Statin Simulation Report

Haodong Li

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Simulation

DGD:

$$\begin{array}{lcl} W_1 & \sim & Unif(-1,1) \\ W_2 & \sim & Unif(-1,1) \\ A & \sim & Bernoulli(p) \text{ where } p = \operatorname{expit}(0.1*W_1*W_2 - 0.4*W_1) \\ \tau & = & W_1^2*(W_1 + 7/5) + (5*W_2/3)^2 \\ \mu_Y & = & A*\tau + W_1*W_2 + 2*W_2^2 - W_1 \\ Y & \sim & N(\mu_Y,1) \end{array}$$

Initial estimating models for Q and g:

- 1) GAM: General Additive Models (Correctly specified based on true DGD of Q and g)
- 2) Earth: Multivariate Adaptive Regression Splines

Note this is not SL, just individual algorithms. For estimations of other terms (τ, τ_s, γ_s) , all use Earth.

CATE estimation

- 1) DR-learner: regress pseudo outcome estimates φ_n^0 on W.
- 2) T-learner: just use $\bar{Q}_n^0(1,W)$ $\bar{Q}_n^0(0,W)$

Truncation:

For TMLE, $\bar{Q}_n^{(0)} \in [0.001, 0.999], g_n \in [0.025, 0.975].$

For EE, $g_n \in [0.025, 0.975]$

Table 1: Configuration of simulations

Table_ID	${\rm Model}_{\rm Q}_{\rm g}$	${\rm CATE_learner}$	CV
Table 2 Table 3	GAM GAM	DR-learner T-learner	No No
Table 3 Table 4	Earth	DR-learner	No
Table 5	Earth	T-learner	No
Table 6 Table 7	GAM GAM	DR-learner T-learner	Yes Yes
Table 8	Earth	DR-learner	Yes
Table 9	Earth	T-learner	Yes

Table 2: Performance of TMLE and EE for Theta (Correct Q and g, DR-learner, no CV)

n	Method	$True_Theta$	Variance	Bias	MSE	Coverage	${\bf Coverage_or}$	CI_width
500	TMLE EE	$0.686 \\ 0.686$	$0.0265 \\ 0.0414$	$0.0302 \\ 0.0758$	$0.0275 \\ 0.0472$	$0.938 \\ 0.914$	$0.948 \\ 0.928$	$0.6386 \\ 0.7010$
1000	$\begin{array}{c} \mathrm{TMLE} \\ \mathrm{EE} \end{array}$	$0.686 \\ 0.686$	$0.0124 \\ 0.0165$	$0.0100 \\ 0.0430$	$0.0125 \\ 0.0183$	$0.950 \\ 0.930$	$0.954 \\ 0.942$	$0.4369 \\ 0.4652$
2000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	$0.0056 \\ 0.0061$	$0.0070 \\ 0.0272$	$0.0057 \\ 0.0068$	$0.958 \\ 0.942$	$0.956 \\ 0.928$	$0.3062 \\ 0.3145$
3000	TMLE EE	0.686 0.686	0.0040 0.0041	0.0112 0.0215	0.0041 0.0046	0.946 0.942	0.940 0.938	$0.2504 \\ 0.2538$
4000	TMLE EE	0.686 0.686	0.0029 0.0031	$0.0054 \\ 0.0106$	0.0030 0.0032	$0.952 \\ 0.954$	0.952 0.952	0.2152 0.2169
5000	TMLE EE	0.686 0.686	0.0023 0.0023	$0.0038 \\ 0.0046$	0.0023 0.0023	$0.956 \\ 0.958$	0.952 0.946	$0.1925 \\ 0.1931$
10000	TMLE EE	$0.686 \\ 0.686$	0.0013 0.0013	$0.0060 \\ 0.0000$	0.0013 0.0013	$0.934 \\ 0.934$	$0.938 \\ 0.946$	0.1364 0.1360
20000	TMLE EE	0.686 0.686	0.0006 0.0006	0.0079 -0.0017	0.0007 0.0006	0.938 0.948	0.940 0.944	0.0966 0.0960

