第 4 章时间序列分析作业

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4.2

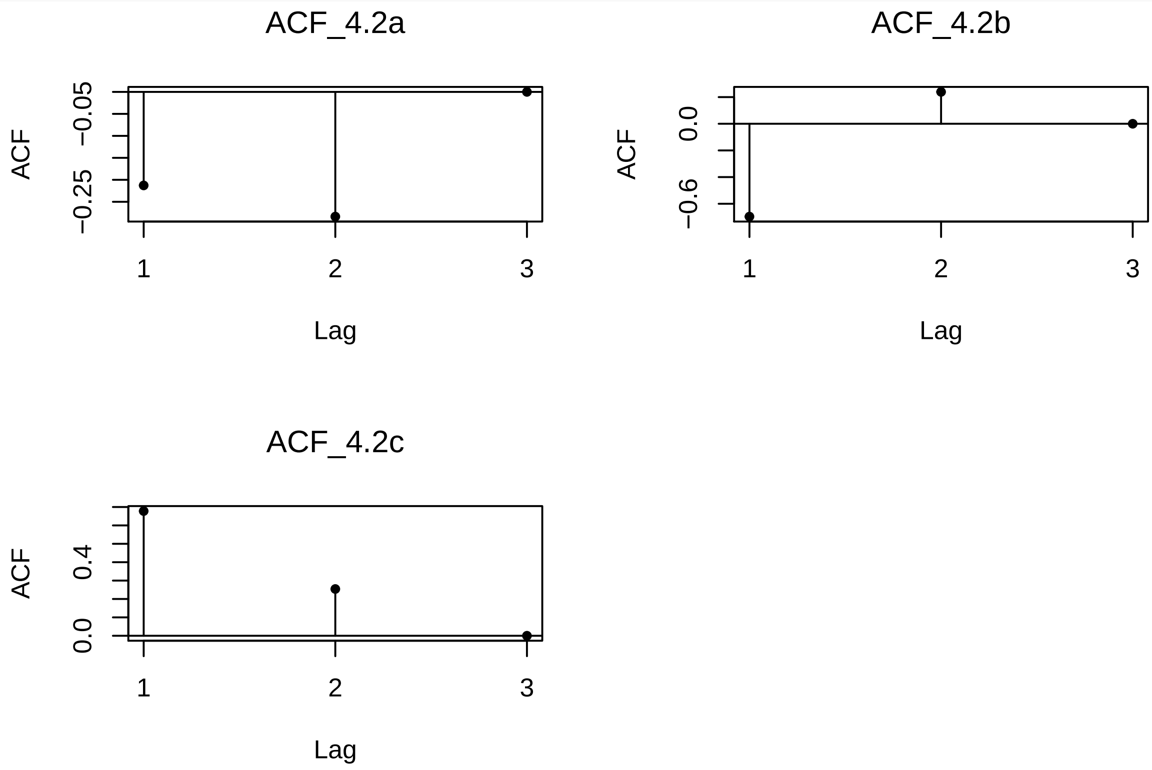
首先借助 *ARMAacf* 函数计算数值

```
ACF_4.2a = ARMAacf(ma = c(-0.5,-0.4));
ACF_4.2b = ARMAacf(ma = c(-1.2,0.7));
ACF_4.2c = ARMAacf(ma = c(1,0.6));
```

然后自定义 *ACFplot* 函数用于绘图

```
ACFplot = function(acf)
{
  plot(y = acf[-1], x = 1:(length(acf)-1),
       main=substitute(acf), xlab = "Lag", ylab = "ACF", type = c("h"),  xaxt="n")
  abline(h = 0)
  axis(at = 1:(length(acf)-1), side = 1)
  points(y = acf[-1], x = 1:(length(acf)-1), pch = 20)
}

op = par(mfrow=c(2,2))
ACFplot(ACF_4.2a)
ACFplot(ACF_4.2b)
ACFplot(ACF_4.2c)
```



4.4

根据 MA(1) 自相关函数公式，代入可得

$$\frac{-\frac{1}{\theta}}{1+(\frac{1}{\theta})^2} = \frac{-\theta}{1+\theta^2} = \rho_1$$

