage, ripple, and starting and running currents for each excitation should be specified. Transient conditions may need to be specified.

5.5.4 Test points

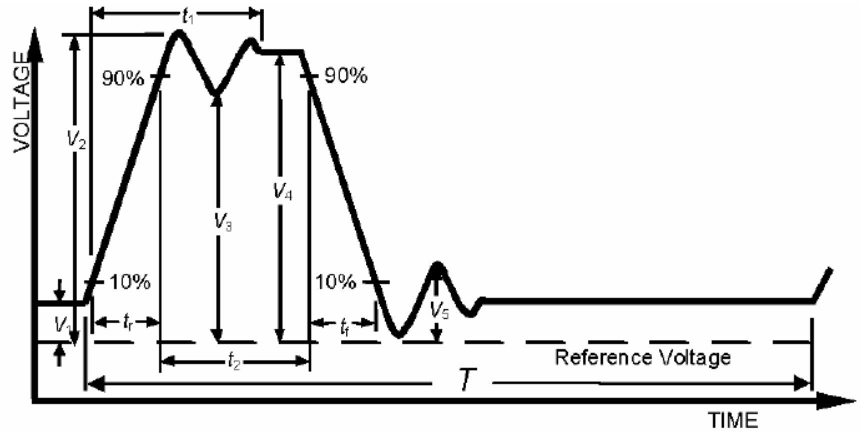
> NOTE—Test points required for monitoring and testing of the laser gyro should be specified. These may include excitation voltages, other control signals, or temperature sensor(s). Any special buffering or scaling requirements should be specified.

5.5.5 Grounding

> NOTE—Electrical grounding design requirements (e.g., requirements for isolation between input, output, and power returns and the grounding requirements for shields, chassis, and critical components) should be specified.

5.5.6 Output signals

> NOTE—Specify the type and characteristics of output signals required. For example:
>
> a) Type: pulses indicating positive angular increments on one signal line and pulses indicating negative angular increments on a second line
>
> b) Characteristics:
>
>   1) Source impedance
>
>   2) Load impedance
>
>   3) Wave shape (see Figure 2)
>
>   4) Maximum pulse rate



	UNITS	MAXIMUM	MINIMUM	REMARKS
V_1	volts	_______	_______	steady state low voltage
V_2	volts	_______	N/A	overshoot voltage
V_3	volts	N/A	_______	undershoot voltage
V_4	volts	_______	_______	steady state high voltage
V_5	volts	_______	N/A	maximum low voltage transient
t_r	seconds	_______	_______	rise time
t_f	seconds	_______	_______	fall time
t_1	seconds	_______	_______	turn-on transient time
t_2	seconds	_______	_______	high voltage on time
t_1/t_2	_______	_______	N/A	ratio of turn-on transient to high voltage on time
T	seconds	_______	_______	waveform period
t_2/T	_______	_______	_______	duty cycle

Figure 2—Wave shape requirements for direct coupled pulse-type signals

5.5.7 Temperature sensors

The output of the temperature sensor(s) shall be _____ ± _____ [V, _____] within the temperature range specified in 5.3.5 or 5.6.4.4. The temperature rise of the sensor due to self-heating shall not exceed _____ °C at _____ mA. The scale factor of the temperature sensor(s) shall be [+, –] _____ ± _____ [V/°C, _____] at operating temperature.

NOTE—Thermistors, thermocouples, or other temperature sensors may be specified. Typically, laser gyros are not designed with internal temperature controls. Temperature sensors may be built into the gyro for use in characterizing its temperature sensitivity. If sensors are required, specify quantity, locations, and characteristics.

5.5.8 Temperature sensors

The output of the temperature sensor(s) shall be _____ ± _____ [V, _____] within the temperature range specified in 5.3.5 or 5.6.4.4. The temperature rise of the sensor due to self-heating shall not exceed _____ °C at _____ mA. The scale factor of the temperature sensor(s) shall be [+, –] _____ ± _____ [V/°C, _____] at operating temperature.

NOTE—Thermistors, thermocouples, or other temperature sensors may be specified. Typically, laser gyros are not designed with internal temperature controls. Temperature sensors may be built into the gyro for use in characterizing its temperature sensitivity. If sensors are required, specify quantity, locations, and characteristics.

5.5.9 Dielectric strength

The leakage current shall not exceed _____ nA when a voltage of _____ ± _____ V rms, at _____ Hz, is applied between isolated interface circuits, and between the case(s) and circuits isolated from the case(s), for _____ ± _____ s.

NOTE—Different voltages may be specified for different circuits. In some instances, lower voltages may be specified for subsequent tests.

5.5.10 Insulation resistance

The insulation resistance between isolated interface circuits and between the case(s) and circuits isolated from the case(s) shall not be less than _____ MΩ measured at _____ ± _____ V dc, applied for _____ ± _____ s.

NOTE—Different voltages may be specified for different circuits.

5.5.11 Electromagnetic interference

The electromagnetic emissions and susceptibilities shall conform to _____.

NOTE—Describe the requirements. In the United States, a common standard is MIL-STD-461.

5.5.12 Magnetic leakage

The magnetic leakage shall not exceed _____ [mT, G] at a distance of _____ ± _____ [m, _____] from the gyro in any direction.

5.5.13 Electromagnetic compatibility

NOTE—Describe the requirements. In the United States, a common standard is MIL-STD-461.

5.6 Environmental requirements

NOTE—This subclause contains environmental requirements only. Test procedures that are to be used, including required combinations of environments, are covered in 6.6.

5.6.1 Storage

The gyro shall conform to all requirements of this specification after exposure to the following environments or specified combinations thereof, while packaged as specified in Clause 7.

NOTE—The procuring organization should list the applicable environments from 5.6.4 and specify the limits for each based on the expected storage conditions.

5.6.2 Transport

The gyro shall conform to all requirements of this specification after exposure to the following environments or specified combinations thereof, while packaged as specified in Clause 7.

NOTE—The procuring organization should list the applicable environments from 5.6.4 and specify the limits for each based on the expected transportation conditions.

5.6.3 Operation

NOTE—In order to properly specify the environmental requirements for a gyro, three different sets of environments or environmental limits, or both, as defined in 5.6.3.1, 5.6.3.2, and 5.6.3.3 may need to be described.

5.6.3.1 Operating environment

The gyro shall conform to all requirements of this specification during exposure to the following environments or specified combinations thereof (except during exposure to _____ where the deviations from this specification are given in the following clauses).

NOTE—Where degraded performance is to be allowed, include the parenthetical phrase in 5.6.3.1 and list in the blank space those environments for which degraded performance is allowed. The procuring organization should list the applicable environments from 5.6.4 and specify the limits for each based on the conditions expected when the gyro is in use. For those environments where degraded performance is allowed, specify the performance deviation in the clause that specifies the environment.

5.6.3.2 Survival environment—Operating

The gyro shall conform to all requirements of this specification after the operating gyro has been exposed to the following environments or specified combinations thereof.

NOTE—The procuring organization should list the applicable environments from 5.6.4 and specify the limits for each based on the conditions expected when the gyro is operating but not in use.

5.6.3.3 Survival environment—Nonoperating

The gyro shall conform to all requirements of this specification after the nonoperating gyro has been exposed to the following environments or specified combinations thereof.

> NOTE—The procuring organization should list the applicable environments from 5.6.4 and specify the limits for each based on the conditions expected when the gyro is not operating. Such conditions are expected to occur when the gyro is mounted in the system and the system is stored or transported in severe environments.

Environmental characteristics shall be as specified in this subclause.

> NOTE—This list is intended as an aid to the selection of the applicable environments under 5.6.1, 5.6.2, and 5.6.3. In this form, 5.6.4 would not be included in a final specification.

5.6.4.1 Vibration

> NOTE 1—Axes should be defined.
>
> NOTE 2—When available, the specific vibration characteristics including dwell frequencies, frequency spectrum, time duration, etc., should be supplied.
>
> NOTE 3—If exposure to random vibration is required, PSD, bandwidth, peak acceleration level, and duration should be specified.
>
> NOTE 4—The linear or angular vibration amplitude should be below levels that may cause permanent performance degradation or damage to the gyro.

5.6.4.1.1 Linear

Sinusoidal: _____ [cm, in] DA (double amplitude) _____ to _____ Hz; _____ [m/s^2, g] peak, _____ to _____ Hz. Sweep rate: _____ min/octave (continuous). Exposure time: _____ [min, _____] per axis.

5.6.4.1.2 Angular

Sinusoidal: _____ rad/s^2 _____ to _____ Hz. Sweep rate shall be _____ min/octave (continuous). Exposure time: _____ [min, _____] per axis.

5.6.4.2 Mechanical shock

_____ [m/s^2, *g*] peak, _____ wave shape, _____ ms, _____ shock(s) per axis.

> NOTE—Gyro axes and direction of shock should be defined. A figure may be included to describe more complex waveforms. A shock spectrum may be specified.

5.6.4.3 Linear acceleration

_____ [m/s^2, *g*], exposure time _____ [min, _____].

> NOTE—Gyro axes and direction of acceleration should be defined.

5.6.4.4 Temperature

_____ °C to _____ °C.

5.6.4.5 Thermal shock

_____°C to _____°C within _____ [s, _____].

NOTE—If exposure to cyclic conditions is required, temperature limits for each level, dwell times, and sequence should be specified. A figure may be included to describe the complex profiles.

5.6.4.6 Thermal radiation

_____W/cm^2 of radiation of wavelength from _____ to _____ m; exposure time: _____ [min, _____].

5.6.4.7 Ambient air pressure

_____ to _____ [Pa, _____].

5.6.4.8 Acoustic noise

_____ dB referenced to 2×10^{-5} Pa, _____ to _____ Hz; exposure time: _____ [min, _____].

NOTE—When available, the specific decibel versus frequency for the application should be supplied. Sweep rate may be specified as _____ min/octave (continuous), if applying sinusoidal acoustic noise.

5.6.4.9 Humidity

_____ % relative humidity for _____ h obtained from steam or distilled water having a pH value of _____ ± _____.

5.6.4.10 Air currents

_____ [m/s, _____].

NOTE—Gyro axes and direction of air flow should be defined.

5.6.4.11 Fungus

NOTE—Specify fungi organisms, length of exposure, and temperature and humidity conditions during exposure.

5.6.4.12 Salt spray

_____ % salt solution; exposure time: _____ [min, _____].

5.6.4.13 Nuclear radiation

_____ [J/kg, rad(Si)] with an exposure of _____ [(J/kg)/s, rad(Si)/s] or combinations, of _____ particles at a fluence of _____/cm^2

5.6.4.14 Magnetic fields

_____ [mT, G]; exposure time: _____ [min, _____].

> NOTE—Gyro axes and direction of field should be defined. If exposure to varying fields is required, the nature of the variation should be described.

5.6.4.15 Electromagnetic fields

Electromagnetic fields shall conform with _____.

> NOTE—In the United States, a common standard is MIL-STD-461.

5.6.4.16 Sand and dust

_____ [g/m^3, _____] at a velocity of _____ ± _____ [m/s, _____]; exposure time: _____ [min, _____]. Relative humidity not to exceed _____ %.

> NOTE—Particle size, shape, and chemical composition should be specified.

5.6.4.17 Solar radiation

_____ ± _____ [W/m^2, _____] of wavelength _____ to _____ m; exposure time: _____ [min, _____].

> NOTE—Distribution of power density versus wavelength should be specified.

5.6.4.18 Rain

_____ ± _____ [mm/h, _____] consisting of droplets having a minimum diameter of _____ mm; exposure time: _____ [min, _____].

5.6.4.19 Excitation variation

The excitation voltage and frequency required shall conform with _____.

> NOTE—In the United States, a common standard is MIL-STD-704C.

5.6.4.20 Life

NOTE—Life may need to be specified under differing environmental conditions.

5.6.4.20.1 Storage

_____ [months, _____].

5.6.4.20.2 Operating

_____ [h, _____].

5.6.4.20.3 Start cycles

_____ minimum.

5.7 Reliability

5.7.1 Reliability program

The reliability program required shall conform with _____.

NOTE—In the United States, a common standard is MIL-STD-785B.

5.7.2 Mean time between failure

The mean time between failure (MTBF) lower _____ % confidence limit shall be a minimum of _____ h.

6. Quality assurance provisions

6.1 Classification of tests

Inspection and testing shall be classified as follows:

a) Acceptance Tests. Those tests performed on gyros submitted for acceptance under contract.
b) Qualification Tests. Those tests performed on gyros submitted for qualification as a satisfactory product.
c) Reliability Tests. Those tests performed to demonstrate the reliability specified in 5.7.

6.2 Acceptance tests

6.2.1 Individual tests

NOTE 1—Each gyro shall be subjected to the following tests as described in 6.6.

NOTE 2—The list of individual tests should be specified by the procuring organization based on individual requirements. A burn-in period under specified conditions may be required before beginning individual tests. Those tests that are usually specified as individual tests are listed as follows:

6.6.1	Examination of product (Mechanical)
6.6.2.1	Insulation resistance
6.6.2.2	Impedances
6.6.3	Leak test
6.6.4	Input power
6.6.5	Turn-on time
6.6.6	Warm-up time
6.6.7	Polarity
6.6.9	Gyro scale factor test series
6.6.10.	Input rate test series
6.6.11.1	Bias
6.6.11.2	Random drift rate
6.6.12.1	Misalignment (Nominal)

NOTE 3—There are other individual tests that are not generally specified but they may be included under individual tests based on specific application. In some cases the gyros are subjected to specific environmental tests.

6.2.2 Sampling plans and tests

6.2.2.1 Sampling plans

NOTE—This subclause is intended to designate a sampling plan if required. Sampling plans are up to the discretion of the procuring organization based upon usage, size of contract, individual requirements, etc.

Gyros selected in accordance with specification _____ shall be subject to the tests specified in 6.2.2.2, which are described in 6.6.

NOTE—In the United States, selection according to MIL-STD-105 is common.

6.2.2.2 Sample tests

NOTE—In addition to the individual tests listed in 6.2.1, the procuring organization should specify those tests (see 6.6) that should be performed on gyros selected by 6.2.2.1. Sampling plan units may be used for delivery unless the procuring agency specifies life tests or other destructive type tests under the sampling plan.

6.2.2.3 Rejection and retest

When one item selected from a production run fails to meet the specification, the procuring organization shall be immediately notified of the failure. At the discretion of the procuring organization no items still on hand or later produced shall be accepted until the extent and cause of failure are determined. After corrections have been made, all necessary tests shall be repeated. For operational and production reasons, individual tests may be continued pending the investigation of a failure.

NOTE—Other requirements suitable to an individual contract may be substituted for this subclause.

6.2.2.4 Defects in items already accepted

The investigation of a test failure could indicate that defects may exist in items already accepted. If so, the manufacturer shall fully advise the procuring organization of defects likely to be found and of methods for correcting them.

6.3 Qualification tests

6.3.1 Qualification test samples

A sample of _____ gyros manufactured in accordance with the requirements of this specification shall be subjected to qualification tests specified herein at an activity designated by the procuring organization.

If the product is later modified in any way, the modified form shall be subjected to and pass those qualification tests designated by the procuring organization.

The qualification test samples shall be identified with the manufacturer's own part number and any other information required by the procuring organization.

6.3.2 Qualification tests

NOTE—The procuring organization should specify from 6.6 those tests, or combination of tests, to be performed on gyros submitted for qualification.

6.4 Reliability tests

NOTE—The reliability tests may be performed at the gyro or subassembly level, or both.

6.4.1 Burn-in

Each gyro shall be subjected to a burn-in period of _____ h under the following conditions.

NOTE—Specify environment and operating conditions. Consideration shall be given to not compromising the useful operating life by excessive test time during burn-in.

6.4.2 Demonstration testing

The MTBF requirements of 5.7.2 shall be demonstrated by testing _____ units for _____ h each, and a minimum of _____ h combined. The stress level test shall conform with _____.

NOTE 1—In the United States, testing in accordance with MIL-STD-781C is common.

NOTE 2—Other methods of demonstration testing may be selected at the discretion of the procuring organization. A demonstration test plan should be prepared to define test conditions, failures, types of tests, etc.

6.5 Test conditions and equipment

NOTE—The procuring organization should specify from Clause 11, the nominal test conditions and test equipment required. The conditions should apply to all tests unless otherwise specified. When a test condition is specified, the complete test condition should be detailed in this specification. The test equipment required should also be listed by name and model, part number, or performance requirement.

6.6 Test methods

NOTE—Instructions for performing specified tests in this subclause are detailed in Clause 12. When a test is specified, the complete test method should be detailed in this specification, including requirements to be met to determine satisfactory performance. A test method should not be listed in 6.6 unless a requirement exists in Clause 5 of this specification.

