An Improved MRC-Rake Symbol Detector for OTFS Modulation Using Expectation Cancellation

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Outline

- **Motivations**
- EC-MRC-Rake detector
- Simulations

Orthogonal Time Frequency Space (OTFS)

- Properties [1]
  - *Symbols are modulated in the delay-Doppler (DD) domain*
  - *Exploits the time-frequency diversity to improve the error rate*
  - *With lower peak-to-average power ratio (PAPR) than OFDM*

- Probability graph based symbol detection in the DD domain [2], [3], [4]
  - *To approximately solve the Maximum A Posteriori (MAP) detection problem using* ***factor graph***
  - ***Damping factor*** *is introduced to mitigate the error rate degradation caused by loopy graph*
  - ***DD sparsity*** *assumption is usually introduced to reduce the implementation complexity*

[1] R. Hadani, S. Radkib, M. Tsatsanis, A. Monk, A. J. Goldsmith, A. F. Molisch and R. Calderbank, "Orthogonal time frequency space modulation," *IEEE WCNC, San Francisco*, CA, USA, Mar. 2017
[2] P. Raviteja, K. T. Phan, Y. Hong, and E. Viterbo, "Interference cancellation and iterative detection for orthogonal time frequency space modulation," *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 6501-6515, Oct. 2018.
[3] W. Yuan, Z. Wei, J. Yuan, and D. W. K. Ng, "A simple variational Bayes detector for orthogonal time frequency space (OTFS) modulation," *IEEE Trans. Veh. Technol.*, vol. 69, no. 7, pp. 7976-7980, Apr. 2020.
[4] Y. Ge, Q. Deng, P. C. Ching, and Z. Ding, "OTFS signaling for uplink NOMA of heterogeneous mobility users," *IEEE Trans. Commun.*, vol. 69, no. 5, pp. 3147-3161, May. 2021.

Orthogonal Time Frequency Space (OTFS)

- Probability graph based symbol detection in the DD domain

  - *To approximately solve the Maximum A Posteriori (MAP) detection problem using* ***factor graph***

    - When the **girth** of the factor graph becomes small, the detectors are more sensitive to the error propagation and easy to fall into **weak local optimums**

  - ***Damping factor*** *is introduced to mitigate the error rate degradation caused by loopy graph*

    - Smaller damping factor brings a smaller information update step, so as to prevent falling into a weak local optimum, but **more iterations** are need

$$x^{(t)} \leftarrow (1-\alpha)x^{(t-1)} + \alpha x^{(t)}$$

  - ***DD sparsity assumption*** *is usually introduced to reduce the implementation complexity*

    - A Doppler component may spread over **multiple resolvable Doppler bins**
    - Not satisfied in the **Doppler-abundant channels**

Proposed EC-MRC-Rake detector

- Expectation Cancellation (EC) based MRC-Rake detector

  - *Based on the MRC-Rake[1] detector implemented in the* ***delay-time (DT) domain***
    - The complexity no longer depends on the number of Doppler taps
    - DT channels can be well estimated using non-parametric algorithms [2]

  - *Using* ***soft interference cancellation****, inspired by the factor graph based detectors*
    - The **error propagations** can be mitigated, thereby the error rate and convergence can be improved

$$\hat{\mathbf{y}}_i = \mathbf{y} - \sum_{j\neq i} \mathbf{H}_j \hat{\mathbf{x}}_j \xrightarrow{\text{Replace With}} \hat{\mathbf{y}}_i = \mathbf{y} - \sum_{j\neq i} \mathbf{H}_j \boldsymbol{\mu}_j$$

  - *Soft information* $\boldsymbol{\mu}$ *is defined using* ***symbol expectation***
    - Symbol expectation is mapped from the bit expectations, i.e., the bit LLRs
    - The estimation approach of equalized gain and variance is proposed to calculate the LLRs
    - Easy to be combined with the turbo equalization structure [3]

$$LLR(b_{i,0}),\ LLR(b_{i,1}),\ \cdots,\ LLR(b_{i,Q}) \xrightarrow{\text{Mapper}} \boldsymbol{\mu}_i$$

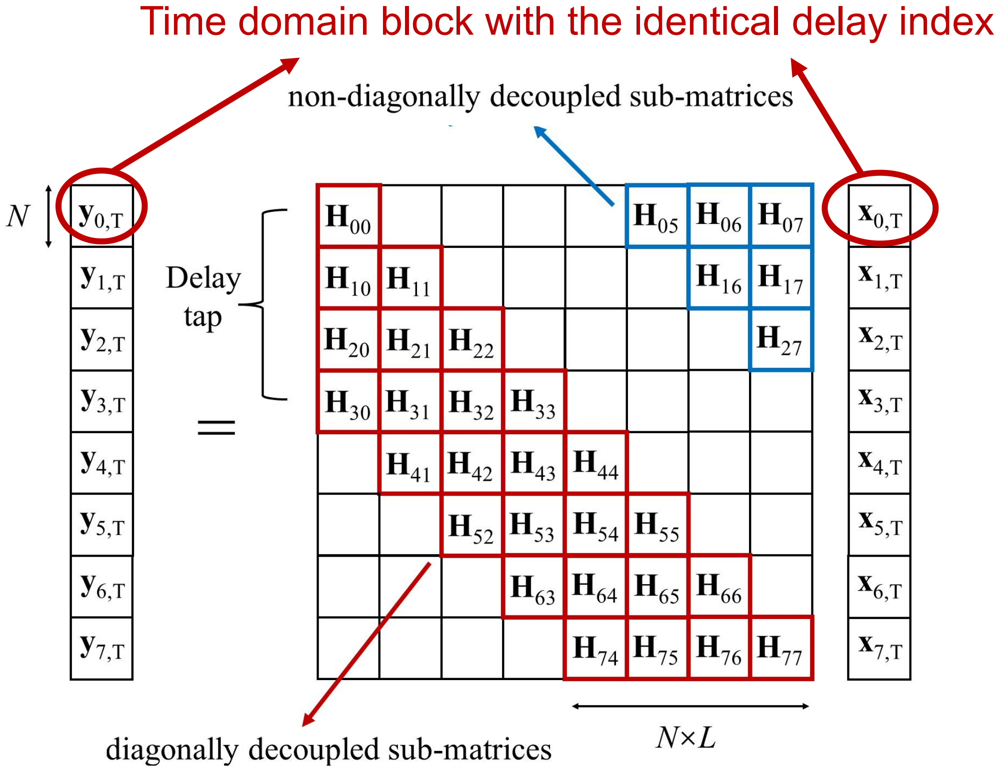
[1] T. Thaj, and E. Viterbo, "Low complexity iterative rake decision feedback equalizer for zero-padded OTFS systems," *IEEE Trans. Veh. Technol.*, vol. 69, no. 12, pp. 15606-15622, Dec. 2020
[2] H. Zhang, J. Li, T. Zhang, and X. Zhu, "An efficient channel estimation scheme for short frame OTFS using impulse-train pilots," *IEEE GLOBECOM*, Rio de Janeiro, Brazil, Dec. 2022
[3] X. Wang, and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, vol. 47, no. 7, pp. 1046-1061, July. 1999.

