Experimental Design to Find the Optimal Razor

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1 Exclusive Summary

The goal of this report is to design a new experiment to help the Procter & Gamble company to develop a new male razor product. We designed a quarter factional factor experiment. Firstly, we used 32 prototypes to test the main effect and interaction effect of those 7 design factors. Secondly, we used 2 prototypes with all middle-level factors to find out the most influential design factors. Finally, we used 6 prototypes to test the effect of quadratic terms of those 3 design factors.

2 Introduction

2.1 General Background

This report aims to design an experiment to help Dr. William Myers from the Procter & Gamble company develop a new male razor product. The main goal of the Procter & Gamble company is to identify which razor prototype can give the best consumers experiences. According to the discussion with our client, two outcome variables are strongly correlated with customer satisfaction. They are the closeness of shave (Y_1) and skin irritation (Y_2.) Moreover, we know a closer shave is typically more irritating to the skin. Therefore, the relationship between the closeness of shave and skin irritation is inverse. In addition, according to the discussion with our client, the Procter & Gamble engineers find out those seven design factors will influence our outcome variables.

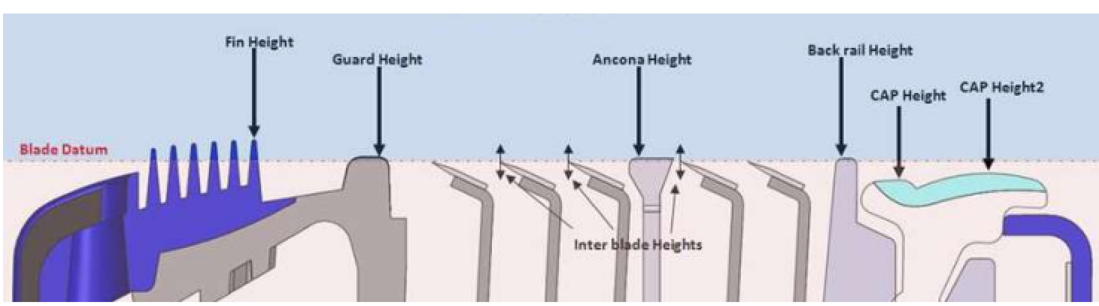

Figure 1: Seven Engineering design factors

2.2 Objectives

The main objective of our client is to design an experiment to explore the relationship between the seven design factors and the two outcome variables up to the second order. In addition, we need to keep the experimental design under design constraints.

3 Design Constraints

Due to the limited budget, we have 40 prototypes in total, which indicates we can only test 40 different combinations of design factors. The following table shows the range of those seven design factors and the increments of those factors no smaller than 0.01.

Factor names (mm)	Lower	Upper
Fin Height	1.5	3.0
Guard Height	0.2	1.2
Ancona Height	1.0	2.0
Inter Blade Heights	0.1	0.5
Back Rail Heights	1.0	3.0
CAP Height	0.0	4.0
CAP Height 2	-3.0	1.0

Moreover, we assume all constraints are independent of each other.

4 Experimental design

To achieve this report's main objective, we would like to use fractional factorial designs to investigate the effect of the seven design factors, quadratic terms, and interactions. Because of the limited sample size, we will consider three different levels for each design factor. We denote the lowest boundary of each factor as -1, the most up boundary of each factor as 1, and the middle of each factor's boundary as 0. For example, we test 0.2 mm guard height as -1, 1.2 mm guard height as 1, and 0.7 mm guard height as 0. Then we will have 3^7 combinations of design factors beyond the sample size of 40. Therefore, we reduce our combinations to 2 level factors. We use alphabet letters to replace design factors' names for easy understanding.

A	B	C	D	E	F	G	...	ABCDE	CDEFG
1	1	1	1	1	1	1	...	1	1
1	1	1	1	1	1	-1	...	1	-1
1	1	1	1	1	-1	1	...	1	-1
1	1	1	1	1	-1	-1	...	1	1
...	...	...	...	...	...	...	...	...	...
-1	1	1	1	1	1	1	...	-1	1
-1	1	1	1	1	1	-1	...	-1	-1
-1	1	1	1	1	-1	1	...	-1	-1
-1	1	1	1	1	-1	-1	...	-1	1

This table has $2^7 = 128$ rows which are still larger than our sample size. We calculate the value of $ABCDE$ and $CDEFG$ by using the product of these five factors. Thus, we use a quarter fractional factor design here. Then we have $\frac{1}{4} * 2^7 = 32$ combinations to test the effect of those seven design factors. For choosing 32 combinations, we choose rows in which the last two columns both be 1. Then we will have the following table, which has 32 rows.

