A Localization Method Avoiding Flip Ambiguities for micro-UAVs with Bounded Distance Measurement Errors

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Abstract—**Localization is a fundamental function in cooperative control of micro unmanned aerial vehicles (UAVs), but is easily affected by flip ambiguities because of measurement errors and flying motions. This study proposes a localization method that can avoid the occurrence of flip ambiguities in bounded distance measurement errors and constrained flying motions; to demonstrate its efficacy, the method is implemented on bilateration and trilateration. For bilateration, an improved bi-boundary model based on the unit disk graph model is created to compensate for the shortage of distance constraints, and two boundaries are estimated as the communication range constraint. The characteristic of the intersections of the communication range and distance constraints is studied to present a unique localization criterion which can avoid the occurrence of flip ambiguities. Similarly, for trilateration, another unique localization criterion for avoiding flip ambiguities is proposed according to the characteristic of the intersections of three distance constraints. The theoretical proof shows that these proposed criteria are correct. A localization algorithm is constructed based on these two criteria. The algorithm is validated using simulations for different scenarios and parameters, and the proposed method is shown to provide excellent localization performance in terms of average estimated error. Our code can be found at: https://github.com/QingbeiGuo/AFALA.git.**

Index Terms—**bi-boundary model, flip ambiguity, bilateration**

I. INTRODUCTION

The use of multiple unmanned aerial vehicles (UAVs) has become very popular in civil and military applications. As a key technical problem, localization of multiple mirco-UAVs finds its application in location tracking, formation flight and cooperative mission, etc. As the most frequently used localization method, GPS suffers from large location errors of 10-30m on average. Therefore, different techniques have been proposed to address the localization problem [1]–[4].

Localization techniques, which are based on distance measurement (e.g., trilateration and bilateration), have always attracted significant research interest [5]–[9]. However, flip ambiguity (FA) is one of the major problems of these localization techniques, especially in the presence of measurement errors [10]–[14]. The measurement error between the measurement distance and the true distance always occur because of external environment noise [5]–[9], that easily causes FA and results in an incorrect estimate [15] [16] [17]. Moreover, once a FA has occurred, it not only affects the current localization but also causes erroneous results for subsequent localizations.

Global rigidity is widely used for localization to ensure that the result is unique [18]–[20]. A network can be uniquely located if and only if its corresponding graph is globally rigid [21]. However, even if the network is globally rigid in the presence of errors, FA may still occur. In trilateration, the three measured distances may locate their connected node to a false side, which causes FA due to errors in the measurements [22], [23].

Bilateration only requires network rigidity conditions to be met, not global rigidity [24]–[26], and reduces the reliance on high node density. However, FA in bilateration deserves more attention. Given that only two distances exist, the localization conditions are so insufficient that additional localization constraints are required. The communication range constraint of the nodes is widely adopted as an additional localization constraint and therefore has an important role in determining the final location from the candidate locations. More importantly, an incorrect choice inevitably results in the occurrence of FA so that the estimation of the communication range of a node becomes very important.

The unit disk graph [27]–[29] is commonly used to model the communication range of a wireless node. Each node has a single circle with a radius equal to a fixed value, and two nodes are connected if the distance between them is below the

specified threshold. However, in reality, the boundary between reachable and unreachable areas cannot be clearly defined most of the time. Communication range between nodes is impeded by external conditions, for instance, buildings in a city [30], and internal factors, such as its energy availability especially after operating for long duration. Therefore, the fixed communication range assumption has its limitation in localization.

Our proposed localization method is intended to address the aforementioned problems. In our method, we assume the distance measurement errors to be bounded [31] and the flying motions to be constrained [32]. An improved bi-boundary model of the communication range is proposed as localization constraints based on the model of unit disk graph. The new model depends on only distances and connectivity, and calculates the double boundaries of communication ranges through the bounded measurement error. Given that both bilateration and trilateration are analyzed, every constraint is regarded as a possible location region. The intersections of two distance constraints form two possible localization regions first, then the third constraint (bi-boundary and distance constraint for bilateration and trilateration, respectively) is used to eliminate one region, which causes FA, by analyzing the characteristic of their intersections. Therefore, the remaining region must contain the true location and the estimated location without the possibility of FA.

Accordingly, the main contributions of this paper are as follows:

- An improved bi-boundary model based on the unit disk graph model is presented by analyzing the connectivity characteristics of a wireless node.
- Based on the double constraints of distance and bi-boundary, a localization criterion that avoids FA in bilateration is proposed.
- Another localization criterion that avoids FA in trilateration is also developed.
- A localization algorithm based on the above two localization criteria, which dramatically improves the location accuracy, is constructed and evaluated through extensive simulations.

The rest of this paper is organized as follows: Section II introduces the related work. Section III formulates the localization problem. Section IV presents an improved bi-boundary model based on the unit disk graph model, describes the localization criteria that avoid FA in bilateration and trilateration, and provides their proofs. Based on these localization criteria, the localization algorithm is developed. In section V, the localization algorithm is validated through comprehensive simulation. Finally, section VI presents the conclusion.

II. Related Work

Various works have investigated the localization and the phenomenon of FA that may hamper the unique localization of distributed nodes, and have proposed different localization methods for reducing or avoiding FA. These methods can be classified into two categories: methods based on global and non-global rigidity properties which take trilateration and bilateration as representatives, respectively. Further details of these methods are provided below.

A unique and anchor-free localization algorithm, which also resolves the FA problem in its second step, has been proposed by Zhang et al. [14]. A novel combination of distance and direction estimation technique is introduced to detect and estimate ranges between neighbors. Using both distance and angle information, we construct unidirectional local coordinate systems to avoid the reflection ambiguity. However, angle measurement is too expensive to be imbedded in most nodes, leading to few practical applications and inspiring our development of a new localization algorithm to solve the FA issue.

Trilateration is the most widely used localization method based on distance measurement [19], [21]. The estimated location of a node is determined using measured distances to three other nodes that have known locations and are not collinear. Trilateration without measurement errors is uniquely localizable; however, trilateration with measurement errors tends to suffer from the effect of FA [23], [33].

To prevent incorrect localization caused by FA, a robust quadrilateral was used to perform trilateration in [16], [34]–[36]. Moore et al. [16] introduced the notion of robust quadrilateral, which is a fully connected quadrilateral with global rigid properties, and used it to reduce the probability of FA. They found all possible robust quadrilaterals and then realized global localization by overlapping any two robust quadrilaterals that have three common nodes. The simulation results showed that the aforementioned method is suitable for the localization of high node density, because it depends on these robust quadrilaterals that require a high node density to meet the feature of global rigidity.

