Final paper

Haoluan Chen

12/15/2021

1 Introduction

Sudoku is a famous, logic-based, number-placement puzzle that has appeared since 1979. The objective of classic sudoku is to fill a nine by nine grid with numbers from 1 to 9 so that each column, each row, and each of the three by three distinct sub-grid contains all of the digits from 1 to 9. Additionally, some numbers will be fixed to some cells in the grid, and the goal is to fill in the rest of the cells. In this paper, I will first show you how simulated annealing can help solve classic sudoku. Then, I will compare geometric, linear, and logarithmic cooling schedules in terms of their ability to solve various sudoku levels and the number of iterations it took to find the solution.

2 Methods

2.1 Set-up in Solving Sudoku

1. For each of the three by three sub-grid, randomly fill in the non-fixed cells such that the sub-grid contains numbers from 1 to 9 exactly once. Then we will have a completed filled grid such that every sub-grid includes numbers from 1 to 9 only once.
2. In each iteration, randomly choose a sub-grid and randomly select two cells non-fixed within the chosen sub-gird. Then swap the value in the selected two cells. This newly filled gird is our new proposal.
3. Keep swapping the cells within the same sub-grid until we reach the solution

2.2 Cost Function

The rules of sudoku are to have numbers from 1 to 0 appear exactly once in each row, column, and sub-grid. Based on my set-up, the criterion that every sub-grid contains numbers from 1 to 9 only once is always satisfied. Thus, we only need to worry about the numbers along the rows and columns. Therefore, I can evaluate the candidate solution by summing up the duplicates in each row and column. When no duplicate in any row and column means I reach my solution, my cost function equals zero.

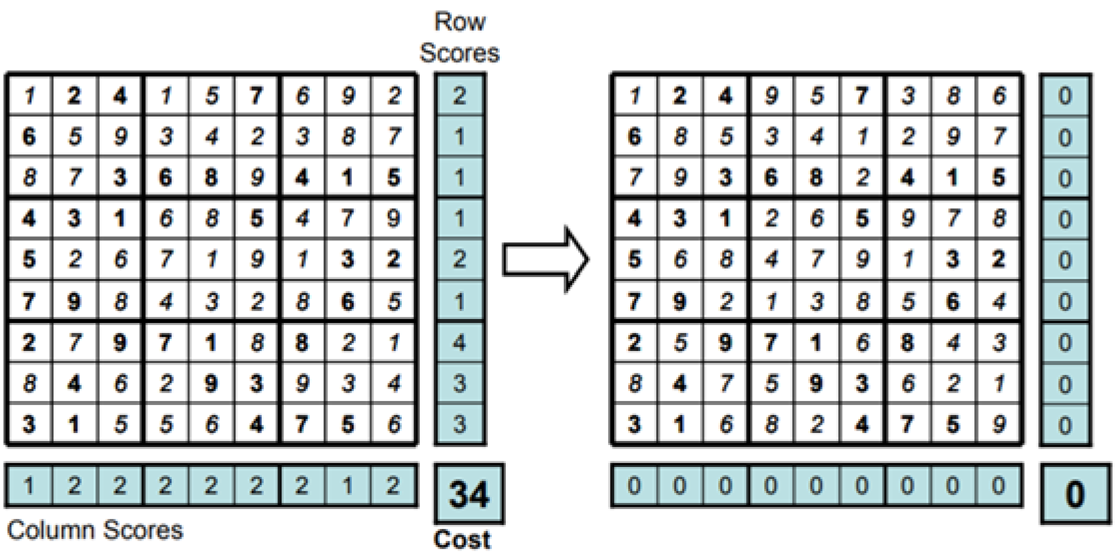


Figure 1: illustration of the cost function (Note the image is from: Lewis, R. Metaheuristics can solve sudoku puzzles. J Heuristics 13, 387–401 (2007).)