因此不变。

4.5

自相关函数计算结果如下：

```
ACF_4.5a = ARMAacf(ar = 0.6, lag.max = 10); ACF_4.5a
```

```
##           0           1           2           3           4           5
## 1.000000000 0.600000000 0.360000000 0.216000000 0.129600000 0.077760000
##           6           7           8           9          10
## 0.046656000 0.027993600 0.016796160 0.010077696 0.006046618
```

```
ACF_4.5b = ARMAacf(ar = -0.6, lag.max = 10); ACF_4.5b
```

```
##            0            1            2            3            4
##  1.000000000 -0.600000000  0.360000000 -0.216000000  0.129600000
##            5            6            7            8            9
## -0.077760000  0.046656000 -0.027993600  0.016796160 -0.010077696
##           10
##  0.006046618
```

```
ACF_4.5c = ARMAacf(ar = 0.95, lag.max = 20); ACF_4.5c
```

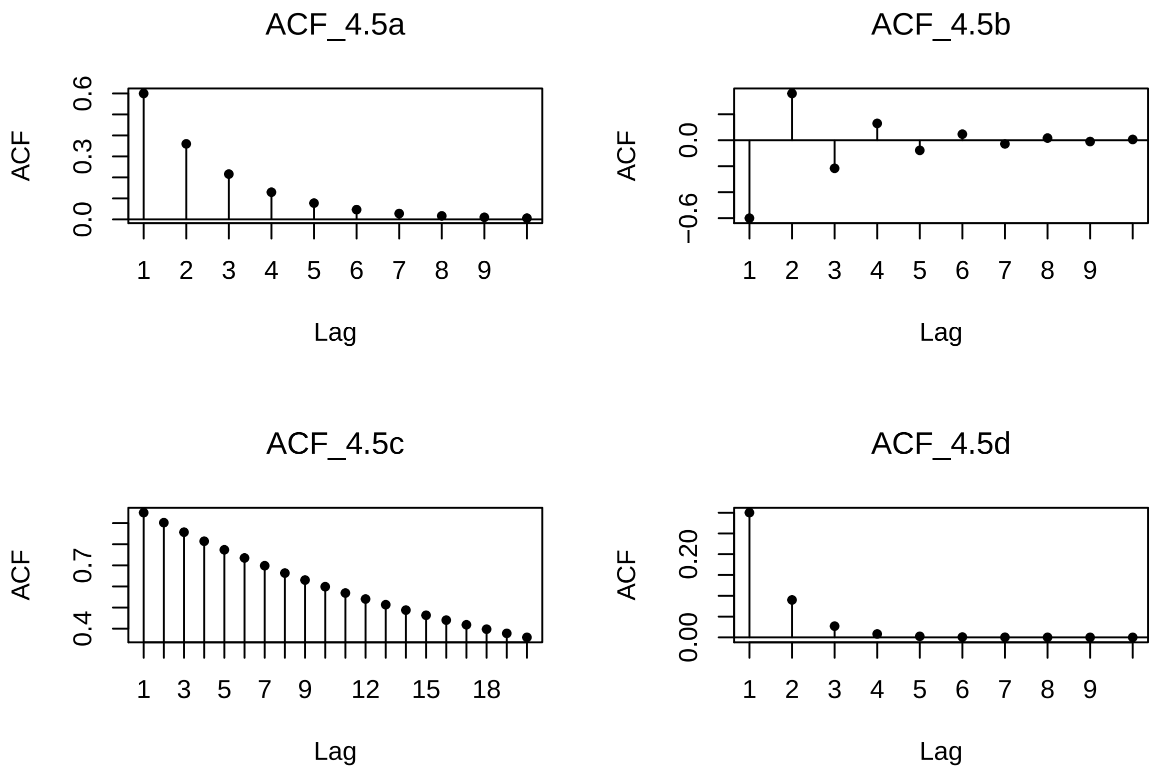
```
##         0         1         2         3         4         5         6
## 1.0000000 0.9500000 0.9025000 0.8573750 0.8145062 0.7737809 0.7350919
##         7         8         9        10        11        12        13
## 0.6983373 0.6634204 0.6302494 0.5987369 0.5688001 0.5403601 0.5133421
##        14        15        16        17        18        19        20
## 0.4876750 0.4632912 0.4401267 0.4181203 0.3972143 0.3773536 0.3584859
```

```
ACF_4.5d = ARMAacf(ar = 0.3, lag.max = 10); ACF_4.5d
```

```
##          0          1          2          3          4          5
## 1.0000e+00 3.0000e-01 9.0000e-02 2.7000e-02 8.1000e-03 2.4300e-03
##          6          7          8          9         10
## 7.2900e-04 2.1870e-04 6.5610e-05 1.9683e-05 5.9049e-06
```