Table 3: Performance of TMLE and EE for Theta (Correct Q and g, T-learner, no CV)

n	Method	${\bf True_Theta}$	Variance	Bias	MSE	Coverage	${\bf Coverage_or}$	CI_width
500	TMLE EE	$0.686 \\ 0.686$	$0.0234 \\ 0.0238$	$0.0276 \\ 0.0442$	$0.0242 \\ 0.0257$	$0.954 \\ 0.944$	$0.954 \\ 0.946$	$0.6279 \\ 0.6111$
1000	TMLE EE	0.686 0.686	0.0113 0.0112	0.0179 0.0285	$0.0116 \\ 0.0120$	$0.954 \\ 0.950$	0.936 0.934	0.4373 0.4287
2000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	$0.0061 \\ 0.0061$	$0.0121 \\ 0.0161$	$0.0062 \\ 0.0064$	$0.952 \\ 0.940$	$0.932 \\ 0.932$	$0.3062 \\ 0.3024$
3000	TMLE EE	0.686 0.686	0.0039 0.0039	$0.0082 \\ 0.0106$	0.0040 0.0040	0.944 0.944	0.948 0.952	0.2488 0.2466
4000	TMLE EE	0.686 0.686	0.0026 0.0026	$0.0057 \\ 0.0073$	0.0027 0.0027	$0.964 \\ 0.964$	$0.952 \\ 0.944$	$0.2154 \\ 0.2137$
5000	TMLE EE	$0.686 \\ 0.686$	$0.0024 \\ 0.0024$	$0.0090 \\ 0.0101$	$0.0025 \\ 0.0025$	$0.956 \\ 0.946$	$0.950 \\ 0.944$	0.1929 0.1915
10000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	$0.0012 \\ 0.0012$	$0.0057 \\ 0.0058$	$0.0012 \\ 0.0012$	$0.952 \\ 0.954$	$0.944 \\ 0.942$	0.1363 0.1358
20000	TMLE EE	0.686 0.686	$0.0005 \\ 0.0005$	0.0044 0.0041	0.0006 0.0006	$0.960 \\ 0.958$	$0.952 \\ 0.952$	0.0962 0.0960

Table 4: Performance of TMLE and EE for Theta (earth est Q and g, DR-learner, no CV)

n	Method	True_Theta	Variance	Bias	MSE	Coverage	Coverage_or	CI_width
500	TMLE	0.686	0.0930	0.0069	0.0930	0.918	0.978	0.7526
	EE	0.686	0.3765	0.1823	0.4097	0.920	0.968	1.1256
1000	TMLE	0.686	0.0274	-0.0071	0.0275	0.932	0.980	0.4766
1000	EE	0.686	0.0467	0.0648	0.0509	0.932	0.972	0.5946
2000	TMLE	0.686	0.0077	-0.0053	0.0077	0.944	0.964	0.3217
2000	EE	0.686	0.0245	0.0496	0.0269	0.952	0.964	0.4007
3000	TMLE	0.686	0.0053	-0.0072	0.0053	0.920	0.960	0.2539
3000	EE	0.686	0.0071	0.0293	0.0080	0.926	0.954	0.2831
4000	TMLE	0.686	0.0032	-0.0148	0.0035	0.934	0.950	0.2171
4000	EE	0.686	0.0056	0.0157	0.0059	0.952	0.976	0.2379
5000	TMLE	0.686	0.0027	-0.0170	0.0029	0.930	0.944	0.1978
3000	EE	0.686	0.0033	0.0075	0.0033	0.958	0.964	0.2069
10000	TMLE	0.686	0.0014	-0.0174	0.0017	0.884	0.922	0.1360
10000	EE	0.686	0.0013	-0.0018	0.0013	0.926	0.944	0.1379
20000	TMLE	0.686	0.0006	-0.0160	0.0009	0.876	0.896	0.0957
20000	EE	0.686	0.0006	-0.0036	0.0006	0.942	0.950	0.0968

