16-720 Homework 2

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Q1.5:

Figure 1: Detected keypoints

Q2.4:

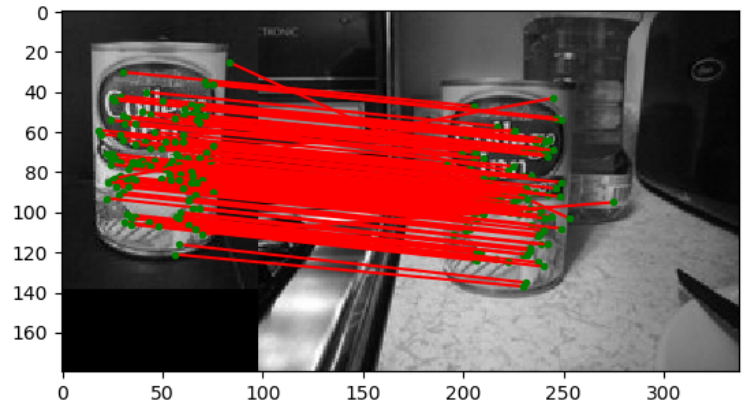

Figure 2: Matches between **model_chickenbroth.jpg** and **chickenbroth_03.jpg**

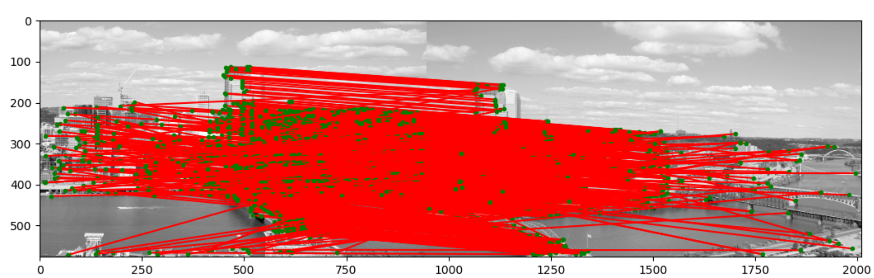

Figure 3: Matches between **incline_L.png** and **incline_R.png**

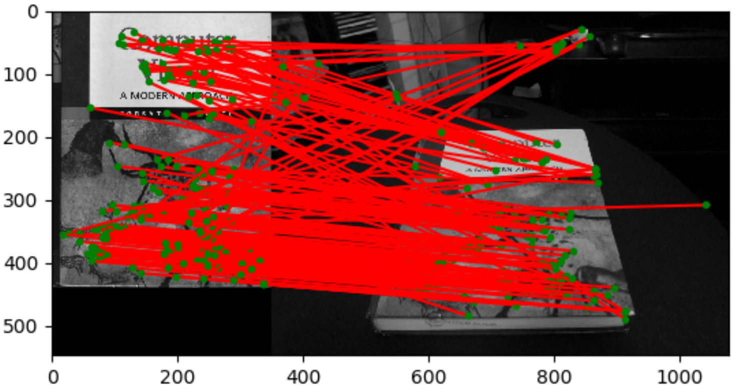

Figure 4: Matches between **pf_scan_scaled.jpg** and **pf_desk.jpg**

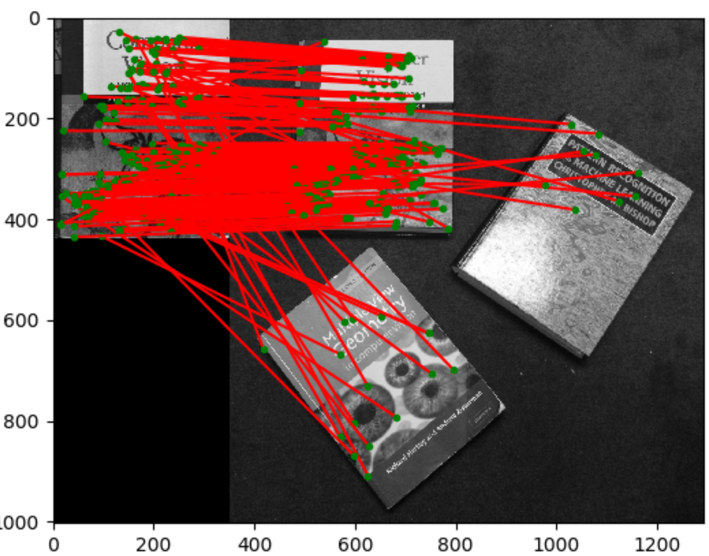

Figure 5: Matches between **pf_scan_scaled.jpg** and **pf_floor.jpg**