画图：

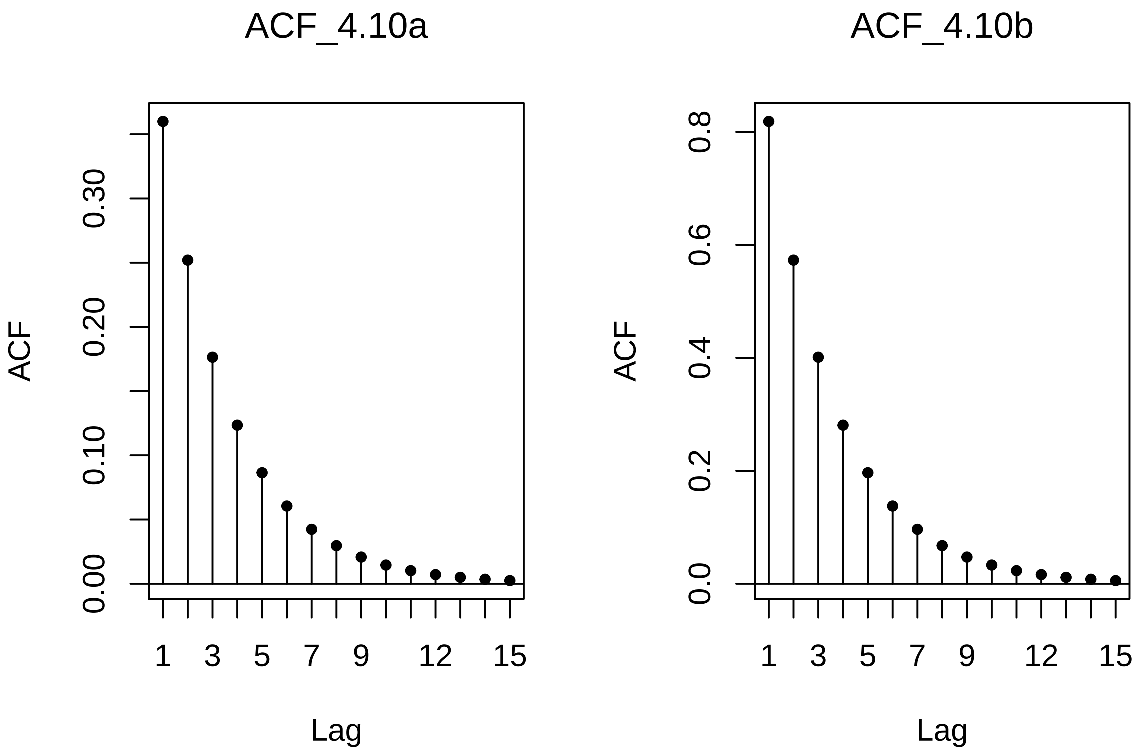
```
op = par(mfrow=c(2,2))
ACFplot(ACF_4.5a)
ACFplot(ACF_4.5b)
ACFplot(ACF_4.5c)
ACFplot(ACF_4.5d)
```



4.10

```
ACF_4.10a = ARMAacf(ar = 0.7, ma = -0.4, lag.max = 15);
ACF_4.10b = ARMAacf(ar = 0.7, ma = 0.4, lag.max = 15);

op = par(mfrow=c(1,2))
ACFplot(ACF_4.10a)
ACFplot(ACF_4.10b)
```



4.16

(a)

将该式代入 Y_{t-1} 得：

$$\begin{aligned}3Y_{t-1}+e_t &= 3*-\sum_{j=1}^{\infty}(1/3)^j e_{t-1+j}+e_t\\ &= -\sum_{j=1}^{\infty}(1/3)^{j-1}e_{t-1+j}+e_t\\ &= -\sum_{k=0}^{\infty}(1/3)^k e_{t+k}+e_t\\ &= -e_t-\sum_{k=1}^{\infty}(1/3)^k e_{t+k}+e_t\\ &= -\sum_{k=1}^{\infty}(1/3)^k e_{t+k}\\ &= Y_t\end{aligned}$$

因此满足方程。

(b)