Table 5: Performance of TMLE and EE for Theta (earth est Q and g, T-learner, no CV)

n	Method	True_Theta	Variance	Bias	MSE	Coverage	Coverage_or	CI_width
500	TMLE EE	$0.686 \\ 0.686$	$0.0239 \\ 0.0254$	0.0121 -0.0018	$0.0240 \\ 0.0255$	$0.958 \\ 0.962$	$0.962 \\ 0.956$	$0.6459 \\ 0.6530$
1000	TMLE EE	$0.686 \\ 0.686$	$0.0114 \\ 0.0122$	0.0093 -0.0005	$0.0115 \\ 0.0122$	$0.958 \\ 0.952$	$0.940 \\ 0.940$	$0.4492 \\ 0.4557$
2000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	$0.0065 \\ 0.0066$	0.0002 -0.0051	$0.0065 \\ 0.0066$	$0.946 \\ 0.942$	$0.948 \\ 0.948$	0.3133 0.3171
3000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	$0.0042 \\ 0.0043$	-0.0053 -0.0088	$0.0042 \\ 0.0044$	$0.934 \\ 0.944$	$0.936 \\ 0.940$	$0.2500 \\ 0.2529$
4000	TMLE EE	0.686 0.686	0.0028 0.0028	-0.0087 -0.0107	0.0029 0.0029	0.942 0.946	0.940 0.942	0.2165 0.2186
5000	TMLE EE	$0.686 \\ 0.686$	$0.0025 \\ 0.0026$	-0.0060 -0.0082	$0.0026 \\ 0.0026$	$0.942 \\ 0.942$	$0.946 \\ 0.946$	$0.1924 \\ 0.1946$
10000	TMLE EE	$0.686 \\ 0.686$	$0.0012 \\ 0.0012$	-0.0099 -0.0108	$0.0013 \\ 0.0013$	$0.942 \\ 0.938$	$0.944 \\ 0.946$	$0.1355 \\ 0.1372$
20000	TMLE EE	$0.686 \\ 0.686$	$0.0006 \\ 0.0006$	-0.0119 -0.0125	$0.0007 \\ 0.0007$	$0.920 \\ 0.924$	0.920 0.916	0.0955 0.0966

Table 6: Performance of TMLE and EE for Theta (Correct Q and g, DR-learner, CV)

n	Method	True_Theta	Variance	Bias	MSE	Coverage	Coverage_or	CI_width
500	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	0.0299 0.0868	0.0302 -0.1703	$0.0308 \\ 0.1158$	$0.964 \\ 0.852$	$0.960 \\ 0.962$	0.6897 0.8408
1000	TMLE EE	0.686 0.686	0.0129 0.0168	0.0162 -0.0946	0.0132 0.0257	0.960 0.870	0.950 0.894	0.4583 0.4990
2000	TMLE EE	0.686 0.686	0.0058 0.0068	0.0072 -0.0512	0.0059 0.0094	0.960 0.902	0.954 0.914	0.3119 0.3246
3000	TMLE EE	0.686 0.686	0.0041 0.0045	0.0094 -0.0306	0.0041 0.0054	0.948 0.908	0.944 0.930	0.2536 0.2594
4000	TMLE EE	0.686 0.686	$0.0030 \\ 0.0032$	0.0040 -0.0231	0.0030 0.0038	0.964 0.918	$0.956 \\ 0.932$	0.2169 0.2202
5000	TMLE EE	$0.686 \\ 0.686$	$0.0023 \\ 0.0024$	0.0023 -0.0206	0.0023 0.0028	$0.958 \\ 0.922$	$0.954 \\ 0.922$	0.1937 0.1956
10000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	0.0013 0.0013	0.0036 -0.0084	0.0013 0.0014	$0.930 \\ 0.918$	$0.946 \\ 0.936$	$0.1366 \\ 0.1367$
20000	TMLE EE	$0.686 \\ 0.686$	0.0006 0.0006	0.0055 -0.0053	0.0006 0.0006	$0.936 \\ 0.942$	0.936 0.946	$0.0966 \\ 0.0962$

