Writing Projects:

1. Describe the Lucas–Lehmer test for determining whether a Mersenne number is prime. Discuss the progress of the GIMPS project in finding Mersenne primes using this test.
2. Explain how probabilistic primality tests are used in practice to produce extremely large numbers that are almost certainly prime. Do such tests have any potential drawbacks?

For question 1:
The Lucas-Lehmer test is the current standard algorithm for testing the primality of Mersenne numbers. It may have limitations in terms of efficiency and accuracy. The Lucas-Lehmer test is a method designed to check if a Mersenne number is also a prime number.

Mersenne numbers are numbers of the form $M_p = 2^p - 1$. P is a prime number of this form.

Steps in the Lucas-Lehmer Test:

1. First set S_0=4 which is an initial seed value.
2. Compute a sequence:
   $S_{n+1} = S_n^2 - 2 \mod M_p$, for $n = 1, 2, \ldots, p-2$
3. After p−2 iterations, if the result S_(p−2) mod M_p=0, then M_p is prime. Otherwise, M_p is composite.

The Great Internet Mersenne Prime Search (GIMPS), a collaborative project aiming to discover new Mersenne primes, discovered the largest known prime number having 24,862,048 digits. Participants use distributed computing power to test ever-larger Mersenne numbers using the Lucas-Lehmer test.

Citation:
Ibrahim, M. (2023). On the Eight Levels theorem and applications towards Lucas-Lehmer primality test for Mersenne primes, I. *Arab Journal of Basic and Applied Sciences*, *30*(1), 267–284. https://doi.org/10.1080/25765299.2023.2204672

For question 2:
The probabilistic primality test is an algorithm for determining whether a number is prime. It is particularly useful for testing very large numbers. These tests (such as the Fermat test and the Miller-Rabin test) rely on mathematical properties and algorithms to check whether a number meets certain criteria to determine whether it is prime.

Practical uses of the probabilistic prime test:
Large Number Efficiency: These tests are computationally efficient and suitable for very large numbers.
There is a certain degree of certainty: while prime numbers are not guaranteed, they can achieve an extremely low probability of error.

Iterative Confidence: By running the test multiple times using different parameters (such as the base in Miller-Rabin).

shortcoming:
Non-zero error rate: they may incorrectly identify composite numbers as prime
Non-deterministic: Unlike deterministic tests, they do not provide absolute proof of prime numbers but rather probabilistic guarantees.
Complex numbers and special cases: Certain specially constructed numbers, such as pseudoprimes and Carmichael numbers, can pass these tests, complicating their reliability

Citation:
A. Oliver L. Atkin. *Probabilistic Primality Testing.* University of Illinois, Chicago. Summary by François Morain.