Plate Report

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1 Formula Derivation

In the stap90, I create two kinds of plate elements, both of which are for thick plate.

The difference between them is the number of the nodes in each element. The first type has only 4 nodes each element, the same as 4Q, while the other has 8. I would explain the reason below.

1.1 Plate4Q

The Plate4Q element is almost the same as 4Q. We use isoparametric element by represent both the transformation of coordination and the w, β_x, β_y with the same bunch of bilinear shape function.

The only disparity is the formula of strain and stress, as shown below.

As for strain:

$$\begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_x y \end{pmatrix} = -z \begin{pmatrix} \frac{\partial \beta_x}{\partial x} \\ \frac{\partial \beta_y}{\partial y} \\ \frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x} \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} \gamma_{xz} \\ \gamma_{yz} \end{pmatrix} = \begin{pmatrix} \frac{\partial w}{\partial x} - \gamma_x \\ \frac{\partial \beta_y}{\partial y} - \gamma_y \end{pmatrix} \tag{2}$$

And for stress:

$$\begin{pmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_x y \end{pmatrix} = -z \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} \begin{pmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \gamma_x y \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} \tau_{xz} \\ \tau_{yz} \end{pmatrix} = \frac{E}{2(1+\nu)} \begin{pmatrix} \gamma_{xz} \\ \gamma_{yz} \end{pmatrix} \tag{4}$$

The last thing we need is the formula for total energy:

$$\begin{aligned} \Pi = & \frac{1}{2} \int_A \int_{-h/2}^{h/2} \begin{pmatrix} \epsilon_{xx} & \epsilon_{yy} & \gamma_x y \end{pmatrix} \begin{pmatrix} \tau_{xx} \\ \tau_{yy} \\ \tau_x y \end{pmatrix} dz dA \\ & + \frac{k}{2} \int_A \int_{-h/2}^{h/2} \begin{pmatrix} \epsilon_{xz} & \epsilon_{yz} \end{pmatrix} \begin{pmatrix} \tau_{xz} \\ \tau_{yz} \end{pmatrix} dz dA \end{aligned} \tag{5}$$

Then we could use the variational principle to gain the specific formula of each term in the equation below.

$$(K_\kappa + K_\gamma)d = f \tag{6}$$

However, I soon find out that this element could not pass any kinds of patch test, since it has represent bending. Therefore, I choose to modify the element to 8 nodes.

1.2 Plate8Q

The increase of the number of nodes barely change anything. The only alternation is using the serendipity shape function for 8 nodes instead of the bilinear one of 4 nodes.

2 Convergence Analysis

The convergence of the two elements are the same as 4Q and 8Q.

Both could exactly reconstruct linear field. And the plate8Q element could only reconstruct a quadratic field only is the coordination transformation is linear.

3 Patch Test

After discussion, our group decides to use a simple quadratic w field to do the patch test.

As we have no idea the deformation formula for a plate, we simplified it to a beam by restricting the β_y for every node and fixed one end of the board to make it a cantilever.

From the formula of a cantilever in the situation that only a torque acts on the free end.

$$\begin{aligned} w &= \frac{Mx^2}{2EI} \\ \beta_x &= \frac{Mx}{EI} \end{aligned} \tag{7}$$

In the patch test, we set the length of the board as 40m, the width as 2m, and the thickness as 0.1m, E as 10^10, ν as 0, and the total M in one end as 80 N.M.