6.6.1 Examination of product (Mechanical)

6.6.2 Examination of product (Electrical)

6.6.2.1 Insulation resistance

6.6.2.2 Impedances

6.6.2.3 Dielectric strength

6.6.3 Leak test

6.6.4 Input power

6.6.5 Turn on time

6.6.6 Warm up time

6.6.7 Polarity

6.6.8 Operating temperature test

6.6.8.1 Temperature sensor characteristics

6.6.8.2 Operating temperature

6.6.9 Gyro scale factor test series

6.6.9.1 Gyro scale factor

6.6.9.2 Gyro scale factor errors

a) Asymmetry
b) Nonlinearity
c) Repeatability
d) Stability

6.6.9.3 Gyro scale factor sensitivities

a) Temperature
b) Temperature Gradient

6.6.10 Input rate test series

6.6.10.1 Maximum input rate

6.6.10.2 Minimum input rate

6.6.11 Drift rate test series

6.6.11.1 Bias

a) Nominal Value
b) Repeatability
c) Warm-up

6.6.11.2 Random drift rate

6.6.11.3 Anti lock residual

6.6.12 IA alignment test series

6.6.12.1 Misalignment (nominal)

6.6.12.2 Misalignment repeatability

6.6.12.3 Alignment sensitivities

6.6.13 Generated fields

6.6.13.1 Electromagnetic interference

6.6.13.2 Magnetic leakage

6.6.14 Environmental test series

6.6.14.1 Temperature

6.6.14.2 Excitation variation

6.6.14.3 Mechanical shock

6.6.14.4 Thermal shock

6.6.14.5 Vibration

6.6.14.5.1 Linear

6.6.14.5.2 Angular

6.6.14.6 Acceleration

6.6.14.6.1 Linear acceleration

6.6.14.6.2 Angular acceleration

6.6.14.7 Life

6.6.14.7.1 Storage

6.6.14.7.2 Operating

6.6.14.7.3 Start cycles

6.6.14.8 Fungus

6.6.14.9 Humidity

6.6.14.10 Salt spray

6.6.14.11 Acoustic noise

6.6.14.12 Thermal radiation

6.6.14.13 Air currents

6.6.14.14 Ambient air pressure

6.6.14.15 Nuclear radiation

6.6.14.16 Magnetic fields

6.6.14.17 Electromagnetic fields

6.6.14.18 Sand and dust

6.6.14.19 Solar radiation

6.6.14.20 Rain

6.6.14.21 Other

NOTE—Specific applications may require combined environmental tests, such as the following:

a) Thermal vacuum (6.6.14.1, 6.6.14.12, and 6.6.14.14)
b) Margin tests (6.6.14.1, 6.6.14.2, and 6.6.14.14)
c) Thermal shock/vacuum (6.6.14.1, 6.6.14.4, and 6.6.14.14)

6.7 Data submittal

NOTE—The format for all data organization and the method of submittal should be specified.

7. Preparation for delivery

NOTE 1—Detailed procedures for delivery should be supplied for the following:

a) Preservation and packaging
b) Packing
c) Marking of shipping containers

NOTE 2—A common United States specification that covers preservation and packaging is MIL-P-116.

8. Notes

8.1 Intended use

NOTE—A description of the application should be supplied if it is considered necessary or helpful.

8.2 Ordering data

NOTE—Procuring documents should specify the following:

a) Title, number, and date of this specification
b) Level of packaging and packing desired
c) Mode of shipment required
d) Whether sampling plan tests are to be conducted
e) Number of preproduction samples to be submitted for qualification testing
f) Reliability program
g) Data package

8.3 Model equation

$$S_0(\Delta N/\Delta t) = [I+E+D][1+10^{-6}\varepsilon_k]^{-1} \quad (1)$$

where

S_0	is the nominal scale factor ("/p)
$(\Delta N/\Delta t)$	is the output pulse rate (p/s)
I	is the inertial input terms ("/s)
E	is the environmentally sensitive terms ("/s)
D	is the drift terms ("/s)
ε_k	is the scale factor error terms (in ppm)
I	is $\omega_{IRA} + \omega_{LRA}\sin\theta_N - \omega_{NRA}\sin\theta_L$
E	is $D_T\Delta T + D_{\nabla T} \bullet \nabla T$
D	is $D_F + D_F(t) + D_R + D_Q$

where

D_R	is $D_{RN} + D_{RB} + D_{RK} + D_{RR}$

NOTE—D_R is composed of a number of random processes of different origins. Assuming that these processes are independent, the power spectral density (PSD) of D_R can be written as follows:

$$\mathrm{PSD}(D_R) = \mathrm{PSD}(D_{RN}) + \mathrm{PSD}(D_{RB}) + \mathrm{PSD}(D_{RK}) + \mathrm{PSD}(D_{RR})$$

where

$PSD(D_{RN})$	is N^2
$PSD(D_{RB})$	is $B^2/2\pi f$
$PSD(D_{RK})$	is $K^2/(2\pi f)^2$
$PSD(D_{RR})$	is $R^2/(2\pi f)^3$

where

f	is the frequency in hertz

The coefficients N, B, K, and R can be evaluated by forming the Allan variance. See 12.12.4.1.3 and Annex C.

ε_k	is $\varepsilon_T \Delta T + \varepsilon_{\nabla T} \bullet \nabla T + f(\omega_I)$
ω_{IRA}, ω_{LRA}, ω_{NRA}	are the components of the inertial input rate resolved into the gyro reference coordinate frame
θ_L	is the misalignment of the IA about the LRA
θ_N	is the misalignment of the IA about the NRA
D_F	is the bias
$D_F(t)$	is the variations of D_F during warm-up period
$D_T \Delta T$	is the drift rate attributable to a change in temperature ΔT, where D_T is the drift rate temperature sensitivity coefficient
$D_{\nabla T} \bullet \nabla T$	is the drift rate attributable to a temperature gradient ∇T, where $D_{\nabla T}$ is the coefficient vector of the temperature gradient drift rate sensitivity vector
$\varepsilon_T \Delta T$	is the scale factor error attributable to a change in temperature ΔT, where ε_T is the scale factor temperature sensitivity coefficient
$\varepsilon_{\nabla T} \bullet \nabla T$	is the scale factor error attributable to a temperature gradient ∇T, where $\varepsilon_{\nabla T}$ is the coefficient vector of the temperature gradient scale factor sensitivity vector
$f(\omega_I)$	is the scale factor nonlinearity
ω_I	is the angular rate about the IA ("/s)
D_{RN}	is the random drift rate attributable to angle random walk where N is the coefficient
D_{RB}	is the random drift rate attributable to bias instability where B is the coefficient
D_{RK}	is the random drift rate attributable to rate random walk where K is the coefficient
D_{RR}	is the random drift rate attributable to rate ramp where R is the coefficient
D_Q	is the combined effect of the apparent equivalent random drift rate attributable to angle quantization, and the apparent equivalent random drift rate attributable to the anti-lock residual, where Q is the combined coefficient

Part II—Test procedures[3]

9. Test procedure overview

This test procedure describes the test requirements for _____ (model number, part number, change letter (if any), other identification), specification number _____, manufactured by _____.

10. Description

The gyro considered in this standard is a ring laser gyro (RLG) that senses angular rate about a single IA. This rate causes equal and opposite optical frequency shifts, due to the Sagnac effect, in two counter-propagating standing light waves in a resonant cavity. The resulting optical signals are combined and processed to provide electrical pulses, each pulse being proportional to an incremental angular displacement. A distinguishing characteristic of the gyro is that the laser gain medium is integral with the resonant cavity. The dynamics of the gyro are expressed in Equation (1).

11. Test conditions and test equipment

11.1 Standard test conditions

Unless otherwise stated, the conditions in 11.1.1, 11.1.2, and 11.1.3 apply.

11.1.1 Ambient environment

11.1.1.1 Atmospheric conditions

a) Pressure: _____ ± _____ [Pa, in Hg].
b) Ambient temperature: _____ ± _____ °C.
c) Relative humidity: _____ to _____ %.

11.1.1.2 Magnetic field

a) Horizontal component: _____ [mT, G] maximum.
b) Vertical component: _____ [mT, G] maximum.

The method of measurement shall be as specified in Clause _____.

[3] The sample forms in Part II of this guide are included for information and example only. Users of this guide may freely reproduce and modify the forms in Part II so that they can be used for their intended purpose, so long as each page is reproduced in its entirety, including any and all copyright language.

11.1.1.3 Radiation

NOTE—The type of radiation and application intensity limits should be listed.

11.1.1.4 Seismic conditions

11.1.1.4.1 Tilt

Stable within _____" with respect to the local vertical.

11.1.1.4.2 Linear vibration

a) Acceleration: _____ *g* maximum.
b) Frequency range: _____ to _____ Hz.

The preceding limits apply to each of the three axes of a coordinate system.

11.1.1.4.3 Angular vibration

a) Acceleration: _____ rad/s^2 maximum.
b) Frequency range: _____ to _____ Hz.

The preceding limits apply to each of the three axes of a coordinate system.

11.1.2 Installation conditions

NOTE—In designing the mounting fixture, consideration should be given to the installation conditions of the application. Reference to a specific mounting block thermal and mechanical design, etc. may be necessary if deemed important to meet performance requirements. Typically, laser gyros are not designed with internal temperature controls and are tested without control of the installation. If temperature control is required, the following should be specified:

a) Unit operating temperature
b) Means of temperature determination
c) Criteria for establishing thermal equilibrium

11.1.3 Electrical excitation and load conditions

11.1.3.1 Circuit excitations

NOTE—Some laser gyros require more than one excitation voltage. For each excitation required, specify the source impedance, voltage level, frequency, ripple, and starting and running currents.

11.1.3.2 Output signals

> NOTE 1—Laser gyro electronics are typically transistor–transistor logic TTL compatible and are the interface between the moving fringe pattern set up by the heterodyning of the opposing laser beams and the output of the gyro. The output typically occurs on two lines coming from the electronics. One form of logic allows one line to have a series of pulses representing positive angular rotation (up counts) about the IA and the other line to have a series of pulses representing negative angular rotation (down counts).
>
> NOTE 2—Another common form of logic allows one line to have a series of pulses representing angular rotation about the IA and the other line to have a series of pulses phase shifted by ± 90° to indicate direction of rotation.
>
> NOTE 3—The type of logic device and the number of unit loads to be driven should be specified.

11.1.3.3 Electrical connections and phasing

Electrical connections and phasing shall be specified on schematic diagram _____.

> NOTE—Grounding, shielding, test points, load requirements, etc., should be specified as needed.

11.1.4 Turn-on procedure

The sequence of operations required to bring the gyro to operating condition shall be _____.

> NOTE—For gyros without built-in anti-lock means, angular motion about the IA should be applied before the gyro is energized; otherwise, damage to the gyro could occur.

11.1.5 Turn-off procedure

The sequence of operations required to turn off the gyro shall be _____.

> NOTE—For gyros without built-in anti-lock means, angular motion about the IA should be continued until the gyro is deenergized, otherwise, damage to the gyro could occur.

11.2 Standard operating and test equipment

11.2.1 General requirements

> NOTE—The accuracy and bandpass characteristics of the test equipment should be compatible with the requirements of the gyro performance specification. Provisions should be made for adequate stabilization of the test equipment. When possible, adequate limitations should be placed on the test equipment to protect the gyro from excessive inputs and loads, such as electrical, mechanical, thermal, etc.

11.2.2 Standard operating equipment

NOTE—Standard operating equipment is the equipment used to provide standard gyro operation and should be listed by name, manufacturer, model, part number, or performance requirements.

11.2.3 Test equipment

NOTE—Test equipment is the equipment used to provide a stimulus or a measurement capability and should be listed by name, manufacturer, model, part number, or performance requirements.

12. Test procedures

12.1 Examination of product (mechanical)

The gyro shall be inspected visually and dimensionally for proper identification, surface finish, and for defects in workmanship to determine that it conforms to the requirements of Clause _____.

12.2 Examination of product (electrical)

The gyro shall be inspected electrically according to the following subclauses to measure insulation resistances, impedances, and dielectric strength.

NOTE—Where preamplifiers or other sensitive devices are included within the gyro case, care should be taken to avoid the application of voltages that could damage those devices.

12.2.1 Insulation resistance test

12.2.1.1 Purpose of the insulation resistance test

The purpose of this test is to measure the insulation resistance between the insulated electrical circuits and between the gyro case and the circuits isolated from the gyro case.

12.2.1.2 Insulation resistance test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, a megohmmeter is required test equipment (see 11.2.3) for this test and should be listed in this subclause.

12.2.1.3 Insulation resistance test—Setup and procedure

Apply _____ ± _____ V dc for a period of _____ ± _____ s between the indicated circuits and between the circuits and the gyro case. The resistance reading shall be recorded.

12.2.1.4 Insulation resistance test—Results

The results shall conform to the requirement of Clause _____.

12.2.2 Impedance test

12.2.2.1 Purpose of the impedance test

The purpose of this test is to measure the impedance of the specified gyro circuits.

12.2.2.2 Impedance test—Equipment

> NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:
>
> a) Impedance bridge with frequency generator adjustable to the specified frequency
>
> b) DC resistance bridge or other suitable resistance measuring system

12.2.2.3 Impedance test—Setup and procedure

The gyro shall be allowed to reach thermal equilibrium at the temperature specified in subclause _____ in order that the final value will not be affected by changing temperature.

Measure all gyro impedances specified in Clause _____. The test current through the circuit shall be as small as practical considering the sensitivity of the measuring instruments. The test current shall be uninterrupted for _____ ± _____ s.

12.2.2.4 Impedance test—Results

The measured impedance quantities shall be recorded and shall conform to the requirements of subclauses _____.

12.2.3 Dielectric strength test

12.2.3.1 Purpose of the dielectric strength test

The purpose of this test is to ascertain that a circuit element or component part of the gyro can operate safely at its rated voltage and withstand momentary overpotentials due to switching, surges, etc., by measuring the leakage current between isolated circuits and between the gyro case and the circuits isolated from the gyro case.

12.2.3.2 Dielectric strength test—Equipment

> NOTE—In addition to the standard operating equipment discussed in 11.2.2, an ac high-voltage source equipped with voltage and current-measuring devices is required test equipment (see 11.2.3) and should be listed in this subclause.

12.2.3.3 Dielectric strength test—Setup and procedure

Apply _____ ± _____ V rms at _____ Hz between the insulated portion and the case ground. The test voltage shall be raised from zero to the specified value as uniformly as possible, at a rate of approximately 500 V rms/s. The test voltage will be maintained for a period of _____ s. At this time, the voltage shall be gradually reduced to avoid surges. During each test, the fault indicator shall be monitored for the leakage current.

12.2.3.4 Dielectric strength test—Results

The results shall conform to the requirements of Clause _____.

12.3 Leak test

12.3.1 Purpose of the leak test

The purpose of this test is to determine if leakage through the gyro case is occurring.

12.3.2 Leak test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Leak detector

b) Vacuum enclosure

12.3.3 Leak test—Setup and procedure

The gyro shall be cleaned of all dirt and grease and placed in a vacuum enclosure at _____ ± _____ [Pa, _____] and stabilized at _____ ± _____ °C gyro temperature. External gas leakage shall then be measured using a leak detector.

12.3.4 Leak test—Results

The measured gas leakage rate shall not exceed _____ cm^3/s.

12.4 Input power test

12.4.1 Purpose of the input power test

The purpose of this test is to measure the input power (current) for each excitation.

12.4.2 Input power test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, power (current) measuring equipment is required test equipment (see 11.2.3) and should be listed in this subclause:

12.4.3 Input power test—Setup and procedure

Connect the power (current) measuring devices and apply power as specified in Clause _____. Record maximum input power (current) for each circuit throughout duration of the test for specified excitation voltage.

12.4.4 Input power test—Results

The results shall conform to the requirements of Clause _____.

12.5 Turn-on time test

12.5.1 Purpose of the turn-on time test

The purpose of this test is to determine the time interval between the application of power and the presence of a useable output of the gyro.

12.5.2 Turn-on time test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Rate table

b) Gyro pulse output measuring equipment

c) Gyro pulse output recording equipment

d) Timing device

12.5.3 Turn-on time test—Setup and procedure

Mount the gyro in the fixture on the rate table so that the IA is parallel to the table rotational axis within _____ arc minutes (′). Connect the gyro to the output measuring equipment. Turn the rate table on and set the rate at _____ [°/s, _____]. Apply power to the gyro and record elapsed time and the gyro output rate.

12.5.4 Turn-on time test—Results

From the recorded data, determine the time interval from the application of power until the indicated rate from the gyro is within _____ [°/s, _____] of the table rate after correcting for earth rate and fixed drift. This time shall conform to the requirements of subclause _____.

12.6 Warm-up time test

12.6.1 Purpose of the warm-up time test

The purpose of this test is to determine the time interval required for the gyro to reach specified performance from the instant that it is energized under specified operating conditions.

12.6.2 Warm-up time test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Means of measuring elapsed time
b) Equipment for measuring the parameter of interest (refer to applicable performance and test subclauses)
c) Recording equipment for the parameter of interest

12.6.3 Warm-up time test—Setup and procedure

Operate the gyro in accordance with the standard test conditions of 11.1, except that the starting conditions shall be _____ and the starting sequence shall be _____.

Energize the gyro and record the parameter of interest as a function of time for _____ [s, min, h].

12.6.4 Warm-up time test—Results

The time for all parameters of interest to reach specified performance shall conform to the requirements of subclause _____.

12.7 Polarity test

12.7.1 Purpose of the polarity test

The purpose of the polarity test is to determine the gyro polarity with reference to the axes defined in subclause _____.

12.7.2 Polarity test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Rate table
b) Means of measuring the gyro output
c) Gyro output recording equipment

12.7.3 Polarity test—Setup and procedure

Mount the gyro on the rate table with the gyro IRA nominally parallel to the table axis. Connect the gyro to the output measuring equipment. Prepare the gyro for test in accordance with the standard test conditions of 11.1. Accelerate the table to _____ °/s with the input vector in the same direction as the positive IRA and record the gyro output polarity. Rotate the table in a similar manner in the opposite direction and again record the gyro output polarity.

12.7.4 Polarity test—Results

The results obtained shall conform to the requirements of subclause _____.

12.8 Operating temperature test series

12.8.1 Temperature sensor characteristics

NOTE—This test applies only when the gyro has externally available terminals for temperature sensor readout.

12.8.1.1 Purpose of the temperature sensor characteristics test

The purpose of this test is to determine the output magnitude of the temperature sensor and its variation with temperature.

12.8.1.2 Temperature sensor characteristics test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Temperature control chamber
b) Temperature sensor output measuring equipment

12.8.1.3 Temperature sensor characteristics test—Setup and procedure

Mount the gyro in a temperature-controlled chamber with the standard test conditions of 11.1. Stabilize the gyro temperature for _____ min at _____ ± _____ °C (operating temperature). Measure the temperature sensor output. Repeat these measurements at the following temperatures: _____ and ± _____ °C.

