

Project

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Appendices

```
nine <- c(1, 2, 3, 4, 5, 6, 7 ,8 ,9)

# randomly fill in each sub-grid

subgrid <- function(grid, n){
  if (n == 1){
    result <- grid[1:3, 1:3]
  }
  if (n==2) {
    result <-grid[1:3, 4:6]
  }
  if (n==3) {
    result <- grid[1:3, 7:9]
  }
  if (n==4) {
    result <-grid[4:6, 1:3]
  }
  if (n==5) {
    result <-grid[4:6, 4:6]
  }
  if (n==6) {
    result <-grid[4:6, 7:9]
  }
  if (n==7) {
    result <-grid[7:9, 1:3]
  }
  if (n==8) {
    result <-grid[7:9, 4:6]
  }
  if (n==9) {
    result <-grid[7:9, 7:9]
  }
  result
}

fixed <- function(sub){
  fixed_numbers = c()
  for (row in 1:3){
    for (col in 1:3){
      if (!is.na(as.numeric(sub[row, col]))){
        fixed_numbers = c(fixed_numbers, as.numeric(sub[row, col]))
      }
    }
  }
  fixed_numbers
}

fill_subgrid <- function(sub, candidate){
  candidate <- sample(candidate, length(candidate), replace = FALSE)
  for (row in 1:3){
    for (col in 1:3){
      if (is.na(as.numeric(sub[row, col]))){


```

```

        sub[row, col] = candidate[1]
        candidate <- candidate[-1]
    }
}
}
sub
}

# create a function that fill in each of the sub-grid
# first get all the numbers in the sub-grid, remove them from the candidate, then sample the non-fixed

comb_subgrid <- function(list_of_subgrid){
  row1 <- cbind(list_of_subgrid[[1]], list_of_subgrid[[2]], list_of_subgrid[[3]])
  row2 <- cbind(list_of_subgrid[[4]], list_of_subgrid[[5]], list_of_subgrid[[6]])
  row3 <- cbind(list_of_subgrid[[7]], list_of_subgrid[[8]], list_of_subgrid[[9]])
  output <- rbind(row1, row2, row3)
  output
}

fill <- function(grid){
  result <- list()
  for (i in 1:9){
    sub <- subgrid(grid, i) # get the subgrid
    fixed_numbers <- fixed(sub) # find fixed numbers
    candidate <- setdiff(nine, fixed_numbers) # find the set difference to get candidate numbers for n
    candidate <- sample(candidate, length(candidate), replace = FALSE)
    for (row in 1:3){
      for (col in 1:3){
        if (is.na(as.numeric(sub[row, col]))){
          sub[row, col] = candidate[1]
          candidate <- candidate[-1]
        }
      }
    }
    result[[i]] <- sub
  }
  comb_subgrid(result)
}

swap <- function(grid, filled_grid){
  # randomly choose a subgrid
  grid_num <- sample(1:9, 1)
  non_fix_index = data.frame(row = c(), col = c())
  sub <- subgrid(grid, grid_num)
  for (row in 1:3){
    for (col in 1:3){

```

```

    if (is.na(as.numeric(sub[row, col]))){
      non_fix_index <- rbind(non_fix_index,c(row, col) )
    }
  }
}

index <- sample_n(non_fix_index,2) # index of the two cell to swap
sub_filled_grid <- subgrid(filled_grid, grid_num)

temp <- sub_filled_grid[index[1,1], index[1,2]]
sub_filled_grid[index[1,1], index[1,2]] <- sub_filled_grid[index[2,1], index[2,2]]
sub_filled_grid[index[2,1], index[2,2]] <- temp

if (grid_num == 1){
  filled_grid[1:3, 1:3] <- sub_filled_grid
}
if (grid_num==2) {
  filled_grid[1:3, 4:6] <-sub_filled_grid
}
if (grid_num==3) {
  filled_grid[1:3, 7:9] <- sub_filled_grid
}
if (grid_num==4) {
  filled_grid[4:6, 1:3] <-sub_filled_grid
}
if (grid_num==5) {
  filled_grid[4:6, 4:6] <-sub_filled_grid
}
if (grid_num==6) {
  filled_grid[4:6, 7:9] <-sub_filled_grid
}
if (grid_num==7) {
  filled_grid[7:9, 1:3] <-sub_filled_grid
}
if (grid_num==8) {
  filled_grid[7:9, 4:6] <-sub_filled_grid
}
if (grid_num==9) {
  filled_grid[7:9, 7:9] <-sub_filled_grid
}

filled_grid

# find non-fixed cell indicated by numbers
}

# Calculate the cost function
# calculate the cost in each row