Kannan et al. [17] pointed out that a node may be estimated at a flipped location caused by measurement errors in a globally rigid graph when its three neighboring nodes with known location are nearly collinear. They proposed a robust criterion based on the robust quadrilateral to calculate the probability of FA, to eliminate all the locations that might have caused it, and to improve localization performance. The simulations show that, compared with the robust quadrilateral, this method decreases not only the average estimation error but also the average number of localized nodes resulting from the robust criterion, which requires more constraints than the robust quadrilateral. Although more constraints can avoid problems of FA, they also limit the location of more nodes.

Akcan et al. [22] proposed a heuristic solution based on the notion of a "safe-triangle" to mitigate the problem of FA in a trilateration network with range noise. First proposed here, a safe-triangle is formed by three nodes with known location, where the distance of any node to the line passing through two other nodes is larger than a set threshold. If a triangle is not a safe-triangle, its nodes are unable to provide trilateration for other nodes. The main aim is to minimize the number of FA. The simulation results show that the algorithm can achieve better performance than trilateration. However, similar to trilateration, the safe-triangle algorithm also requires a high node density.

To reduce localization dependence on global rigidity, many algorithms [24], [25], [27] are proposed for network local-

ization using bilateration which requires rigid conditions but not necessarily globally rigid. Based on [24], Goldenberg *et al.* [25] proposed the "Sweeps" algorithm for network localization using bilateration. The Sweeps algorithm achieves one of the best performance in terms of number of localized nodes using bilateration. However, the algorithm requires a bilateration ordering to exist in the network, a condition which does not always exist in a sparse network; thus, it may fail in many localizable networks [25], [26].

Oliva *et al.* [27] introduced a model of shadow edges to extend trilateration to bilateration, which can produce effective solutions despite the lack of localization condition. The algorithm shows better performance than trilateration, and is able to localize the network even when trilateration fails. However, this method is based on unit disk graphs and only limited to noiseless environments.

The objective of our research is to avoid the occurrence of FA and achieve the accurate localization network using trilateration and bilateration with the errors including bounded measurement errors and constrained motions. A localization algorithm is proposed based on two classes of localization criteria that avoid the FA problem. In our algorithm, the two classes of localization criteria are derived by gradually analyzing the intersection characteristic of all the constrains. Then, trilateration and bilateration are combined to take advantage of the best of them, thus making them applicable in various kinds of networks that are sparse or not.

III. PROBLEM FORMULATION

Without loss of generality, the localization problem of a micro-UAV network modelled as a mobile sensor network can be formulated in 2D space [37], [38]. Multiple micro-UAVs, which consists of a set of n nodes, denoted as s_1 to s_n, are flying in a physical region. Each node has a limited communication range and establishes a wireless link with a neighboring node, which is called its neighboring node only if they are within the communication range of each other. A node is assumed to be capable of estimating the distance to the neighboring node using distance measurement technology. Given the constraints on energy consumption and implementation environment, most nodes do not know their locations except anchors which can obtain their own locations by using GPS. In this study, the micro-UAV network is analyzed without anchor nodes. The communication range of node s_i is denoted by r_i, and its estimated communication range by $\hat{r}_i$, its true location by p_i, and its estimated location by $\hat{p}_i$. The true distance between any two neighboring nodes s_i and s_j is denoted by d_{ij}, and the measured distance by $\hat{d}_{ij}$, where $i, j = 1,2,\ldots, n$.

For a connected network, we assume that the distances between neighboring nodes can be acquired in a time unit [39]. The speed of a node s_i is the distance travelled by the node per time unit [32], denoted as d_i, and d_{max} is the maximum distances travelled by these nodes in a time unit. Therefore, in each time unit, the micro-UAV network can be described by a model of undirected graph $G_t = (V, E)$ with a nonempty vertex set $V = \{1,2,\ldots, n\}$ and edge set E, where each vertex $i \in V$ uniquely represents a node s_i, and each edge $e(i, j) \in E$ is uniquely associated with a node pair (s_i, s_j), for which s_i and s_j are neighbors, and $\hat{d}_{ij}$ is known. The measurement error of $e(i, j)$ is denoted by e_{ij} such that $e_{ij} = d_{ij} - \hat{d}_{ij}$, and e_{max} is the maximum measurement errors. The network topology can and will change in the different time units because of the measurement errors and the flying trajectories. However, during the localization process, s_i has a possible localization region set $R_i = \{r_{i1}, r_{i2}\}$ and corresponding candidate location set $P_i = \{p_{i1}, p_{i2}\}$ because of the FA phenomenon. While the true location p_i is difficult to be determined due to the errors consisting of the measurement errors and the flying distances, R_i and P_i can be determined by the measured distances and the errors. FA occurs when the estimated location and the true location are not in the same localization region. Therefore, to determine the true localization region becomes a key problem in deciding whether FA occurs. By gradually analyzing the characteristic of the intersections corresponding to all the constraints, our method eliminates the localization region in which each location causes FA instead of choosing which one contains the true location. Therefore, the remaining region is identified as the true localization region, and the corresponding candidate location is regarded as the estimated location with no possibility of FA.

IV. LOCALIZATION CRITERIA AND LOCALIZATION ALGORITHM

A. Bi-boundary Model of Communication Range Constraint

In this section, an improved bi-boundary model based on the unit disk graph is presented to model the topology of wireless sensor networks with the errors which consist of the bounded measurement errors and the constrained motions. Now, we first focus on the effect of only the measurement errors, and the flying motions will be introduced in Section IV.D. In this bi-boundary model, each node has double concentric circles with two different radii, and the space between two circles represents an uncertainty in the communication range. The inner and outer boundaries represent the lower limit of the reachable range and the upper limit of the unreachable range, respectively; thus, this bi-boundary design is more suitable for the actual environment. The calculation of the two boundaries is independent of the measuring technique and instead completely depends on the knowledge of the distances, the measurement errors and connectivity between nodes, without maintaining the individual parameter of the communication range. The process of estimating the upper and lower boundaries of a wireless node is given in detail below.