NOTE—For temperature controlled gyros, one of the temperatures should be above and one below the operating temperature.

During this test, the power to the sensor shall not exceed _____ W. Record chamber temperatures and sensor output.

12.8.1.4 Temperature sensor characteristics test—Results

The output magnitude of the sensor at the specified temperature shall be _____ ± _____ [Ω/°C, V/°C, _____].

12.8.2 Operating temperature test

NOTE—This test applies only to temperature controlled gyros.

12.8.2.1 Purpose of the operating temperature test

The purpose of this test is to ensure that the gyro operating temperature is within the operating temperature range as indicated by the temperature sensor output.

12.8.2.2 Operating temperature test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Temperature measuring equipment
b) Temperature chambers (if required)
c) Temperature sensor output measuring equipment
d) Temperature recording equipment

12.8.2.3 Operating temperature test—Setup and procedure

Operate the gyro in accordance with the standard test conditions of 11.1. The gyro shall be allowed to reach thermal equilibrium for _____ [min, ____] before the temperature measurement is made. Measure and record the temperature sensor output.

12.8.2.4 Operating temperature test—Results

The gyro operating temperature shall be within the operating temperature range specified in subclause _____.

12.9 Gyro scale factor test series

12.9.1 Purpose of the gyro scale factor test series

The purpose of this test series is to measure gyro scale factor, gyro scale factor errors, and gyro scale factor sensitivities.

12.9.2 Gyro scale factor test series—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Rate table with angular readout
b) Gyro pulse output measuring equipment
c) Gyro pulse output recording equipment
d) Anti-lock means compensation equipment
e) Gyro electronics and power supplies
f) Timing device
g) Environmental temperature control equipment

12.9.3 Gyro scale factor test series—Setup and procedure

12.9.3.1 Gyro scale factor and scale factor errors

a) Gyros with built-in anti-lock means: align the rate table rotation axis to within _____ ′ of vertical. Mount the gyro in the fixture on the rate table so that the IA is parallel to the rotational axis within _____ ′. Connect the gyro to the output measuring equipment and operate the gyro in accordance with 11.1. Perform a zero table-rate measurement and record the number and polarity of the output pulses. Apply table rates of ± _____ [°/s, _____] and record the number and polarity of the accumulated output pulses obtained during *n* integral revolutions of the rate table in each direction. Also record the time interval for the *n* revolutions.

NOTE—Sufficient measurements should be made to provide for accurate determination of the scale factor and scale factor errors over the input rate range. A zero table-rate measurement is required to remove the effects of earth rate and uncompensated drift rate from the scale factor data. The zero table-rate test time should correspond with the time required for the number of integral table revolutions. In selecting the test time, consideration should be given to the error contributions of the table angle uncertainty, gyro output quantization, IA misalignment, and gyro output noise.

b) Gyros without built-in anti-lock means: align the rate table rotation axis to within _____ arc minutes of vertical. Mount the gyro in the fixture on the rate table so that the IA is parallel to the table rotational axis within _____ arc minutes. Connect the gyro to the output measuring equipment. Apply a table rate of _____ [°/s, _____]. Operate the gyro in accordance with 11.1. Record the polarity and number of output pulses accumulated during *n* integral revolutions of the rate table. Reverse the direction of rotation of the table and repeat the measurements. Repeat the above procedure for a total number of _____ cycles.

NOTE—Sufficient measurements should be made to provide for accurate determination of the scale factor and scale factor errors over the input rate range. In selecting the number of revolutions and number of cycles, consideration should be given to the error contributions of the table angle uncertainty, gyro output quantization, IA misalignment, and gyro output noise. The data from this test can also be used to obtain bias and random drift parameters.

12.9.3.2 Gyro scale factor sensitivities

12.9.3.2.1 Temperature

Using the environmental control equipment, repeat 12.9.3.1 at temperatures _____ ± _____ °C, after a minimum of _____ [h, ____] dwell at the specified temperatures.

NOTE—Measurements need not be taken at all of the rates of 12.9.3.1.

12.9.3.2.2 Temperature gradient

Using the environmental temperature control equipment, establish a temperature gradient of _____ °C across the gyro case in the IA direction. Using the means of sensing gyro case temperatures, measure and record the actual temperature gradients. Repeat 12.9.3.1 for temperature gradients of _____ °C. The tests shall also be performed for temperature gradients across [LA, NA, _____].

NOTE—Measurements need not be taken at all of the rates of 12.9.3.1.

12.9.4 Gyro scale factor test series—Results

12.9.4.1 Gyro scale factor

NOTE 1—The nominal gyro scale factor, S_o, may be specified at one rate, may be the mean value over several rates, or a model equation may be specified.

NOTE 2—Polarity convention should be in accordance with 5.3.8. Typically, a positive scale factor will occur when positive counts are obtained for a positive input rate.

a) Gyros with built-in anti-lock means: from the test data taken in item a) of 12.9.3.1 for each direction and table rate, compute the gyro scale factor, *S*, by dividing the total angular displacement (in arc seconds [″]) during *n* revolutions of the table by the accumulated number of gyro output pulses corrected for zero table rate. The gyro scale factor shall conform to the requirements of subclause _____.

b) Gyros without built-in anti-lock means: from the test data taken in item b) of 12.9.3.1 at each table rate, compute the gyro scale factor, *S*, for each cycle as follows:

$$S = \left[\frac{2592000n}{N_{\text{pos}} - N_{\text{neg}}} \right] \tag{2}$$

where

S	is the gyro scale factor ("/p)
n	is the number of integral revolutions per half-cycle
N_{pos}	is the number of pulses (including polarity) for *n* revolutions about the positive IA
N_{neg}	is the number of pulses (including polarity) for *n* revolutions about the negative IA

The gyro scale factor shall conform to the requirements of subclause _____.

12.9.4.2 Gyro scale factor errors

12.9.4.2.1 Asymmetry

Compute the asymmetry at specified rates as the ratio of the difference in magnitudes of scale factor measured for positive and negative inputs to one-half the sum of the magnitudes. Asymmetry shall conform to the requirements of subclause _____.

12.9.4.2.2 Nonlinearity

At each rate specified in 12.9.3.1, compute the deviation from the scale factor obtained above. Compute the nonlinearity as the [maximum, 1σ, _____] of the deviations expressed in ppm. The nonlinearity shall conform to the requirements of subclause _____.

NOTE—Alternate methods include weighting, calculation over a limited rate range, and computing nonlinearity after compensation.

12.9.4.2.3 Repeatability

Compute the changes in scale factor [ppm, %], [maximum spread, 1σ, _____] that occur as measured under the same operating conditions (e.g., temperature) when changes in operating or nonoperating conditions (e.g., power, random vibration) have occurred. The gyro scale factor repeatability shall conform to the requirements of subclause _____.

12.9.4.2.4 Stability

Compute the changes in scale factor [ppm, %], [maximum spread, 1σ, _____] that occur within specified periods of fixed operating conditions. The gyro scale factor stability shall conform to the requirements of subclause _____.

12.9.4.3 Gyro scale factor sensitivities

12.9.4.3.1 Temperature

From the test data taken in 12.9.3.2.1, compute the temperature sensitivity as the maximum scale factor change (from that at the specified operating temperature) divided by the temperature range. The temperature sensitivity of the gyro scale factor shall conform to the requirements of subclause _____.

12.9.4.3.2 Temperature gradient

From the test data taken in 12.9.3.2.2, compute the temperature gradient sensitivity as the maximum scale factor change over the specified temperature gradient range, divided by the temperature gradient. The temperature gradient sensitivity of the gyro scale factor shall conform to the requirements of subclause _____.

12.10 Input rate test series

12.10.1 Purpose of the input rate test series

The purpose of this test series is to verify that the scale factor nonlinearity requirements are not exceeded when the input rate limits (and minimum input rates, if applicable) are included.

12.10.2 Input rate test series—Equipment

NOTE—This test series has the same equipment requirements as specified in 12.9.

12.10.3 Input rate test series—Setup and procedure

Install and operate the gyro in accordance with 12.9.3. The data points selected shall include the input rate limits requirement of subclause _____ (and the minimum input rate requirement of subclause _____, if applicable).

12.10.4 Input rate test series—Results

12.10.4.1 Maximum input rate

From the test data taken in 12.10.3, compute the nonlinearity of gyro scale factor, with the maximum input rate of Clause _____ included in the set of data, using the same method described in 12.9.4.1 and 12.9.4.2.2. The gyro scale factor nonlinearity shall conform to the requirements of subclause _____.

12.10.4.2 Minimum input rate

From the test data taken in 12.10.3, compute the nonlinearity of gyro scale factor, with the minimum input rate limit of Clause _____ included in the set of data, using the same method described in 12.9.4.1 and 12.9.4.2.2. The gyro scale factor nonlinearity shall conform to the requirements of subclause _____.

12.11 Anti-lock residual and quantization noise test

12.11.1 Purpose of the anti-lock residual and quantization noise test

The purpose of this test series is to quantify the noise associated with anti-lock residual and with quantization.

12.11.2 Anti-lock residual and quantization noise test—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Gyro pulse output measuring equipment
b) Anti-lock means compensation equipment
c) Gyro pulse output recording equipment
d) Gyro electronics and power supplies
e) Timing device

12.11.3 Anti-lock residual and quantization noise test—Setup and procedure

Mount the gyro so that [IA, LA, NA] is [vertical, horizontal, polar, _____] within _____ [″, _____]. Connect the gyro to the pulse output measuring equipment in accordance with the standard test conditions of 11.1. Energize the gyro and timing device simultaneously and record the gyro output. The test shall be _____ [min, _____] in duration with output pulses accumulated over _____ [s, _____] intervals.

NOTE—Care should be taken during these tests so that the means of reducing anti-lock residual and quantization is consistent with end-use methods. Also, the pulse accumulation period selected should be sufficiently short so as to enhance the combined effect of anti-lock residual and quantization noise.

12.11.4 Anti-lock residual and quantization noise test—Results

From the test data taken in 12.11.4 after the warm-up time, compute the combined anti-lock residual and quantization noise coefficient Q by forming the following Allan variance estimates:

$$\sigma_{\Omega}^{2}(nT_{o}) = S^{2}\left[2n^{2}T_{o}^{2}(M-2n)\right]^{-1}\sum_{m=1}^{M-2n}\left(N_{m+2n}-2N_{m+n}+N_{m}\right)^{2}$$

for $n = 1, 2, 3, \ldots, n_{max} \leq (M–1)/2$ and fitting the results to the polynomials

$$\sigma_{\Omega}^{2} = \frac{R^{2}n^{2}T_{o}^{2}}{2} + \frac{K^{2}nT_{o}}{3} + B^{2}\left[\frac{2}{\pi}\right]\ln(2) + \frac{N^{2}}{nT_{o}} + \frac{3Q^{2}}{n^{2}T_{o}^{2}} \tag{3}$$

in the least squares sense

where

S	is the gyro scale factor
$1/T_o$	is the data sample rate
MT_o	is the data record length
N_m	is the total output pulses accumulated at time mT_o
R, K, B, N	are the random drift coefficients
Q	is the combined quantization and anti-lock noise coefficient

Note that, depending on the test duration, the random drift coefficients R, K, and B may be unimportant in the polynomial fit process.

12.12 Drift rate test series

12.12.1 Purpose of the drift rate test series

The purpose of this test series is to measure the coefficients, repeatabilities, and sensitivities of the bias, random drift, and environmentally sensitive terms.

12.12.2 Drift rate test series—Equipment

NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:

a) Precision positioning means
b) Gyro pulse output measuring equipment
c) Gyro pulse output recording equipment
d) Anti-lock means compensation equipment
e) Gyro electronics and power supplies
f) Timing device
g) Environmental temperature control equipment
h) Magnetic field generating equipment

12.12.3 Drift rate test series—Setup and procedure

12.12.3.1 Bias and random drift

NOTE—In general, the data record length (test duration) should be sufficient to determine performance characteristics at desired confidence levels. The data sample rate should be at least twice the highest frequency of interest. If prefiltering of the raw data is required to minimize the effect of quantization the test results will be different than if prefiltering is not used. PSDs are useful to isolate and identify specific frequency components that may be present in the gyro output. In the test results subclause, the appropriate data reduction procedure should be used. See Annex C for a discussion of data reduction techniques.

a) Gyros with built-in anti-lock means: mount the gyro so that the [IA, LA, NA] is [vertical, horizontal, polar, _____] within _____ [″, _____]. Connect the gyro to the pulse output measuring equipment and energize the gyro in accordance with the standard test conditions of 11.1. Record the number and polarity of the gyro output pulses for a period of _____ [h, _____] with pulses accumulated over _____ [s, _____] sample intervals.

NOTE—Care should be taken to minimize the combined effect of quantization and anti-lock-means compensation on measurements of the parameters of interest. One technique to minimize the anti-lock residual is to accumulate gyro pulses over an integral number of dither cycles. Another technique is to use long sample periods.

b) Gyros without built-in anti-lock means

NOTE—The test procedure is identical to that for measuring scale factor in item b) of 12.9.3.1. Both scale factor and bias/drift rate parameters should be obtained from the same test data.

12.12.3.2 Repeatability

Repeat the procedure of 12.12.3.1 _____ times. Between each iteration, cycle power and/or change the environment and then restabilize at the previous environmental conditions.

12.12.3.3 Environmentally sensitive drift

12.12.3.3.1 Temperature

Using the environmental temperature control equipment, stabilize the gyro at _____ ± _____ °C. Perform the procedure of 12.12.3.1. Repeat this procedure at additional temperature(s) of _____ ± _____ °C.

12.12.3.3.2 Temperature gradient

Using the environmental temperature control equipment, establish a temperature gradient of _____ °C across the gyro case in the _____ direction. Using the means of sensing gyro case temperatures, measure and record the actual temperature gradients. Repeat 12.12.3.1 for temperature gradients of _____ °C. The tests shall also be performed for temperature gradients across _____.

12.12.3.3.3 Magnetic

Using the magnetic field generating equipment, repeat 12.12.3.1 at steady-state flux densities of ± _____ [mT, G] directed along the gyro IA. The tests shall also be performed with the magnetic field directed along the _____ gyro axis.

12.12.3.3.4 Other sensitivities

12.12.4 Drift rate test results

The data processing applied to the drift rate test series data from 12.12.3 includes least squares estimation, prefiltered and unfiltered Allan variance computation, and PSD analysis methods, for the purpose of determining bias and random drift coefficient.

NOTE 1—Annex B presents an overview of dynamic modeling methods including the Allan variance method.

NOTE 2—Agreement between the supplier and the user concerning the modeling method and the data reduction and analysis process is recommended.

12.12.4.1 Warm-up, bias, and random drift

12.12.4.1.1 Warm-up, $D_F(t)$

a) Gyros with built-in anti-lock means: from the test data taken in item a) of 12.12.3.1 during warm-up, compute the bias, D_F, for each sample interval by dividing the accumulated number of pulses in each sample interval by the corresponding sample time, multiplying by the gyro scale factor, and removing the component of earth rate along the IA.

b) Gyros without built-in anti-lock means: from the test data taken in item b) of 12.12.3.1 during warm-up, compute the bias, D_F, for each *cycle* as follows:

$$D_F = \left[\frac{1296000n}{\tau}\right]\left[\frac{N_{pos} + N_{neg}}{N_{pos} - N_{neg}}\right] - \omega_E \tag{4}$$

where

n	is the number of integral revolutions in per half cycle
N_{pos}	is the number of pulses (including polarity) for n revolutions about the positive IA
N_{neg}	is the number of pulses (including polarity) for n revolutions about the negative IA
τ	is the time for n integral revolutions per half cycle
ω_E	is the component of earth rate along the IA

Compute $D_F(t)$ by a least-squares fit to the D_F sample (cycle) data. The results shall conform to the requirements of subclause _____.

12.12.4.1.2 Bias, D_F

a) Gyros with built-in anti-lock means: from the test data taken in item a) of 12.12.3.1 after warm-up, compute the bias, D_F, for each sample interval by dividing the accumulated number of pulses in each sample interval by the corresponding sample time, multiplying by the gyro scale factor, and removing the component of earth rate along the IA.

b) Gyros without built-in anti-lock means. From the test data taken in item b) of 12.12.3.1 after warm-up, compute the bias, D_F, for each cycle as follows:

$$D_F = \left[\frac{1296000n}{\tau}\right]\left[\frac{N_{pos} + N_{neg}}{N_{pos} - N_{neg}}\right] - \omega_E \tag{5}$$

where

n	is the number of integral revolutions in per half cycle
N_{pos}	is the number of pulses (including polarity) for n revolutions about the positive IA
N_{neg}	is the number of pulses (including polarity) for n revolutions about the negative IA
τ	is the time for n integral revolutions per half cycle
ω_E	is the component of earth rate along the IA

Compute D_F by obtaining the average of all the D_F sample (cycle) data. The results shall conform to the requirements of Clause _____.