```

```

# calculate the cost in each col
# then sum it up
rowcost <- function(filled_grid){
  cost <- 0
  for (r in 1:9){ # go over each row
    row <- c()
    for (i in 1:9){
      row <- c(row,filled_grid[r,i])
    }
    cost = cost + (9 - length(unique(row)))
  }
  cost
}

colcost <- function(filled_grid){
  cost <- 0
  for (col in 1:9){
    cost = cost + (9 - length(unique(filled_grid[,col])))
  }
  cost
}

cost <- function(filled_grid){
  cost = rowcost(filled_grid) + colcost(filled_grid)
  cost
}

# swap the non-fixed cell, probably need to keep the orginal table around

solve_sudoku_linear_cooling <- function(M, initialt, grid){
  iteration <- 0
  finaltemp <- 0.01
  costlist <- c()
  stuck <- 0
  reheat <- 0
  solution <- FALSE
  filled_grid <- fill(grid)
  tempdiff = (initialt-finaltemp)/M      # for linearcooling
  t <- initialt

  for (i in 1:M){
    iteration = iteration + 1
    U <- runif(1)
    t = t - tempdiff # linear cooling
    precost <- cost(filled_grid)
    proposal <- swap(grid, filled_grid)
    costpro <- cost(proposal)
    accept_num = 0
    if (costpro < precost | U < exp(-(costpro-precost)/t)){
      filled_grid = proposal
      accept_num = accept_num + 1
      stuck = 0
    }
  }
}

```

```

    else {stuck = stuck + 1}

    if (stuck >= 80){
        t = t + 1
        stuck = 0
        reheat = reheat + 1
    }
    if (cost(filled_grid) == 0){
        solution = T
        costlist = c(costlist, precost)
        break
    }
    costlist = c(costlist, precost)

}

cat("Ran simulated annealing with linear cooling for", iteration, "iterations.\n");
if (solution){
    cat("Solution found", ".\n");
}
else {
    cat("Solution not found", ".\n");
    cat("Minimum cost reached", min(costlist), ".\n");
}
cat("Final temperature is", t, "\n");
cat("Number of reheat in the run", reheat, "\n");

plot(costlist, main ="Cost for Each Iteration", xlab = "Iteration", ylab = "Cost")
filled_grid
}

solve_sudoku_exp_cooling <- function(M, initialt, grid, cooling_rate){
    iteration <- 0
    finaltemp <- 0.01
    costlist <- c()
    stuck <- 0
    reheat <- 0
    solution <- FALSE
    filled_grid <- fill(grid)
    t <- initialt

    for (i in 1:M){
        iteration = iteration + 1
        U <- runif(1)
        t = cooling_rate * t
        precost <- cost(filled_grid)
        proposal <- swap(grid, filled_grid)
        costpro <- cost(proposal)
        accept_num = 0
        if (costpro < precost | U < exp(-(costpro-precost)/t)){
            filled_grid = proposal
            accept_num = accept_num + 1
            stuck = 0
        }
    }
}

```

```

    else {stuck = stuck + 1}

    if (stuck >= 80){
      t = t + 5
      stuck = 0
      reheat = reheat + 1
    }
    if (cost(filled_grid) == 0){
      solution = T
      costlist = c(costlist, precost)
      break
    }
    costlist = c(costlist, precost)

  }

cat("Ran simulated annealing with geometric cooling for", iteration, "iterations.\n");
if (solution){
  cat("Solution found", ".\n");
}
else {
  cat("Solution not found", ".\n");
  cat("Minimum cost reached", min(costlist), ".\n");
}
cat("Final temperature is", t, "\n");
cat("Number of reheat in the run", reheat, "\n");
plot(costlist, main ="Cost for Each Iteration", xlab = "Iteration", ylab = "Cost")
filled_grid
}

solve_sudoku_log_cooling <- function(M, initialt, grid){
  iteration <- 0
  finaltemp <- 0.01
  costlist <- c()
  stuck <- 0
  reheat <- 0
  solution <- FALSE
  filled_grid <- fill(grid)
  t <- initialt

  for (i in 1:M){
    iteration = iteration + 1
    U <- runif(1)
    t = 2/log(1+i)
    precost <- cost(filled_grid)
    proposal <- swap(grid, filled_grid)
    costpro <- cost(proposal)
    accept_num = 0
    if (costpro < precost | U < exp(-(costpro-precost)/t)){
      filled_grid = proposal
      accept_num = accept_num + 1
      stuck = 0
    }
    else {stuck = stuck + 1}
  }

}