Let $\varepsilon > 0$ be a threshold of the distance variation, such that $|e_{ij}| = |d_{ij} - \hat{d}_{ij}| \leq \varepsilon = e_{max}$. Thus,

$$d_{ij} \in [\hat{d}_{ij} - \varepsilon, \hat{d}_{ij} + \varepsilon]. \quad (1)$$

Following the aforementioned analysis on the neighboring node and communication noise, for every node s_i, the estimated communication range $\hat{r}_i$ can be bounded as

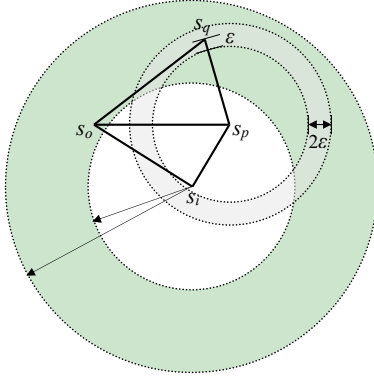$$\hat{r}_i \geq max(D_i^1) - \varepsilon, \quad (2)$$

Fig. 1. Example of estimating the communication range. For node s_i, the minimum of the estimated communication range is $\hat{d}_{io} - \varepsilon$, and the maximum is $\hat{d}_{pi} + \hat{d}_{pq} + 2\varepsilon$.

where D_i^k represents the measured distance set between the nodes that are at least k-hop away from node s_i, where k = 1,2,…,n. In particular, $D_i^1 = \{\hat{d}_{ij} \mid e(i,j) \in E\}$, $D_i^2 = \{\hat{d}_{il} + \hat{d}_{lj} \mid e(i,j) \notin E, e(i,j) \in E, e(i,j) \in E, \text{and } l \neq i, l \neq j\}$.

To further analyze r_i, the scenario is illustrated in Fig. 1. Given the unknown location of s_i which is connected with both s_o and s_p but is not connected with s_q, $\hat{d}_{iq}$ cannot be directly calculated using p_i and p_q; thus, it cannot contribute to the estimation of r_i. The sum of any two sides of a triangle is greater than the third side, so that r_i is less than the sum of any 2-hop distance from s_i to s_q. Therefore, considering the factor of the measurement errors, the following condition must be satisfied:

$$\hat{r}_i < min(D_i^2) + 2\varepsilon. \tag{3}$$

Finally, the estimated communication range $\hat{r}_i$, which satisfies equations (2) and (3), is bounded as follows:

$$max(D_i^1) - \varepsilon \leq \hat{r}_i < min(D_i^2) + 2\varepsilon. \tag{4}$$

The connection property of a node therefore affects its estimated result. Moreover, higher node density, which increases the degree of node connectivity, makes the estimate more precise.

B. FA Avoidance Criteria for Localization using Bilateration

In this section, we introduce the scenario of using bilateration and present a localization criteria for avoiding FA using the bi-boundary and distance constraints. Fig. 2 represents the corresponding graph of localization without flip ambiguities using bilateration. Fig. 2 (a) shows a normal case without near-collinear and overlapping occurrences. Fig. 2 (b) and (c) depict a near-collinear case and an overlapping case, respectively. Moreover, a compositive case with near-collinear and overlapping occurrences is shown in Fig. 2 (d). For each case above, we will follow the same process because our proposed localization method adapts to all the cases. The location of node s_o is assumed to be the origin, and another node s_p is selected to form the positive x-axis, and the third node s_q is located in the upper half-plane. The intersections of the black annulus and the red annuli form the upper region r_{q1}, which is the possible localization region of s_q. The possible localization regions of s_i are the two regions r_{i1} and r_{i2}

consisting of the intersections of the gray annulus and the blue annuli. The three regions of r_{r1}, r_{r2} and r_{q1} are highlighted with horizontal stripes, vertical stripes and mesh, respectively. u and v are any points of r_{q1} and r_{i1}, respectively, and their communication ranges are denoted by the green annulus and the yellow annulus, respectively. Obviously, both u and v are within the communication range of each other, and hence they must be adjacent. Furthermore, each point of r_{i1} clearly is within the communication range of u; thus, for u, the whole region r_{i1} is not a possible localization region of s_i. Note that due to the uncertainty of the true location p_p, there exist countless red annuli and blue annuli whose centers are at every point in between $(\hat{d}_{op} - \varepsilon, 0)$ and $(\hat{d}_{op} + \varepsilon, 0)$, but only two red annuli and two blue annuli at both ends are shown to simplify the figure.

The true distances d_{op}, d_{oq} and d_{pq} among the three nodes can be bounded as follows:

$$d_{op} \in [\hat{d}_{op} - \varepsilon, \hat{d}_{op} + \varepsilon], \tag{5}$$

$$d_{oq} \in [\hat{d}_{oq} - \varepsilon, \hat{d}_{oq} + \varepsilon], \tag{6}$$

$$d_{pq} \in [\hat{d}_{pq} - \varepsilon, \hat{d}_{pq} + \varepsilon]. \tag{7}$$

It is obvious that the true location p_p of s_p is between $(\hat{d}_{op} - \varepsilon, 0)$ and $(\hat{d}_{op} + \varepsilon, 0)$. The true location p_q of node s_q is calculated based on p_o, p_p, d_{oq} and d_{pq}. Based on equations (6) and (7), the two close intervals form two annulus regions, which can be defined using $\hat{d}_{oq}$ and $\hat{d}_{pq}$ as

$$r_{oq} = \{(x,y)| \\ (\hat{d}_{oq} - \varepsilon)^2 \leq (x - x_o)^2 + (y - y_o)^2 \leq (\hat{d}_{oq} + \varepsilon)^2\}, \tag{8}$$

$$r_{pq} = \{(x,y)| \\ (\hat{d}_{pq} - \varepsilon)^2 \leq (x - x_p)^2 + (y - y_p)^2 \leq (\hat{d}_{pq} + \varepsilon)^2\}, \tag{9}$$

where (x_o, y_o) and (x_p, y_p) are the location coordinates of s_o and s_p, respectively. Since s_o is assumed to be the origin, equation (8) can be also written as follows:

$$r_{oq} = \{(x,y)|(\hat{d}_{oq} - \varepsilon)^2 \leq x^2 + y^2 \leq (\hat{d}_{oq} + \varepsilon)^2\}. \tag{10}$$

However, as equation (5) shows, the true location p_p is uncertain. Thus, equation (9) can be also written as follows:

$$r_{pq} = \{(x,y) \mid \\ ((\hat{d}_{pq} - \varepsilon)^2 \leq (x - \hat{d}_{op} + \varepsilon)^2 + y^2 \leq (\hat{d}_{pq} + \varepsilon)^2) \vee \tag{11} \\ ((\hat{d}_{pq} - \varepsilon)^2 \leq (x - \hat{d}_{op} - \varepsilon)^2 + y^2 \leq (\hat{d}_{pq} + \varepsilon)^2)\}.$$

