APPENDIX-3
SEMI-LOG and LOG-LOG GRAPHS

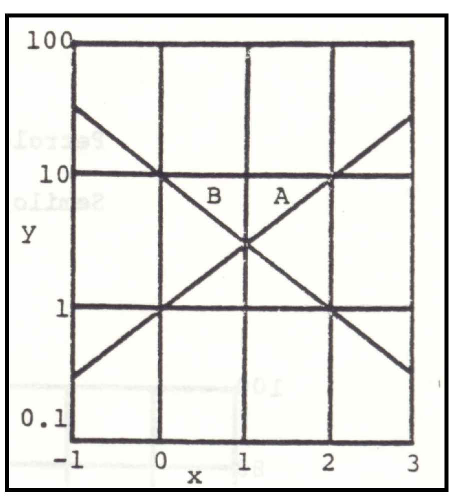
Logarithmic scales are used in plotting graphs primarily because they permit coverage of a wide range of values and yet allow interpretation in the lower range of values. Compare, for example, the Cartesian coordinate graph and the semi-log graph of the same function. The semi-log plot makes evident how **y** is varying as a function of x for $-1<x<0$, whereas in the Cartesian plot the curve is all compressed on the **x** axis in this range. A second important feature of logarithmic scales is that many systems have relationships between variables or groups of variables, which plot as straight lines or slightly curve lines on semi-log or log-log graph paper. Where this is the case, curve fitting, extrapolation and interpolation are facilitated.

Straight Lines on Semi-log Graph Paper

An equation of the following form plots as a straight line on semi-log graph paper.

$$\mathbf{y = a\ e^{bx}}$$

The variables in the exponent, here **x**, is plotted on the linear scale and the other variable, here **y**, is plotted on the log scale. The symbol **e** is the Naperian base 2.718. For x = 0, y = a, which is the y-intercept. For line A in the graph below, a = 1.0 and for line B, a = 10.0 The slope **b** may be found by the two-point method from the simultaneous solution of the above equation.



An alternate form of these equations using common logarithm is:

$$\log y_1 = \log a + bx_1 \log e$$

$$\log y_2 = \log a + bx_2 \log e$$

Subtracting the above equation and writing log e = 0.4343 yields;

$$b = (\log y_2 - \log y_1) / [0.4343 (x_2 - x_1)]$$

If y_1 and y_2 are taken at points such that $y_2 = 10 y_1$, since $\log 10y_1 - \log y_1 = \log 10 = 1.0$ then;

$$b = 2.303 / \Delta x \text{ (per log cycle)}$$

For line A, b = 2.303 / 2 = **1.15**

For line B; b = 2.303 / (-2) = -**1.15**

The equations for lines A and B are thus;

A : $y = 1.0^{e1.15x}$

B : $y = 10^{e-1.15x}$

Note that both curves have the same numerical slope, (0.4343 x 1.15)

Another form of the base equation ($y = a e^{bx}$), using common logarithm is ;

$$\log y = mx + b$$

where **m** is the slope. Note that **b** has a different meaning and value in the above equation from that of the base equation. Application of the two-point method;

$$m = (\log y_2 - \log y_1) / (x_2 - x_1)$$

For line B, using the points (0,10) and (2,1)

$$m = (\log 1 - \log 10) / (2 - 0) = -0.5$$

The intercept **b** may be found by substituting m = -0.5 in the above equation for any point , e.g. (2,1)

$$b = \log y - mx$$

$$b = \log 1 - (-0.5)\, 2 = 1.0$$

Thus the equation of line B is,

$$\log y = 1 - 0.5\,x$$

This may be converted into the equivalent form of the base equation as follows:

$$\ln (y/10) = 2.303\,(-0.5x) = -1.15x$$

$$y = 10\,e^{-1.15x}$$

Straight Lines on Log-log Graph Paper

Straight lines on log-log graph paper represent equations of the following form.

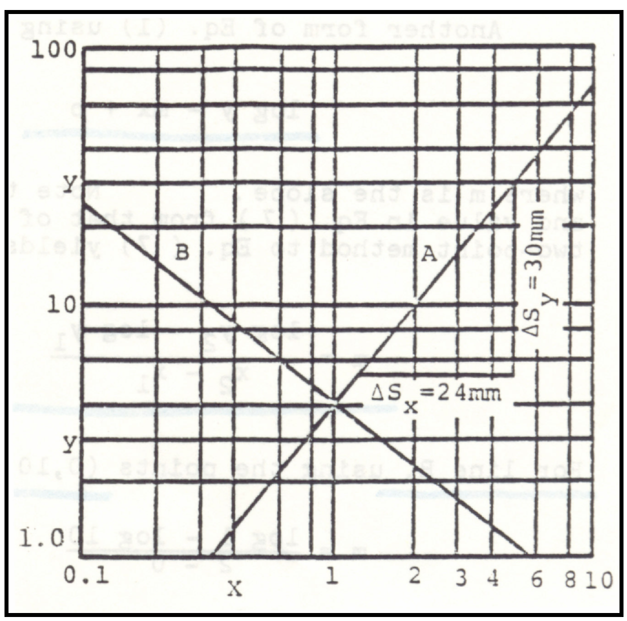
$$y = a\,x^{b}$$

or in the equivalent form using either common or Naperian logarithms.