Outline

- Motivations
- **EC-MRC-Raker detector**
- Simulations

Proposed EC-MRC-Rake detector

- Input-output relationship in the DT domain [1]



- *Block-wise input-output (received block with delay index ℓ):*

$$\mathbf{y}_{\ell,\mathrm{T}} = \sum_{\ell' \in \mathcal{L}} \mathbf{H}_{\ell \langle \ell - \ell' \rangle_M} \mathbf{x}_{\langle \ell - \ell' \rangle_M, \mathrm{T}} + \mathbf{w}_{\ell,\mathrm{T}}$$

  - ➢ Delay domain **block-wise summation**
  - ➢ Delay domain **block-wise interference cancellation** and **MRC**

- *Inner-block input-output (reduced-CP OTFS):*

$$\mathbf{H}_{\ell \langle \ell - \ell' \rangle_M} \mathbf{x}_{\langle \ell - \ell' \rangle_M, \mathrm{T}} = \begin{cases} \mathrm{diag}\left\{ \mathbf{\Pi}_N^{-1} \mathbf{H}_{\ell \langle \ell - \ell' \rangle_M} \right\} \odot \mathbf{x}_{\langle \ell - \ell' \rangle_M, \mathrm{T}}, & \ell' \leq \ell \\ \mathbf{\Pi}_N \left(\mathrm{diag}\left\{ \mathbf{\Pi}_N^{-1} \mathbf{H}_{\ell \langle \ell - \ell' \rangle_M} \right\} \odot \mathbf{x}_{\langle \ell - \ell' \rangle_M, \mathrm{T}} \right), & \ell' > \ell \end{cases}$$

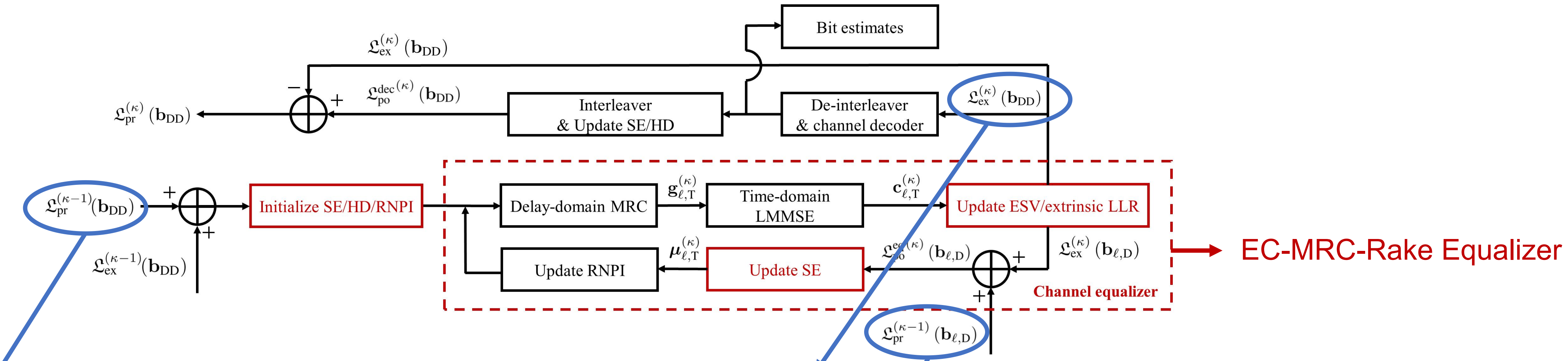
  - ➢ Time domain **element-wise multiplications**
  - ➢ Time domain **one-tap equalization**

[1] T. Thaj, E. Viterbo, and Y. Hong “General I/O relations and low-complexity universal MRC detection for all OTFS variants,” *IEEE Access*, vol. 10, pp. 96026-96037, Sept. 2022.

Proposed EC-MRC-Rake detector

- Turbo equalization structure

  - *Decoding gain is exploited in the iterative channel equalizer*

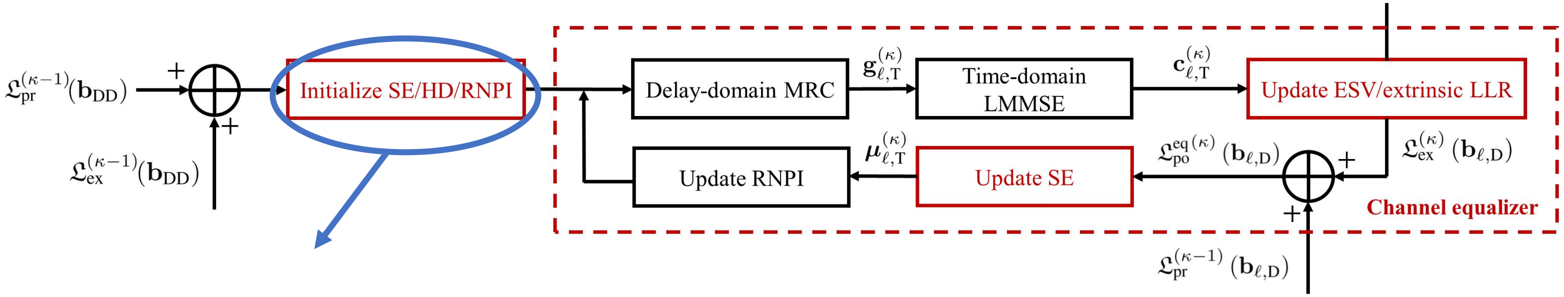


➢ Channel decoder **corrects** the equalizing incremental information (i.e., extrinsic LLR) in the current iteration