The intersections of r_{oq} and r_{pq} form the two regions r_{q1} and r_{q2}. The two regions are certainly symmetrical with respect to the edge $e(o,p)$, but they can be adjacent or nonadjacent in various situations [17]. To ensure that they are separated, the edge $e(o,p)$ is added as the boundary between them. The half-plane that contains s_q is denoted by H, and the complementary half-plane is denoted by H'. Hence, H and H' can be written as follows:

$$H = \{(x,y)|y > 0\}, \tag{12}$$

$$H' = \{(x,y)|y \leq 0\}. \tag{13}$$

(a) Noraml case (b) Near-collinear case (c) Overlapping case (d) Both near-collinear case and overlapping case
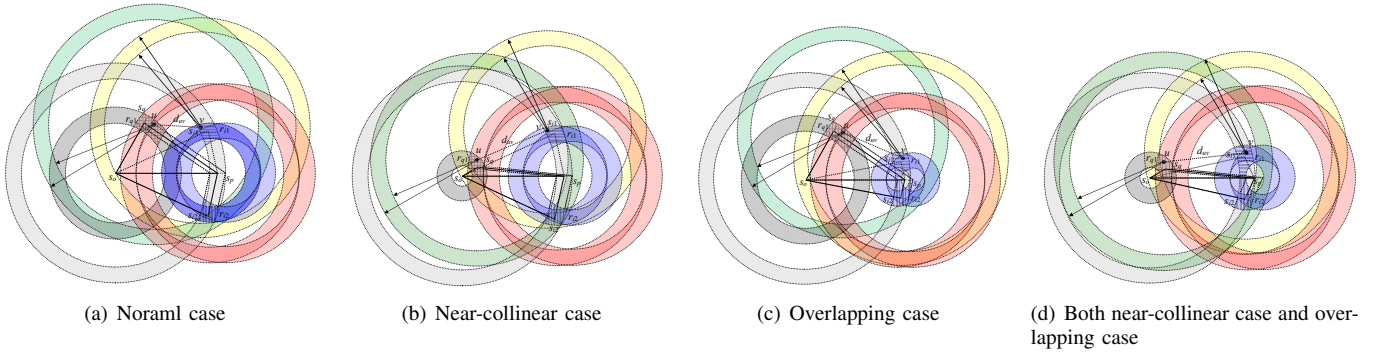
Fig. 2. Localization without flip ambiguities using bilateration.

Thus, the possible localization region of node s_q can be defined as follows:

$$r_{q1} = \{(x,y)|(x,y) \in r_{oq} \cap r_{pq} \cap H\}. \tag{14}$$

Similarly, this method can also be extended to the analysis of the possible localization regions of s_i. The two annulus regions related with s_i can be defined as follows:

$$r_{oi} = \{(x,y)|(\hat{d}_{oi} - \varepsilon)^2 \leq x^2 + y^2 \leq (\hat{d}_{oi} + \varepsilon)^2\}, \tag{15}$$

$$r_{pi} = \{(x,y)|$$
$$((\hat{d}_{pi} - \varepsilon)^2 \leq (x - \hat{d}_{op} + \varepsilon)^2 + y^2 \leq (\hat{d}_{pi} + \varepsilon)^2) \vee \tag{16}$$
$$((\hat{d}_{pi} - \varepsilon)^2 \leq (x - \hat{d}_{op} - \varepsilon)^2 + y^2 \leq (\hat{d}_{pi} + \varepsilon)^2)\}.$$

Thus, the two possible localization regions of s_i can be defined as follows:

$$r_{i1} = \{(x,y)|(x,y) \in r_{oi} \cap r_{pi} \cap H\}, \tag{17}$$

$$r_{i2} = \{(x,y)|(x,y) \in r_{oi} \cap r_{pi} \cap H'\}. \tag{18}$$

The true location of node s_i is in one of the two regions r_{i1} or r_{i2}. To obtain an unambiguous estimated location, additional knowledge is required to determine which region contains the true location. In this paper, the communication ranges of the nodes s_q and s_i are used to address the ambiguity problem. According to the aforementioned bi-boundary model, the communication ranges of s_q and s_i can be bounded as follows:

$$r_q \in [max(D_q^1) - \varepsilon, min(D_q^2) + 2\varepsilon], \tag{19}$$

$$r_i \in [max(D_i^1) - \varepsilon, min(D_i^2) + 2\varepsilon]. \tag{20}$$

Hence, the maximum communication distance, within which s_q and s_i can certainly communicate with each other, is defined by:

$$D_{qi} = min(max(D_q^1) - \varepsilon, max(D_i^1) - \varepsilon). \tag{21}$$

Lastly, Theorem 1 proves whether node s_i can be uniquely localizable in $\triangle opq$.

Theorem 1. *Given a localized triangle $\triangle opq$, where its three localized nodes s_o, s_p and s_q are adjacent to each other, and an unknown node s_i, such that the edges $e(o,i)$ and $e(p,i)$ exist, but $e(q,i)$ does not exist. Let $\varepsilon = e_{max} > 0$ be a*

threshold of the distance measurement errors. s_i can be a unique localization, and its sufficient conditions are as follows:

(1) $\forall x \in r_{q1}$, $\forall y \in r_{i1}$, $d_{xy} < D_{qi}$,
(2) $\exists u \in r_{q1}$, $\exists v \in r_{i2}$, $d_{uv} > D_{qi}$.

Proof. r_{i1} and r_{i2} are two possible localization regions of s_i based on constraints of the two distance $\hat{d}_{oi}$ and $\hat{d}_{pi}$. If every pair of points between r_{q1} and r_{i1} are adjacent while at least one pair of points between r_{q1} and r_{i2} are beyond D_{qi}, then the whole region r_{i1} is excluded as a possible localization region of s_i. Therefore, the region r_{i2} is the only possible region which contains the true location p_i, and no possibility exists for the occurrence of FA. In this case, both nodes s_i and s_q are clearly on the opposite of the edge $e(o,p)$. □

C. FA Avoidance Criteria for Localization using Trilateration

We now address the scenario where trilateration is used without flip ambiguities, and Fig. 3 shows the corresponding graph of this scenario. Just like Fig. 2, a normal case is depicted without near-collinear and overlapping occurrences in Fig. 3 (a). Fig. 3 (b) and (c) show a near-collinear case and an overlapping case, respectively. And then, Fig. 3 (d) shows a compositive case with near-collinear and overlapping occurrences. Due to the feasibility of our method to all the cases, we will also follow the same process. The detection of the FA occurrences is now analyzed. The possible localization region of s_q is the upper region r_{q1} consisting of the intersections of the black annulus and the red annuli. The intersections of the gray annulus and the blue annuli form two regions r_{i1} and r_{i2}, which are the possible localization regions of s_i. The three regions highlighted with horizontal stripes, vertical stripes and mesh indicate r_{r1}, r_{r2} and r_{q1}, respectively. u and v are any points of r_{q1} and r_{i1}, respectively. Since $d_{uv} \notin [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$, v is without the constraints of the distance $\hat{d}_{qi}$, and each point of r_{i1} is clearly without the constraint of $\hat{d}_{qi}$. Therefore, for u, the whole region r_{i1} is not a possible localization region of s_i.