Table 7: Performance of TMLE and EE for Theta (Correct Q and g, T-learner, CV)

n	Method	True_Theta	Variance	Bias	MSE	Coverage	Coverage_or	CI_width
500	TMLE EE	$0.686 \\ 0.686$	$0.0252 \\ 0.0277$	-0.0295 -0.0506	$0.0261 \\ 0.0302$	$0.926 \\ 0.918$	$0.950 \\ 0.932$	$0.6457 \\ 0.6667$
1000	TMLE EE	$0.686 \\ 0.686$	$0.0111 \\ 0.0118$	-0.0136 -0.0207	0.0113 0.0122	$0.950 \\ 0.946$	$0.950 \\ 0.944$	$0.4395 \\ 0.4456$
2000	TMLE EE	$0.686 \\ 0.686$	$0.0061 \\ 0.0061$	-0.0055 -0.0086	$0.0062 \\ 0.0062$	$0.942 \\ 0.938$	$0.944 \\ 0.940$	$0.3068 \\ 0.3085$
3000	TMLE EE	0.686 0.686	0.0039 0.0040	-0.0049 -0.0067	$0.0040 \\ 0.0040$	0.940 0.938	0.938 0.944	0.2488 0.2496
4000	TMLE EE	0.686 0.686	0.0027 0.0026	-0.0052 -0.0059	0.0027 0.0027	$0.952 \\ 0.954$	0.938 0.940	0.2153 0.2159
5000	TMLE EE	$0.686 \\ 0.686$	$0.0025 \\ 0.0025$	-0.0016 -0.0008	$0.0025 \\ 0.0025$	$0.950 \\ 0.952$	$0.964 \\ 0.958$	$0.1928 \\ 0.1931$
10000	TMLE EE	$0.686 \\ 0.686$	$0.0012 \\ 0.0012$	-0.0009 -0.0001	$0.0012 \\ 0.0012$	$0.952 \\ 0.954$	$0.954 \\ 0.954$	$0.1363 \\ 0.1364$
20000	TMLE EE	$0.686 \\ 0.686$	$0.0005 \\ 0.0005$	$0.0006 \\ 0.0008$	$0.0005 \\ 0.0005$	$0.958 \\ 0.958$	0.950 0.950	0.0962 0.0962

Table 8: Performance of TMLE and EE for Theta (earth est Q and g, DR-learner, CV)

n	Method	True_Theta	Variance	Bias	MSE	Coverage	Coverage_or	CI_width
500	$\begin{array}{c} \mathrm{TMLE} \\ \mathrm{EE} \end{array}$	$0.686 \\ 0.686$	0.0913 0.5247	0.0484 -0.2821	$0.0936 \\ 0.6043$	$0.948 \\ 0.844$	$0.968 \\ 0.948$	0.9364 1.4933
1000	TMLE EE	0.686 0.686	0.0209 0.1500	0.0283 -0.1901	0.0217 0.1861	$0.952 \\ 0.850$	0.954 0.944	$0.5865 \\ 0.8569$
2000	TMLE EE	0.686 0.686	0.0092 0.0345	0.0108 -0.0549	0.0094 0.0375	$0.946 \\ 0.884$	0.950 0.970	$0.3653 \\ 0.4765$
3000	TMLE EE	0.686 0.686	0.0053 0.0151	0.0006 -0.0443	$0.0053 \\ 0.0171$	0.938 0.898	0.954 0.960	0.2753 0.3244
4000	TMLE EE	0.686 0.686	$0.0034 \\ 0.0085$	-0.0102 -0.0356	0.0035 0.0098	0.940 0.912	0.956 0.966	0.2270 0.2580
5000	TMLE EE	$0.686 \\ 0.686$	$0.0030 \\ 0.0047$	-0.0125 -0.0235	$0.0032 \\ 0.0053$	$0.930 \\ 0.936$	$0.954 \\ 0.966$	$0.2065 \\ 0.2282$
10000	${ m TMLE}$ ${ m EE}$	$0.686 \\ 0.686$	$0.0014 \\ 0.0016$	-0.0179 -0.0108	$0.0017 \\ 0.0017$	$0.878 \\ 0.918$	$0.920 \\ 0.944$	$0.1369 \\ 0.1412$
20000	TMLE EE	$0.686 \\ 0.686$	$0.0007 \\ 0.0007$	-0.0170 -0.0075	0.0009 0.0007	$0.870 \\ 0.932$	0.900 0.950	0.0966 0.0969