$$\log y = \log a + b \log x$$

For $x = 1$, $\log x = \ln x = 0$, and therefore $y = a$, which is the y-intercept. The constants **a** and **b** may be found simultaneous solution of the above equations. The slope may be found readily from the graph by taking the ratio of the linear distances, Δs_y and Δs_x, using any linear scale for measurement. This is true because by the two-point method:

$$b = (\log y_2 - \log y_1) / (\log x_2 - \log x_1) = \Delta s_y / \Delta s_x$$



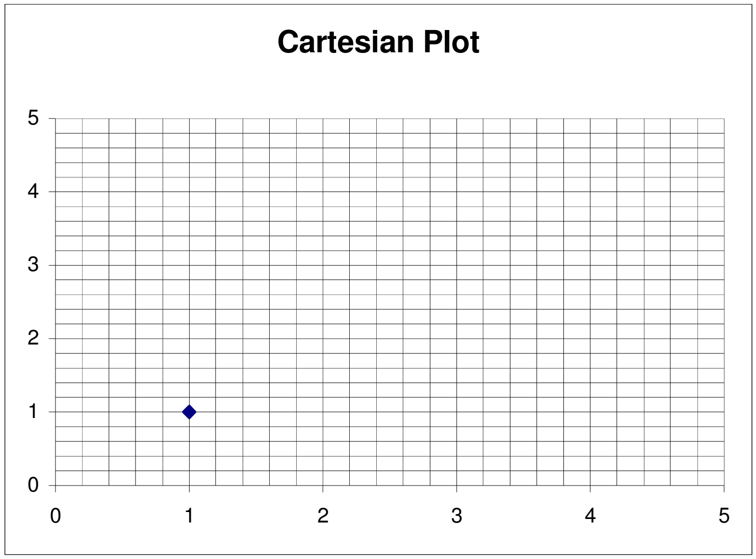
The above graph shows two lines, both of which pass through the point (1,4). Thus the intercept value a is 4.0 for both lines.

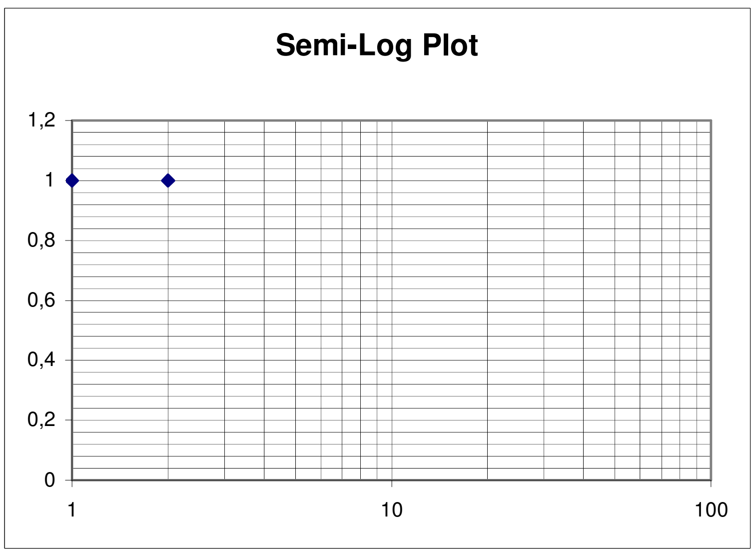
$$\mathbf{b = \Delta s_y / \Delta s_x = 30\ mm / 24\ mm = 1.25}$$

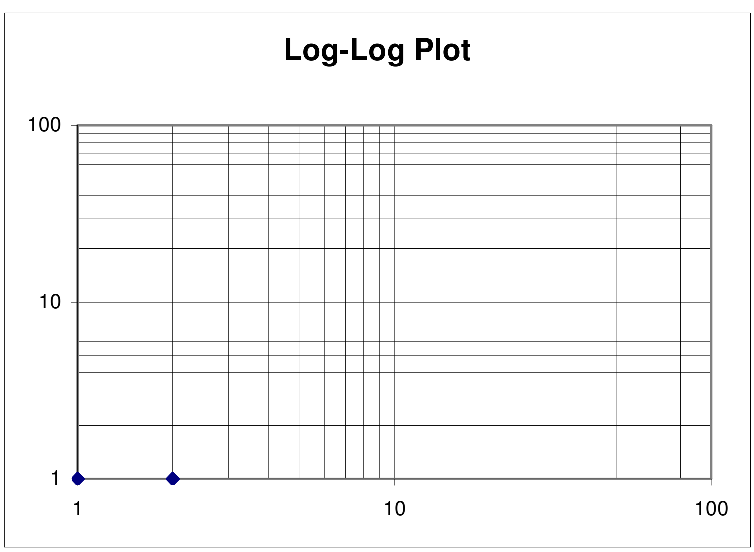
Line B happens to be perpendicular to line A; therefore the value of **b** for line B is the reciprocal of that for line A, i.e, 24/30 or 0.80. Thus the equations for the lines are:

A: $y = 4\ x^{1.25}$ or $\log y = \log 4 + 1.25 \log x$

B: $y = 4\ x^{-0.80}$ or $\log y = \log 4 - 0.80 \log x$


Cartesian Plot
5
4
3
2
1
0
0
1
2
3
4
5


Semi-Log Plot
1,2
1
0,8
0,6
0,4
0,2
0
1
10
100


Log-Log Plot
100
10
1
1
10
100