Similar to the analysis for the scenario using bilateration, the two annuli r_{oq} and r_{pq} are also respectively defined as follows:

$$r_{oq} = \{(x,y)|(\hat{d}_{oq} - \varepsilon)^2 \leq x^2 + y^2 \leq (\hat{d}_{oq} + \varepsilon)^2\}, \tag{22}$$

(a) Noraml case (b) Near-collinear case (c) Overlapping case (d) Both near-collinear case and over-lapping case
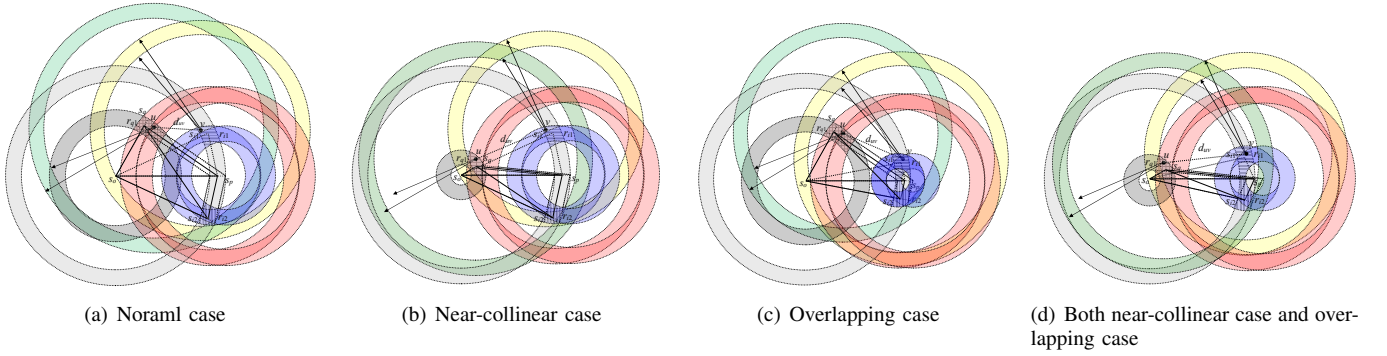
Fig. 3. Localization without flip ambiguities using trilateration.

$$r_{pq} = \{(x,y)|$$
$$((\hat{d}_{pq} - \varepsilon)^2 \le (x - \hat{d}_{op} + \varepsilon)^2 + y^2 \le (\hat{d}_{pq} + \varepsilon)^2) \vee \quad (23)$$
$$((\hat{d}_{pq} - \varepsilon)^2 \le (x - \hat{d}_{op} - \varepsilon)^2 + y^2 \le (\hat{d}_{pq} + \varepsilon)^2)\}.$$

Thus, the possible localization region of node s_q is also defined as follows:

$$r_{q1} = \{(x,y)|(x,y) \in r_{oq} \cap r_{pq} \cap H\}. \quad (24)$$

Similarly, using the two measured distances $\hat{d}_{oi}$ and $\hat{d}_{pi}$ that are affected by noises, the two annuli r_{oi} and r_{pi} are also respectively defined as follows:

$$r_{oi} = \{(x,y)|(\hat{d}_{oi} - \varepsilon)^2 \le x^2 + y^2 \le (\hat{d}_{oi} + \varepsilon)^2\}, \quad (25)$$

$$r_{pi} = \{(x,y)|$$
$$((\hat{d}_{pi} - \varepsilon)^2 \le (x - \hat{d}_{op} + \varepsilon)^2 + y^2 \le (\hat{d}_{pi} + \varepsilon)^2) \vee \quad (26)$$
$$((\hat{d}_{pi} - \varepsilon)^2 \le (x - \hat{d}_{op} - \varepsilon)^2 + y^2 \le (\hat{d}_{pi} + \varepsilon)^2)\}.$$

Following which, the two possible localization regions of s_i are also defined as follows:

$$r_{i1} = \{(x,y)|(x,y) \in r_{oi} \cap r_{pi} \cap H\}, \quad (27)$$

$$r_{i2} = \{(x,y)|(x,y) \in r_{oi} \cap r_{pi} \cap H'\}. \quad (28)$$

The true location p_i must be located in one of the two separated regions r_{i1} and r_{i2}, but incorrect calculation can cause FA; thus, to address the problem of FA and derive the estimated location, we use a third annulus r_{qi}, which is defined by the following equation:

$$r_{qi} = \{(x,y)|(\hat{d}_{qi} - \varepsilon)^2 \le (x - x_q)^2 + (y - y_q)^2 \le (\hat{d}_{qi} + \varepsilon)^2\}. \quad (29)$$

To summarize, we have the following theorem:

Theorem 2. *Given a localized triangle $\triangle opq$, where its three localized nodes s_o, s_p and s_q are adjacent to each other, and an unknown node s_i, such that the edges $e(o,i)$, $e(p,i)$ and $e(q,i)$ exist. Let $\varepsilon = e_{max} > 0$ be a threshold of the distance measurement errors. s_i can be a unique localization, and its sufficient conditions are as follows:*

(1) $\forall x \in r_{q1}$, $\forall y \in r_{i1}$, $d_{xy} \notin [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$,
(2) $\exists u \in r_{q1}$, $\exists v \in r_{i2}$, $d_{uv} \in [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$.
Or;
(1) $\forall x \in r_{q1}$, $\forall y \in r_{i2}$, $d_{xy} \notin [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$,
(2) $\exists u \in r_{q1}$, $\exists v \in r_{i1}$, $d_{uv} \in [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$.

Proof. Following the same analysis in Theorem 1, r_{i1} and r_{i2} are two possible localization regions of s_i based on constraints of the two distances $\hat{d}_{oi}$ and $\hat{d}_{pi}$. There are at least one pair of points, whose distances are under the constraints of the third distance $\hat{d}_{qi}$, between r_{q1} and r_{i2} (or r_{i1}), whereas there is no any pair of points, whose distance is without the constraints of the third distance $\hat{d}_{qi}$, between r_{q1} and r_{i1} (or r_{i2}). Thus, the whole region r_{i1} (or r_{i2}) is eliminated from the possible localization regions of s_i, and the other region r_{i2} (or r_{i1}) is a unique estimated region that causes no FA. $\square$

D. Localization Considering Velocities and Directions

Considering the motion velocities and directions of micro-UAVs, we further introduce our localization criteria avoiding FA in trilateration and bilateration. For a connected network, we first assume that the location update can be completed in one time unit. Since the velocities and directions vary, they are represented as the motion distances per time unit [32]. The size and sign of the motion distances are used to denote the magnitude and direction of motions. Here, positive value indicates that a node moves in the opposite direction, leading to an increase of distances between nodes, while negative value indicates the motion towards each other. When the distances between neighboring nodes are acquired together in a time unit, the possible locations of each node fall into a larger region due to their motions.