2.3 Simulated Annealing and Temperature

With our cost function, this sudoku puzzle is now a minimization problem, which we can solve by simulated annealing algorithm. The idea of simulated annealing is to randomly search through the possible number placement so that the new proposed number placement will have a lower cost. However, the algorithm may be stuck in some local minimum instead of finding the global minimum. This is where the temperature comes in. In each iteration, any two non-fixed cells within the same sub-grid are swapped. Then, this new proposal is accepted if:

a. The cost of the new proposal is lower than the previous candidate, or
b. It is accepted with a probability: $exp(-\delta/t)$ where δ is the proposed change in the cost function and t is the temperature parameter. The proposal is accepted if any of the two conditions are met.

For a large t value, the algorithm will accept and explore a lot of different number placement, which is good at the beginning of the search. For a small t value, the algorithm will only accept if it improves the cost function; this is good at the end of the search because it is close to the minimum of the cost function. So, we choose an initial t value, and it should be slowly reduced as we get close to the minimum. The rate we decrease the temperature is called the cooling schedule.

2.4 Cooling Schedule

There are many cooling schedules that we can use for simulated annealing. I will experiment with geometric, linear, and logarithmic cooling schedules. Then, I will evaluate each performance by its ability to solve sudoku in three different levels of difficulty(easy, medium, and evil in https://www.websudoku.com/) and the number of iterations it took to find the solution.

2.4.1 Geometric cooling schedule

In geometric cooling schedule, the temperature is reduced by $t_{i+1} = \alpha * t_i$, where α is called the cooling rate, and 0< α <1. For a large value of α, the temperature will drop very slowly, but the temperature will drop much quicker for a lower α.

2.4.2 Linear cooling schedule

In linear cooling schedule, the temperature is reduced by $t_{i+1} = t_i - d$, where d $>$ 0, and d is chosen so that t is still positive for any iteration of the run.

2.4.3 Logarithmic cooling schedule

In logarithmic cooling schedule, the temperature is reduced by $t_{i+1} = c/log(1 + i)$, where i is the number of iterations and c is a parameter.