➢ Channel equalizer **uses** the decoding incremental information as a priori information in the next iteration

Proposed EC-MRC-Rake detector

- EC-MRC-Rake equalizer



➢ *SE (symbol expectation) : used in the soft interference cancellation*

Bit-to-Symbol Mapper in 3GPP 38.211

$$LLR(b_{\ell k,0}),\cdots,LLR(b_{\ell k,Q}) \xRightarrow{\mathbb{E}\{b_{\ell k,i}\}=\Pr\{b_{\ell k,i}=1\}=\frac{1}{1+e^{LLR(b_{\ell k,i})}}} \mathbb{E}\{b_{\ell k,0}\},\cdots,\mathbb{E}\{b_{\ell k,Q}\} \xRightarrow{\mathcal{F}(\mathbb{E}\{b_{\ell k,0}\},\cdots,\mathbb{E}\{b_{\ell k,Q}\})} \mu_{\ell k}$$

Bit LLRs | Bit Expectations | Symbol Expectation (**delay-Doppler** domain)

➢ *HD (hard decision) : used in the estimation of equalized symbol variance*

➢ *RNPI (residual noise-plus-interference) : used in the interference cancellation to reduce the complexity*

$$\tilde{\boldsymbol{\varepsilon}}_{\mathrm{DT}} = \mathbf{y}_{\mathrm{DT}} - \mathbf{H}_{\mathrm{DT}}\boldsymbol{\mu}_{\mathrm{DT}}$$

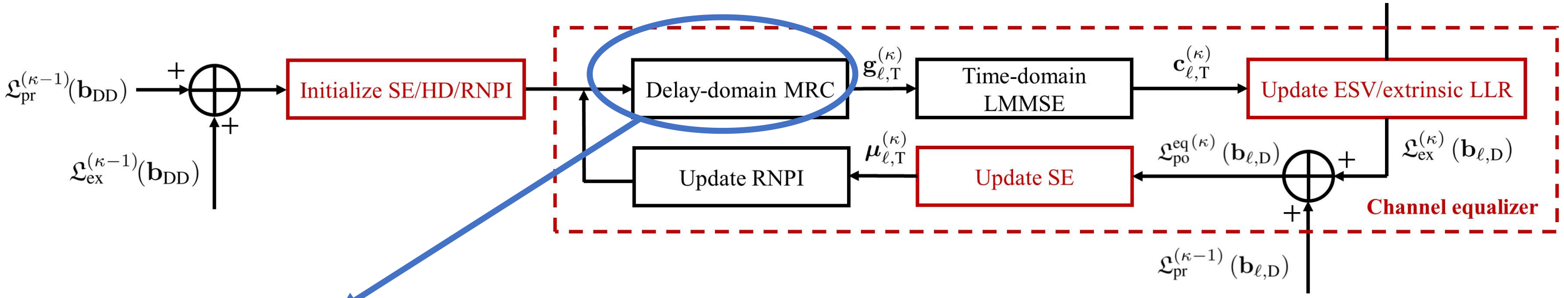
$\mathbf{y}_{\mathrm{DT}}$ Received signal in the **delay-time** domain

$\mathbf{H}_{\mathrm{DT}}$ **Delay-time** channel matrix

$\boldsymbol{\mu}_{\mathrm{DT}}$ Symbol expectation in the **delay-time** domain

Proposed EC-MRC-Rake detector

- EC-MRC-Rake equalizer



- *Block-wise interference cancellation* ***(inter-delay-interference cancellation)***

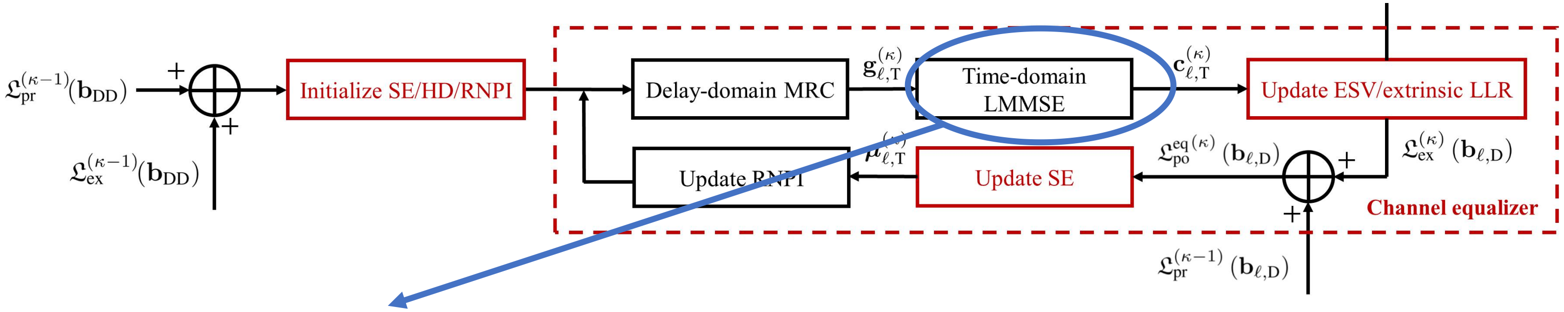
(DT domain) $$\Delta \mathbf{y}_{\langle \ell+\ell'\rangle_M,\mathrm{T}} = \overbrace{\mathbf{y}_{\langle\ell+\ell'\rangle_M,\mathrm{T}} - \underbrace{\sum_{\ell''\in\mathcal{L},\ell''\neq\ell'} \mathbf{H}_{\langle \ell+\ell'\rangle_M \langle \ell+\ell'-\ell''\rangle_M}\boldsymbol{\mu}_{\langle\ell+\ell'-\ell''\rangle_M,\mathrm{T}}}_{\text{soft inteference reconstruction}}}^{\text{soft interference cancellation}} = \underbrace{\tilde{\boldsymbol{\varepsilon}}_{\langle\ell+\ell'\rangle_M,\mathrm{T}} + \mathbf{H}_{\langle\ell+\ell'\rangle_M \ell}\boldsymbol{\mu}_{\ell,\mathrm{T}}}_{\text{low}-\text{complexity realization}}$$