Combining the factor of measurement errors mentioned above, the distance d_{ij} between any two neighboring nodes s_i and s_j can be re-constrained as follows:

$$d_{ij} \in [\hat{d}_{ij} - (e_{max} + d_{max}), \hat{d}_{ij} + (e_{max} + d_{max})]. \quad (30)$$

These two parameters, measurement error and motion, are regarded as the influential factors on distance-based localizations. Therefore, the dynamic localization problem can be formulated with $\varepsilon = e_{max} + d_{max}$. According to theorem 1 and 2, two lemmas in trilateration and bilateration can be induced.

Lemma 1. *Given a localized triangle $\triangle opq$, where its three localized nodes s_o, s_p and s_q are adjacent to each other, and an unknown node s_i, such that the edges $e(o,i)$ and $e(p,i)$*

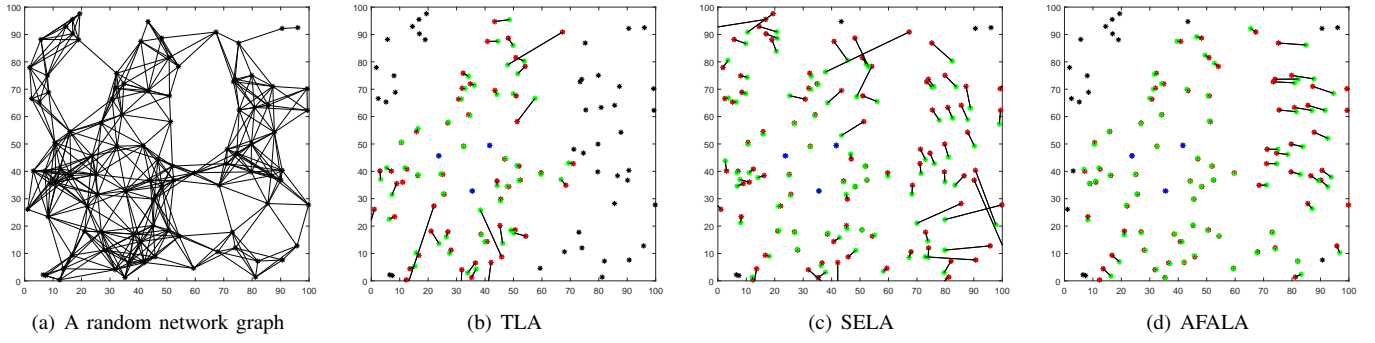(a) A random network graph   (b) TLA   (c) SELA   (d) AFALA

Fig. 4. Comparison of TLA, SELA and AFALA on a random graph with $r=20$, $\delta=0.02$ and $\varepsilon=\delta$, where green and red tic marks denote the true and estimated locations, respectively.
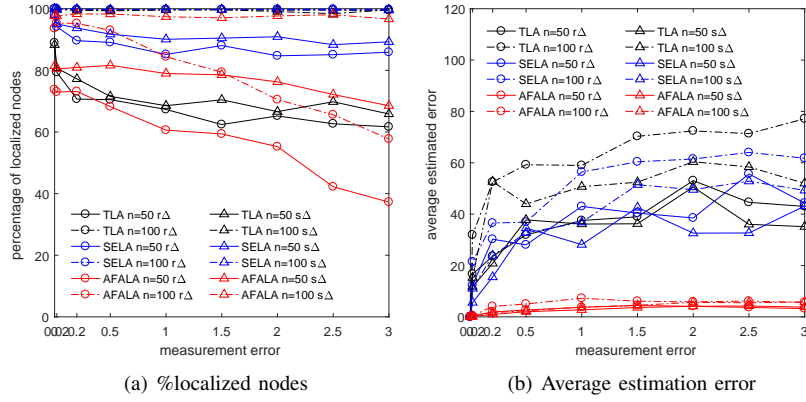

(a) %localized nodes   (b) Average estimation error

Fig. 5. Impact of initial triangle. $r = 30$, $d_{max} = 0$. r$\triangle$ and s$\triangle$ denote the random and special triangle, respectively.
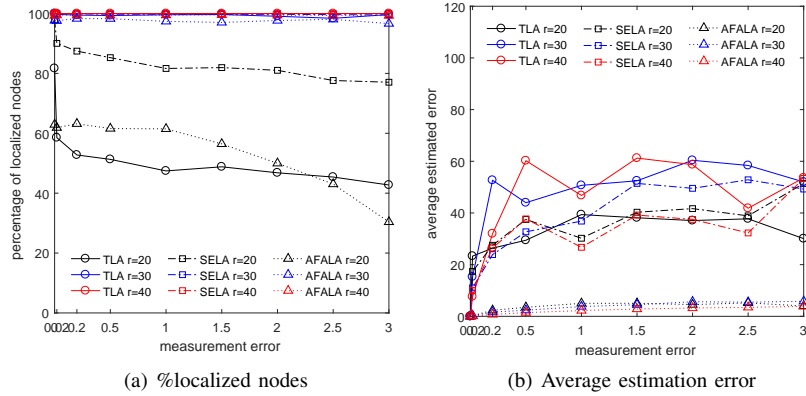

(a) %localized nodes   (b) Average estimation error

Fig. 6. Impact of measurement error. $n = 100$, s$\triangle$, $d_{max} = 0$.

(a) %localized nodes   (b) Average estimation error
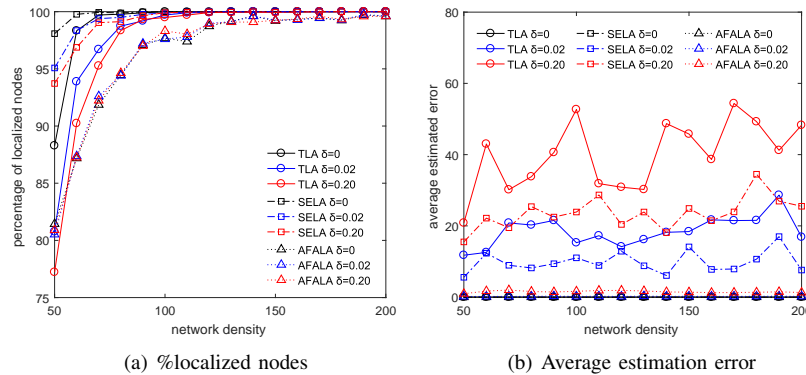
Fig. 7. Impact of network density. s$\triangle$, $r = 30$, $d_{max} = 0$.