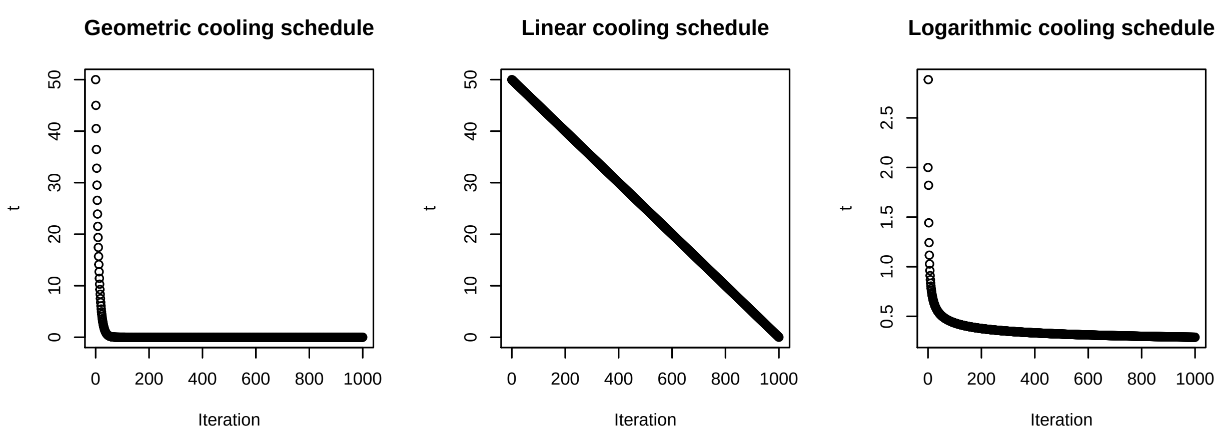


Figure 2: Cooling Schedule

2.5 Reheat process

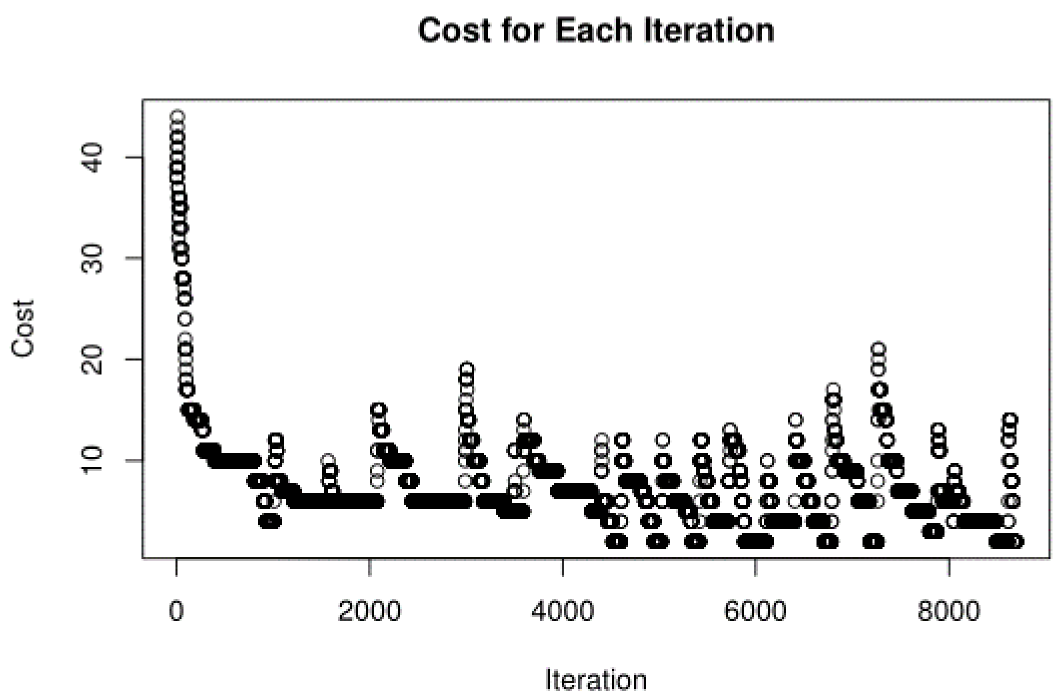
Lastly, I added a reheat process to prevent the algorithm from stuck in a local minimum. The reheat process works as follows: if there is no improvement in the cost for a fixed number of iterations (It was set to 80 for my algorithm), we increase t by some fixed number(reheat temperature). So, the algorithm started to accept more and explore different number placement.

3 Results

I allow each of them to run 10000 iterations to solve four different level sudoku 10 times for all three cooling schedules. Then I will check the solve rate for each cooling schedule and the number of iteration it took.

3.1 Geometric cooling schedule

In the geometric cooling schedule, I set the cooling rate to be 0.9 and the reheat temperature to be 5.




```
## Ran simulated annealing with geometric cooling for 2850 iterations.
## Solution found .
## Final temperature is 1.51849e-09
## Number of reheat in the run 5
```

Figure 3: Example run using Geometric cooling schedule

As we see from the result that geometric cooling can solve easy levels pretty consistently with 9 out of 10 times and an average iteration of 4208. However, as the difficulty increase, the algorithm started to have problems in solving sudoku within 10000 iteration. The solve rate dropped significantly, only 1 out of 10 times for medium level, 2 out of 10 times for evil level, and 0 times for hard level. But, this also means that the algorithm is able to solve the sudoku if we give a lot more iteration.

Level	Solution	Iteration took
Easy	9/10	4208
Medium	1/10	8811
Hard	0/10	N/A
Evil	2/10	5159

Figure 4: Figure 4 Geometric cooling schedule result

3.2 Linear cooling schedule

In geometric cooling schedule, I set reheat temperature to be 1.