Table 9: Performance of TMLE and EE for Theta (earth est Q and g, T-learner, CV)

n	Method	True_Theta	Variance	Bias	MSE	Coverage	Coverage_or	CI_width
500	TMLE EE	$0.686 \\ 0.686$	$0.0287 \\ 0.0436$	-0.0312 -0.0780	0.0297 0.0497	$0.916 \\ 0.910$	$0.942 \\ 0.950$	$0.6818 \\ 0.7574$
1000	TMLE EE	$0.686 \\ 0.686$	$0.0134 \\ 0.0155$	-0.0229 -0.0501	$0.0139 \\ 0.0180$	$0.928 \\ 0.924$	$0.944 \\ 0.930$	$0.4649 \\ 0.4948$
2000	TMLE EE	$0.686 \\ 0.686$	$0.0063 \\ 0.0067$	-0.0168 -0.0295	$0.0066 \\ 0.0075$	$0.942 \\ 0.940$	$0.946 \\ 0.942$	0.3184 0.3299
3000	TMLE EE	0.686 0.686	0.0043 0.0046	-0.0128 -0.0203	$0.0045 \\ 0.0050$	$0.928 \\ 0.932$	0.946 0.950	0.2541 0.2610
4000	TMLE EE	0.686 0.686	$0.0030 \\ 0.0031$	-0.0176 -0.0230	0.0034 0.0036	0.934 0.934	0.944 0.936	0.2166 0.2214
5000	TMLE EE	$0.686 \\ 0.686$	$0.0024 \\ 0.0025$	-0.0195 -0.0226	$0.0028 \\ 0.0030$	$0.926 \\ 0.930$	$0.938 \\ 0.928$	$0.1932 \\ 0.1969$
10000	TMLE EE	$0.686 \\ 0.686$	$0.0013 \\ 0.0013$	-0.0181 -0.0189	$0.0017 \\ 0.0017$	$0.886 \\ 0.892$	$0.912 \\ 0.904$	$0.1352 \\ 0.1372$
20000	TMLE EE	$0.686 \\ 0.686$	$0.0006 \\ 0.0006$	-0.0168 -0.0173	0.0009 0.0009	0.884 0.894	0.902 0.896	0.0953 0.0967

Table 10: Performance of SS and TMLE for Theta (earth est Q and g, DR-learner, no CV)

n	Method	${\bf True_Theta}$	Variance	Bias	MSE
1000	SS TMLE	$0.686 \\ 0.686$	$0.4599 \\ 0.0274$	4.3097 -0.0071	$19.0335 \\ 0.0275$

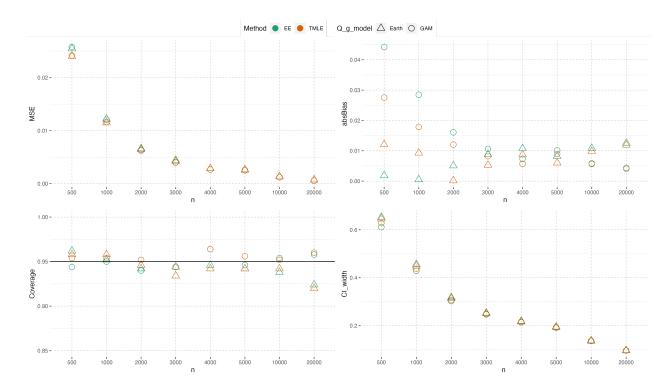


Figure 1: Main performance metrics (T-learner, no CV)

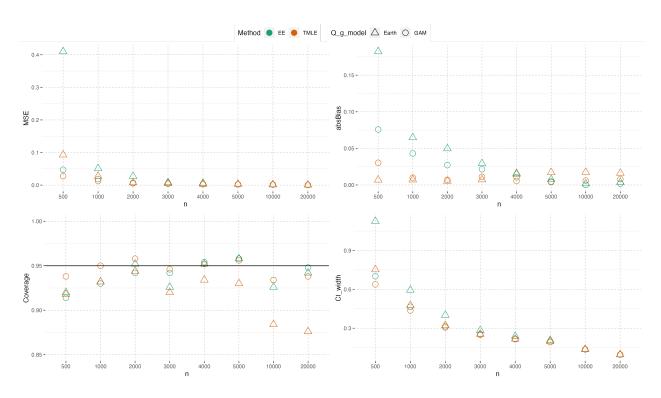


Figure 2: Main performance metrics (DR-learner, no CV)