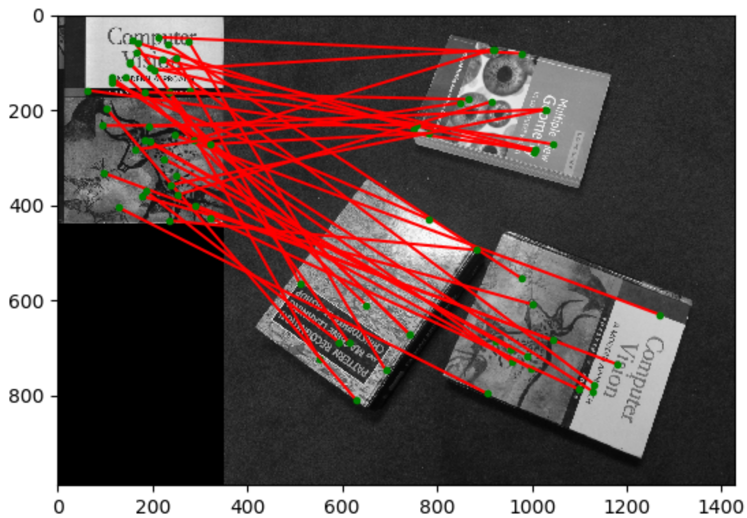

Figure 6: Matches between **pf_scan_scaled.jpg** and **pf_floor_rot.jpg**

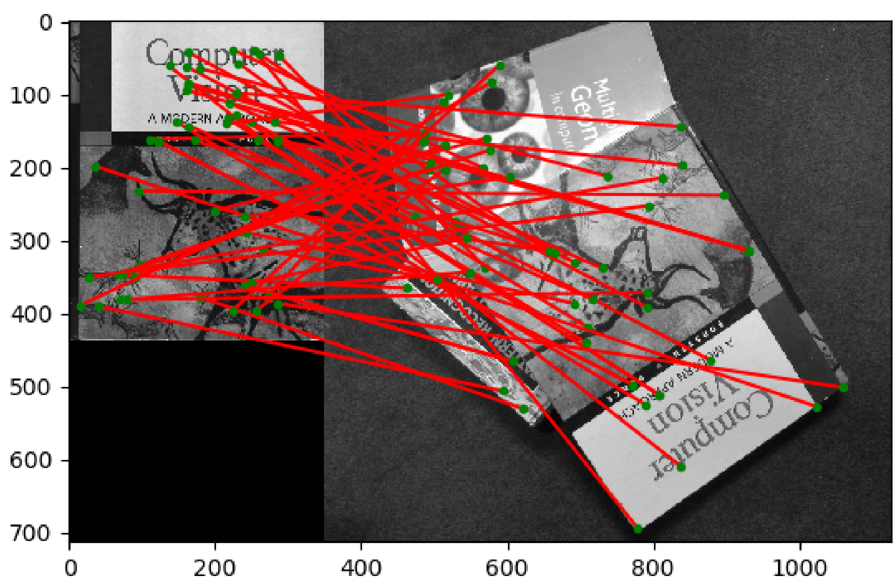

Figure 7: Matches between **pf_scan_scaled.jpg** and **pf_pile.jpg**

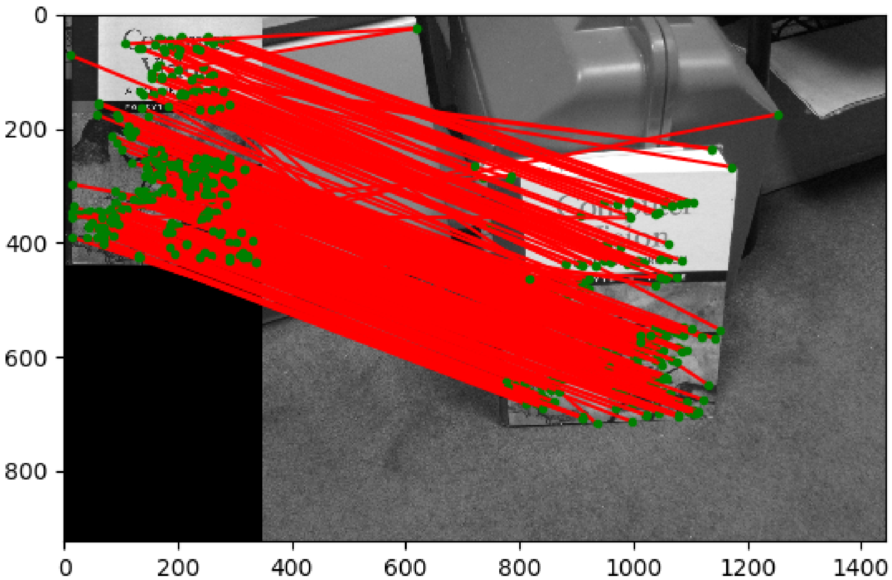

Figure 8: Matches between **pf_scan_scaled.jpg** and **pf_stand.jpg**

Discussion:

It can be observed that: the greater the rotation is between two images, the worse the result it performs.

Q2.5:

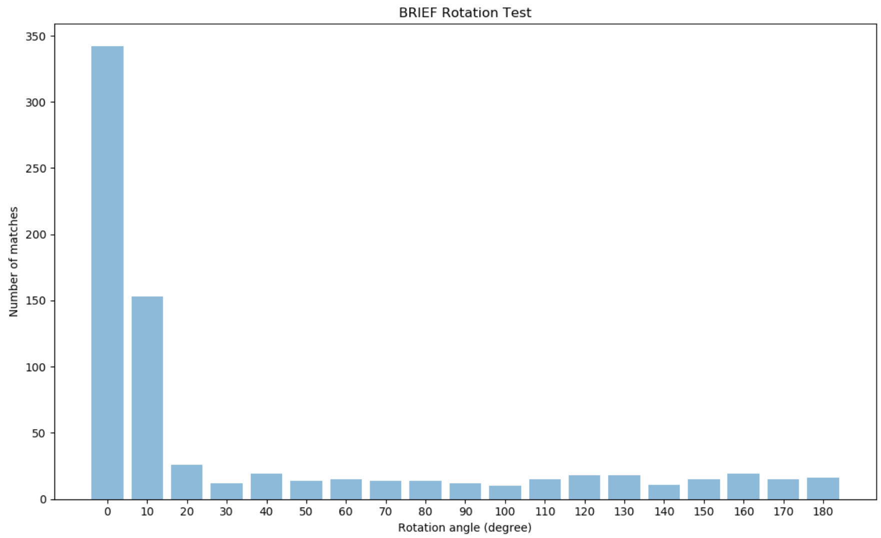

Figure 9: BRIEF Rotation Test

From the bar graph we can observe that: rotation significantly affects the performance of the BRIEF descriptor. Even with a 10 20 degree of rotation angle between two identical images, the number of correctly matched points decreases a lot.

One possible reason could be that: since BRIEF uses a rectangular patch to compute the descriptors, and if the images are rotated, the patches are not rotated together. Thus, corresponding points' correspondence becomes significantly low while rotation happens.