12.12.4.1.3 Random, D_R

a) Method 1, no prefilter: From the test data taken in item a) of 12.12.3.1 or item b) of 12.12.3.1 after warm-up where no prefilter was used, compute the random drift coefficients *R*, *K*, *B*, and *N* by forming the following Allan variance estimates:

$$\sigma_\Omega^2(nT_o) = S^2\left[2n^2T_o^2(M-2n)\right]^{-1}\sum_{m=1}^{M-2n}\left(N_{m+2n} - 2N_{m+n} + N_m\right)^2$$

for $n = 1, 2, 3, ..., n_{max} \leq (M-1)/2$ and fitting the results to the polynomials

$$\sigma_\Omega^2 = \frac{R^2n^2T_o^2}{2} + \frac{K^2nT_o}{3} + B^2\left[\frac{2}{\pi}\right]\ln(2) + \frac{N^2}{nT_o} + \frac{3Q^2}{n^2T_o^2} \quad (6)$$

in the least squares sense

b) Method 2, with prefilter: From the test data taken in item a) of 12.12.3.1 or item b) of 12.12.3.1 after warm-up where a prefilter was used, compute the random drift coefficients *R*, *K*, *B*, and *N* after forming the Allan variance estimate using the prefiltered test data as follows:

$$\sigma_\Omega^2(nT_o) = S^2\left[2n^2T_o^2(P-2n)\right]^{-1}\sum_{m=1}^{P-2n}\left(N_{m+2n} - 2N_{m+n} + N_m\right)^2$$

for $n = 1, 2, 3, ..., n_{max} \leq (P-1)/2$ and fitting the results to the polynomials

$$\sigma_\Omega^2 = \frac{R^2n^2T_o^2}{2} + \frac{K^2nT_o}{3} + B^2\left[\frac{2}{\pi}\right]\ln(2) + \frac{N^2}{nT_o} + \frac{3Q^2}{n^2T_o^2h} \quad (7)$$

in the least squares sense

where

h	is number of data points used to form an average (filtered) value
$P=M/h$	is total number of averaged pulse measurements
N_i	is total output pulses accumulated at time iT_o
N_m	$=1/h \sum_{i=(m-1)h+1}^{mh} N_i$
N_m	is average value of h consecutive pulse measurements

12.12.4.2 Repeatability

Repeat the procedure of 12.12.4.1 _____ times to compute the changes in bias that occur between specified periods of operation. The results of the bias repeatability [°/h, _____] [maximum spread, 1σ, _____] shall conform to the requirements of subclause _____.

12.12.4.3 Environmentally sensitive drift

12.12.4.3.1 Temperature

From the test data taken in 12.12.3.3.1 after warm-up, calculate the gyro bias as in 12.12.4.1. Compute the bias temperature sensitivity as the slope of a least squares fit of bias over the test temperature range. The bias temperature sensitivity shall conform to the requirements of subclause _____.

> NOTE—It is assumed that the data from gyros with a supplied bias thermal model are compensated with that model prior to the above analysis.

12.12.4.3.2 Temperature gradient

From the test data taken in 12.12.3.3.2 after warm-up, calculate the gyro bias as in 12.12.4.1. Compute the bias temperature gradient sensitivity as the slope of a least squares fit of the bias over the test temperature gradient range. The bias temperature gradient sensitivity shall conform to the requirements of subclause _____.

12.12.4.3.3 Magnetic

From the test data taken in 12.12.3.3.3, compute the magnetic sensitivity as the maximum bias change over the specified field range. The magnetic sensitivity of the gyro bias shall conform to the requirements of subclause _____ for each axis specified.

12.12.4.3.4 Other sensitivities

12.13 IA alignment characteristics

12.13.1 Purpose of the IA alignment characteristics test

The purpose of this test is to measure the misalignment of the IA to the IRA. This misalignment is defined by components of the IA along the NRA and LRA.

12.13.2 IA alignment characteristics test—Equipment

> NOTE—In addition to the standard operating equipment discussed in 11.2.2, the following test equipment (see 11.2.3) is required for this test and should be listed in this subclause:
>
> a) Rate table with angular readout
>
> b) Gyro pulse output measuring and recording equipment
>
> c) Precision alignment test fixture: A right angle test fixture for gyros with built-in anti-lock means or an adjustable test fixture for gyros without built-in anti-lock means

12.13.3 IA alignment characteristics test—Setup and procedure

NOTE—For the following tests, test fixture error may be eliminated by rotating the gyro with respect to the test fixture 180° about the IRA and repeating the tests.

12.13.3.1 Misalignment of the IA about the NRA

a) Gyros with built-in anti-lock means: Mount the gyro on the right angle test fixture on the rate table so that the LRA is parallel to the rate table rotational axis within _____ ″. Operate the gyro in accordance with the standard test conditions of 11.1. Apply a positive table rate of _____ ± _____ °/s. Record the gyro pulse output for *n* revolutions. Repeat the test with a negative input rate whose magnitude is within ± _____ °/s of the positive rate.

b) Gyros without built-in anti-lock means: Mount the gyro on the adjustable test fixture on the rate table so that the LRA is at an angle φ with respect to the rate table rotational axis within _____ ″ and the NRA is perpendicular to the rate table rotational axis within _____ ″. The angle φ and table rotational rate shall be chosen to ensure that angular rates about the IA exceed the specified minimum input rate. Apply a positive table rate of _____ ± _____ °/s, then operate the gyro in accordance with the standard test conditions of 11.1. Record the gyro pulse output for *n* revolutions. Repeat the test with a negative input rate whose magnitude is within ± _____ °/s of the positive rate.

12.13.3.2 Misalignment of the IA about the LRA

a) Gyros with built-in anti-lock means: Mount the gyro on the right angle test fixture on the rate table so that the NRA is parallel to the rate table rotational axis within _____ ″. Operate the gyro in accordance with the standard test conditions of 11.1. Apply a positive table rate of _____ ± _____ °/s.

   Record the gyro pulse output for *n* revolutions. Repeat the test with a negative input rate whose magnitude is within ± _____ °/s of the positive rate.

b) Gyros without built-in anti-lock means: Mount the gyro on the adjustable test fixture on the rate table so that the NRA is at an angle φ with respect to the rate table rotational axis within _____ ″ and the LRA is perpendicular to the rate table rotational axis within _____ ″. The angle φ and the table rotational rate shall be chosen to ensure that angular rates about the IA exceed the specified minimum input rate. Apply a positive table rate of _____ ± _____ °/s, then operate the gyro in accordance with the standard test conditions of 11.1. Record the gyro pulse output for *n* revolutions. Repeat the test with a negative input rate whose magnitude is within ± _____ °/s of the positive rate.

12.13.3.3 Repeatability

Repeat the procedures of 12.13.3.1 and 12.13.3.2 _____ times. In between each iteration, cycle power and/or change the environment and then restabilize at the previous environmental conditions.

12.13.4 IA alignment characteristics test—Results

12.13.4.1 IA misalignment

From the data obtained in 12.13.3 using the appropriate gyro scale factor obtained from 12.9.4.1, calculate the misalignment θ as follows:

$$\theta = \sin^{-1}\left[\frac{S(N_{pos} - N_{neg})}{2592000n}\right] - \phi \tag{8}$$

where

$$\theta \begin{cases} \text{is } \theta_N \text{ for misalignment of the IA about the NRA for rotations about the LRA (see 12.13.3.1)} \\ \text{is } -\theta_L \text{ for misalignment of the IA about the LRA for rotations about the NRA (see 12.13.3.2)} \end{cases}$$

φ is 0 for gyros with built-in anti-lock means

Θ shall conform to the requirements of subclause _____

12.13.4.2 Repeatability

From the data obtained in 12.13.3.3 repeat the calculations of 12.13.4.1. The misalignment angle shall agree with that obtained in 12.13.4.1 within the requirements of subclause _____.

12.14 Generated fields

12.14.1 Electromagnetic interference

The purpose of this test is to measure the electromagnetic interference generated by the gyro.

NOTE 1—MIL-STD-462 is a reference commonly used in the United States to describe test procedures and equipment required for this test.

NOTE 2—To conform to the format used in the remainder of this standard, the test should be outlined in the following manner:

12.14.1.1	Purpose
12.14.1.2	Test Equipment
12.14.1.3	Test Setup and Procedures
12.14.1.4	Test Results

12.14.2 Magnetic leakage test

12.14.2.1 Purpose of the magnetic leakage test

The purpose of this test is to determine the magnetic flux leakage of the gyro.

12.14.2.2 Magnetic leakage test—Equipment

NOTE—In addition to the standard operating equipment given in 11.2.2, magnetic field measuring equipment is required test equipment (see 11.2.3) for this test and should be listed in this subclause.

12.14.2.3 Ambient magnetic field test—Setup and procedure

12.14.2.3.1 Without gyro

Measure and record the magnetic and direction of the ambient magnetic field at the following positions surrounding the gyro test fixture without the gyro present: _____.

12.14.2.3.2 Gyro mounted

Mount the gyro in the test fixture. Repeat the measurements of 12.14.2.3.1.

12.14.2.3.3 Operating

Operate the gyro in accordance with the standard test conditions of 11.1. Repeat the measurements of 12.14.2.3.1.

12.14.2.4 Magnetic leakage test—Results

12.14.2.4.1 Nonoperating

The magnetic leakage for the nonoperating gyro shall be found by subtracting the measurements of 12.14.2.3.1 from the measurements of 12.14.2.3.2 for each position specified. The results shall conform to the requirements of subclause _____.

12.14.2.4.2 Operating

The magnetic leakage for the operating gyro shall be found by subtracting the measurements of 12.14.2.3.1 from the measurements of 12.14.2.3.3 for each position specified. The results shall conform to the requirements of subclause _____.

NOTE—Care should be taken in the design of the fixture to minimize its effect on the measurements.

12.15 Environment test series

These tests are to verify that the gyro performs as specified during or after subjection to environments outside of the standard operating conditions, or both, but within the specified environmental limits.

NOTE 1—Procedures for most environmental tests are covered by existing industry, government, and military documents, e.g., MIL-STD-810. Selection criteria should include the following:

a) Importance of the stability and sensitivity of the parameter in a given environment
b) Practicability of running the test with existing equipment

NOTE 2—Testing should be limited to that required by application. To conform to the format used in the remainder of this standard the tests should be outlined in the following manner:

1 Name of test
1.1 Purpose
1.2 Test equipment

Annex A

(informative)

Examples of laser gyro design features

Examples of laser gyro design features include the following:

a) Gain medium
   1) Helium-neon
b) Optical resonator
   1) Mirrors
   2) Prisms
c) Pathlength control
   1) Piezo-optical
   2) Thermo-optical
d) Combining optics
   1) Mirrors
   2) Prisms
   3) Partial reflectors (beam splitters)
   4) Waveplates
e) Anti-lock means
   1) Magneto-optical (Faraday cell, radio frequency, magnetic mirror)
   2) Mechanical (dither)
   3) Mechanical-optical (mirror dither)
f) Power supplies
   1) Signal electronics
   2) High-voltage discharge(s)
   3) Piezoelectric drive(s)
   4) Anti-lock

Annex B

(informative)

Dynamic and stochastic modeling overview

B.1 Introduction

This annex introduces dynamic and stochastic modeling, as applied to gyro modeling and performance evaluation. This is intended to be a tutorial overview [B12][1].

The general form of the model consists of a mathematical statement of the physical plant equations, an error model consisting of a perturbation model and environmental sensitivities, a stochastic model describing random drift behavior, and a measurement model consisting of a linear combination of the output states and additive measurement noise. This is related in a generic form of a gyro model equation, consisting of the response to inertial inputs, environmental sensitivities, drift rate, and scale factor error contributors.

Methods of determining input–output characteristics (dynamic modeling, system identification) are discussed. Stochastic modeling via time series analysis is introduced, including frequency domain and time domain methods. Emphasis is placed on application of the Allan variance and PSD. An approach to test and analysis is presented. Data acquisition, data reduction, preprocessing, and evaluation of results are discussed.

The general nonlinear problem was posed by Norbert Wiener during the early 1940s [B47]: given the yet-to-be analyzed system, which he defined as a black box, identify and characterize the system in terms of bodies of known structures, or what he called white boxes.

The solution to the linear problem uses various time and frequency domain techniques to find an operational equivalent of the black box, which may then be constructed by combining certain canonical forms of these white boxes. Although the model structure may be different from the true structure, the input–output properties are to be equivalent.

B.1.1 Historical review

The foundation of modeling dates back to approximately 1800, with Gauss's method of least-squares estimation [B14]. Current methods of determining the steady-state input–output characteristics of a variety of devices are based on this approach.

By 1905, Albert Einstein and Willard Gibbs had independently conceived methods of statistical physics. By 1910, Fisher applied the use of the probability density function to maximum likelihood estimation [B14]. In 1930, Wiener [B47] made the first significant use of frequency domain analysis and in 1940 established the beginnings of modern optimization theory [B14]. During World War II, game theory [B45] and operations research [B30] were conceived. These involved some of the earliest applications of modern modeling techniques.

During the 1950s, stochastic process theory and differential game theory were developing. By 1960, Kalman conceived a time domain approach to optimal recursive filter design [B10], [B14], [B23], [B41], [B44]. By 1963, signal identification [B14] and frequency domain time series analysis (TSA) [B4], [B14], [B18], [B34], [B44] methods were developed. In 1965, Tukey and Cooley published their famous paper on

[1] The numbers in brackets correspond to those of the bibliography in Annex D.

the fast Fourier transform [B14]. In 1966, David Allan proposed a simple variance analysis method to the study of oscillator stability [B1], [B2], [B3], [B15], [B19], [B22], [B28], [B29], [B46]. The method has since been applied to gyro drift analysis [B26], [B27], [B37], [B40], [B42]. Parameter identification methods were known by 1968 [B8], [B14].

During the decade that followed, time domain and frequency domain characterization [B6], [B9], [B10], [B27], [B36] of sensors gained importance. By 1970, the Box-Jenkins method of Time Series Analysis was developed [B5], [B14], [B39] together with system identification and adaptive Kalman filter techniques [B14]. During the 1980s, nonlinear multiple-input, multiple-output stochastic optimal control/estimation gained interest. In the 1990s, artificial intelligence and expert systems ideas were finding application to modeling.

B.1.2 Unit model

The term *unit* refers to an operational entity that performs a well-defined, unique function. It can be a sensor, system, or other device. The unit model may be broken down into several parts, as illustrated in Figure B.1. One part, the plant, or physical model, is described by either differential or algebraic equations that express the physics of its operation. This is the deterministic part of the plant that is being addressed with dynamic modeling techniques. The second part, the error model, consists of a perturbational model, which includes the sensitivity to the variation of parameters in the plant equations, and an environmental model, which includes the sensitivities to environmental disturbances. The third part is the stochastic model, which includes the random drift observed under otherwise benign operating conditions. The measurement model consists of a linear combination of the system states and additive output noise.

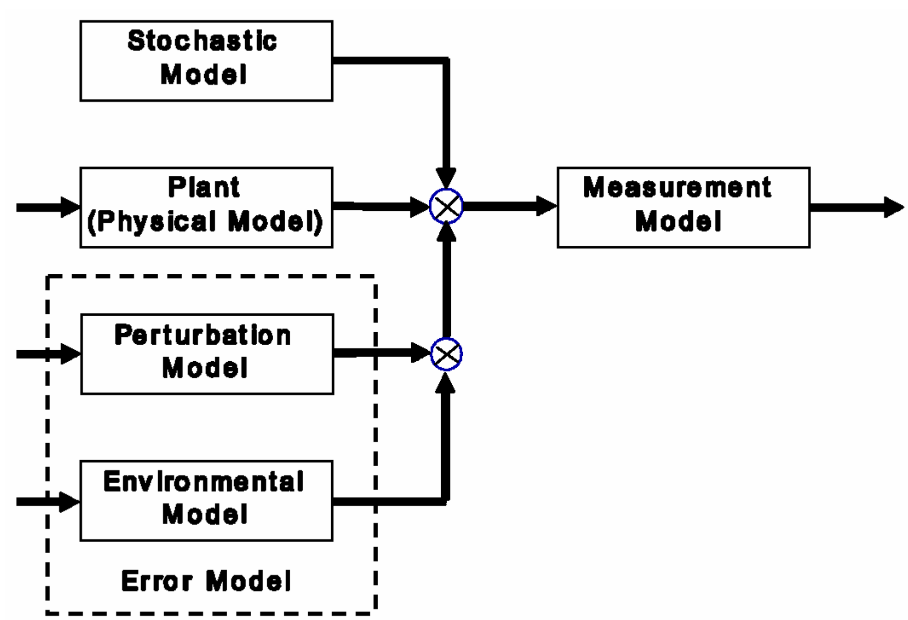


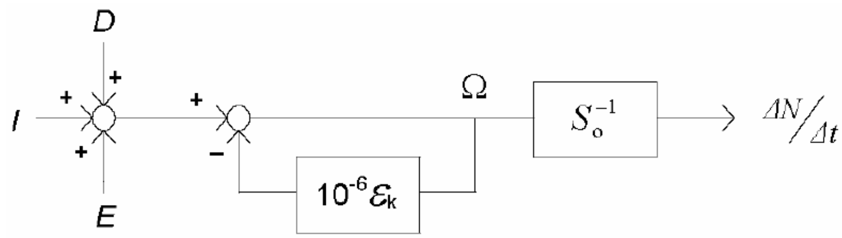
Figure B.1—System model

The idea is that if the unit under test could be so modeled, optimal use of the model to evaluate or possibly improve performance at some higher level could be achieved. That is, through optimal filtering, system error could be reduced to the limit of minimum residual white noise. The model also can be used for performance prediction and evaluation relative to a specification [B10], [B13], [B14], [B41].

B.1.3 Gyro model equation

A generic model equation [B6], [B48] that applies to many types of sensors is shown in Figure B.2. It consists of inertial (including misalignment), environmental, and random (including quantization) contributors.

This approach to compartmentalizing gyro model equations is introduced to better organize the various model components.



$S_o(\Delta N/\Delta t)$	is $[I + E + D]\,(1 + 10^{-6}\varepsilon_k)^{-1}$
S_o	is nominal scale factor
$\Delta N/\Delta t$	is output pulse rate
I	is inertial input terms
E	is environmentally sensitive terms
D	is drift terms
ε_k	is scale factor error terms (ppm)
Ω	is equivalent gyro rate output

Figure B.2—Generic model equation

B.2 Modeling

Some of the important applications of modeling occur in simulation studies, performance evaluation, and Kalman filter design [B6]. The basic difference between dynamic and stochastic modeling is as follows: in dynamic modeling, given one or more inputs (input vector) and one or more outputs (output vector), it is desired to determine the input–output relationships from both time series. Applications include those where random noise is summing at the output.