```

```

if (stuck >= 80){
  t = t + 10
  stuck = 0
  reheat = reheat + 1
}
if (cost(filled_grid) == 0){
  solution = T
  costlist = c(costlist, precost)
  break
}
costlist = c(costlist, precost)

}

cat("Ran simulated annealing with logarithmic cooling for", iteration, "iterations.\n");
if (solution){
  cat("Solution found", ".\n");
}
else {
  cat("Solution not found", ".\n");
  cat("Minimum cost reached", min(costlist), ".\n");
}
cat("Final temperature is", t, "\n");
cat("Number of reheat in the run", reheat, "\n");
plot(costlist, main ="Cost for Each Iteration", xlab = "Iteration", ylab = "Cost")
filled_grid
}

ex1 <- read_excel("ex1.xlsx")
ex1

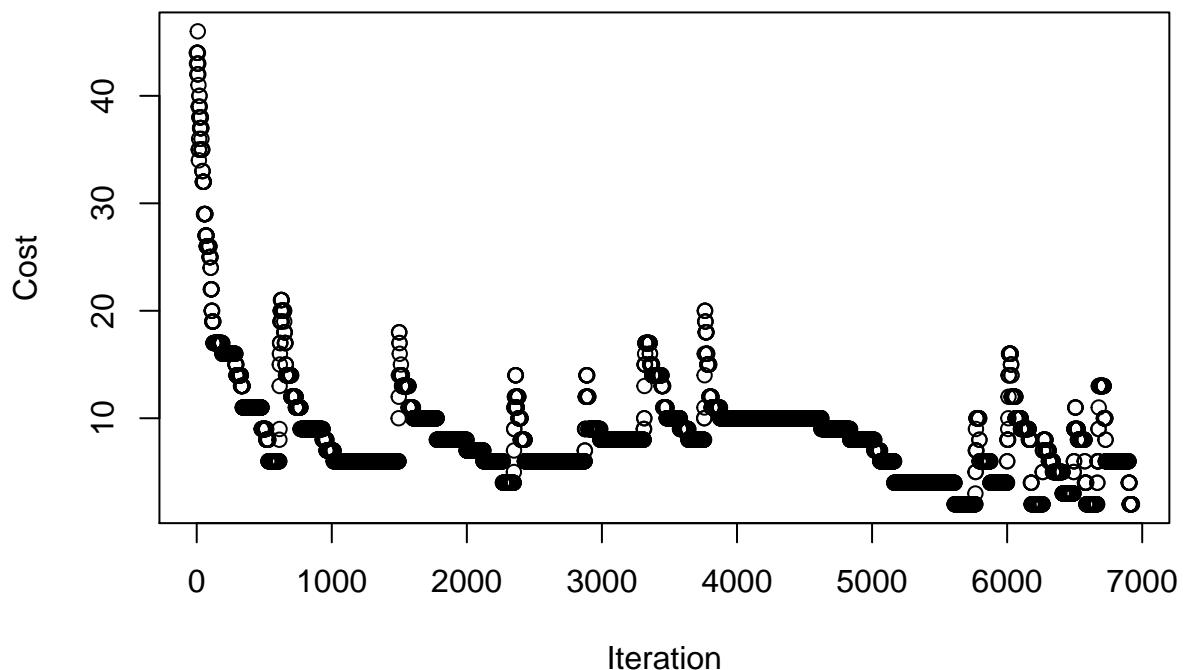
## # A tibble: 9 x 9
##   col1  col2  col3  col4  col5  col6  col7  col8  col9
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     NA     2     NA     5     NA     NA     3     NA     NA