Q3.1:

(a):

For homography we have:

$$\lambda \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \tag{1}$$

Expanding the equations we get:

$$x(h_{31}u + h_{32}v + h_{33}) = h_{11}u + h_{12}v + h_{13}$$
$$y(h_{31}u + h_{32}v + h_{33}) = h_{21}u + h_{22}v + h_{23}$$

and given N point pair correspondences $\tilde{\boldsymbol{x}}_{\boldsymbol{n}}$ and $\tilde{\boldsymbol{u}}_{\boldsymbol{n}}$, for each pair $\tilde{x}_k$ and $\tilde{u}_k$:

$$\begin{bmatrix} 0 & 0 & 0 & -u_k & -v_k & -1 & y_k u_k & y_k v_k & y_k \\ u_k & v_k & 1 & 0 & 0 & 0 & -x_k u_k & -x_k v_k & -x_k \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \mathbf{0} \tag{2}$$

Putting everything together:

$$\begin{bmatrix} 0 & 0 & 0 & -u_1 & -v_1 & -1 & y_1 u_1 & y_1 v_1 & y_1 \\ u_1 & v_1 & 1 & 0 & 0 & 0 & -x_1 u_1 & -x_1 v_1 & -x_1 \\ 0 & 0 & 0 & -u_2 & -v_2 & -1 & y_2 u_2 & y_2 v_2 & y_2 \\ u_2 & v_2 & 1 & 0 & 0 & 0 & -x_2 u_2 & -x_2 v_2 & -x_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -u_N & -v_N & -1 & y_N u_N & y_N v_N & y_N \\ u_N & v_N & 1 & 0 & 0 & 0 & -x_N u_N & -x_N v_N & -x_N \end{bmatrix} \begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix} = \mathbf{0} \tag{3}$$

where the left big matrix is $\boldsymbol{A}$.

(b): There are 9 elements in $\mathbf{h}$.

(c): 4 point pairs (correspondences) are required to solve this system. Since the homography $\mathbf{H}$ has 8 degrees of freedom, and each point pair correspondence gives two equations.

(d):

We want to find a solution to $\boldsymbol{h}$ which satisfies:

$$\boldsymbol{A}\boldsymbol{h} = \mathbf{0}$$

with constraint

$$||\boldsymbol{h}|| = 1$$

We can use SVD to solve this problem.

Take SVD on A:

$$\boldsymbol{A} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^T$$

From orthonormality of $\boldsymbol{U}$ and $\boldsymbol{V}$ follows that:

$$||\boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^T\boldsymbol{h}|| = ||\boldsymbol{S}\boldsymbol{V}^T\boldsymbol{h}||$$

and

$$||\boldsymbol{V}^T\boldsymbol{h}|| = ||\boldsymbol{h}||$$

Then substitute $\boldsymbol{y} = \boldsymbol{V}^T\boldsymbol{h}$. Now we minimize $||\boldsymbol{S}\boldsymbol{y}||$ subject to $||\boldsymbol{y}|| = 1$

And since $\boldsymbol{S}$ is diagonal and the elements along the diagonal line are sorted in descending order, then obviously the solution is: $\boldsymbol{y} = [0, 0, ..., 1]^T$

From substitution we know that $\boldsymbol{h} = \boldsymbol{V}\boldsymbol{y}$, therefore $\boldsymbol{h}$ is the last column of the matrix $\boldsymbol{V}$.

Q6.3

Figure 10: Stitched Panorama

Q7.2

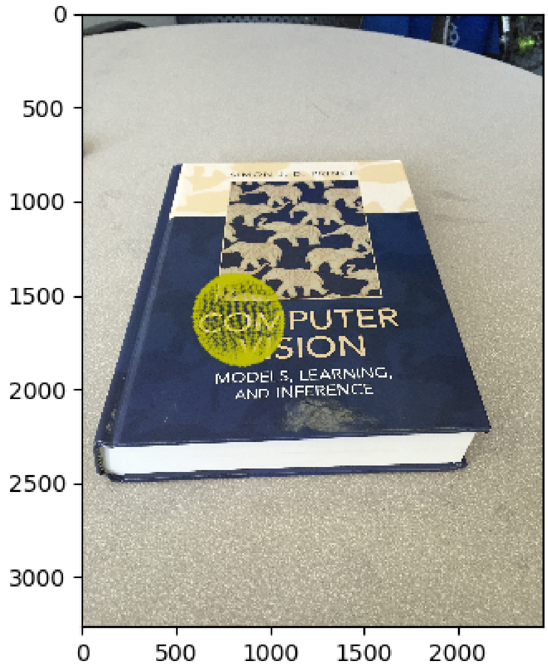

Figure 11: Resulted AR Image