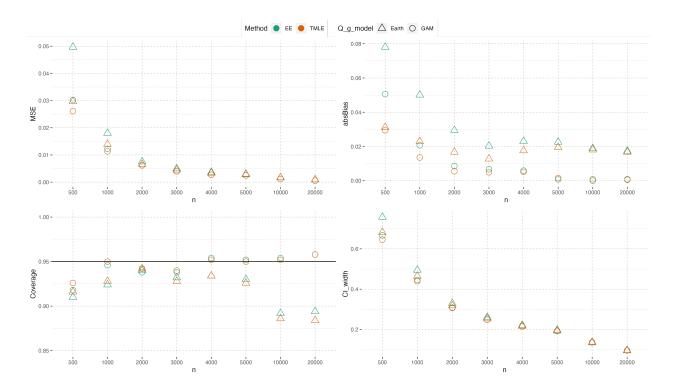


Figure 3: Main performance metrics (T-learner, CV)

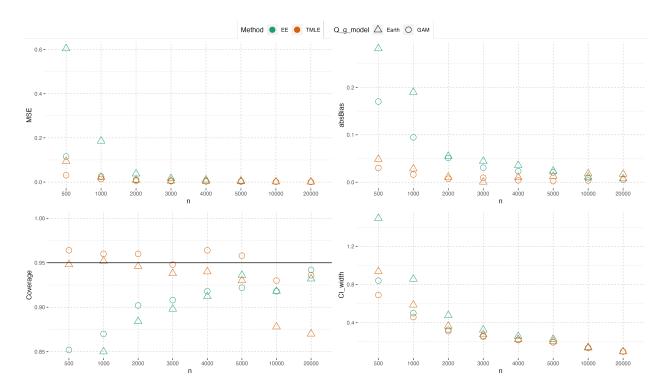


Figure 4: Main performance metrics (DR-learner, CV)

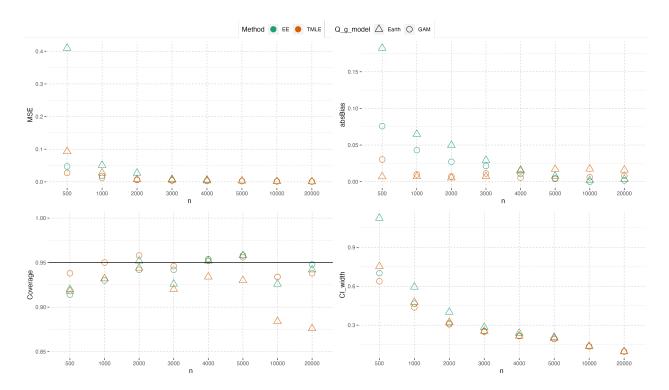


Figure 5: Main performance metrics (DR-learner, no CV)

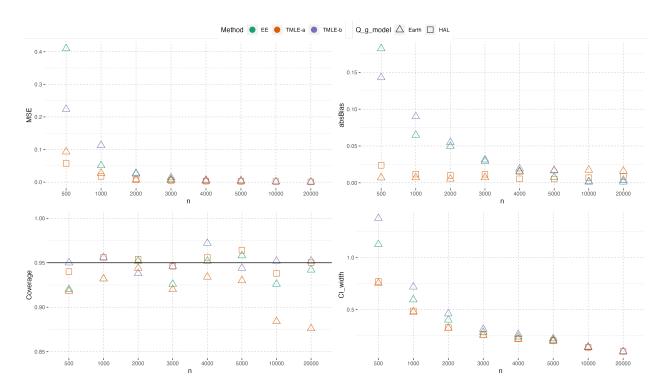


Figure 6: Main performance metrics (DR-learner, no CV)