Therefore,

$$I = 2 \times \int_{-0.05}^{0.05} z^2 dz = \frac{1}{6000} \tag{8}$$

Put those numbers back to (7), we have,

$$\begin{aligned} w &= 2.4 \times 10^{-5} x^2 \\ \beta_x &= 4.8 \times 10^{-5} x \end{aligned} \tag{9}$$

And the result from the two kinds of elements are:

The 4 nodes element:

D I S P L A C E M E N T S

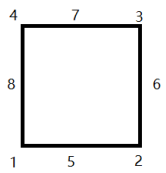
NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
1	0.00000E+00	0.00000E+00	0.00000E+00
2	0.00000E+00	0.00000E+00	0.00000E+00
3	0.00000E+00	0.00000E+00	0.00000E+00
4	0.57597E-06	0.57597E-07	0.00000E+00
5	0.57597E-06	0.57597E-07	0.00000E+00
6	0.57597E-06	0.57597E-07	0.00000E+00
7	0.23039E-05	0.11519E-06	0.00000E+00
8	0.23039E-05	0.11519E-06	0.00000E+00
9	0.23039E-05	0.11519E-06	0.00000E+00

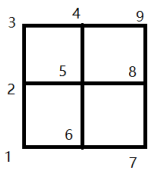
The 8 nodes elements:

D I S P L A C E M E N T S

NODE	X-DISPLACEMENT	Y-DISPLACEMENT	Z-DISPLACEMENT
1	0.00000E+00	0.00000E+00	0.00000E+00
2	0.38400E-01	0.19200E-02	0.00000E+00
3	0.38400E-01	0.19200E-02	0.00000E+00
4	0.00000E+00	0.00000E+00	0.00000E+00
5	0.96000E-02	0.96000E-03	0.00000E+00
6	0.38400E-01	0.19200E-02	0.00000E+00
7	0.96000E-02	0.96000E-03	0.00000E+00
8	0.00000E+00	0.00000E+00	0.00000E+00

The nodes are indexed in the following way.



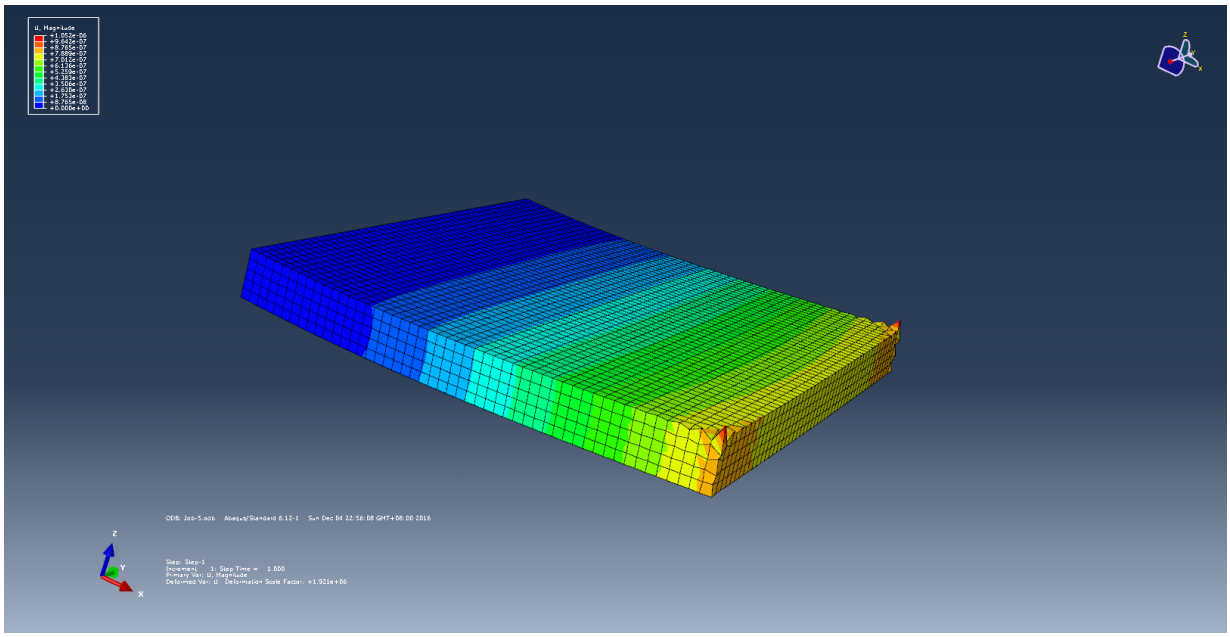


As we could see, since the 4 nodes element could not represent bending, the result of it was poor, while the 8 nodes element successfully passed the patch test.