A	B	C	D	E	F	G	...	ABCDE	CDEFG
1	1	1	1	1	1	1	...	1	1
1	1	1	1	1	-1	-1	...	1	1
1	1	-1	-1	-1	-1	1	...	1	1
1	-1	-1	1	1	-1	1	...	1	1
...	...	...	...	...	...	...	...	1	1
-1	1	1	1	-1	-1	1	...	1	1
-1	1	1	1	-1	1	-1	...	1	1
-1	1	-1	1	1	1	-1	...	1	1
-1	1	-1	1	1	-1	1	...	1	1

According to the result given by NIST Engineering Statistics Handbook[1], we know the effects of AB and $BCDEF$ are confounded. It means we can determine the two factors' interaction effect such as AB instead of determining the higher-order interaction effect such as $BCDEF$. Then we would like to build the following equation.

$$Y_1 = \beta_0+\beta_1 A+\beta_2 B+\beta_3 C+\beta_4 D+\beta_5 E+\beta_6 F+\beta_7 G+\beta_8 AB+\beta_9 AC++\beta_{10} AD+\cdots+\beta_{28} EG+\beta_{29} FG+e$$

$$Y_2 = \beta_0+\beta_1 A+\beta_2 B+\beta_3 C+\beta_4 D+\beta_5 E+\beta_6 F+\beta_7 G+\beta_8 AB+\beta_9 AC++\beta_{10} AD+\cdots+\beta_{28} EG+\beta_{29} FG+e$$

A : Fin Height
B : Guard Height
C : Ancona Height
D : Inter Blade Heights
E : Back Rail Heights
F : CAP Height
G : CAP Height 2

To determine the effect of quadratic terms, we can start by using extra 2 prototypes with the following setup.

Index of prototypes	A	B	C	D	E	F	G
33th	0	0	0	0	0	0	0
34th	0	0	0	0	0	0	0

Those 0 means we measure two outcome variables of the 33rd and 34th prototypes if they all have middle-level design factors. Those two prototypes can help us determine which 3 design factors are most important for Y_1 and Y_2 by calculating the difference in the outcome value. Then we can use those 3 design factors to build a three-dimensional axis.

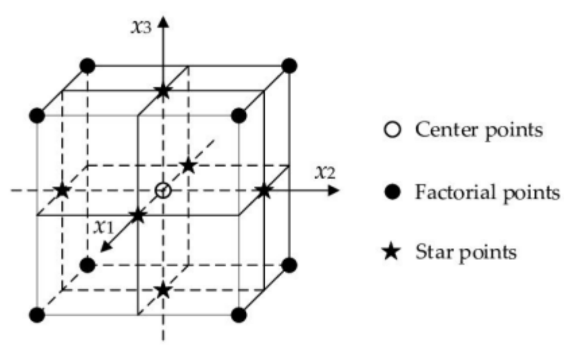

Figure 2: The idea of testing quadratic terms

For figure 2, we can test quadratic terms of those 3 design factors $(x1, x2, x3)$ by placing our prototype in the star points. Then we will have this following experimental design table.

x1	x2	x3
1	0	0
-1	0	0
0	1	0
0	-1	0
0	0	1
0	0	-1

We can build our regression model as:

$$Y_1 = \beta_0 + \beta_1 x1 + \beta_2 x2 + \beta_3 x3 + \beta_4 x1^2 + \beta_5 x2^2 + \beta_6 x3^2 + e$$

$$Y_2 = \beta_0 + \beta_1 x1 + \beta_2 x2 + \beta_3 x3 + \beta_4 x1^2 + \beta_5 x2^2 + \beta_6 x3^2 + e$$

Using the previous experimental design table and regression model, we will know the effect of quadratic terms of 3 design factors $(x1, x2, x3)$.

5 Conclusion

Procter & Gamble wants to create a new male razor product, and the purpose of this report is to design an experiment to help them do so. The experiment is called a quarter factional factor experiment. Firstly, we used 32 prototypes to test the main effect and interaction effect of those 7 design factors. Secondly, we used 2 prototypes with all middle-level factors to find out the most influential design factors. Finally, we used 6 prototypes to test the effect of quadratic terms of those 3 design factors.

References

[1] URL: https://www.itl.nist.gov/div898/handbook/pri/section3/pri334.htm.