In stochastic modeling, on the other hand, there may be no direct access to an input. A model is hypothesized that, as though excited by white noise, has the same output characteristics as the unit under test. Such models are not generally unique, so certain canonical forms are chosen. For example, David Allan of the National Institute of Standards and Technology used a power series for the PSD and the corresponding variance analysis in the time domain for the analysis of oscillator stability [B1]. This type of variance analysis is discussed in Annex C. The idea is that one or more white noise sources of strength N_i^2 drive the canonical transfer function(s), resulting in the same statistical and spectral properties as the actual device (black box model). This is also the objective of the gyro drift analysis.

B.2.1 Dynamic modeling

Exclusive of environmental sensitivities, dynamic modeling of an optical gyro involves only scale factor, bias, and misalignment. This model is determined using regression methods, with the gyro being forced with deterministic inputs.

Spinning wheel gyroscopes, such as the two-degree-of-freedom gyro or single-degree-of-freedom gyros that exhibit very high two-degree-of-freedom effects [B3], [B5] are far more interesting subjects for dynamic modeling. The latter are gyros that have significant finite spring constants on the cross axis, or large angular momentum relative to the spring constant. There is then a large interaction from the angular momentum on the response of the gyro. The result is a two-degree-of-freedom model, which is similar to the model equation for the dynamically tuned gyro. The dynamics have a fourth-order characteristic response. The four eigenvalues are dependent on the angular momentum of the gyro. The resulting frequency-dependent transfer function may be analyzed by methods described below.

Many methods of estimating transfer functions are presented in Sinha and Kuszta [B39]. Of the classical approaches, for example, the frequency response method is one of the earlier methods used [B4]. Prior to development of the FFT, the input signal was stepped through discrete frequencies while measuring the relative amplitude and phase from input to output. With current methods, white noise is inserted at a rate proportional summing point. The open and closed loop transfer functions are computed using digital processing. This method allows for estimation in the presence of uncorrelated additive noise.

Time response methods are another classical means (using step and impulse response) to estimate the transfer function. One method, called deconvolution [B39], determines the impulse response from the input and output by using the convolution integral (discrete form) in terms of the sampled data. A matrix equation describes the output at each point of time as a function of the input. The solution to the equation is the impulse response of the black box unit under test.

Another approach to using the impulse (or step) response method models the discrete form of the transfer function (Z-transform) as the ratio of two power series [B39]. Two matrix equations are derived that express the coefficients in terms of the output impulse response. The denominator coefficients may be solved in terms of the impulse response data, and the numerator coefficients may then be solved in terms of the denominator coefficients and the output data.

Several approaches model the output in terms of a difference equation corresponding to the discrete time transfer function [B39]. The output is expressed parametrically in terms of its past values (the autoregressive part) together with present and past values of the input (the moving average part). A matrix equation relates the parameter vector, which comprises the transfer coefficients, to a concatenated set of the input, output data (expressed in a matrix), and output vector comprising another set of the sequential output data. The problem has been variously formulated for noisy data using least-squares or maximum likelihood estimation methods. Recursive forms have been derived for online estimation.

Another technique is to introduce a white noise input and analyze the output relative to the input. This time-domain method is called the correlation method [B4], [B39]. The cross correlation between the input and the output is computed, from which the impulse response is deduced. This method is limited to stationary time series from linear time invariant systems. The main problem with this approach is that as the model becomes more complicated, it becomes more difficult to identify the cross correlation function. The contributors are particularly difficult to decompose into constituent parts when they cover a large dynamic range or overlap in frequency content. If the form of the model is unknown, identification is a difficult job. This is one of the problems with the Box-Jenkins time domain method of time series analysis, and most of the above methods of dynamic modeling.

Frequency domain approaches (also referred to as spectral decomposition) [B44] are usually a better tool for model investigation. The Fourier transform of the correlation function is the PSD. Means of calculating the PSD from the raw data using the FFT are more commonly used. Real time processing with ensemble averaging has considerably improved the ability to analyze transfer functions.

With the frequency domain approach, the transfer function may be estimated from the cross-power spectral density of the output with the input, divided by the PSD of the input [B4]. This gives both the magnitude and phase of the transfer function.

B.2.2 Stochastic modeling

The idea of applying white noise and constructing the transfer function in this manner is important to stochastic modeling. The reason for that is, if the input is white noise, you can estimate the transfer function of a linear, minimum-phase, time-invariant system simply from the power spectrum of the output. Instead of getting the cross PSD between input and output, the transfer function can be estimated from the power spectrum of the output alone. The phase information is uniquely determined from the magnitude response.

Thus, for a linear time-invariant system, by having knowledge of the output only, and assuming white noise inputs, it is possible to characterize the unknown model. Many of the methods are very similar to some of the dynamic modeling methods except that the input is unobservable. The frequency domain approach of using the PSD to estimate transfer functions is straight forward. Even certain pathological cases, such as bias instability [B1], [B26], [B27], [B35], [B42], which looks like a $1/f$ process [B24] (flicker rate noise), and angle quantization noise (characteristically different from continuous white angle noise) [B7], [B26], [B32] can be discerned with careful analysis technique.

As in the case of dynamic modeling, several time domain methods have been devised for stochastic modeling. The correlation function approach [B4], [B32], [B34], [B39] is the dual of the PSD approach, being related as Fourier transform pairs. Similar to the corresponding dynamic modeling method, the equivalent impulse response may be deduced from the autocovariance sequence computed from the output data. One approach models the covariance function as sums of exponentials and damped sinusoids, using least squares estimation to obtain model parameters. This is analogous to expressing the frequency response function in terms of partial fraction expansion.

Another correlation method relates the autocovariance sequence to coefficients of a difference equation, expressed as an autoregressive moving average (ARMA) process. This method was expounded by Box and Jenkins [B5], [B14], [B39]. Correlation methods are very model sensitive and not well suited to dealing with odd power law processes, higher order processes, or wide dynamic range. They work best with *a priori* knowledge based on a model of few terms.

Yet another class of time domain methods, several variance techniques have been devised. They are basically very similar, and primarily differ in that various signal processing, by way of weighting functions, window functions, etc., are incorporated into the analysis algorithms in order to achieve a particular desired result of improving the model characterizations. Many of these are discussed in Rutman [B35]. The two simplest are the Allan variance, and modified Allan variance [B2], [B22], which are discussed in the next clause together with the PSD.

The adaptive Kalman filter is another means of system identification [B14]. The noise covariance and dynamics may be estimated if the form of the model is known. This may be combined with a model adjustment or learning model approach for more flexibility.

B.2.3 Gyro random drift model

Noise contributors in typical gyro models [B14], [B26], [B27], [B36], [B37], [B41], [B42] include white angle noise, quantization noise [B14], [B32] white rate noise, correlated (Markov) random drift, bias instability ($1/f$ or flicker rate), rate random walk, flicker rate ramp (ramp instability), and random rate ramp. Correlated (Markov) drift rate has been recently reported in optical rotation sensors, but is more common in spinning wheel gyros. White angle noise has also been observed in dithered laser gyros at both the gyro and system level. Bias instability (flicker rate) and ramp instability (flicker ramp, flicker acceleration) behave like evolutionary (nonstationary) processes.

Normally, the PSD of a random process is expected to exhibit even-order log-log slopes of –2, 0, +2, and so on, indicating even powers of frequency (+2 slope corresponds to +6 dB per octave). However, the $1/f$ flicker process has a –1 slope PSD (–3 dB per octave). It occurs in certain types of distributed parameter type models; for example, a hypothetical resistance-capacitance (R-C) transmission line excited with white noise current will exhibit a $1/f$ noise voltage at the input [B7]. Because flicker noise is not readily expressed in terms of ordinary state equations, it is sometimes approximated by a Markov model or a multiple stage ARMA (autoregressive moving average) model. Rate random walk is a long-term, very low frequency phenomenon. Even lower frequency is flicker ramp, which can be thought of as instability in the slope of rate ramp, and is equivalent to the integral of $1/f$ noise.

Other model contributors include deterministic ramp (different from flicker ramp), usually removed together with the bias prior to processing, and periodic signals that ought to be removed through filtering, better selection of oscillator frequencies, electromagnetic interference EMI reduction, etc.

B.3 Preferred means of analysis

Of the less restrictive methods of analysis, the PSD and Allan variance methods have more general application to investigation of stochastic models. Thus, they have been adopted as preferred means of analysis in the inertial systems community.

B.3.1 Power spectral density

The PSD is the most commonly used representation of the spectral decomposition of a time series. It is a powerful tool for analyzing or characterizing data, and stochastic modeling. The PSD, or spectrum analysis, is also better suited to analyzing periodic or aperiodic signals than other methods.

To summarize the basic relationships for stationary processes, the two-sided PSD, $S(\omega)$, and covariance, $K(\tau)$, are Fourier transform pairs, related by the following equation:

$$S(\omega)=\int_{-\infty}^{+\infty} e^{-j\omega\tau} K(\tau)\,d\tau$$

$$K(\tau)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{j\omega\tau} S(\omega)\,d\omega$$

$$K_{xy}(\tau)=< x(t)y(t+\tau)>$$

It can be shown [B34] that for nonstationary process, the average covariance $\overline{K}(\tau)$ and average power spectrum $\overline{S}(\omega)$ are related in the same way.

The transfer function form of the stochastic model may be estimated directly from the PSD of the output data (on the assumption of an equivalent white noise driving function). Similar to curve-fitting a Bode plot, the transfer function may be estimated using the pole-zero form, partial fraction expansion, power series, ARMA model spectral estimation, etc.

B.3.1.1 Useful properties

For linear systems, the output PSD is the product of the input PSD and the magnitude squared of the system transfer function. If state space methods are used, the PSD matrices of the input and output are related by the system transfer function matrix by the following equation:

$$S_{yy}(\omega)=H(j\omega)S_{xx}(\omega)H^{*T}(j\omega)$$

where

H^{*T} is the complex conjugate transpose of H

Thus, for the special case of white noise input, the output PSD directly gives the system transfer function.

The PSD and autocorrelation function are Fourier transform pairs. The Fourier transform representation of the PSD is directly related to the bilateral Laplace transform derived from the transfer function of the corresponding stochastic model. The corresponding Allan variance of a stochastic process may be uniquely derived from its PSD; however, there is no general inversion formula. The same is true of the relationship between a process probability density function and its PSD.

The white noise covariances of process and measurement noise pertaining to the (continuous) Kalman filter theory are identically the corresponding two-sided PSDs white noise strengths expressed in units squared per hertz.

For a process to have finite power, its PSD must eventually terminate in a negative slope at high frequencies. This property must be produced to satisfy the Nyquist sampling criterion for sampled data. This is discussed further in B.4.2. Likewise, a PSD cannot continue to rise (without limit) toward zero frequency (over a finite time interval). In practice, this is limited by the finite length of the time series.

Certain process, such as periodic, “narrow band,” or quantization noise are better described by their energy spectrum or an integrated PSD, since their PSD amplitudes are dependent on the resolution bandwidth, a function of sampling rate. This property is used to distinguish, for example, quantization noise from white angle noise. Both have a +2 slope rate PSD, but whereas the amplitude of the white angle noise PSD is independent of the sample rate (resolution bandwidth), the amplitude of the quantization noise PSD is directly proportional (approximately) to the sample period (at the lower frequencies), as can be seen by rewriting the expression for quantization noise from Annex C, as shown in the following equation:

$$S(\omega) = \omega^2 Q^2 T \left(\frac{\sin \omega \frac{T}{2}}{\omega \frac{T}{2}} \right)^4$$

$$\approx \omega^2 Q^2 T \qquad \qquad for\ \ T << \frac{1}{\omega}$$

The time average PSD of a nonstationary process has the properties of a PSD of a stationary process [B34]. With present real-time PSD analysis, evolutionary spectra can be represented as either two-dimensional or time-varying PSDs. For nonstationary processes, the covariance function is a function of two time variables (e.g., the age variable and the “running time” variable). In the frequency domain, a two-dimensional PSD is defined as the double Fourier transform of the covariance function and is a function of two frequency variables.

For reference, the relationships are given by the following equation:

$$K_{xy}(t_1, t_2) = \overline{x(t_1) y(t_2)} = < x(t_1) y(t_2) >$$

Alternative representations are shown in the following equation:

$$\overline{x(t_1) y(t_2)}$$

$$< x(t_1) y(t_2) >$$

The two-sided, two-dimensional PSD $\Gamma(\omega_1,\omega_2)$ and the general covariance function $K(t_1,t_2)$ are double Fourier transform pairs, related by the following equation:

$$\Gamma(\omega_1,\omega_2)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}K(t_1,t_2)\,e^{-j(\omega_1 t_1-\omega_2 t_2)}\,dt_1\,dt_2$$

$$K(t_1,t_2)=\frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\Gamma(\omega_1,\omega_2)\,e^{j(\omega_1 t_1-\omega_2 t_2)}\,d\,\omega_1\,d\,\omega_2$$

For further discussion on nonstationary processes, see Papoulis [B32].

For certain types of spectra, where wide dynamic range is required, such as when periodic content is present, application of a "window" function as part of the FFT processing can improve resolution. Windows such as the Hamming and Von Hann (Hanning) windows reduce the impact of Gibb's phenomenon resulting from truncation of a time series. Thus, for example, two adjacent spectral peaks of significantly different amplitudes may be separated, at the expense of somewhat broadening them. These issues are discussed further in B.3.2 and [B4], [B17], and [B35]. PSD properties of the various contributors are given in Table B.1.

B.3.2 Allan variance

The two-sided (i.e., first difference) Allan variance was developed in the mid 1960s and adopted by the time and frequency standards community for the characterization of phase and frequency instability of precision oscillators. Because of the close analogies to inertial sensors, the method has been adapted to random drift characterization of a variety of devices. Annex C supplies more detail than the following discussion.

The old method of specifying drift in terms of a single rms number, even when associated with a correlation time, was inadequate for predicting system performance, leading to some very conservative means of specification. Later, frequency domain methods proved superior for evaluating performance, but difficult for nonsystem analysts to understand.

The Allan variance is a reasonable compromise. Simply put, it is a method of representing rms random drift error as a function of averaging time. It is simple to compute, much better than having a single rms drift number to apply to a system error analysis, and relatively simple to interpret and understand. It is not well suited to rigorous analysis, but a reasonable second step in the modeling process (after the quick look, discussed in B.4.3). Its most useful application is in the specification and estimation of random drift coefficients in a previously formulated model equation.

The basic Allan variance relations are summarized as follows:

$$\theta(t)=\int^{t}\Omega(t')dt'$$

$$\overline{\Omega}_k(\tau)=\frac{\theta(t_k+\tau)-\theta(t_k)}{\tau}$$

Then the Allan variance is defined by the following equation:

$$\sigma_\Omega^2(\tau)=\frac{1}{2}\overline{\left(\overline{\Omega}_{k+1}-\overline{\Omega}_k\right)^2}=\frac{1}{2}<\left(\overline{\Omega}_{k+1}-\overline{\Omega}_k\right)^2>$$

and is related to the two-sided PSD, $S_\Omega(f)$, given by the following equation:

$$\sigma_\Omega^2(\tau) = 4\int_0^\infty s_\Omega(f)\frac{\sin^4 \pi f\tau}{(\pi f\tau)^2}\,df$$

There is no inversion formula.

One of the most notable deficiencies of the standard Allan variance is the nonunique characterization of white angle, flicker angle and quantization noise at the high-frequency end, and random rate ramp versus flicker ramp at the low frequency end. These, however, may be sorted out by using the modified Allan variance and/or prewhitening methods discussed in B.4.3. Other approaches include applying a prefilter to the time series (such as a triangular filter), or alternatively incorporating it into the Allan variance computation, or applying it as a window. The theory and use of windows is discussed by Hamming [B17]. This approach is equivalent to some of the other variance methods discussed by Rutman [B35].

If the standard Allan variance is viewed as an application of a variable rectangular window to the time series, then the modified Allan variance may be viewed as the application of a variable triangular window to the time series. Maximum efficiency is obtained by using an algorithm that uses maximum overlap. Some of these issues are discussed by Stovall [B40], where a triangular prefilter with maximum overlap is suggested. The more obvious advantage of the higher order filtering is in dealing with these “high pass” processes (those with positive PSD slopes) by terminating the filtered process in a controlled low pass characteristic, insuring a finite variance.

Allan variance properties of the various contributors are given in Table B.1.

B.4 Test, data processing, and analysis considerations

This clause discusses methods of test, data acquisition, and data processing. Several suggestions are made to improve the efficiency of these operations and the subsequent analysis.

B.4.1 Approach

General test conditions and equipment are discussed in Clause 11, and test procedures are discussed in Clause 12. It is important to control the influence of external environments on the test. External sources of error should be removed or compensated for in an appropriate prefilter. Differential techniques (such as back-to-back sensor test) are also effective. Error detection/correction is also an important step of the data acquisition process.

Sample rates, anti-aliasing filtering, and record lengths are to be chosen with regard to various system considerations [B4]. The longer term effects of random drift on performance with regard to the mission profile should be separated out using signal processing techniques prior to data acquisition, and the high-frequency data should be analyzed separately.

B.4.2 Data acquisition

In terms of economy, efficiency is achieved by combining data reduction methods into the data acquisition process [B31], [B43], thus transforming the data into a more usable form. Generally, the sample rate is selected to be at least twice the sample bandwidth (limited by filtering to the highest frequency of interest), however, Papoulis [B32] shows that six samples per bandwidth cycle are required to characterize a signal from its past values only. The record length should be at least several times the required performance interval, as dictated by the mission profile. Coverage of too large a temporal dynamic range, however, is uneconomical, impractical and unnecessary; collecting 0.1 ms data for 1000 h yields a large, unwieldy number of data points.