- *Block-wise maximal ratio combining*

(DT domain) $$\mathbf{g}_{\ell,\mathrm{T}} = \underbrace{\sum_{\ell'\in\mathcal{L}} \mathbf{H}^{\mathrm{H}}_{\langle\ell+\ell'\rangle_M \ell}\Delta\mathbf{y}_{\langle\ell+\ell'\rangle_M,\mathrm{T}}}_{\text{delay}-\text{block combining}} = \left(\sum_{\ell'\in\mathcal{L}} \mathbf{H}^{\mathrm{H}}_{\langle\ell+\ell'\rangle_M \ell}\mathbf{H}_{\langle\ell+\ell'\rangle_M \ell}\right)\boldsymbol{\mu}_{\ell,\mathrm{T}} + \sum_{\ell'\in\mathcal{L}} \mathbf{H}^{\mathrm{H}}_{\langle\ell+\ell'\rangle_M \ell}\tilde{\boldsymbol{\varepsilon}}_{\langle\ell+\ell'\rangle_M,\mathrm{T}}$$

Proposed EC-MRC-Rake detector

- EC-MRC-Rake equalizer



➢ *One-tap equalization* ***(inter-Doppler interference cancellation)***

MRC output :
(DT domain)

$$\mathbf{g}_{\ell,\mathrm{T}} = \underbrace{\left(\sum_{\ell'\in\mathcal{L}} \mathbf{H}^{\mathrm{H}}_{\langle \ell+\ell'\rangle_M \ell}\mathbf{H}_{\langle \ell+\ell'\rangle_M \ell}\right)}_{\text{diagonal matrix } \mathbf{K}_{\ell,\mathrm{T}}}\boldsymbol{\mu}_{\ell,\mathrm{T}} + \underbrace{\sum_{\ell'\in\mathcal{L}} \mathbf{H}^{\mathrm{H}}_{\langle \ell+\ell'\rangle_M \ell}\tilde{\boldsymbol{\varepsilon}}_{\langle \ell+\ell'\rangle_M,\mathrm{T}}}_{\triangleq \Delta\mathbf{g}_{\ell,\mathrm{T}}}$$

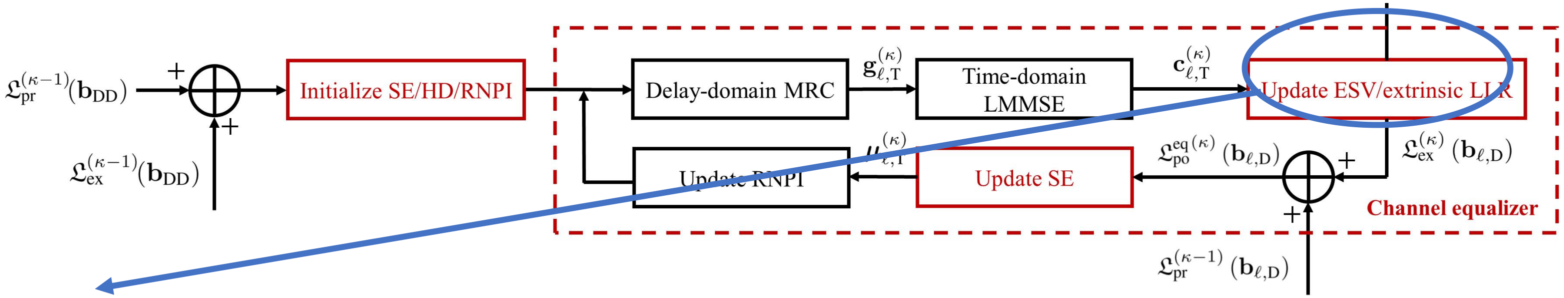
Equalized output :
(DT domain)

$$\mathbf{c}_{\ell,\mathrm{T}} = \underbrace{\mathbf{g}_{\ell,\mathrm{T}} \oslash \left(\mathrm{diag}\{\mathbf{K}_{\ell,\mathrm{T}}\} + \sigma_w^2 \mathbf{1}_N\right)}_{\text{time-domain one-tap equalizer}}$$

$$= \underbrace{\left(\mathrm{diag}\{\mathbf{K}_{\ell,\mathrm{T}}\} \oslash \left(\mathrm{diag}\{\mathbf{K}_{\ell,\mathrm{T}}\} + \sigma_w^2 \mathbf{1}_N\right)\right)}_{\triangleq \boldsymbol{\beta}_{\ell,\mathrm{T}}} \odot \boldsymbol{\mu}_{\ell,\mathrm{T}} + \underbrace{\left(\mathbf{1}_N \oslash \left(\mathrm{diag}\{\mathbf{K}_{\ell,\mathrm{T}}\} + \sigma_w^2 \mathbf{1}_N\right)\right)}_{\triangleq \mathbf{d}_{\ell,\mathrm{T}}} \odot \Delta\mathbf{g}_{\ell,\mathrm{T}}$$

Proposed EC-MRC-Rake detector

- EC-MRC-Rake equalizer



➢ *Equalized output*

(DT domain) $$\mathbf{c}_{\ell,\mathrm{T}} = \boldsymbol{\beta}_{\ell,\mathrm{T}} \odot \mathbf{x}_{\ell,\mathrm{T}} + \underbrace{\boldsymbol{\beta}_{\ell,\mathrm{T}} \odot \left(\boldsymbol{\mu}_{\ell,\mathrm{T}} - \mathbf{x}_{\ell,\mathrm{T}}\right) + \mathbf{d}_{\ell,\mathrm{T}} \odot \Delta\mathbf{g}_{\ell,\mathrm{T}}}_{\mathbf{e}_{\ell,\mathrm{T}}}$$