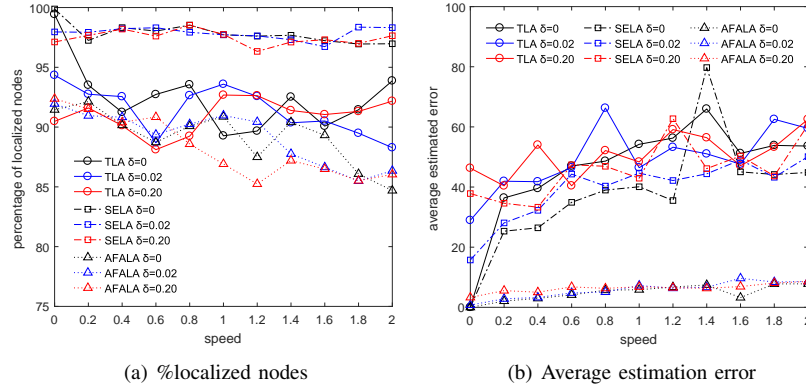(a) %localized nodes (b) Average estimation error

Fig. 8. Impact of node motion. $n = 150$, s$\triangle$, $r = 20$.

exist, but $e(q,i)$ does not exist. Let $\varepsilon = e_{max} + d_{max} > 0$ be a threshold of the distance variations. s_i can be a unique localization, and its sufficient conditions are as follows:

(1) $\forall x \in r_{q1}$, $\forall y \in r_{i1}$, $d_{xy} < D_{qi}$,
(2) $\exists u \in r_{q1}$, $\exists v \in r_{i2}$, $d_{uv} > D_{qi}$.

Lemma 2. *Given a localized triangle $\triangle opq$, where its three localized nodes s_o, s_p and s_q are adjacent to each other, and an unknown node s_i, such that the edges $e(o,i)$, $e(p,i)$ and $e(q,i)$ exist. Let $\varepsilon = e_{max} + d_{max} > 0$ be a threshold of the distance variations. s_i can be a unique localization, and its sufficient conditions are as follows:*

(1) $\forall x \in r_{q1}$, $\forall y \in r_{i1}$, $d_{xy} \notin [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$,
(2) $\exists u \in r_{q1}$, $\exists v \in r_{i2}$, $d_{uv} \in [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$.
Or,
(1) $\forall x \in r_{q1}$, $\forall y \in r_{i2}$, $d_{xy} \notin [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$,
(2) $\exists u \in r_{q1}$, $\exists v \in r_{i1}$, $d_{uv} \in [\hat{d}_{qi} - \varepsilon, \hat{d}_{qi} + \varepsilon]$.

E. Localization Algorithm

Based on the aforementioned criteria, we now present our *Avoiding Flip Ambiguities Localization Algorithm* (AFALA). The algorithm first selects a random triangle or a special triangle, which is an acute triangle with every edge greater than four fifths of the communication ranges, as the initial triangle and adds its three nodes to a set of localized nodes. For any unlocalized node, if it is connected to these three localized nodes and the condition of Theorem 1 or Lemma 1 is satisfied, or it is connected to two localized nodes and the condition of Theorem 2 or Lemma 2 is satisfied, it is localized and is moved from the unlocalized nodes set to the localized nodes set. The process is iterated until no new node can join the set of localized nodes. Details are shown in Algorithm 1.

V. PERFORMANCE VALIDATION

In this section, the localization performance of the proposed algorithm is compared with TLA [21] and SELA [27] in terms of the percentage of localizable nodes and the average estimation error, using simulations implemented in MATLAB 7.14.0.739 (R2012a). Simulations are conducted in a square

unit area of 100 *units* by 100 *units*, where micro-UAVs are uniformly distributed. 100 random instances of micro-UAV network are administered in each group trial, and the average result is taken to ensure more accurate result. The experimental parameter settings are as follows:

- The number n of micro-UAVs deployed in networks ranges from 50 to 200;
- A random or special triangle is randomly chosen as the initial triangle;
- The communication radius r is set to be 20, 30 and 40 *units*;
- The measurement errors $e_{ij} \sim N(0, \delta^2)$, such that $\hat{d}_{ij} = |d_{ij} + e_{ij}|$, where δ is set to be 0, 0.02, 0.2, 0.5, 1, 1.5, 2, 2.5 and 3 *units*;
- The motion speed of each micro-UAV is randomly chosen from $[-d_{max}, d_{max}]$ units per time unit, where the maximum motion speed d_{max} is set to be 0:0.2:2 *units* per time unit;
- The motion factor is considered or not by the threshold value ε which is set to be δ or $\delta + d_{max}$. In order to keep the errors within the maximum allowable bounds, if $|d_{ij} - \hat{d}_{ij}| > \varepsilon$, then $|d_{ij} - \hat{d}_{ij}| = \varepsilon$.

Fig. 4 presents a representative localization scenario with $r = 20$ and $\delta = 0.2$. Fig. 4 (a) shows the original graph of the network. Fig. 4 (b), (c) and (d) present the localization results of the TLA, SELA and AFALA algorithms in the same scenario. The three blue nodes are randomly chosen as the nodes of the initial triangle. The green and red nodes are the true and estimated locations, respectively, which are connected with black lines, while the remaining black parts are the unlocated nodes. Obviously, the SELA and AFALA algorithms locate more nodes than TLA because of the utilization of bilateration. However, many FAs occur in the TLA and SELA algorithms, while no FAs occur in AFALA. That is because our algorithm, AFALA, fully consider the effect of measurement error on FA. Therefore, our algorithm locates more accurately than TLA and SELA.

We further demonstrate the localization performance in terms of percentage of localized nodes and average estimation error by comparing AFALA with TLA and SELA from aspects such as initial triangle, measurement error, network density and node motion.

Algorithm 1 the AFALA algorithm based on both localization criteria.

Require:
    A set of unlocalized nodes, Su;
    A set of localized triangles, St;
    The threshold value of distance variations, ε;

Ensure:
    A set of localized nodes, Sl;

