DIMENSIONAL ANALYSIS

<u>Quantities:</u>

Most engineering problems are solved using mathematical approaches. Mathematics is, in fact, one of the primary disciplines necessary to becoming an engineer. One of the first and most important aspects of understanding engineering problems and models is thus to understand the nature of different mathematical "quantities".

In general, mathematical quantities can be grouped into three main categories:

Variables — quantities that vary <u>within</u> a given problem.

Parameters —quantities which are fixed for a given physical system, but which may change from problem to problem.

Constants — quantities that do not vary from problem to problem.

Variables are often further categorized as *independent* or *dependent*. Independent variables are those that can be arbitrarily designated within a problem. The most common independent variables are time or spatial location. Other common independent variables might be pressure, temperature, etc.

Dependent variables are usually dependent upon at least one independent variable. Independent variables may take on any value "spanning" the problem of interest. Generally, although not always, it is dependent variables that are solved for when searching for a problem solution.

Parameters can either be single-valued or multi-valued, however, they should be "known", or at least have known relationships with other quantities. For example, if a given system can be treated as isothermal, temperature would be a parameter — fixed in a given problem, but variable from problem-to-problem. Also, if a known fluid is specified for a given problem it's viscosity would be a parameter, even if allowed to vary with pressure and temperature. The "parameter" in this case would be the functional relationship of viscosity as a function of pressure and temperature. Sometimes parameters are solved for in a problem.

Parameters are sometimes subdivided as to: a) design parameters (those that may be fixed or altered by the designer), and b) descriptive parameters (those that describe a given system, but are out of the control of the designer). Both variables and parameters may be stochastic, that is, they could have a probabilistic representation.

Some parameters that remain the same from system to system are treated as constants. For example, the acceleration due to gravity is essentially the same over the surface of the earth and for most problems may be treated as a constant. However, should the problem move to the moon, gravitational acceleration would have to be treated as a parameter. Other constants, such as pi and the speed of light, are true constants that never vary.

Any given quantity can be a variable, parameter, or constant depending on the particular problem.

Specification of an engineering quantity typically includes: a) definition, b) symbol, c) value(s), and d) units.

Symbols: In general, all symbols used in equations, figures, and tables should be defined upon their first use in any written document. If it is necessary to use the same symbol with two different meanings, both must be defined. Greek letters are often used for certain quantities. Standard symbols used in petroleum engineering can be found in the SPE Letter and Computer Symbols Standard (1993).

<u>Units, unit systems:</u>

In engineering, the value of a quantity is not completely specified without units. In petroleum engineering, two different unit systems are used: a) what is sometimes referred to as *oilfield* (or English) units, and b) SI units. SI (which stands for Le System International d'Unités or

The International System of Units) is a standardized metric system that has been adopted by the Society of Petroleum Engineers (SPE) and other engineering societies. The SI system of units and its relationship with oilfield units is discussed in detail in two articles from the *Journal of Petroleum Technology* (Campbell and Campbell, 1985). A partial list of units conversions is also included in this docment. The complete units standard is available in the *Petroleum Engineering Handbook* (1987)[4].

SI is not identical with any of the former cgs, or mks systems of metric units but is closely related to them and is an extension of and improvement over them. SI is based on seven well defined "base units" that quantify seven base quantities that by convention are regarded as dimensionally independent. SI has chosen the seven base quantities and base units listed below.

SI base quantities and units

Base quantity or "Dimension"	SI Unit	SI Unit Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

In addition, there are two "supplementary units"; radian (rad) for plane angle and steradian (sr) for solid angle. Following table contains the definitions of the base and supplementary units.

Definition of SI base and supplementary units

Unit	Definition
Meter	The meter is the length to 1 650 763.73 wavelength in vacuum of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom
Kilogram	The kilogram is the unit of mass (and is the coherent SI unit); it is equal to the mass of the international prototype of the kilogram
Second	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom
Ampere	The ampere is the constant current that, if maintained in two straight, parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7}N·m of length
Kelvin	The Kelvin, a unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water
Mole	The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kg of carbon-12
Candela	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz (Hz) and that has a radiant intensity in that direction of 1/683 watt per steradian
Radian	The radian is the plane angle between two radii that cut off on the circumference of a circle an arc equal in length to the radius
Steradian	The steradian is the solid angle that, having vertex at the center of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere

SI "derived units" are a third class, formed by combining, as needed, base units, supplementary units, and other derived units according to the algebraic relations linking the corresponding quantities. The symbols for derived units that do not have their own individual symbols are obtained by using the mathematical signs for multiplication and division, together with appropriate exponents (e.g., SI velocity, meter per second, m/s or $m \cdot s^{-1}$; SI angular velocity, radian per second, rad/s or $rad \cdot s^{-1}$).

The following figure shows the basic relationship between units. The base units are shown in the left column. The next column shows four forms of length and time terms used to define the derived units shown in the right column.