## 2     1     NA     4     NA     2     6     NA     NA     8
## 3     8     NA     NA     7     1     9     NA     2     5
## 4     NA     NA     NA     NA     NA     NA     NA     3     NA
## 5     NA     9     8     4     NA     3     5     7     NA
## 6     NA     6     NA     NA     NA     NA     NA     NA     NA
## 7     9     4     NA     6     3     1     NA     NA     2
## 8     6     NA     NA     2     8     NA     9     NA     3
## 9     NA     NA     2     NA     NA     4     NA     6     NA

for (i in 1:10){
  solve_sudoku_exp_cooling(10000, 50, ex1, 0.9)
}

## Ran simulated annealing with geometric cooling for 6924 iterations.
## Solution found .
## Final temperature is 7.043361e-12
## Number of reheat in the run 11

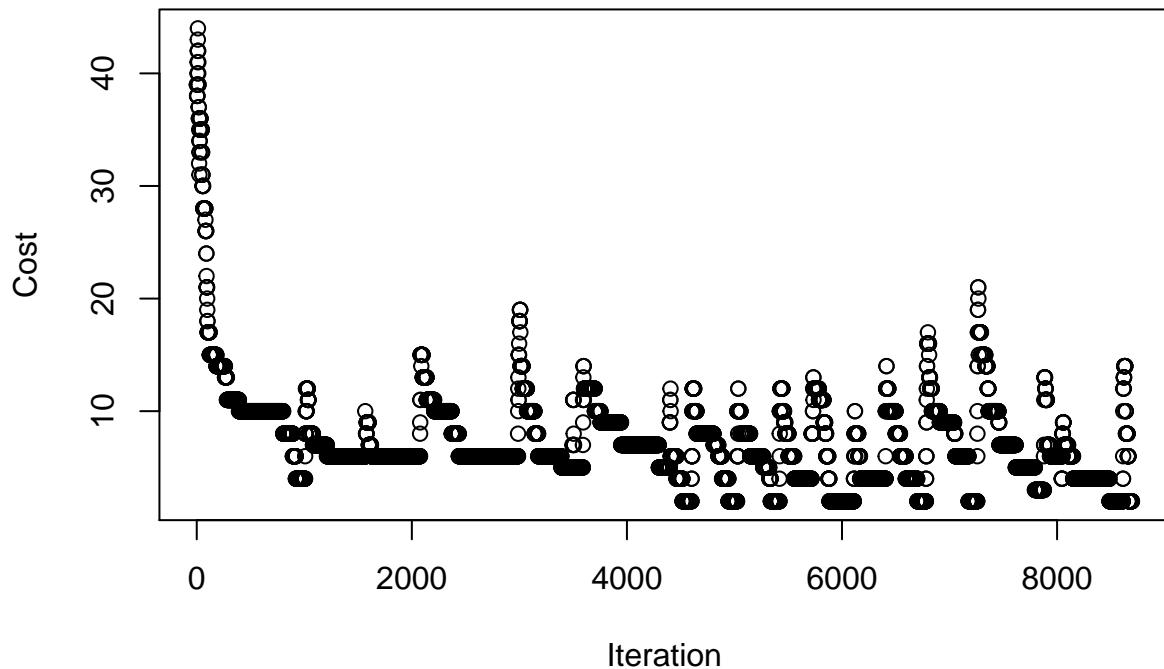
```

