Problem 1

a .

With normalized formulas:
mean 1.04897
variance 5.42722
skewness 0.879298
kurtosis 23.06998

b .

With pandas
mean 1.04897
variance 5.42722
skewness 0.88193
kurtosis 23.24425

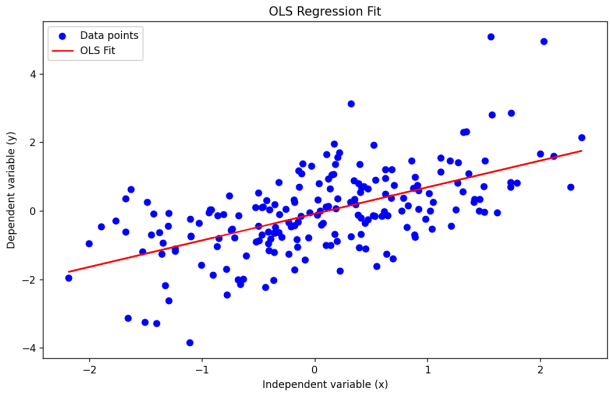
c .

Pandas is not biased since the difference among pandas results and results from normailzed formulas re very small.

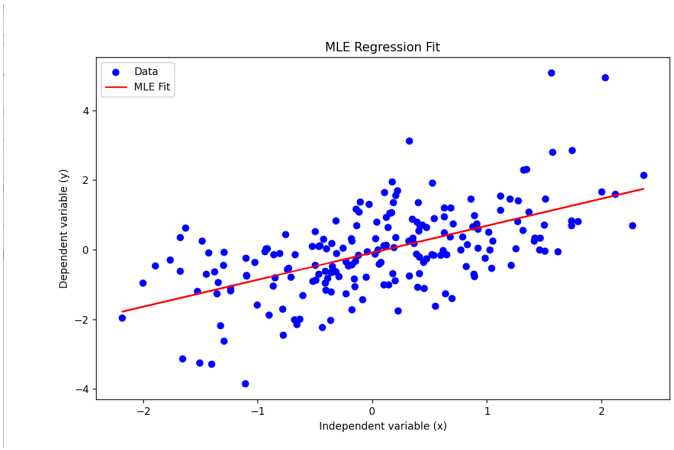
Problem 2

a.

The OLS estimation for the beta coefficient (excluding the intercept) is approximately **0.7753**
The standard error of the OLS estimation is approximately **0.0758**



The MLE estimation for the beta coefficient is exactly the same as the OLS estimate, at approximately **0.7753**
The estimated standard deviation of the residuals from the MLE is approximately **1.0038**



Explanation:

b .

The T-distribution model seems to be the better fit with lower AIC and BIC values.

Adjusted R-Squared:

Normal: 0.3423

T-Distribution: 0.3330

The normal model has a slightly higher Adjusted R-Squared, suggesting a better fit in terms of the variance explained by the model.

AIC:

Normal: 573.08

T-Distribution: 568.59

The T-distribution model has a lower AIC, suggesting it is the better model when considering the trade-off between goodness of fit and model complexity.

BIC:

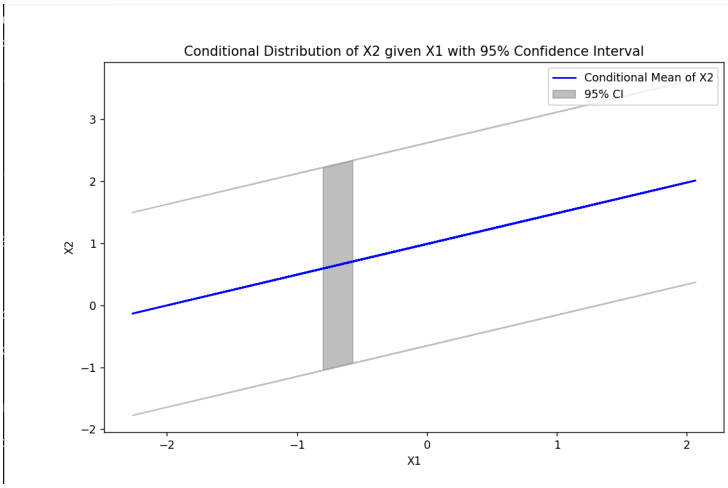
Normal: 579.67

T-Distribution: 578.48

The T-distribution model also has a lower BIC, indicating it is the better model.

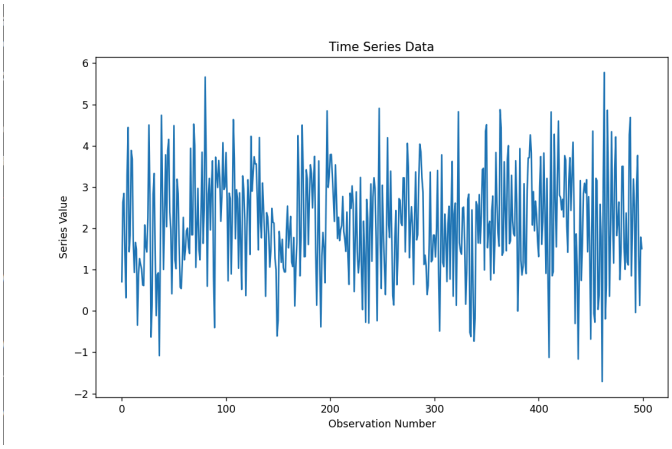
c .

