Problem 1:

Initial parameters:

num_simulations = 10000   : Number of simulations to perform

sigma = 0.5 : standard deviation for the normal distribution of returns

initial_price = 100 : Example initial price P_t-1

With the formulas of the three types of returns in Notes: Classical Brownian Motion, Arithmetic Return System, Geometric Brownian Motion. I got following result:

Classical Brownian Motion:

Mean: 100.00488632834954, Standard Deviation: 0.4993929218440242

Arithmetic Return System:

Mean: 100.48863283495524, Standard Deviation: 49.939292184402426

Log Return or Geometric Brownian Motion:

Mean: 113.90641029354727, Standard Deviation: 61.31809242979602

Problem 2:

With confidence level set to 95% and lambda set to 0.94 as the weight given to more recent observations in EWMV calculations

Var with 5 decimals kept

VaR (Normal Distribution): -0.05428

VaR (EWM): -0.03013

VaR (MLE T Distribution): -0.04313

VaR (AR(1) Model): -0.05389

VaR (Historical Simulation): -0.03948

Problem 3:

Historical VaR (Arithmetic) in $ for each portfolio:

Portfolio A: $-16987.48

Portfolio B: $-10980.36

Portfolio C: $-22143.33

Total Portfolio Historical VaR (Arithmetic) in $: $-49437.19

Lognormal VaR in $ for each portfolio:

Portfolio A: $-19635.93

Portfolio B: $-11524.31

Portfolio C: $-24772.48

Total Portfolio Lognormal VaR in $: $-54381.62

Discuss:

Historical VaR directly uses past returns to determine potential future losses without assuming the returns follow any specific distribution. It's grounded in actual market behavior, providing a pragmatic view of risk based on historical performance.

Lognormal VaR assumes returns follow a lognormal distribution, a common assumption for financial assets over time. This method models risk based on the statistical properties of log-transformed returns, capturing effects like skewness and the non-negativity of asset prices.

Portfolios A, B, and C show higher potential losses under the Lognormal method than Historical, suggesting the lognormal distribution might overestimate risk compared to actual historical data.

Selection:

These methods were chosen to compare a model-free approach with a model-based one, providing insights into risk estimations under different assumptions.

Effect:

The choice between Historical and Lognormal VaR affects risk estimates, with the Lognormal method generally predicting higher risk.