Table B.1—Properties of noise and drift process in frequency and tau domain

Stochastic model contributor				Asymptotic properties of PSD		PSD	Asymptotic properties of root Allan variance		Allan variance
	Equivalent white noise input								
Nomenclature	**Generic**	**This standard**	**Other name**	$S_\theta(f)$	$S_\Omega(f)$	$S_\Omega(f)$	$\sigma_\theta(\tau)$	$\sigma_\Omega(\tau)$	$\sigma_\Omega^2(\tau)$
White angle	N_θ^2	Φ^2	Angle measurement noise	0	+2	$(2\pi f)^2\,\Phi^2$	$-\frac{1}{2}$	$-1, (-\frac{3}{2})$ See Note 1	$\frac{3\Phi^2}{\tau^2} f_n$
Angle quantization	—	Q^2	White angle energy spectrum	0	+2	$\frac{4Q^2}{\tau}\sin^2 \pi f\tau$ See Note 3	0	–1	$\frac{3Q^2}{\tau^2}$ See Note 4
Flicker angle	$N^2_{F\theta}$	—	Pink angle noise	–1	+1	$2\pi f N^2_{F\theta}$	0	–1	$\frac{N^2_{F\theta}}{2\pi\tau^2}\left[3(2+\ln 2\pi f_n\tau)-\ln 2\right]$
Angle random walk, white rate noise	N_Ω^2	N^2	Red angle noise	–2	0	N^2	$+\frac{1}{2}$	$-\frac{1}{2}$	$\frac{N^2}{\tau}$
Rate quantization	—	$\dot{Q}^2$	Discrete white rate noise or white rate energy spectral density	–2	0	$\dot{Q}^2 T\left[\frac{\sin \omega T/2}{\omega T/2}\right]^2$	+1	0	—
Flicker rate	$N_{F\Omega}^2$	B^2	Bias rate instability or pink rate noise	–3	–1	$\frac{B^2}{2\pi f}$	+1	0	$\frac{2B^2\ln 2}{\pi}$
Markov rate	$N_{c\Omega}^2$	q^2	Correlated drift rate	–2, –4	0, –2	$\frac{(q_c\tau_c)^2}{1+(2\pi f\,\tau_c)^2}$	$+\frac{3}{2}, +\frac{1}{2}$	$+\frac{1}{2}, -\frac{1}{2}$	$\frac{(q_c\tau_c)^2}{T}\left[1-\frac{\tau_c}{2\tau}\left(3-4e^{-\frac{\tau}{\tau_c}}+e^{-\frac{2\tau}{\tau_c}}\right)\right]$
Rate random walk	$N_{\dot{\Omega}}^2$	K^2	Red rate noise	–4	–2	$\frac{K^2}{(2\pi f)^2}$	$+\frac{3}{2}$	$+\frac{1}{2}$	$\frac{K^2\tau}{3}$

Table B.1—Properties of noise and drift process in frequency and tau domain *(continued)*

Stochastic model contributor				Asymptotic properties of PSD		PSD	Asymptotic properties of root Allan variance		Allan variance
	Equivalent white noise input								
Nomenclature	Generic	This standard	Other name	$S_\theta(f)$	$S_\Omega(f)$	$S_\Omega(f)$	$\sigma_\theta(\tau)$	$\sigma_\Omega(\tau)$	$\sigma_\Omega^2(\tau)$
Flicker angular acceleration	$N_{F\dot{\Omega}}^2$	R^2	Ramp instability or pink angular acceleration noise	–5	–3	$\frac{R^2}{(2\pi f)^3}$	+2	+1	—
Random bias	$\Omega(0)$	B_0^2	Bias or fixed drift	See Note 2	See Note 2	$B_0^2\delta(f)$	See Note 2	See Note 2	—
Random ramp	$\dot{\Omega}(0)$	R_0^2	Rate ramp	See Note 2	See Note 2	—	See Note 2	See Note 2	$\frac{R_0^2\tau^2}{2}$
Periodic rate	Ω_0	—	Harmonic	Discrete spectra	Discrete spectra	$\frac{1}{2}\Omega_0^2\delta(f-f_0)$	See Note 2	See Note 2	$\Omega_0^2\left[\frac{\sin^2\pi f_0\tau}{\pi f_0\tau}\right]^2$

NOTE 1—mod $\sigma(\tau)$ (modified root Allan variance)

NOTE 2—Remove by regression or by filtering.

NOTE 3—At the highest frequencies, there could be a rolloff of the PSD due to signal processing in the sensor electronics.

NOTE 4—At the shortest averaging times, there could be attenuation of the Allen variance due to signal processing in the sensor electronics.

The recommended approach is to limit the time/frequency domain dynamic range (of record length to sample period) to about three orders of magnitude. This is done by dividing the total range into overlapping intervals of geometrically increasing lengths. Thus, the high frequency data is acquired for a short period of time. Lower frequency data is filtered (integrated) and acquired for a longer period. (e.g., 100 μs data collected for 0.1 s, 0.01 s data for 10 s, 1 s data for 1000 s, and 100 s data for 10^5 s). Signal processing is used to remove the undesired effects outside the bandwidth of interest. It is noted that frequency domain (PSD) analysis is particularly appropriate to investigating high-frequency phenomena as well as long-term random drift, whereas time domain analysis should be confined to the low frequency phenomena, where it is often simpler to use.

The appropriate sampling rates/record lengths should be chosen to overlap about one decade of frequency (time), consideration being given to the particular characterization of the process. Prior characterization of the process is necessary for proper determination of data acquisition parameters. One approach is to choose the geometric means of the record length and sample period to correspond to the geometric mean of the corresponding frequency or time domain range of the dominant characteristic of interest. For example, if bias instability (flicker) is to be observed in the interval 10^4 to 10^6 s (1.0 μHz to 100 μHz) the geometric mean is 10^5 s (10 μHz); the corresponding sample period and record length would be ~ $10^3 \sqrt{10}$ s and ~ $10^6 \sqrt{10}$ s, respectively. However, if the mission profile provided for a fix (or calibration) at 10^5 s, a record length of $10^5 \sqrt{10}$ s might be satisfactory, thus requiring only two decades of time (or frequency). Another aspect of this idea relates to "prewhitening," which is usually done as part of a postprocessing operation after the data is collected but prior to detailed analysis.

Quantization noise, discussed in Britting *et al.* [B7] is equivalent to random binary transmission, as discussed in Papoulis [B32]. Quantization noise can limit the ability to estimate model coefficients efficiently. It can also have a deleterious effect on short term performance. Since quantization is often dictated by sensor design consideration, such as RLG scale factor, or otherwise constrained to unsatisfactory levels, its effect must be handled externally by signal processing. This can be done as part of the data acquisition/reduction process, or in postprocessing. It usually entails processing through a second or higher order digital filter, with calculations scaled to higher precision.

B.4.3 Postprocessing

Postprocessing of the acquired data includes the quick look, data editing, trend removal, digital filtering of other deterministic signal (i.e., periodic), and other prewhitening signal processing.

The quick-look is a cursory visualization of each record of the data in the form of graphical time series. Bad data is edited out and replaced by interpolated or simulated data. Trends are observed for removal and separate analysis by least squares estimation techniques. Possible periodic content is observed for removal by other filtering techniques. Such removal is a first step in the prewhitening processing.

Best estimates of the model are achieved when the estimation error (residuals, innovations) is a white noise process. Some analysis techniques, such as an adaptive Kalman filter can be designed to do this. Another approach requires a priori knowledge of the stochastic model [B31], [B43].

In this approach, an inverse filter is mechanized in the signal processing to prewhiten the data (see Figure B.3). This may simply entail applying either an integration operation (as with angle white noise) or a differencing operation (as with rate random walk). As a practical matter, additional high frequency poles must be added to band limit the resulting dynamic range required for subsequent analysis.

B.4.4 Time series analysis considerations

Subsequent to data acquisition and postprocessing, the resulting processed data records are prepared for time series analysis. Whichever method is used, time- or frequency-domain analysis, there are some suggested guidelines to follow.

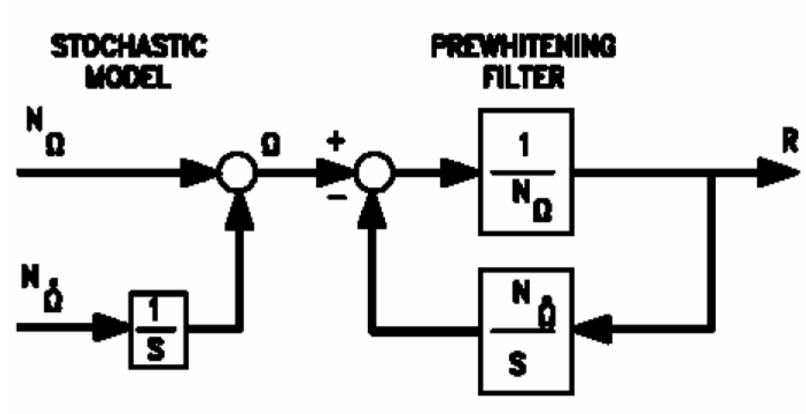


Figure B.3—Example of prewhitening (simplified)

$$S_\Omega(\omega) = N_\Omega^2 + \frac{N_{\dot{\Omega}}^2}{\omega^2}$$

$$H(s) = \frac{\dfrac{1}{\hat{N}_\Omega}}{1 + \dfrac{\hat{N}_{\dot{\Omega}}}{s}\dfrac{1}{\hat{N}_\Omega}} = \frac{\dfrac{s}{\hat{N}_{\dot{\Omega}}}}{1 + s\dfrac{\hat{N}_\Omega}{\hat{N}_{\dot{\Omega}}}} = \frac{1}{\hat{N}_\Omega + \dfrac{\hat{N}_{\dot{\Omega}}}{s}}$$

$$S_R(\omega) = H_{R\Omega}(j\omega)\, S_\Omega(\omega)\, H_{R\Omega}^{*T}(j\omega) = \left|H_{R\Omega}(j\omega)\right|^2 S_\Omega(\omega)$$

$$= \left[\frac{1}{\hat{N}_\Omega^2 + \dfrac{\hat{N}_{\dot{\Omega}}^2}{\omega^2}}\right]\left[N_\Omega^2 + \frac{N_{\dot{\Omega}}^2}{\omega^2}\right]$$

$$= 1 \qquad if \begin{pmatrix} \hat{N}_\Omega = N_\Omega \\ \hat{N}_{\dot{\Omega}} = N_{\dot{\Omega}} \end{pmatrix}$$

Each data record so constructed contains piecewise information to be extracted and reassembled in a composite analysis, which will then display the full dynamic range of the desired result. In reassembling the computed information in the appropriate chart, it may be necessary to undo some of the previous operations to put the output data into the proper format and proper units. If, for example, prewhitening processing was applied, the corresponding postdarkening operation will be necessary to reconstruct the characteristic signatures of the analyzed data.

Prior to this operation, however, the model coefficients are first estimated. A first approximation (second approximation if prewhitening has been performed) can be estimated by sketching in the asymptotes to the charted data analysis, and computing approximate model coefficients. A prewhitening filter may be derived and the error coefficients from the residuals may be reestimated. Alternatively, a more rigorous weighted least squares procedure, which weights by the inverse error covariance, may be used. Error bounds on the coefficient should also be computed to establish the goodness-of-fit.

With the coefficients thus determined, and the composite data analysis assembled on the appropriate chart, the resulting characteristic curve fit can be superimposed on the charted data, together with the error bounds to give a detailed representation of the stochastic model characterization. The coefficients are also used to construct a block diagram (Figure B.4) and generate the detailed state equations describing the stochastic model.

Figure B.5 and Figure B.6 illustrate the resulting piecewise asymptotic representation of the corresponding Allan variance and PSD of a hypothetical gyro with the following parameters:

$N = 0.001°/\mathrm{h}^{1/2}$	rate white noise
$B = 0.001°/\mathrm{h}$	bias instability
$K = 0.0001°/\mathrm{h}^{3/2}$	rate random walk
$Q = 0.577''$	quantization noise (result of 2”/pulse gyro scale factor)

B.5 Conclusion

Inertial systems design and performance prediction depends on accurate knowledge of sensor level behavior.

This annex has attempted to provide a brief introduction and roadmap to the study of a very extensive subject of dynamic and stochastic modeling as it applies to this purpose.

Through better understanding of the modeling process and standardization of test and reporting of data, the inertial system and gyro designer can more effectively meet their goals.