4 Application

I use the Abaqus to check the 2 element.

The problem is that two concentrate force act on the corner of a square board.



The maxima of displacement is 1.05E-6

In the plate4Q element, I use 16 elements. The answer is shown below.

D I S P L A C E M E N T S

NODE	W	BETA_X	BETA_Y
1	0.00000E+00	0.00000E+00	0.00000E+00
2	0.21448E-06	0.42636E-07	0.25127E-08
3	0.21975E-06	0.40684E-07	-0.25891E-21
4	0.00000E+00	0.00000E+00	0.00000E+00
5	0.53146E-07	0.21840E-07	0.22433E-08
6	0.22105E-06	0.41457E-07	0.41694E-09
7	0.60161E-07	0.22520E-07	-0.74446E-22
8	0.00000E+00	0.00000E+00	0.00000E+00
9	0.74715E-06	0.59310E-07	-0.85034E-08
10	0.69327E-06	0.51158E-07	-0.24485E-21
11	0.46051E-06	0.52729E-07	-0.23656E-08
12	0.70919E-06	0.53985E-07	-0.48149E-08
13	0.44462E-06	0.47277E-07	-0.26801E-21
14	0.74715E-06	0.59310E-07	0.85034E-08
15	0.21448E-06	0.42636E-07	-0.25127E-08
16	0.70919E-06	0.53985E-07	0.48149E-08
17	0.46051E-06	0.52729E-07	0.23656E-08
18	0.22105E-06	0.41457E-07	-0.41694E-09
19	0.00000E+00	0.00000E+00	0.00000E+00
20	0.53146E-07	0.21840E-07	-0.22433E-08
21	0.00000E+00	0.00000E+00	0.00000E+00

In the plate8Q element, I use 4 elements. The answer is shown below.

D I S P L A C E M E N T S

NODE	W	BETA_X	BETA_Y
1	0.00000E+00	0.00000E+00	0.00000E+00
2	0.21348E-07	0.83680E-08	0.76908E-09
3	0.81883E-07	0.15238E-07	0.22491E-09
4	0.17267E-06	0.20285E-07	-0.18742E-08
5	0.28399E-06	0.22341E-07	-0.57413E-08
6	0.25545E-06	0.18586E-07	-0.38969E-08
7	0.16413E-06	0.17803E-07	-0.15528E-08
8	0.82044E-07	0.14474E-07	-0.10578E-09
9	0.23186E-07	0.85076E-08	0.11981E-09
10	0.00000E+00	0.00000E+00	0.00000E+00
11	0.00000E+00	0.00000E+00	0.00000E+00
12	0.23345E-07	0.85465E-08	0.82718E-24
13	0.81689E-07	0.14027E-07	0.00000E+00
14	0.15986E-06	0.16652E-07	-0.66174E-23
15	0.24493E-06	0.17056E-07	-0.26470E-22
16	0.25545E-06	0.18586E-07	0.38969E-08
17	0.16413E-06	0.17803E-07	0.15528E-08
18	0.82044E-07	0.14474E-07	0.10578E-09
19	0.23186E-07	0.85076E-08	-0.11981E-09
20	0.00000E+00	0.00000E+00	0.00000E+00
21	0.00000E+00	0.00000E+00	0.00000E+00
22	0.21348E-07	0.83680E-08	-0.76908E-09
23	0.81883E-07	0.15238E-07	-0.22491E-09
24	0.17267E-06	0.20285E-07	0.18742E-08
25	0.28399E-06	0.22341E-07	0.57413E-08

Since the elements are still few, it is nice to have the same scale. And the plate8Q is better.