1: choose an initial triangle $\triangle opq$ into St;
2: add the three nodes of $\triangle opq$ to Sl;
3: **repeat**
4:    **for** each node $s_i \in Su$ **do**
5:      **for** each localized triangle $\triangle opq \in St$ **do**
6:        **if** s_i connects to two nodes s_o and s_p **then**
7:           $r_{q1} = r_{oq} \cap r_{pq} \cap H$;
8:           $r_{i1} = r_{oi} \cap r_{pi} \cap H$;
9:           $r_{i2} = r_{oi} \cap r_{pi} \cap H'$;
10:         $[G_{q1}, G_{i1}, G_{i2}]$ = the set of grid point coordinate of r_{q1}, r_{i1} and r_{i2}, respectively;
11:        **if** s_i connects to the node s_q **then**
12:          **if** $((\forall x \in G_{q1}, \forall y \in G_{i1}, d_{xy} < \hat{d}_{qi} - \varepsilon \wedge d_{xy} > \hat{d}_{qi} + \varepsilon)$ AND $(\exists u \in G_{q1}, \exists v \in G_{i2}, \hat{d}_{qi} - \varepsilon < d_{uv} < \hat{d}_{qi} + \varepsilon))$
             OR $((\exists x \in G_{q1}, \exists y \in G_{i1}, \hat{d}_{qi} - \varepsilon < d_{xy} < \hat{d}_{qi} + \varepsilon)$ AND $(\forall u \in G_{q1}, \forall v \in G_{i2}, d_{uv} < \hat{d}_{qi} - \varepsilon \wedge d_{uv} > \hat{d}_{qi} + \varepsilon))$ **then**
13:            calculate the estimated location $\hat{p}_i$;
14:            move s_i from Su to Sl;
15:            update the distances of the localized subgraph
16:            add $\triangle opi$ to St;
17:          **end if**
18:        **else**
19:          $D_{qi} = min(max(D_q^1) - \varepsilon, max(D_i^1) - \varepsilon)$;
20:          **if** $(\forall x \in G_{q1}, \forall y \in G_{i1}, d_{xy} < D_{qi})$ AND $(\exists u \in G_{q1}, \exists v \in G_{i2}, d_{uv} > D_{qi})$ **then**
21:            calculate the estimated location $\hat{p}_i$;
22:            move s_i from Su to Sl;
23:            update the distances of the localized subgraph
24:            add $\triangle opi$ to St;
25:          **end if**
26:        **end if**
27:       **end if**
28:      **end for**
29:    **end for**
30: **until** the element number of Sl no longer changes
31: **return** Sl;

A. Initial Triangle: Random & Special Triangle

As the starting point of algorithms, the choice of initial triangles determines whether algorithms can localize their first new node. If the node fails to be localized, that means the localization process was over before initiating. Fig. 5 shows the effect of random and special triangle on the localization performance. As is shown in Fig. 5 (a), for every network density, these three algorithms (especially AFALA) with special triangle distinctly localizes more nodes than with random triangle under the same measurement errors. Furthermore, with the increase of measurement errors, the number of nodes localized by AFALA algorithm with random triangle rapidly drops. In contrast, the special triangle slowly decreases the number of localized nodes. Especially, when network density is equal to 100, the number of localized nodes with special triangle almost remains stable, even reaching to about 100%. Theoretically, the special triangle more easily satisfies the localization criteria of AFALA, thus improving the number of localized nodes.

Fig. 5 (b) shows the effect of random and special triangle on the average estimation error. Obviously, the initial triangle tends to have less impact on AFALA than both TLA and SELA. Therefore, the choice of the initial triangle significantly affects the localization result of these methods, and special triangle achieves better localization performance than random triangle in terms of the localization ratio.

B. Measurement Error

We demonstrate the impact of measurement errors on the localization performance in terms of percentage of localized nodes and average estimation error in Fig. 6. Fig. 6 (a) shows the percentage of localized nodes against the measurement error. For $r = 20$, the AFALA and SELA algorithms locate more nodes than TLA when the measurement error is small, especially SELA. However, as the measurement error increases, the percentage of localized nodes decreases markedly in AFALA. That is mainly because the strict localization conditions of AFALA are difficult to satisfy with small communication radius and large measurement errors. For the large communication radius $r = 30$ and 40, the performance gap among the three algorithms decreases gradually. Especially for $r = 40$, they are able to localize almost all the nodes. That is because the large communication radius increases the possibility of using trilateration and reduces the key differences among these algorithms. For the case of AFALA, the localization conditions are also easily satisfied, leading to good outcomes.

Fig. 6 (b) shows the average estimation error against the measurement error where AFALA performs significantly better than TLA and SELA. For every value of r, it is apparent that the average estimation error is much less in AFALA than in TLA and SELA. Thus, the proposed algorithm exhibits excellent localization performance in terms of the average estimation error. This confirms the efficacy of AFALA as it addresses the critical issue of measurement errors, a key source of FA, to fulfil the criteria for accurate localization.

C. Network Density

Fig. 7 illustrates the impact of network density on the localization performance in different localization algorithms. Fig. 7 (a) shows the impact of network density on the localization ratio. For TLA, SELA and AFALA, the number of localized nodes significantly increases with the increase of network density. SELA achieves the fastest growth due to its most flexible localization conditions. On the contrary, AFALA obtains the lowest growth because of its most rigorous conditions. However, the gap among them is gradually reduced with the increase of network density. When network density is larger than 120, AFALA localizes almost all the nodes, reaching nearly the same localization ratio of TLA and SELA.

Fig. 7 (b) shows the impact of network density on the average estimation error. Although these three algorithms are comparable in the percentage of localized nodes, there is an obvious difference among them in the average estimation error. As is shown in Fig. 7 (b), the performance curves fluctuate with a fixed estimation error. For any one of these network densities, AFALA outperforms both TLA and SELA by large margins when the measurement error occurs.

D. Node Motion

We demonstrate the effect of node motion on the localization performance of TLA, SELA and AFALA in Fig. 8. As is shown in Fig. 8 (a), compared with TLA, our AFALA locates similar number of nodes, and the motion speed tends to slightly decrease the number of located nodes. That is because our localization criteria depend on the measurement distances, the measurement error and the motion speed to estimate the size of intersection regions for the avoidance of flip ambiguities, which makes it sensitive to these factors.

Fig. 8 (b) shows the impact of motion speed on the average estimation error. The average measurement error increases as the maximum motion speed increases. Our localization criteria avoiding flip ambiguities improve the localization accuracy, thus making AFALA to achieve much less average measurement error than TLA and SELA. Therefore, AFALA outperforms TLA and SELA in terms of localization accuracy without obvious loss of localization ratio.

VI. Conclusion

In this paper, we have proposed a localization algorithm for swarms micro-UAVs that aims to reduce the occurrence of flip ambiguities (FA). This is critical for collaborative flight of micro-UAVs to prevent collisions that can arise from localization errors caused by FA. For both bilateration and trilateration, under conditions of bounded errors, we analyzed the FA phenomenons using the characteristics of intersecting regions and derived the localization criteria for avoiding FA theoretically. Using these criteria, we developed a corresponding localization algorithm, which we call the *Avoiding Flip Ambiguities Localization Algorithm* (AFALA). Using simulations implemented in MATLAB, we compared the performance of AFALA against other well-known localization methods, viz. TLA and SELA, to demonstrate its efficiency from four aspects of initial triangle, measurement error, network density and node motion.

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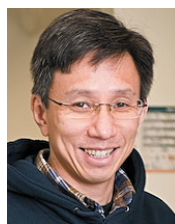
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