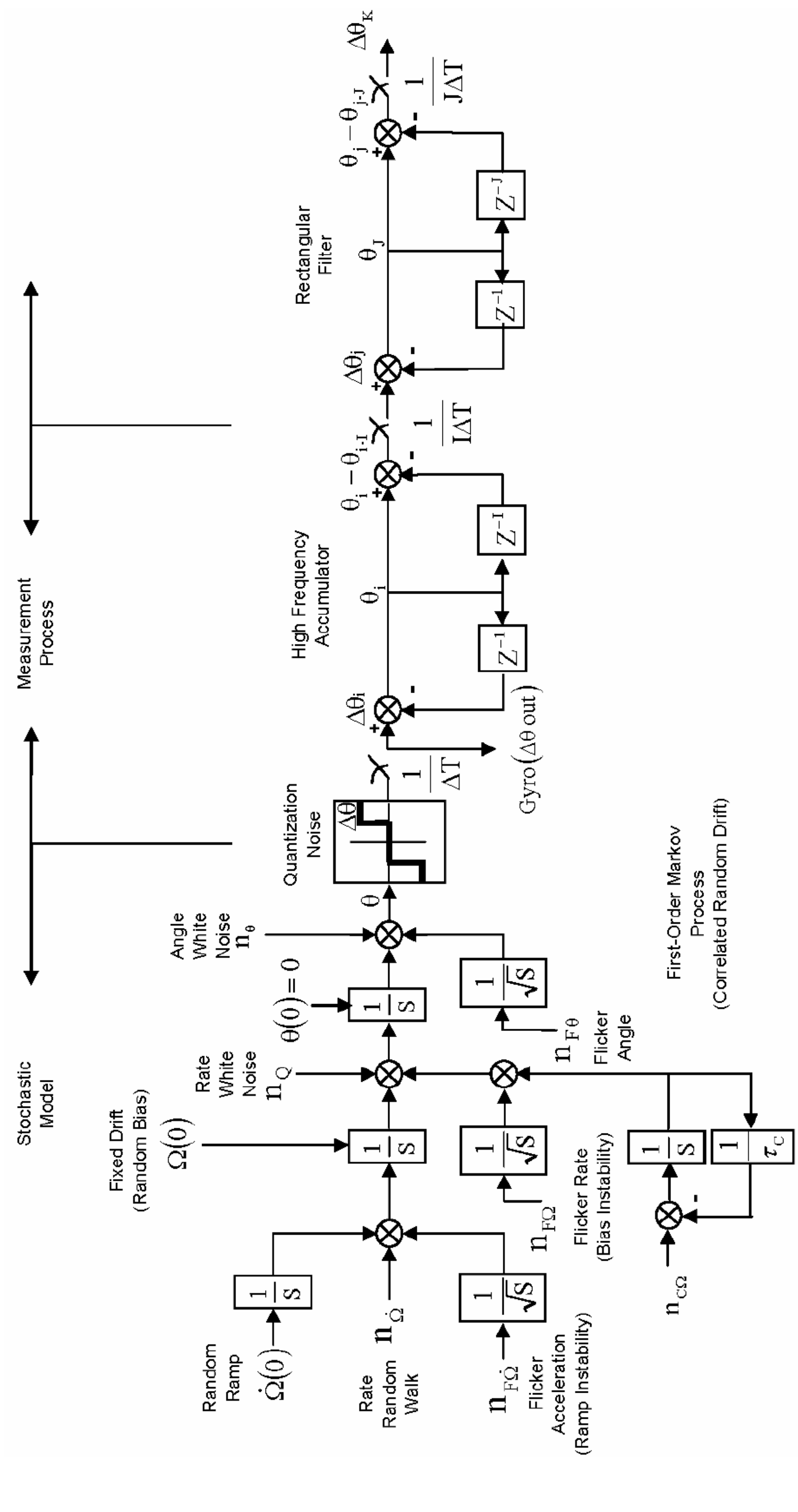


Figure B.4—Block diagram of stochastic model through data acquisition

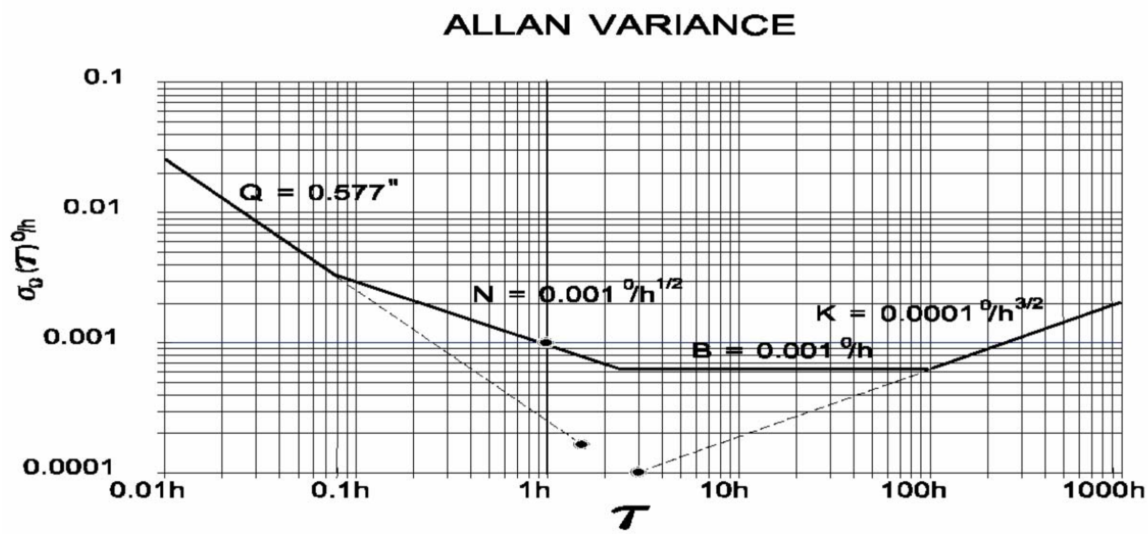


Figure B.5—Piecewise representation of hypothetical gyro in Allan variance

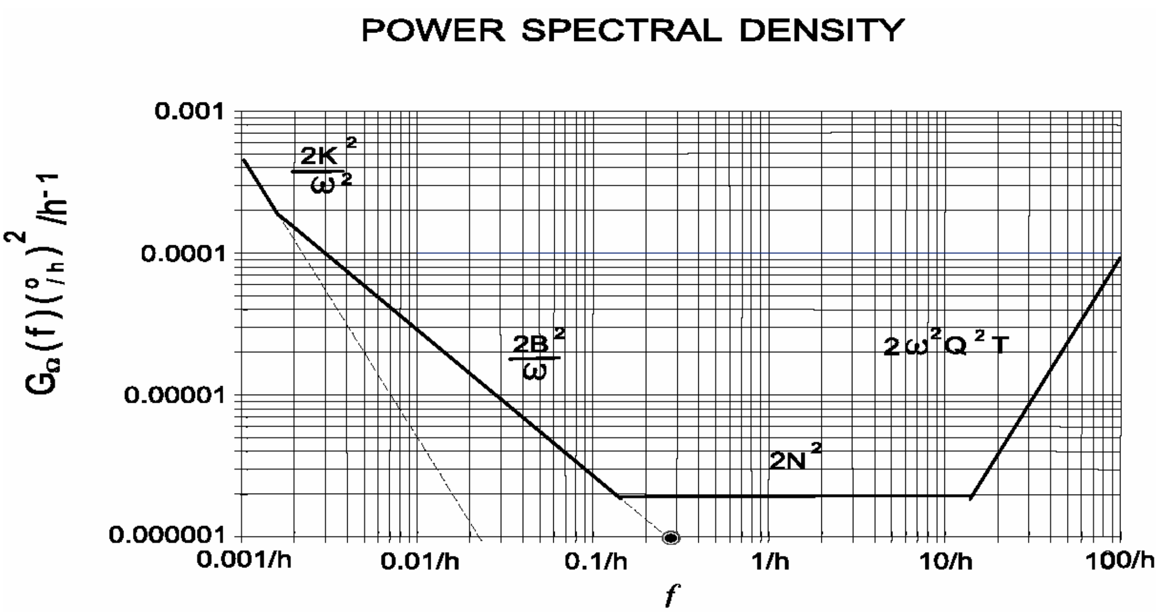


Figure B.6—Piecewise representation of hypothetical gyro in PSD form

Annex C

(informative)

An overview of the Allan variance method of RLG noise analysis

C.1 Allan variance background

Allan variance is a time domain analysis technique originally developed to study the frequency stability of oscillators [B1]. It can be used to determine the character of the underlying random processes that give rise to the data noise. As such, it helps identify the source of a given noise term present in the data; whether it is inherently in the instrument or in the absence of any plausible mechanism within the instrument, its origin should be sought in the test setup. The Allan variance, adopted in this standard, may be used as a stand alone method of data analysis or to complement any of the frequency domain analysis techniques. It should be mentioned that the technique can be applied to the noise study of any instrument. Its value, however, depends on the degree of understanding of the physics of the instrument. The following is an overview of the Allan variance and its adaptation to the noise properties of RLGs, as described in [B42].

In the Allan variance method of data analysis, the uncertainty in the data is assumed to be generated by noise sources of specific character. The magnitude of each noise source covariance is then estimated from the data. The definition of the Allan variance and a discussion of its use in frequency and time metrology is presented in [B1] and [B21].

In this annex, Allan's definition and results are related to the five basic RLG noise terms and are expressed in a notation appropriate for gyro data reduction. The five noise terms are angle random walk, rate random walk, bias instability, quantization noise, and rate ramp.

Consider N samples of data[1] from a gyro with a sample time of τ_0. Form data clusters of durations τ_0, $2\tau_0$, ..., $m\tau_0$ ($m < (N-1)/2$) and obtain averages of the sum of the data points contained in each cluster over the length of that cluster. The Allan variance is defined as the two-sample variance of the data cluster averages as a function of cluster time.

To be specific, the Allan variance can be defined either in terms of the output rate, $\Omega(t)$, or the output angle (the quantity actually measured), as follows:

$$\theta(t) = \int^{t} \Omega(t')\,dt'$$

The lower integration limit is not specified as only angle differences are employed in the definitions. Angle measurements are made at discrete times given by $t = k\tau_0$, $k = 1, 2, 3, ..., N$. Accordingly, the notation is simplified by writing $\theta_k = \theta(k\tau_0)$.

The average rate between times t_k and $t_k + \tau$ is given by the following equation:

$$\overline{\Omega}_k(\tau) = \frac{\theta_{k+m} - \theta_k}{\tau}$$

[1] Sometimes referred to a time series or data streams.

where

τ is equal to $m\tau_o$

The Allan variance is defined as follows:

$$\sigma^2(\tau) = \frac{1}{2}\,\pi\,\left(\overline{\Omega}_{k+m} - \overline{\Omega}_k\right)^2\,\phi$$

$$= \frac{1}{2\,\tau^2}\,\pi\,\left(\theta_{k+2m} - 2\,\theta_{k+m} + \theta_k\right)^2\,\phi$$

where

< > is the ensemble average

The rate Allan variance is estimated as follows:

$$\sigma_\Omega^2(\tau) = \frac{1}{2\,\tau^2(N-2m)} \sum_{k=1}^{N-2m} \left(\theta_{k+2m} - 2\,\theta_{k+m} + \theta_k\right)^2$$

The Allan variance obtained by performing the prescribed operations, is related to the PSD of the noise terms in the original data set. The relationship between Allan variance and the two-sided PSD is given by the following equation:

$$\sigma^2(\tau) = 4\int_0^\infty S_\Omega(f)\,\frac{\sin^4(\pi f\tau)}{(\pi f\tau)^2}\,df \tag{C.1}$$

where

$S_\Omega(f)$ is the two-sided rate noise PSD.[2]

Equation (C.1) is the key result that will be used throughout to characterize the rate noise PSD from the Allan variance calculations. Its physical interpretation is that the Allan variance is proportional to the total noise power of the gyro rate output when passed through a filter with the transfer function of the form $\sin^4(x)/(x)^2$. This particular transfer function is the result of the method used to create and operate on the clusters.

It is seen from Equation (C.1) and the above interpretation that the filter bandpass depends on τ. This suggests that different types of random processes can be examined by adjusting the filter bandpass, namely by varying τ. Thus the Allan variance provides a means of identifying and quantifying various noise terms that exist in the data. It is normally plotted as the square root of the Allan variance versus τ, $[\sigma(\tau)]$, on a log-log plot.

The following paragraphs show the application of Equation (C.1) to a number of noise terms that are either known to exist in the RLG or otherwise influence its data. Detailed derivations are given in [B42]. The physical origin of each noise term will be discussed.

[2] Note that $S_\Omega(f)$ is the PSD of stationary random processes. For nonstationary processes, such as flicker noise, the time average PSD should be used.

C.1.1 Angle random walk

The two main sources for this error follow:

a) Randomized dither: In dithered RLGs, the dither amplitude is randomized each half-dither cycle. This is to avoid the buildup of angular errors during the zero rate crossings of the sinusoidal dither [B25]. Accomplishing this, however, is at the expense of creating a random walkin the gyro phase angle. Laser gyros that do not employ dither as an anti-lock means do not exhibit this type of random walk.

b) Spontaneous emission of photons: A smaller component of the RLG angle random walk is caused by the spontaneously emitted photons that are always present in the lasing action. Its magnitude is smaller than the dither-induced random walk by a factor of 2 to 10. The angle random walk due to spontaneously emitted photons is called the quantum limit [B11], [B38].

Other high-frequency noise terms that have correlation time much shorter than the sample time can also contribute to the gyro angle random walk. However, most of these sources can be eliminated by design. These noise terms are all characterized by a white noise spectrum on the gyro rate output.

The associated rate PSD is represented by the following equation:

$$S_{\Omega}\left(f\right)= N^{2} \tag{C.2}$$

where

N is the angle random walk coefficient.

Substitution of Equation (C.2) in Equation (C.1) and performing the integration yields the following equation:

$$\sigma^{2}\left(\tau\right)=\frac{N^{2}}{\tau} \tag{C.3}$$

As shown in Figure C.1, Equation (C.3) indicates that a log-log plot of $\sigma(\tau)$ versus τ has a slope of –1/2. Furthermore, the numerical value of N can be obtained directly by reading the slope line at $\tau = 1$.

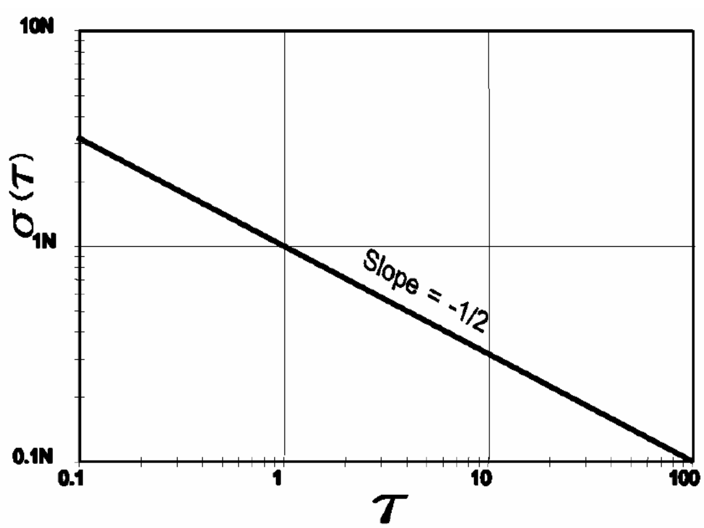


Figure C.1—*σ(τ)* plot for angle random walk

C.1.2 Bias instability

The origin of this noise is in the RLG discharge assembly, the electronics, or other components susceptible to random flickering [B24]. Because of its low-frequency nature, it shows up as the bias fluctuations in the data. The rate PSD associated with this noise follows:

$$S_\Omega(f) = \begin{cases} \left(\dfrac{B^2}{2\pi}\right)\dfrac{1}{f} & f \le f_0 \\ 0 & f > f_0 \end{cases} \tag{C.4}$$

where

B	is the bias instability coefficient
f_0	is the cut-off frequency

Substitution of Equation (C.4) in Equation (C.1) and performing the integration yields the following equation:

$$\sigma^2(\tau) = \frac{2B^2}{\pi}\left[\begin{array}{l} \ln 2 + -\dfrac{\sin^3 x}{2x^2}(\sin x + 4x\cos x) \\ \quad + Ci(2x) - Ci(4x) \end{array}\right] \tag{C.5}$$

where

x	is $\pi f_0 \tau$
Ci	is the cosine-integral function [B16]

Figure C.2 represents a log-log plot of Equation (C.5) that shows that the Allan variance for bias instability reaches a plateau for τ much longer than the inverse cut off frequency. Thus, the flat region of the plot can be examined to estimate the limit of the bias instability as well as the cut-off frequency of the underlying flicker noise.

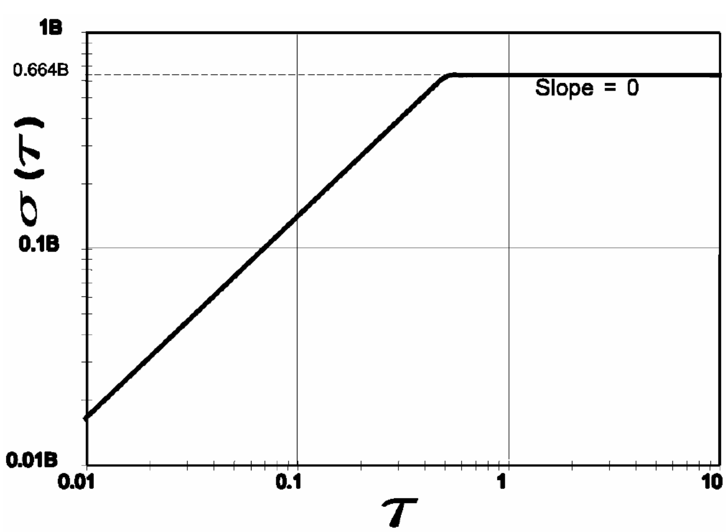


Figure C.2—σ(τ) plot for bias instability (for f_0 = 1)

C.1.3 Rate random walk

This is a random process of uncertain origin, possibly a limiting case of an exponentially correlated noise with a very long correlation time. The mechanical gyro as well as rate biased laser gyros exhibit this noise term.

The rate PSD associated with this noise follows:

$$S_{\Omega}(f) = \left(\frac{K}{2\pi}\right)^2 \frac{1}{f^2} \tag{C.6}$$

where

K is the rate random walk coefficient

Substitution of Equation (C.6) in Equation (C.1) and performing the integration yields the following equation:

$$\sigma^2(\tau) = \frac{K^2\tau}{3} \tag{C.7}$$

This indicates that rate random walk is represented by a slope of +1/2 on a log-log plot of $\sigma(\tau)$ versus τ, as shown in Figure C.3. The magnitude of this noise can be read off the slope line at $\tau = 3$.

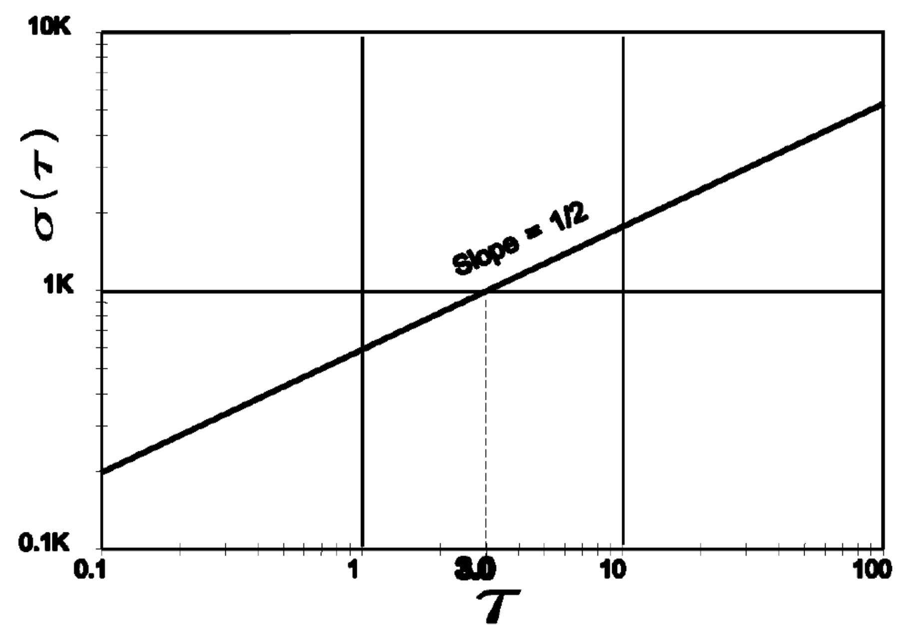


Figure C.3—$\sigma(\tau)$ plot for rate random walk

C.1.4 Rate ramp

This is more of a deterministic error rather than a random noise. Its presence in the data may indicate a very slow monotonic change of the RLG intensity persisting over a long period of time. It could also be due to a very small acceleration of the platform in the same direction and persisting over a long period of time. It appears as a genuine input to the RLG given by the following equation:

$$\Omega = Rt \tag{C.8}$$

where

R is the rate ramp coefficient

By forming and operating on the clusters of data containing an input given by Equation (C.8), we obtain the following equation:

$$\sigma^2(\tau) = \frac{R^2\tau^2}{2} \tag{C.9}$$

This indicates that the rate ramp noise has a slope of +1 in the log-log plot of $\sigma(\tau)$ versus τ, as shown in Figure C.4. The magnitude of rate ramp R can be obtained from the slope line at $\tau = \sqrt{2}$.

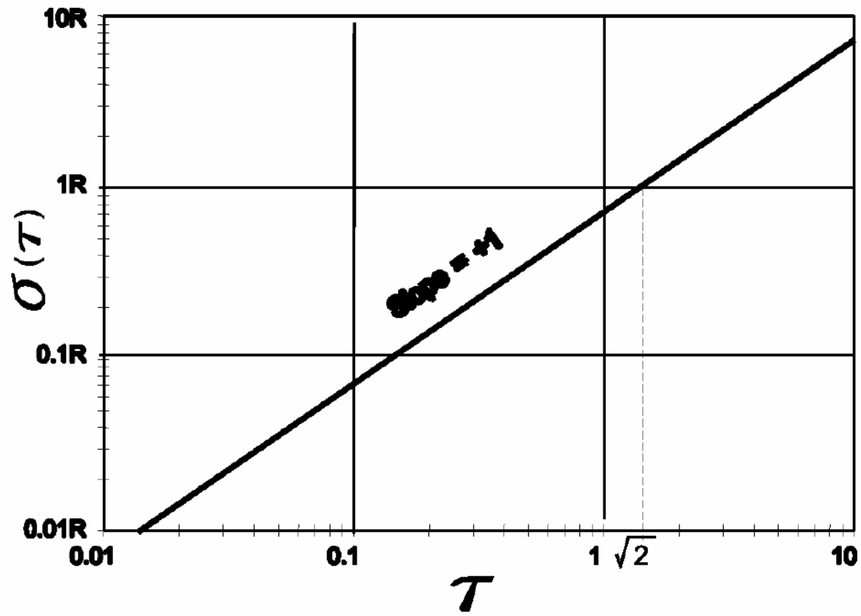


Figure C.4—$\sigma(\tau)$ plot for rate ramp

The rate PSD associated with this noise follows:

$$S_\Omega(f) = \frac{R^2}{(2\pi f)^3} \tag{C.10}$$

The user should be aware that there may be a flicker acceleration noise with a $1/(f)^3$ PSD that leads to the same Allan variance τ dependence. See Annex B.