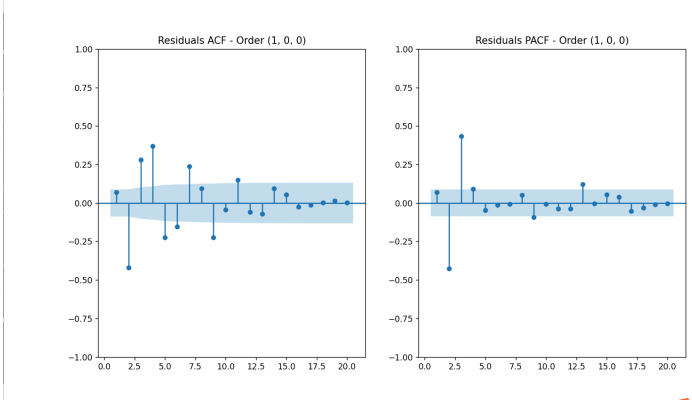
X2 is normally distributed. X1 is normally distributed because the problem says "X = [X1,X2] follows the multivariate normal distribution" , thus X1 is normally distributed. Since the distribution of one variable(X1) given another(X2) in a multivariate normal distribution is also normally distributed.

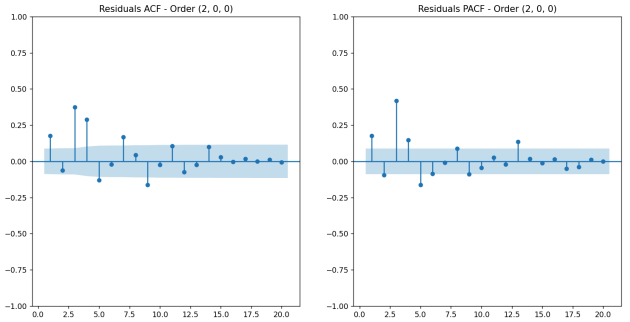


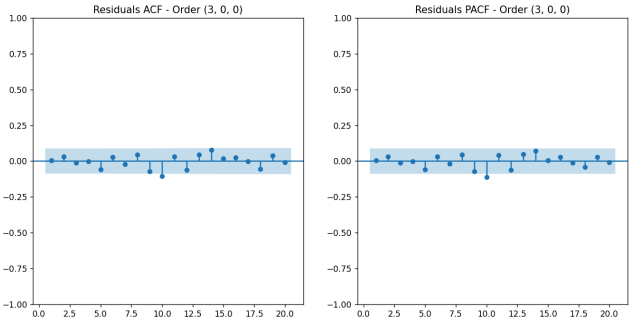
Problem 3

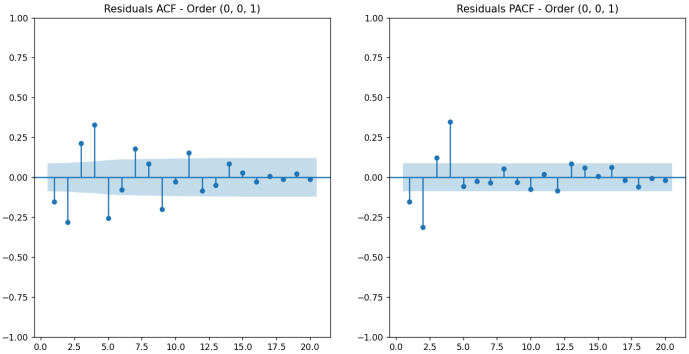
The **AR(3)** model has the lowest AIC (1436.66) and a relatively low BIC (1457.73), it is a good fit according to the AIC criterion. Moreover, according to the graph, it has the smallest volatility.

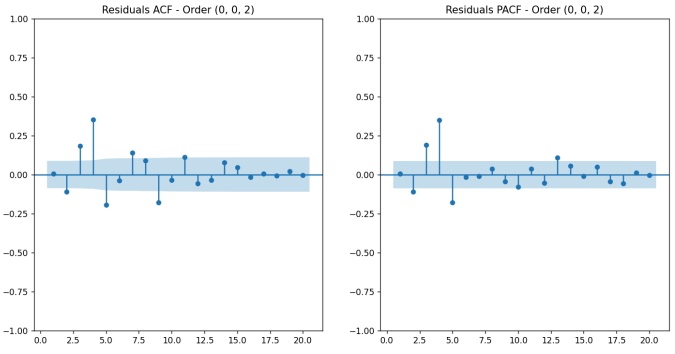


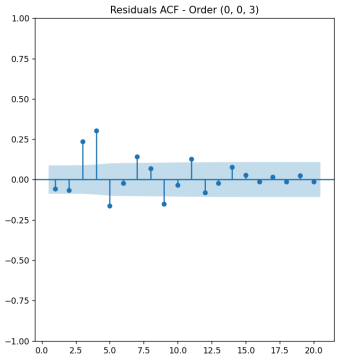


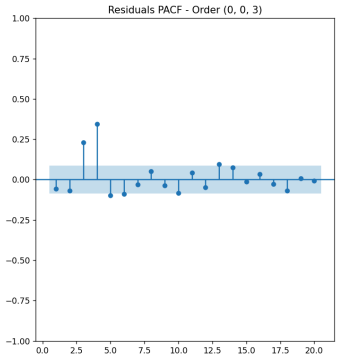

Residuals ACF - Order (2, 0, 0)
Residuals PACF - Order (2, 0, 0)


Residuals ACF - Order (3, 0, 0)
Residuals PACF - Order (3, 0, 0)


Residuals ACF - Order (0, 0, 1)
Residuals PACF - Order (0, 0, 1)


Residuals ACF - Order (0, 0, 2)
Residuals PACF - Order (0, 0, 2)





	AIC	BIC
AR(1)	1644.655505	1657.299329
AR(2)	1581.079266	1597.937698
AR(3)	1436.659807	1457.732847
MA(1)	1567.403626	1580.047451
MA(2)	1537.941206	1554.799639
MA(3)	1536.867709	1557.940749