期望与协方差的计算如下：

$$E(Y_t)=0$$

$$
\begin{aligned}
Cov(Y_t, Y_{t-k}) &= Cov(-\sum_{j=1}^{\infty}(1/3)^j e_{t+j}, -\sum_{j=1}^{\infty}(1/3)^j e_{t-k+j}) \\
&= Cov(-\sum_{j=1}^{\infty}(1/3)^j e_{t+j}, -\sum_{j=k+1}^{\infty}(1/3)^j e_{t-k+j}) \\
&= Cov(-\sum_{j=1}^{\infty}(1/3)^j e_{t+j}, -\sum_{i=1}^{\infty}(1/3)^{i+k} e_{t+i}) \\
&= \sigma_e^2((\frac{1}{3})^{k+2} + (\frac{1}{3})^{k+4} + \cdots) \\
&= \sigma_e^2 \frac{(\frac{1}{3})^{k+2}}{1-(\frac{1}{3})^2} \\
&= \frac{\sigma_e^2}{8}(\frac{1}{3})^k
\end{aligned}
$$

可知满足平稳性的定义

(c)

该解令人不满意的地方在于 Y_t 是取决于未来的误差项，这在实际中无法应用。

4.17

(a)

将该式代入 Y_{t-1} 得：

$$
\begin{aligned}
\frac{1}{2}Y_{t-1} + e_t &= \frac{1}{2} * (10(\frac{1}{2})^{t-1} + e_{t-1} + \frac{1}{2}e_{t-2} + \cdots) + e_t \\
&= 10(\frac{1}{2})^t + e_t + \frac{1}{2}e_{t-1} + (\frac{1}{2})^2 e_{t-2} + \cdots \\
&= Y_t
\end{aligned}
$$

因此是方程的解。

(b)

由于

$$E(Y_t) = E(10(\frac{1}{2})^t + e_t + \frac{1}{2}e_{t-1} + (\frac{1}{2})^2 e_{t-2} + \cdots) = 10(\frac{1}{2})^t$$

因此均值对时间并不是常数，所以该解不平稳。

4.21

(a)

$$Var(Y_t) = \sigma_e^2 + \sigma_e^2 + \frac{1}{4}\sigma_e^2 = \frac{9}{4}\sigma_e^2$$

$$
\begin{aligned}
Cov(Y_t, Y_{t-1}) &= Cov(e_{t-1} - e_{t-2} + 0.5e_{t-3}, e_{t-2} - e_{t-3} + 0.5e_{t-4}) \\
&= -\sigma_e^2 - \frac{1}{2}\sigma_e^2 \\
&= \frac{3}{2}\sigma_e^2
\end{aligned}
$$

$$\begin{aligned}
Cov(Y_t, Y_{t-2}) &= Cov(e_{t-1} - e_{t-2} + 0.5e_{t-3}, e_{t-3} - e_{t-4} + 0.5e_{t-5}) \\
&= 0.5\sigma_e^2
\end{aligned}$$

其余协方差均为 0

(b)

$\{Y_t\}$ 与 MA(2) 有相同的统计特征。原因在于误差项是我们无法观测到的，所以就无法区分 $Y_t = e_t - e_{t-1} + 0.5e_{t-2}$ 与 $Y_t = e'_t - e'_{t-1} + 0.5e'_{t-2}$, 其中 $e'_t = e_{t-1}$。通过计算均值与协方差函数也会发现二者完全相同。

4.23

(a)

$$\begin{aligned}
Cov(b_t, b_{t-k}) &= Cov(Y_t - \phi Y_{t+1}, Y_{t-k} - \phi Y_{t-k+1}) \\
&= Cov(Y_t, Y_{t-k}) - \phi Cov(Y_{t+1}, Y_{t-k}) - \phi Cov(Y_t, Y_{t-k+1}) + \phi^2 Cov(Y_{t+1}, Y_{t-k+1}) \\
&= \gamma_k - \phi\gamma_{k+1} - \phi\gamma_{k-1} + \phi^2\gamma_k \\
&= \frac{\sigma_e^2}{1-\phi^2}(\phi^k - \phi \cdot \phi^{k+1} - \phi \cdot \phi^{k-1} + \phi^2 \cdot \phi^k) \\
&= 0
\end{aligned}$$

(b)

$$\begin{aligned}
Cov(b_t, Y_{t+k}) &= Cov(Y_t - \phi Y_{t+1}, Y_{t+k}) \\
&= Cov(Y_t, Y_{t+k}) - \phi Cov(Y_{t+1}, Y_{t+k}) \\
&= \gamma_k - \phi\gamma_{k-1} \\
&= \frac{\sigma_e^2}{1-\phi^2}(\phi^k - \phi \cdot \phi^{k-1}) \\
&= 0
\end{aligned}$$