C.1.5 Quantization noise

This noise is strictly due to the digital nature of the RLG output. The readout electronics registers a count only when the gyro phase changes by a predetermined amount, e.g., 2π, $\pi/2$, ….

The angle PSD for such a process, given in [B33], is shown in the following equation:

$$S_\theta(f) = \tau Q^2 \left(\frac{\sin^2(\pi f \tau)}{(\pi f \tau)^2} \right)$$
$$\approx \tau Q^2 \quad for \;\; f < \frac{1}{2\tau} \tag{C.11}$$

where

Q is the quantization noise coefficient

Its theoretical limit is equal to $S/\sqrt{12}$

where

S is the gyro scale factor, for tests with fixed and uniform sampling times

The rate PSD is related to the angle PSD through the following equation:

$$S_\Omega(2\pi f) = (2\pi f)^2 \, S_\theta(2\pi f) \tag{C.12}$$

and is

$$S_\Omega(f) = \frac{4Q^2}{\tau} \sin^2(\pi f \tau)$$
$$\approx (2\pi f)^2 \, Q^2 \tau \quad for \;\; f < \frac{1}{2\tau} \tag{C.13}$$

Substitution of Equation (C.13) in Equation (C.1) and performing the integration yields the following equation:

$$\sigma^2(\tau) = \frac{3Q^2}{\tau^2} \tag{C.14}$$

This indicates that the quantization noise is represented by a slope of –1 in a log-log plot of $\sigma(\tau)$ versus τ, as shown in Figure C.5. The magnitude of this noise can be read off the slope line at $\tau = \sqrt{3}$.

The user should be aware that there are other noise terms with different spectral characteristics, such as flicker angle noise and white angle noise that lead to the same Allan variance τ dependence. See Annex B for a discussion of these noise terms.

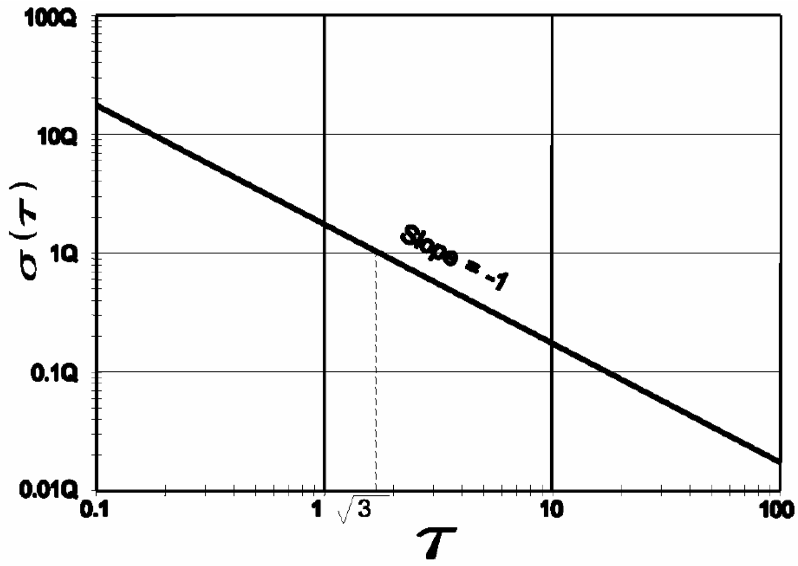


Figure C.5—σ(τ) plot for quantization noise

C.1.6 Other noise terms

In addition to the noise terms explicitly included within the gyro model equation, the following terms may also be present in the gyro output data:

a) Exponentially correlated (Markov) noise: This noise is characterized by an exponential decaying function with a finite correlation time. A potential source of this noise term is randomized mechanical dither. This is due to the resonant nature of the dither mechanism that does not allow all frequencies being imparted to the gyro body with equal amplitude. Thus, in reality the randomized dither introduces a correlated noise to the gyro whose correlation time can be shown to be related to the mechanical Q of the dither spring.

The rate PSD for such a process follows:

$$S_\Omega(f) = \frac{(q_c T_c)^2}{1 + (2\pi f T_c)^2} \tag{C.15}$$

where

q_c is the noise amplitude
T_c is the correlation time

Substitution of Equation (C.15) in Equation (C.1) and performing the integration yields the following equation:

$$\sigma^2(\tau) = \frac{(q_c T_c)^2}{\tau}\left[1 - \frac{T_c}{2\tau}\left(3 - 4e^{\frac{-\tau}{T_c}} + e^{\frac{-2\tau}{T_c}}\right)\right] \tag{C.16}$$

Figure C.6 shows a log-log plot of Equation (C.16). It is instructive to examine various limits of this equation for τ much longer than the correlation time. It is found that:

$$\sigma^2(\tau) \Rightarrow \frac{(q_c T_c)^2}{\tau} \qquad \tau >> T_c \tag{C.17}$$

which is the Allan variance for the angle random walk with the identification $N = q_c T_c$ in the opposite limit, Equation (C.17) reduces to the following equation:

$$\sigma^2(\tau) \Rightarrow \frac{q_c^2}{3}\tau \qquad \tau << T_c \tag{C.18}$$

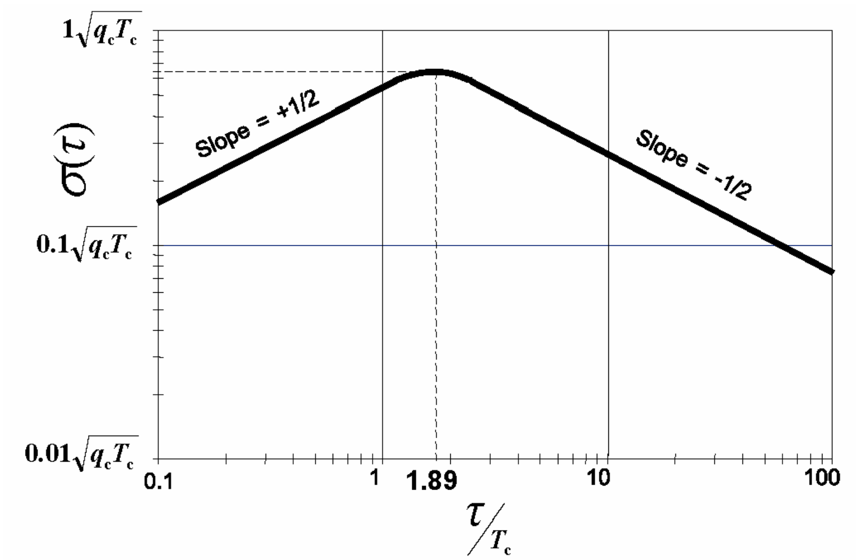


Figure C.6—$\sigma(\tau)$ plot for correlated noise

b) Sinusoidal noise: The PSD of this noise is characterized by a number of distinct frequencies. High frequency noise may originate from plasma oscillations in the laser discharge. A low frequency source could be the slow motion of the test platform due to periodic environmental changes. A representation of the PSD of this noise containing a single frequency is given in the following equation:

$$S_\Omega(f) = \frac{1}{2}\Omega_0^2\left[\delta(f - f_0) + \delta(f + f_0)\right] \tag{C.19}$$

where

Ω_0 is the amplitude
f_0 is the frequency
$\delta(x)$ is the Dirac delta function

Multiple frequency sinusoidal errors can be similarly represented by a sum of terms, such as Equation (C.19), at their respective frequencies and amplitudes. Substitution of Equation (C.19) in Equation (C.1) and performing the integration yields the following equation:

$$\sigma^2(\tau) = \Omega_0^2 \left(\frac{\sin^2 \pi f_0 \tau}{\pi f_0 \tau} \right)^2 \tag{C.20}$$

Figure C.7 shows a log-log plot of Equation (C.20). Identification and estimation of this noise in RLG data requires the observation of several peaks. As is seen, however, the amplitudes of consecutive peaks fall off rapidly and may be masked by higher order peaks of other frequencies making observations difficult.

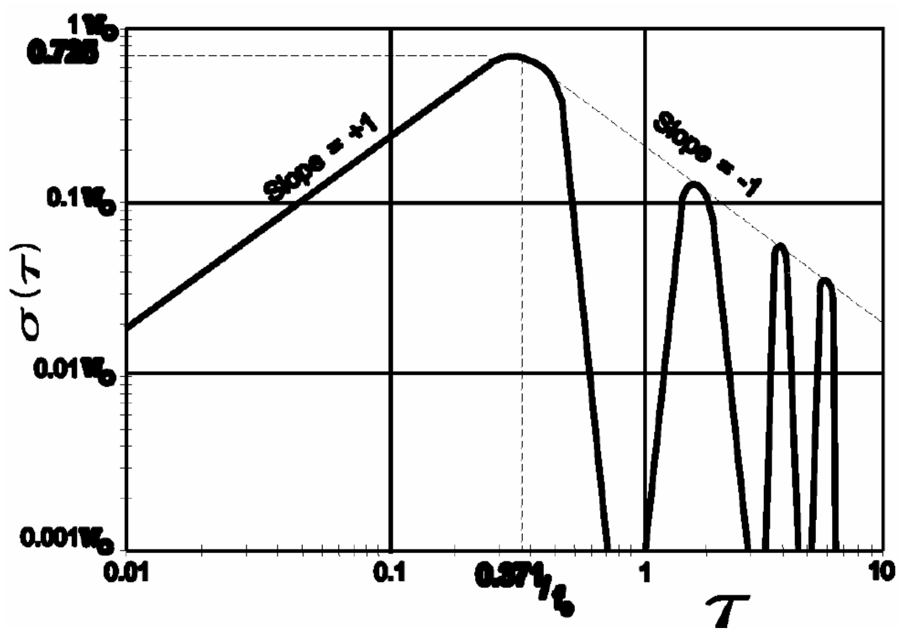


Figure C.7—$\sigma(\tau)$ plot for sinusoidal error

C.1.7 Combined effects of all processes

In general, any number of the random processes discussed previously (as well as others) can be present in the data. Thus, a typical Allan variance plot looks like the one shown in Figure C.8. Experience shows that, in most cases, different noise terms appear in different regions of τ. This allows easy identification of various random processes that exist in the data. If it can be assumed that the existing random processes are all statistically independent then it can be shown that the Allan variance at any given τ is the sum of Allan variances due to the individual random processes at the same τ. In other words it follows that:

$$\sigma_{\text{tot}}^2 = \sigma_{\text{ARW}}^2(\tau) + \sigma_{\text{quant}}^2(\tau) + \sigma_{\text{BiasInst}}^2(\tau) + \ldots \tag{C.21}$$

Thus estimating the amplitude of a given random noise in any region of τ requires knowledge of the amplitudes of the other random noises in the same region.

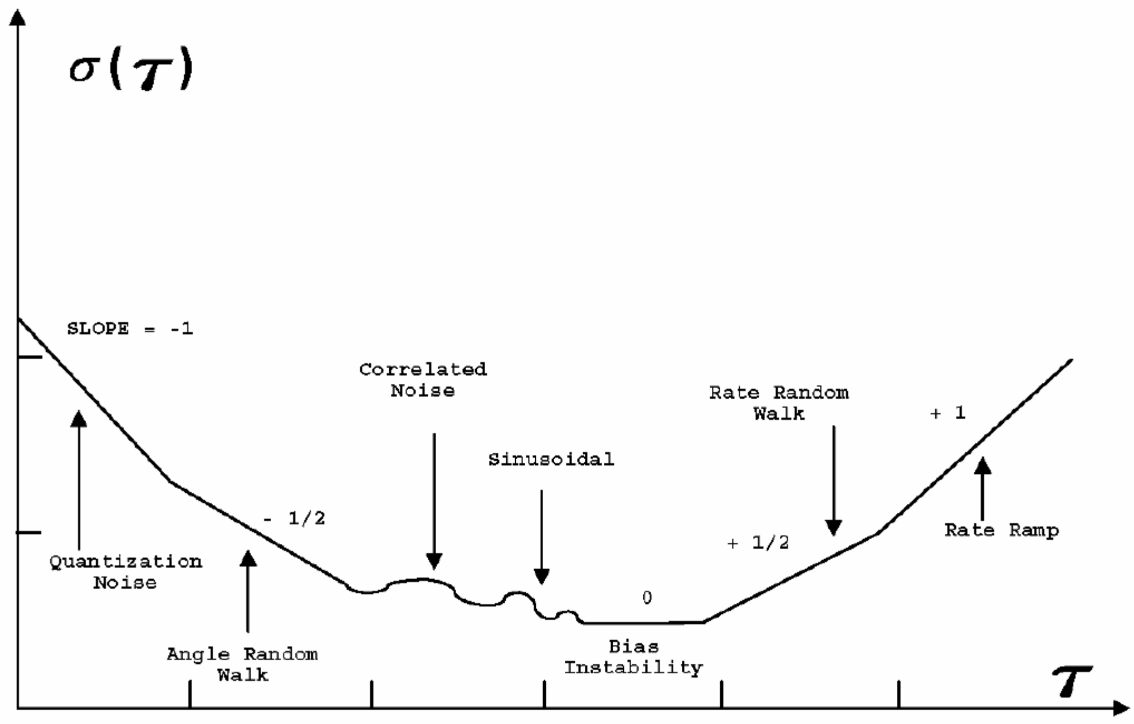


Figure C.8—Sample plot of Allan variance analysis results

C.2 Estimation accuracy and test design

A finite number of clusters can be generated from any finite set of data. Allan variance of any noise term is estimated using the total number of clusters of a given length that can be created. Estimation accuracy of the Allan variance for a given τ, on the other hand, depends on the number of independent clusters within the data set.

It can be shown that the percentage error, σ, in estimating $\sigma(\tau)$ when using clusters containing K data points from a data set of N points is given by the following equation:

$$\sigma = \frac{1}{\sqrt{2\left(\frac{N}{K} - 1\right)}} \qquad \text{(C.22)}$$

Equation (C.22) shows that the estimation errors in the regions of short (long) τ are small (large) as the number of independent clusters in these regions is large (small). In fact, this equation can be used to design a test to observe a particular noise of certain characteristics to within a given accuracy. For example, to verify the existence of a random process with a characteristic time of 24 h in the data to within an error of 25%. We first set $\sigma = 0.25$ in Equation (C.22) and obtain the following equation:

$$K_{\max} = \frac{N}{9} \qquad \text{(C.23)}$$

Since the suspected characteristic time is 24 h, we create clusters of the same length. Thus, the total test length needed for such a test is 24 h × 9 = 216 h.

C.3 Tabulation of some variance analysis

A summary comparison of some variance analyses for noise processes is made in Table C.1. This table presents only a sample of analyses available, and is not meant to be a survey of all analyses. The polynomial variance terms in the left-hand column are identified using gyro terminology. The individual terms relating to each author's publication are given with the same symbology as contained in that author's publication, including the definitions of symbols. For ease in recognition of similarities, the coefficients of interest are shown as the first symbol in each polynomial expression. For example, the variance coefficient for the rate random walk term in the IEEE/GAP analysis is K^2.

Table C.1—Summary comparison of publishing variance analysis for noise processes

	Allan [B1]	This std—Method 1 without prefilter	This std—Method 2 with prefilter	Sargent and Wyman [B36]	Tehrani [B42]
	Rate domain	**Rate domain**	**Rate domain**	**Angle domain**	**Rate domain**
Terms in variance expression for noise processes	τ = sampling time = $m\tau_o$ where τ_o = sample time of original measurements	$1/T_o$ = data sample rate $n = 1, 2, 3, \ldots$	h = number of data points in T_o used to form an average (filtered) value	γ = time interval separating raw data points. $(2L + 1)$ = number of data points combined into an average data point τ = time length of data span = $(2L + 1)\gamma n$ $n = 1, 2, 3, \ldots$	$Q = qT_c$ T_c = correlation time T = cluster time v_o = cut-off frequency for $1/v$ rate noise
Rate ramp	Not addressed	$R^2\left[\frac{(nT_O)^2}{2}\right]$	$R^2\left[\frac{(nT_O)^2}{2}\right]$	$R^2\left[\tau^4\right]$	$R^2\left[T^2\right]$
Rate random walk	$h_{-2}\left[\frac{(2\pi)^2\tau}{6}\right]$	$K^2\left[\frac{nT_O}{3}\right]$	$K^2\left[\frac{nT_O}{3}\right]$	$K^2\left[\begin{matrix}\frac{2\tau^3}{3} - \frac{2L(L+1)\gamma^2\tau}{3} \\ \frac{L(L+1)(12L^2+12L+1)\gamma^3}{15(2L+1)}\end{matrix}\right]$	$q^2\left[\frac{2T}{3}\right]$ $for\ T \ll T_O$
Bias instability	$h_{-1}\left[2\ \ln 2\right]$	$B^2\left[\frac{2}{\pi}\right]$	$B^2\left[\frac{2}{\pi}\right]$	Analysis considers only time dependent rate terms.	$B^2\left[\frac{4}{\pi}\ln 2\right]$ $for\ T \gg \frac{1}{v_O}$
Angle random walk	$h_O\left[\frac{1}{2\tau}\right]$	$N^2\left[\frac{1}{nT_O}\right]$	$N^2\left[\frac{1}{nT_O}\right]$	$\sigma^2\left[2\tau - \frac{4\gamma L(L+1)}{(2L+1)}\right]$	$Q^2\left[\frac{2}{T}\right]$
Quantization noise	Not addressed	$Q^2\left[\frac{3}{(nT_O)^2}\right]$	$Q^2\left[\frac{3}{h(nT_O)^2}\right]$	$\Phi^2\left[\frac{6}{2L+1}\right]$	Not addressed

Annex D

(informative)

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