Each circle representing a derived unit has one or more lines entering or leaving it. Each line is connected to a base unit or to another derived unit. A solid line indicates multiplication; a dashed (broken) line indicates division. Notice the circle for FORCE in the upper right corner. It has two solid lines entering it from mass (kg) and acceleration (m/s^2). This means that force in newton (N) is the product of these two quantities.

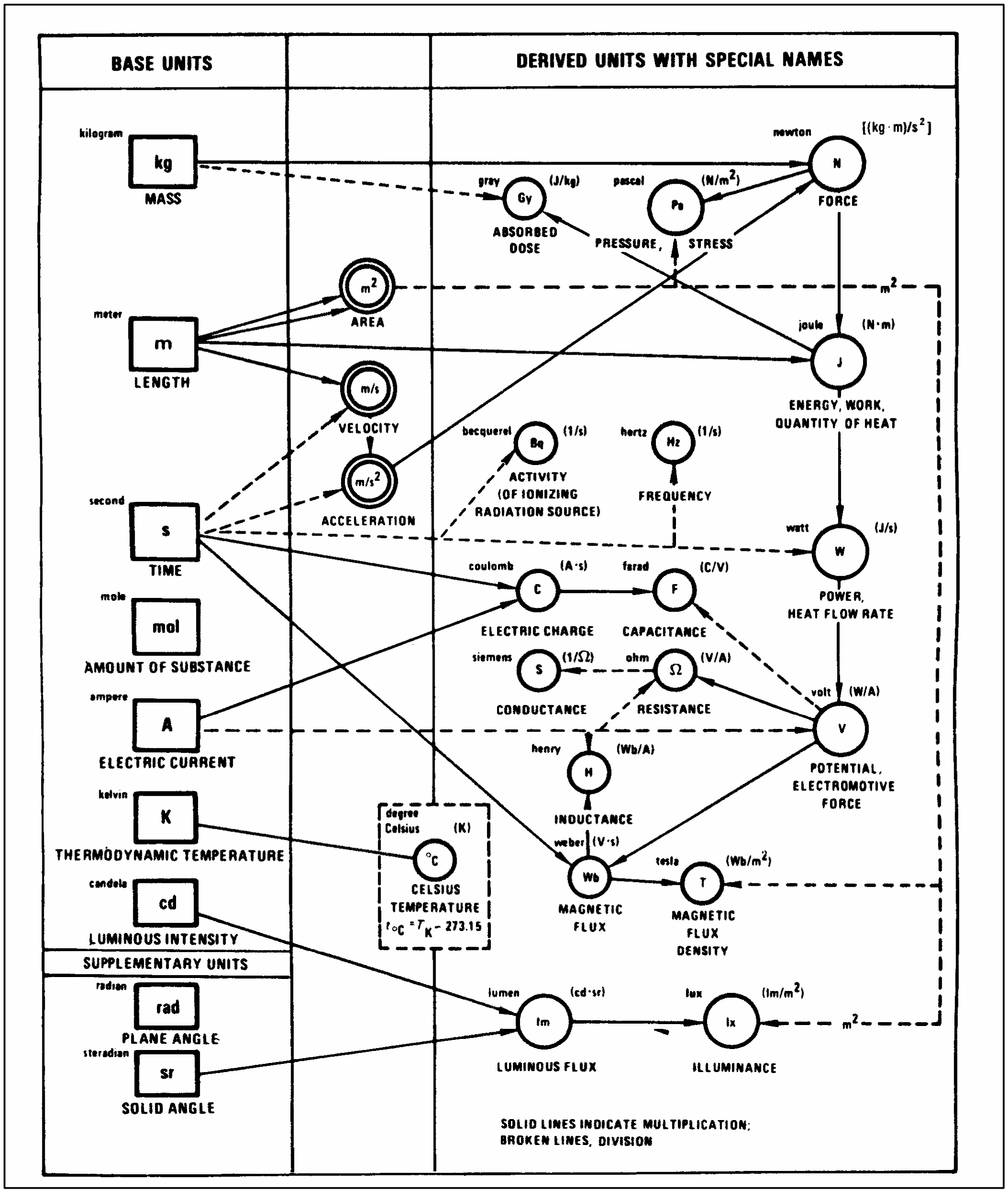
Notice also that there are two solid lines leaving the FORCE circle. One goes to the PRESSURE circle and one to the ENERGY, WORK circle. Consider pressure, which by definition is force per unit area. The PRESSURE circle has two lines entering it, a solid one from FORCE and a dashed one from AREA. This means that pressure is force divided by area: pascal (P) equals N/m^2.

Basic dimension symbols

Base Unit	Length	Mass	Time	Temperature
Symbol	L	m	t	T

Basic dimensions for commonly used units

Unit	Force	Pressure	Area	Volume	Density	Work
Symbol	mL/t^2	m/Lt^2	L^2	L^3	m/L^3	mL^2/t^2



Relationship between SI metric units[2].

Oil field Units

Most of the customary units used in petroleum engineering are not used in the SI metric system. These customary units are known as oil field units and the most common oil field units are listed in following table with their corresponding SI units.