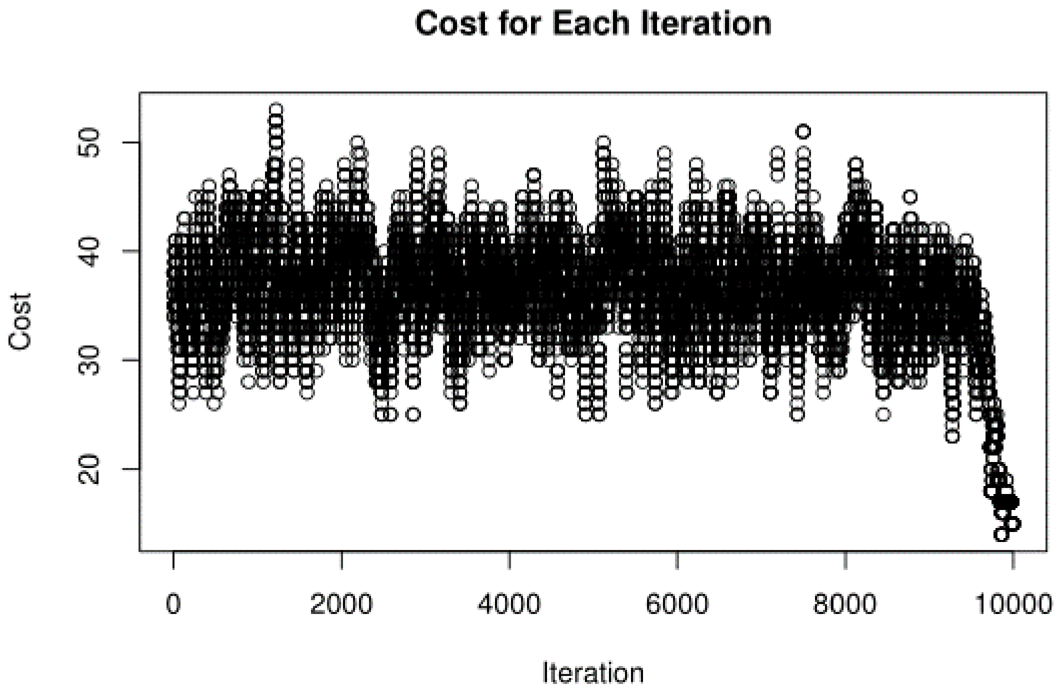
For linear cooling schedule, it is not good in solving sudoku. It spent way too much time exploring different number placement initially, and it only starts to try to reduce the cost at the very end. It also does not take advantage of the reheat process because the reduction in temperature is too slow, so the algorithm usually accepts the new proposal.

3.3 Logarithmic cooling schedule

In logarithmic cooling schedule, I set the reheat temperature to be 10 and the c parameter to be 2.

As we see in figure 2, the logarithmic cooling schedule reduces very fast in the beginning and slows down at the end. So, we see a rapid reduction in the cost, and the algorithm is slowly swapping between cells to find the minimum. Even when we reheat the temperature, the cost does not change significantly in the geometric cooling.

Based on the result, we see a similar pattern in geometric cooling. For an easy level, logarithmic can solve sudoku pretty consistently. But it requires a lot more iteration for more difficult sudoku. However, it managed