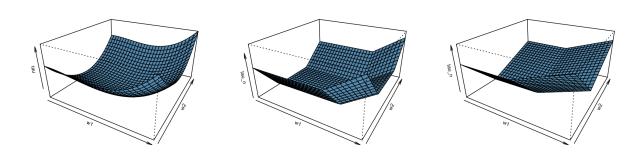


Figure 7: CATE estimation with Earth (n = 500)

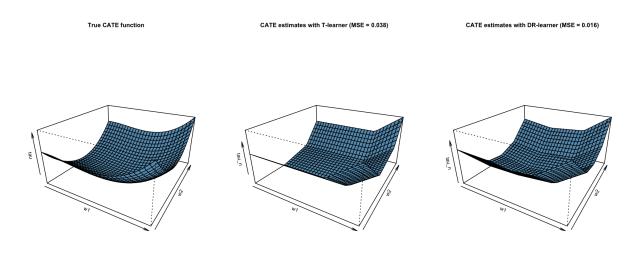
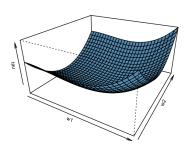
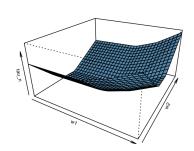


Figure 8: CATE estimation with Earth (n = 5000)





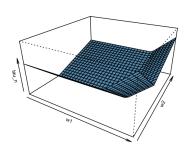


Figure 9: CATE estimation with HAL (n = 500)

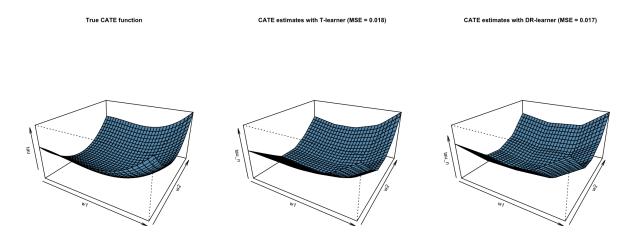
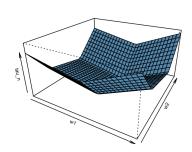


Figure 10: CATE estimation with HAL (n = 5000)

True CATE function



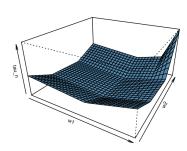


Figure 11: CATE estimation with Earth (n = 500, seed 2)

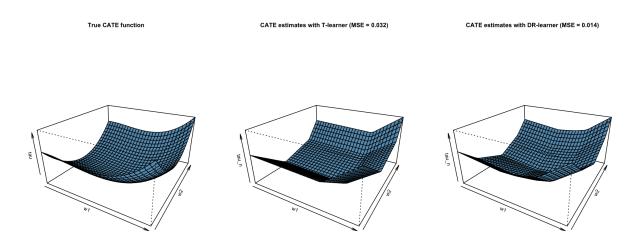


Figure 12: CATE estimation with Earth (n = 5000, seed 2)

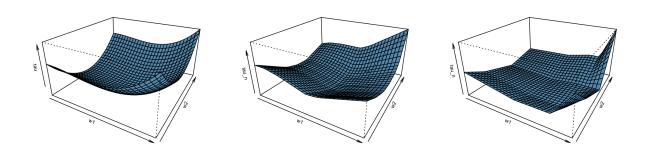


Figure 13: CATE estimation with HAL (n = 500, seed 2)

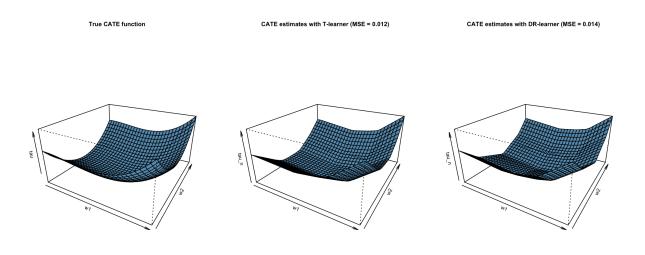


Figure 14: CATE estimation with HAL (n = 5000, seed 2)