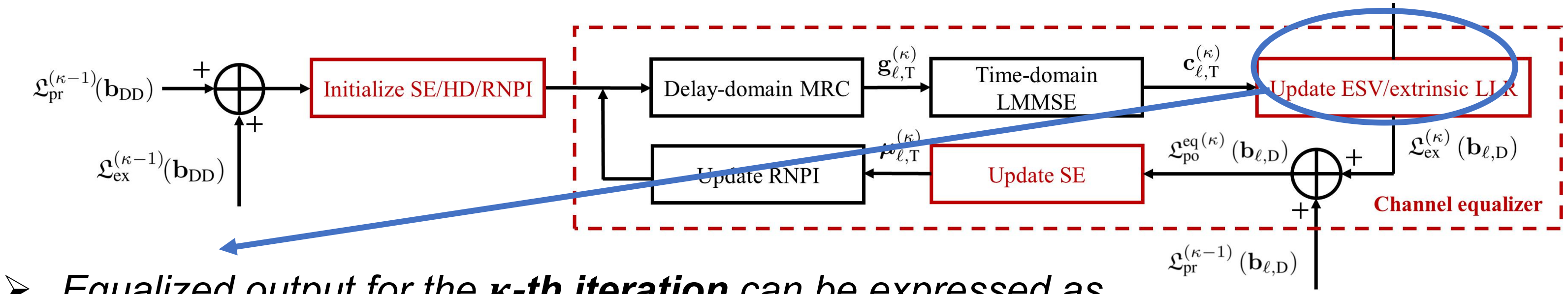
➢ *Extrinsic LLR*

(DD domain) $$LLR_{\mathrm{ex}}\left(b_{\ell k,i}\right) = ln\left(\frac{\sum_{s_{(i)}=0, s\in\mathcal{S}} \exp\left(-\frac{1}{\sigma_{\ell k}^2}|c_{\ell k} - \beta_{\ell k} s|^2\right)}{\sum_{s_{(i)}=1, s\in\mathcal{S}} \exp\left(-\frac{1}{\sigma_{\ell k}^2}|c_{\ell k} - \beta_{\ell k} s|^2\right)}\right)$$

Equalized symbol variance (ESV) Equalized symbol gain (ESG)

Proposed EC-MRC-Rake detector

- EC-MRC-Rake equalizer



➢ *Equalized output for the **κ-th iteration** can be expressed as*

(DD domain) $$\mathbf{c}^{(\kappa)}_{\ell,\mathrm{D}} = \boldsymbol{\beta}_{\ell,\mathrm{D}} \odot \hat{\mathbf{x}}^{(\kappa-1)}_{\ell,\mathrm{D}} + \hat{\mathbf{e}}^{(\kappa)}_{\ell,\mathrm{D}}$$

➢ *Equalized symbol gain (ESG): Calculated referring to the **convolution theorem***

(DD domain) $$\beta_{\ell k} = \frac{1}{N}\sum_{n=0}^{N-1}\beta_{\ell n} \triangleq \beta_{\ell}$$

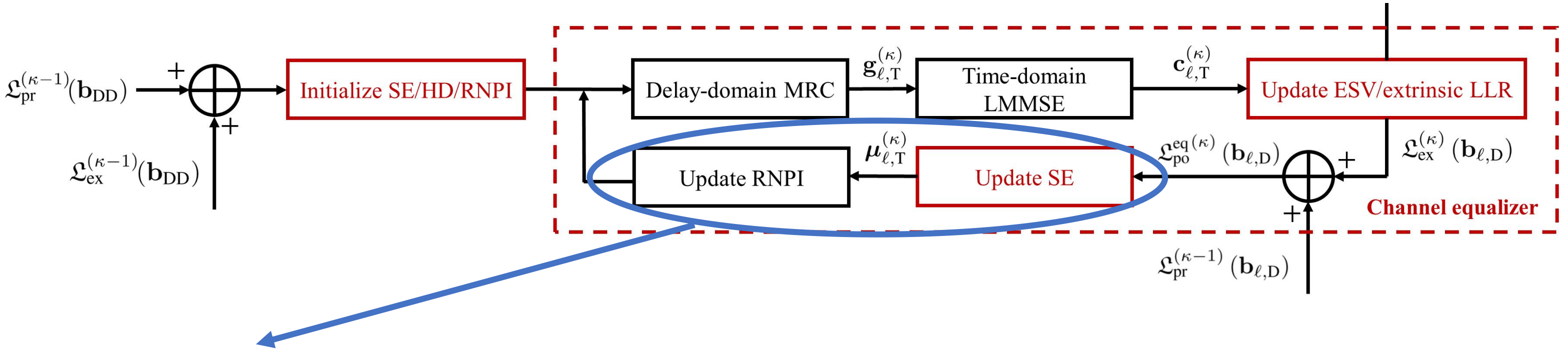
➢ *Equalized symbol variance (ESV): Estimated **sequentially***

(DD domain) $$\widehat{\sigma^2}^{(\kappa)}_{\ell k} = \frac{\ell}{\ell+1}\widehat{\sigma^2}^{(\kappa)}_{(\ell-1)k} + \frac{1}{(\ell+1)N}\sum_{k=0}^{N-1}\left|c^{(\kappa)}_{\ell k} - \beta_{\ell}\hat{x}^{(\kappa-1)}_{\ell k}\right|^2$$

Average along the Doppler-axis since **the symbols with the identical delay index go through the homologous DD channel**

Proposed EC-MRC-Rake detector

- EC-MRC-Rake equalizer



➢ *Updating symbol expectations (SE)*

➢ *Updating residual noise-plus-interference*

⇒ ***Equalizing the next signal block*** $\mathbf{x}_{\ell+1,\mathrm{T}}$ ***using the latest updated information***

Proposed EC-MRC-Rake detector

- EC-MRC-Rake turbo detector

TABLE I: Complexity comparisons in each iteration

EP detector [7] (where $P \le D \le N\,\lvert\mathcal{L}\rvert$)	
$\times$	$MN\,(3D\,\lvert\mathcal{S}\rvert + 6D + 6\,\lvert\mathcal{S}\rvert)$
$\div$	$MN\,(6D + \log_2\lvert\mathcal{S}\rvert + 3)$
exp	$MN\;2\,\lvert\mathcal{S}\rvert$
ln	$MN\;\log_2\lvert\mathcal{S}\rvert$
MRC-Rake detector [11]	
$\times$	$MN\left(4\,\lvert\mathcal{L}\rvert + \frac{3}{2}\log_2 N + 3\,\lvert\mathcal{S}\rvert + 2\log_2\lvert\mathcal{S}\rvert\right)$
$\div$	$MN\,(\lvert\mathcal{S}\rvert + 2)$
exp	$MN\;\lvert\mathcal{S}\rvert$
ln	$MN\;\log_2\lvert\mathcal{S}\rvert$
EC-MRC-Rake detector [proposed]	
$\times$	$MN\left(3\,\lvert\mathcal{L}\rvert + \frac{3}{2}\log_2 N + 3\,\lvert\mathcal{S}\rvert + 4\log_2\lvert\mathcal{S}\rvert - 1\right)$
$\div$	$MN\,(\lvert\mathcal{S}\rvert + \log_2\lvert\mathcal{S}\rvert + 2)$
exp	$MN\,(\lvert\mathcal{S}\rvert + \log_2\lvert\mathcal{S}\rvert)$
ln	$MN\;\log_2\lvert\mathcal{S}\rvert$