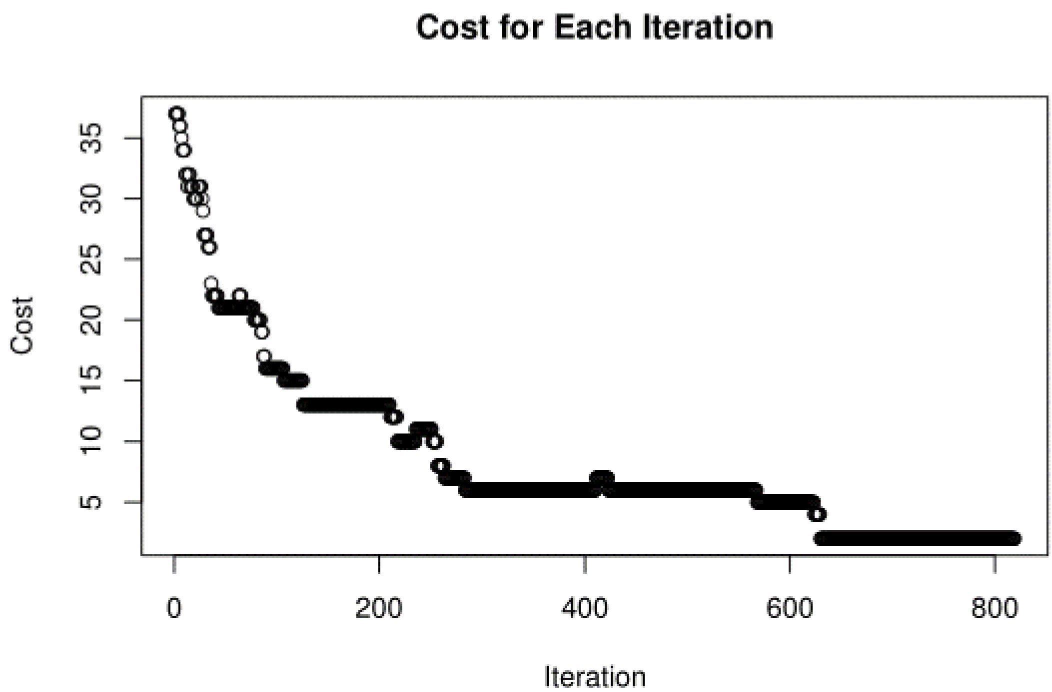



```
## Ran simulated annealing with linear cooling for 10000 iterations.
## Solution not found .
## Minimum cost reached 12 .
## Final temperature is 0.01
## Number of reheat in the run 0
```

Figure 5: Example run using Geometric cooling schedule

Level	Solution	Iteration Took
Easy	0/10	N/A
Medium	0/10	N/A
Hard	0/10	N/A
Evil	0/10	N/A

Figure 6: Linear cooling schedule result




```
## Ran simulated annealing with logarithmic cooling for 2709 iterations.
## Solution found .
## Final temperature is 0.2530139
## Number of reheat in the run 3
```

Figure 7: Example run using Logarithmic cooling schedule

to solve the hard level once with 7684 iterations, but it did not solve the evil level within 10000 iterations.

Level	Solution	Iteration
Easy	8/10	3186
Medium	1/10	3297
Hard	1/10	7684
Evil	0/10	N/A

Figure 8: Logarithmic cooling schedule result

Discussion

Based on the result, we see that the cooling schedule significantly impacts the ability to solve sudoku. Linear cooling schedule is not able to solve sudoku. Both geometric and logarithmic cooling schedules can solve sudoku if we give it enough time. I think the geometric and logarithmic cooling schedules performed similarly because the cooling rate starts to reduce fast at the beginning and slowly at the end. This is the optimal characteristic of a cooling schedule in solving sudoku. It quickly gets to a stage to reduce the cost and does not waste iteration on randomly swapping any two non-fixed cells that do not reduce cost. We also see that the reheat process plays a crucial role in solving the sudoku. In terms of the cost function, the reheat prevents the algorithm from stuck in a local minimum. In terms of sudoku, it randomly swaps some non-fixed cells and tries hard to reduce the cost again. If we are lucky that we got some of the numbers in the correct spot during our initial reheat process, it will be more likely to find the result. However, there is still a lot of randomness in our algorithm, but if we give enough time for any cooling schedule like geometric and logarithmic cooling schedule, it should be able to find the sudoku solution reasonably quickly.

References

Lewis, R. (2007). Metaheuristics can solve sudoku puzzles. Journal of heuristics, 13(4), 387-401.