Common oil field and SI units

Quantity	**Symbol**	**Dimension**	**Oilfield Units**	**SI Units**
Mass	*m*	m	lbm	kg
Moles	*n*	n	lbmol	kmol
Force	*F*	mL/t^2	lbf	N
Length	*L*	L	ft	m
Area	*A*	L^2	acres	m^2
Volume – liquids	*V*	L^3	bbl	m^3
Volume – gases	*V*	L^3	ft^3	m^3
Pressure	*p*	m/Lt^2	psi	kPa
Temperature	*T*	T	R	K
Flow rate – liquids	*q*	L^3/t	bbl/d	m^3/d
Flow rate – gases	*q*	L^3/t	ft^3/d	m^3/d
Viscosity	□	m/Lt	cp	mPa·s
Permeability	*k*	L^2	md	□m^2

Unit Conversions: Even many experienced engineers have difficulty with the use and conversion of units. This problem seems to be particularly troublesome in petroleum engineering because of the many non-standard units used in the discipline. Often equations are given in textbooks only in oilfield units. When trying to use other units or trying to use combinations of equations, problems often arise. This write-up will contain a few simple rules to avoid common pitfalls.

Conversion of numbers from one set of units to another is usually fairly straightforward. A technique which has been taught for many years to engineering students is to place unit conversion factors above and below a horizontal line and "canceling" units until the desired set is achieved. The following is an example showing how to convert from psi to kPa.

$$\left[\frac{1\ \text{lbf}}{\text{in}^2}\right]\left[\frac{12\ \text{in}}{\text{ft}}\right]^2\left[\frac{4.448\ \text{N}}{\text{lbf}}\right]\left[\frac{\text{ft}}{0.3048\ \text{m}}\right]^2\left[\frac{\text{m}^2\ \text{Pa}}{1\ \text{N}}\right]\left[\frac{\text{kPa}}{1000\ \text{Pa}}\right]=6.895\ \text{kPa}$$

Dealing with units in equations always seems to create difficulties. However, if one rule is remembered, this can be done easily and correctly: ***Convert numbers, not equations!*** The best way to show this is by example. Consider Darcy's law in steady-state linear form:

$$q=\frac{kA\Delta p}{\mu L}$$

where,

q	=	flowrate, cm^3/sec
k	=	rock permeability, darcies
A	=	cross-sectional area to flow, cm^2
Δp	=	flowing pressure difference, atm
μ	=	fluid viscosity, cp
L	=	flow length, cm

	SI	Oilfield
Mass	1 kg	2.2046225 lbm
Length	0.3048 m	1 ft
		12 in
Area	4,046.873 m^2	1 acre = 43,560 ft^2
Volume	1 m^3	6.2898106 bbl
		1 bbl = 5.614583 ft^3
Temperature	1 K	1.8 R
	K = °C+273.15	R = °F+459.67
	°C = (°F-32)/1.8	°F = 1.8 °C+32
Pressure	6.894757 kPa	1 psi
	1 MPa	145.03774 psi
	101.325 kPa	1 atm = 14.69595 psi
	1 bar = 100 kPa	14.503774 psi
Dynamic viscosity	1 mPa·s	1 cp
Density	1000 kg/m^3	62.42797 lbm/ft^3
		8.345405 lbm/gal
Water density @ 60°F/1atm	999.04 kg/m^3	62.368 lbm/ft^3
Energy	1.055056 kJ	1 btu
	1 kWh	3412.14 btu
		1 btu=778.169 ft·lbf
Force	4.448 N	1 lbf
Power	745.700 W	1 hp=550 ft·lbf/s
Universal gas constant	$8.31441 \dfrac{\mathrm{kPa \cdot m^3}}{\mathrm{kmol \cdot K}}$	$10.7315 \dfrac{\mathrm{psia \cdot ft^3}}{\mathrm{lbmol \cdot R}}$
	$8.31441 \dfrac{\mathrm{kJ}}{\mathrm{kmol \cdot K}}$	$1.98586 \dfrac{\mathrm{btu}}{\mathrm{lbmol \cdot R}}$
Molecular weight of air	28.9625 kg/kmol	28.9625 lbm/lbmol
Gravitational conversion constant	$9.80665 \dfrac{\mathrm{kg \cdot m}}{\mathrm{s^2 \cdot kgf}}$	$32.1740 \dfrac{\mathrm{lbm \cdot ft}}{\mathrm{s^2 \cdot lbf}}$
Permeability	1 □m^2	1013.25 md
	$1\ \mu\mathrm{m}^2 = 0.0864 \dfrac{\mathrm{m^2 \cdot mPa \cdot s}}{\mathrm{d \cdot kPa}}$	$158.0206\ \mathrm{md} = 1 \dfrac{\mathrm{ft^2 \cdot cp}}{\mathrm{d \cdot psi}}$
		$887.220\ \mathrm{md} = 1 \dfrac{\mathrm{bbl \cdot cp}}{\mathrm{ft \cdot d \cdot psi}}$