(No approximations for LLR computations here. If the Max-Log approximation is adopted, the computational complexity of both MRC-Rake and EC-MRC-Rake detectors can be reduced)

- M : number of delay samples
- N : number of Doppler
- $\lvert\mathcal{S}\rvert$: number of elements in modulation alphabet
- P : number of resolvable path
- $\lvert\mathcal{L}\rvert$: number of resolvable delay taps
- D : number of non-zero entries in each row of DD matrix

Outline

- Motivations
- EC-MRC-Rake detector
- **Simulations**

Numerical Results

- Simulation conditions

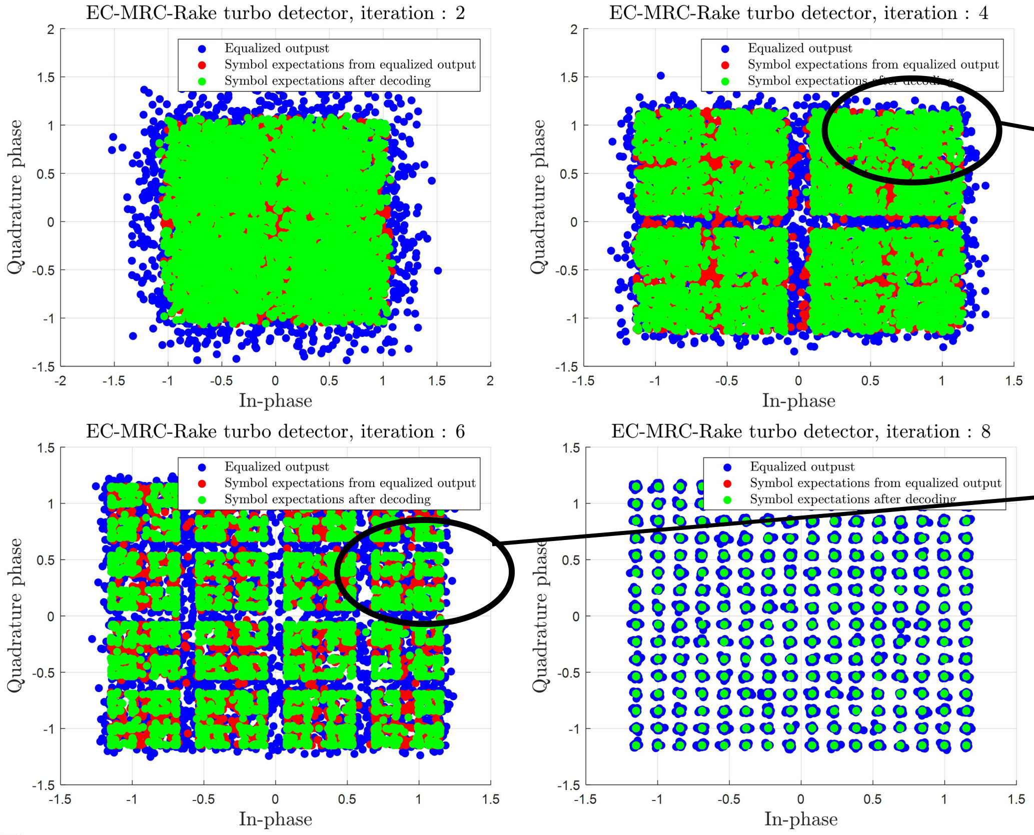
Num. of subcarriers	512
Subcarrier spacing	30KHz
LDPC coder	PEG, 2048, 0.5
Channel model	EVA / ETU
Doppler model	Jakes
Num. of Monte Carlo tests	100000 (stopped until 400 error code blocks for each SNR point)
Num. of Doppler samples	**8 / 32**
Num. of Doppler taps each EVA delay component	**1**

Fractional in the Doppler domain

Short frame OTFS system towards the low-latency communications

Numerical Results

- Iterative evolutions of I/Qs using EC-MRC-Rake turbo detector



512×8, EVA, $E_s/N_0 = 33$dB, $k_{p,max} = 0.25N$
($v_{\max} = 2300$km/h for $\Delta f = 30$KHz and $f_c = 3.5$GHz)

➢ *Comparison between the Blue and Red points*

Symbol expectations (SEs) can be regarded as the filtering results from the equalized output

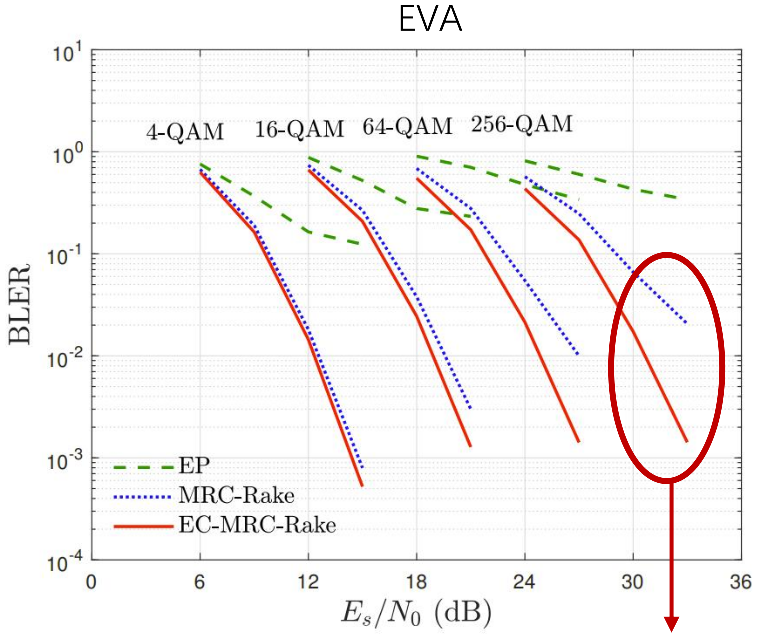
➢ *Comparison between the Red and Green points*

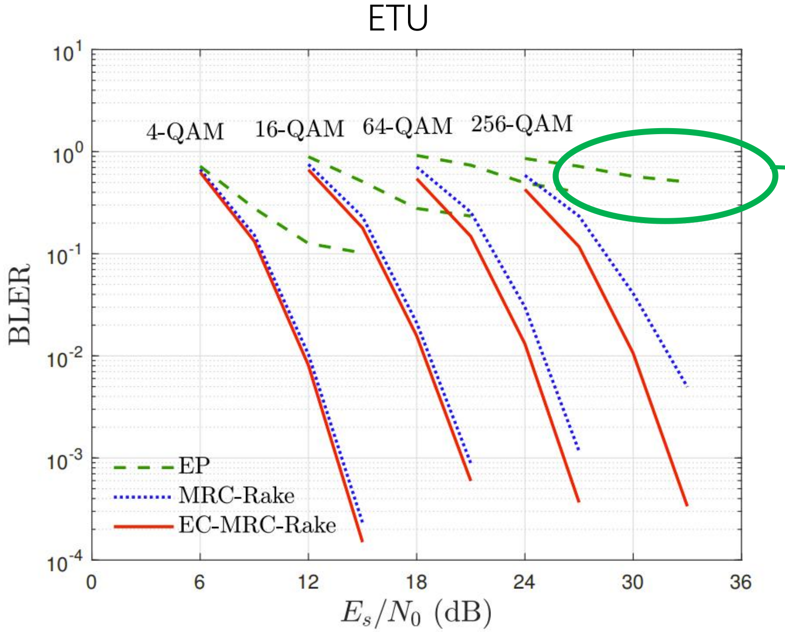
Channel decoding provides additional SNR gain to make SEs closer to the constellation points

Numerical Results

- BLERs vs. E_s/N_0 under different QAM configurations

512×8, $k_{p,max} = 0.05N$ ($v_{\max} = 460$km/h for $\Delta f = 30$KHz and $f_c = 3.5$GHz)





The configured maximum iteration number is not enough to give full play to the EP detector's performance

Expectation cancellation (EC) shows significant **error rate improvement**, especially when high-order QAMs are adopted

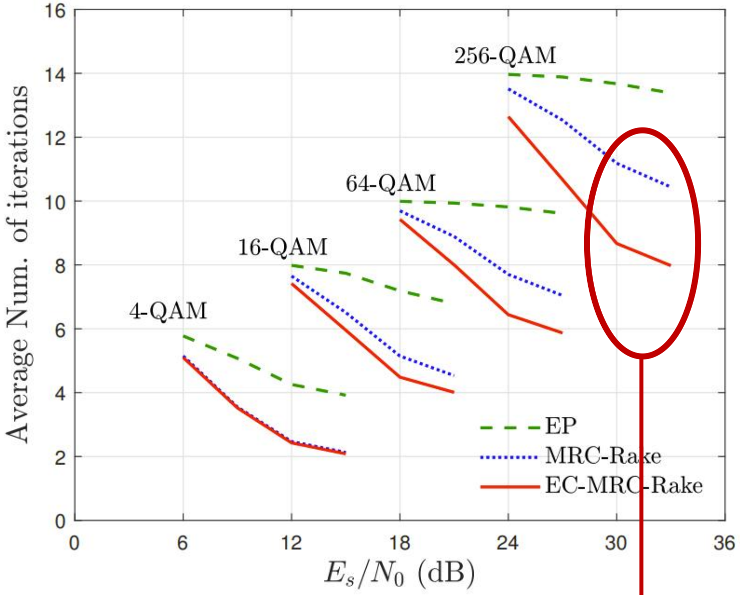
The higher the QAM order, the smaller the tolerance of error propagations

Numerical Results

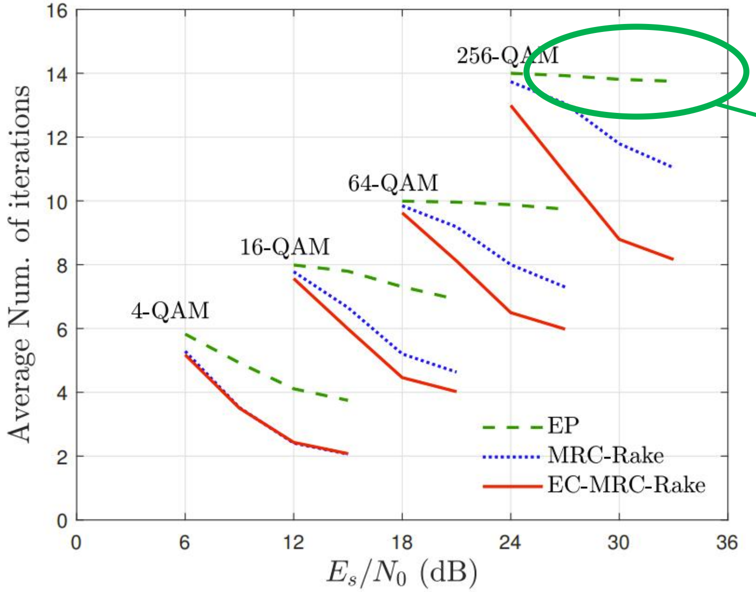
- Average iteration number vs. E_s/N_0 under different QAM configurations

512×8, $k_{p,max} = 0.05N$ ($v_{\max} = 460$km/h for $\Delta f = 30$KHz and $f_c = 3.5$GHz)

EVA



ETU



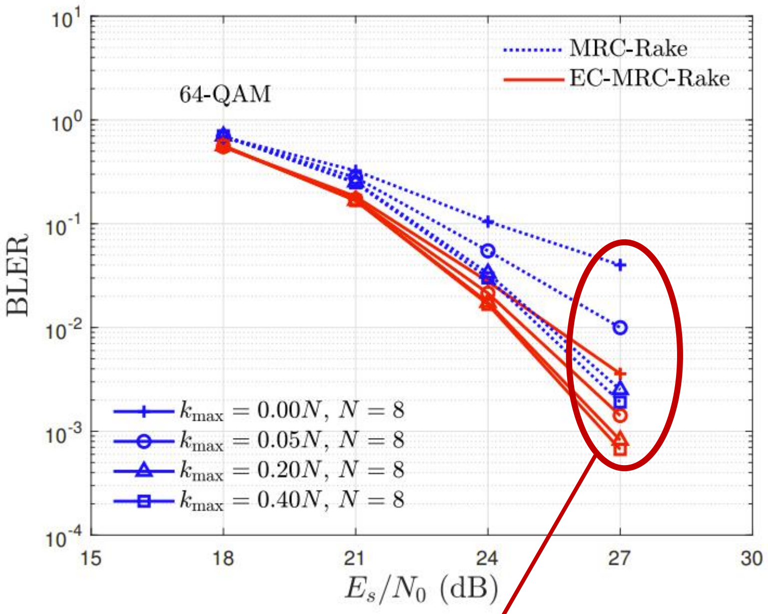
The configured maximum number of iterations is not enough to give full play to the EP detector's performance

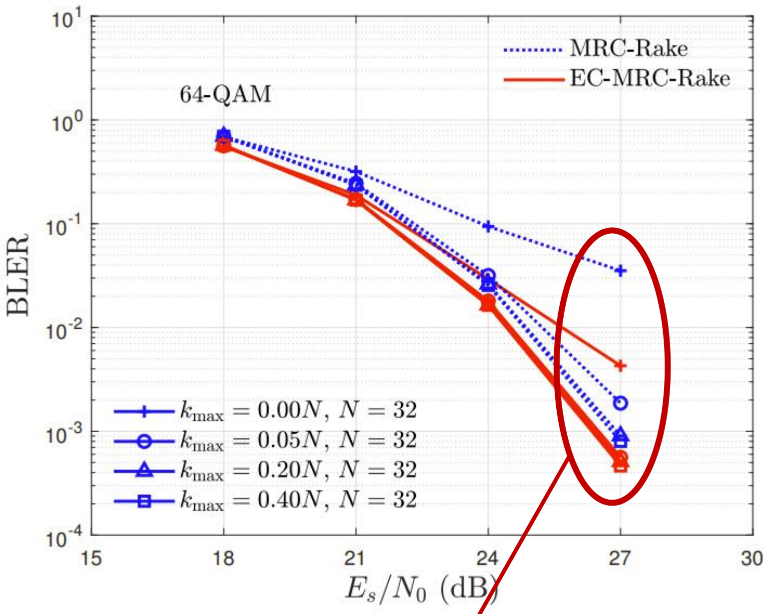
Expectation cancellation (EC) shows significant **convergence improvement**, especially when high-order QAMs are adopted

Numerical Results

- BLERs vs. E_s/N_0 under different $k_{p,max}$ and N

512×8, EVA ($k_{p,max} = 0.05N$ corresponds to $v_{\max} = 460$km/h for $\Delta f = 30$KHz and $f_c = 3.5$GHz)





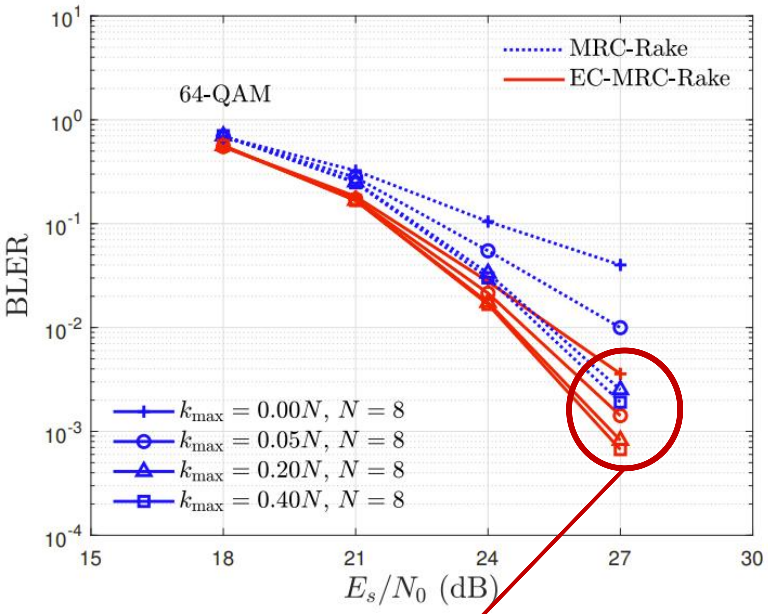
**Note: In the simulations, the resolvable path at delay 0 contains 2 Doppler components, while the paths at other delays only contain single Doppler component*

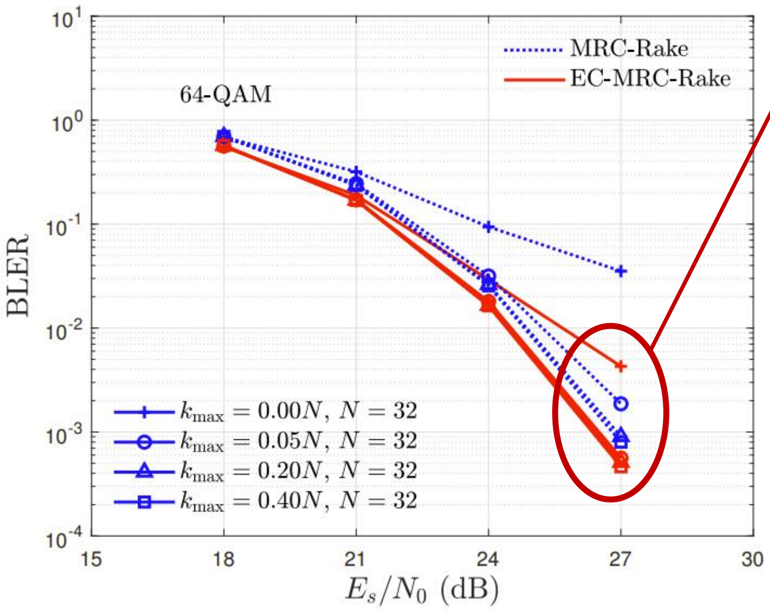
Better diversity exploiting after introducing *the expectation cancellation (EC)*

Numerical Results

- BLERs vs. E_s/N_0 under different $k_{p,max}$ and N

512×8, EVA ($k_{p,max} = 0.05N$ corresponds to $v_{\max} = 460$km/h for $\Delta f = 30$KHz and $f_c = 3.5$GHz)





No higher diversity gain for higher $k_{p,\max}$ under $N = 32$ since the Doppler components at delay 0 are sufficiently separated in the DD plane under $N = 32$

**Note: In the simulations, the resolvable path at delay 0 contains 2 Doppler components, while the paths at other delays only contain single Doppler component*

Higher diversity gain for higher $k_{p,\max}$ under $N = 8$
since the Doppler components at delay 0 tend to be more separated in the Doppler domain for higher $k_{p,\max}$

Thanks for your attention !

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