Cost for Each Iteration



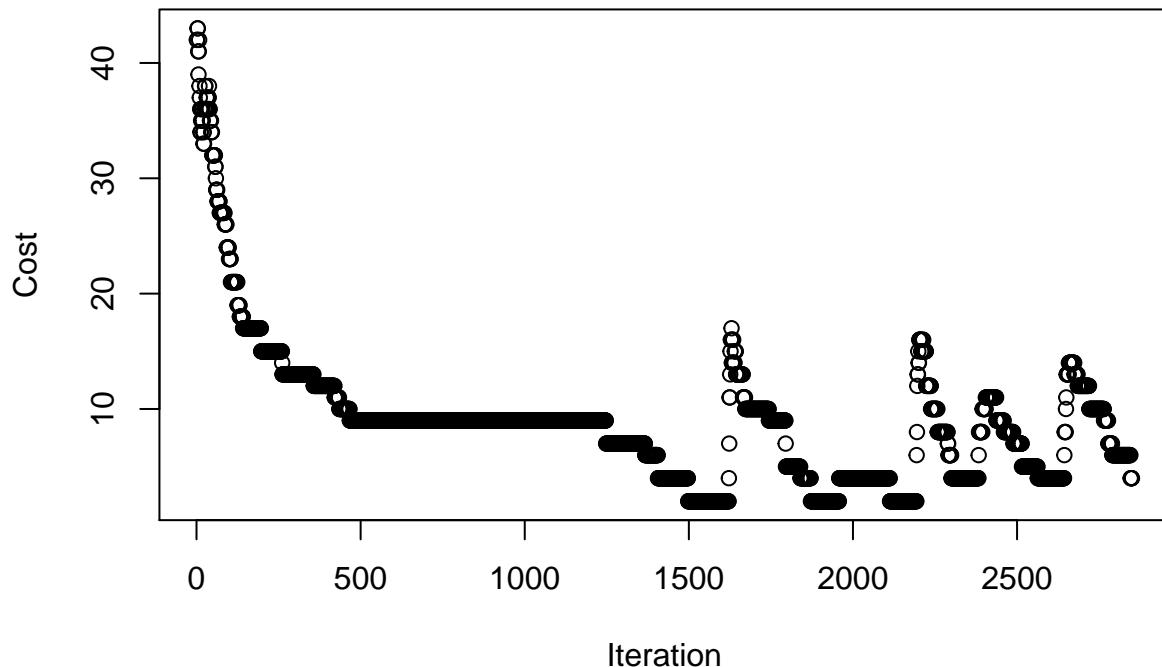
```
## Ran simulated annealing with geometric cooling for 8696 iterations.  
## Solution found .  
## Final temperature is 0.0005805315  
## Number of reheat in the run 18
```

Cost for Each Iteration



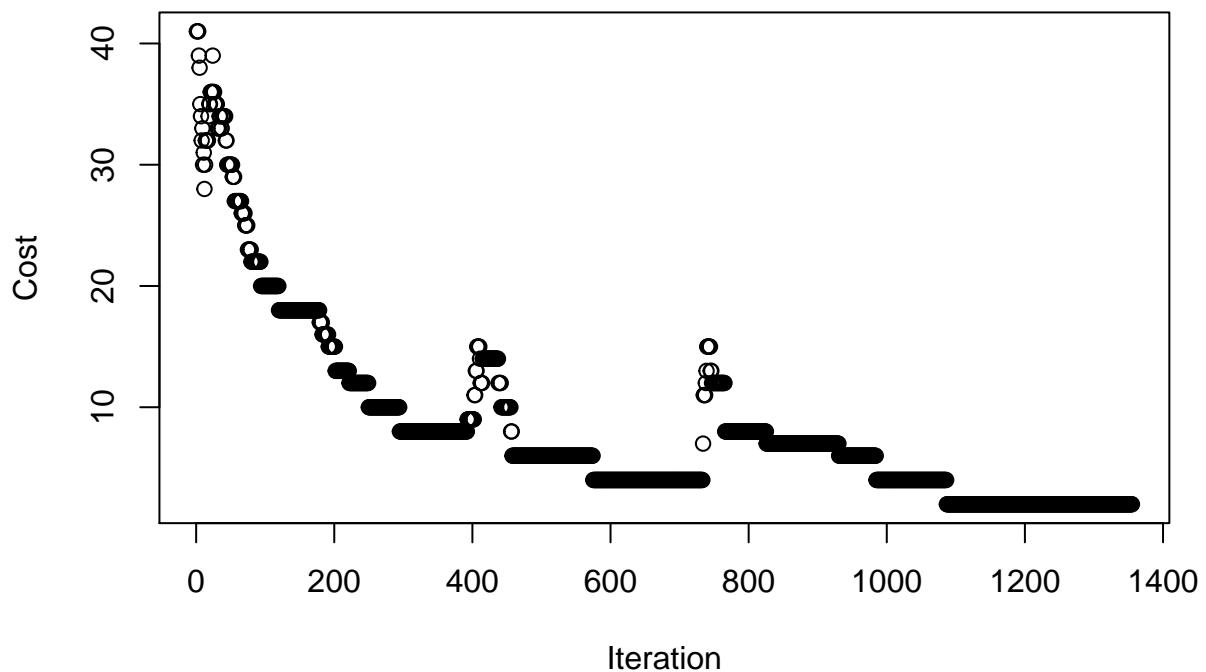
```
## Ran simulated annealing with geometric cooling for 2850 iterations.  
## Solution found .  
## Final temperature is 1.51849e-09  
## Number of reheat in the run 5
```

Cost for Each Iteration



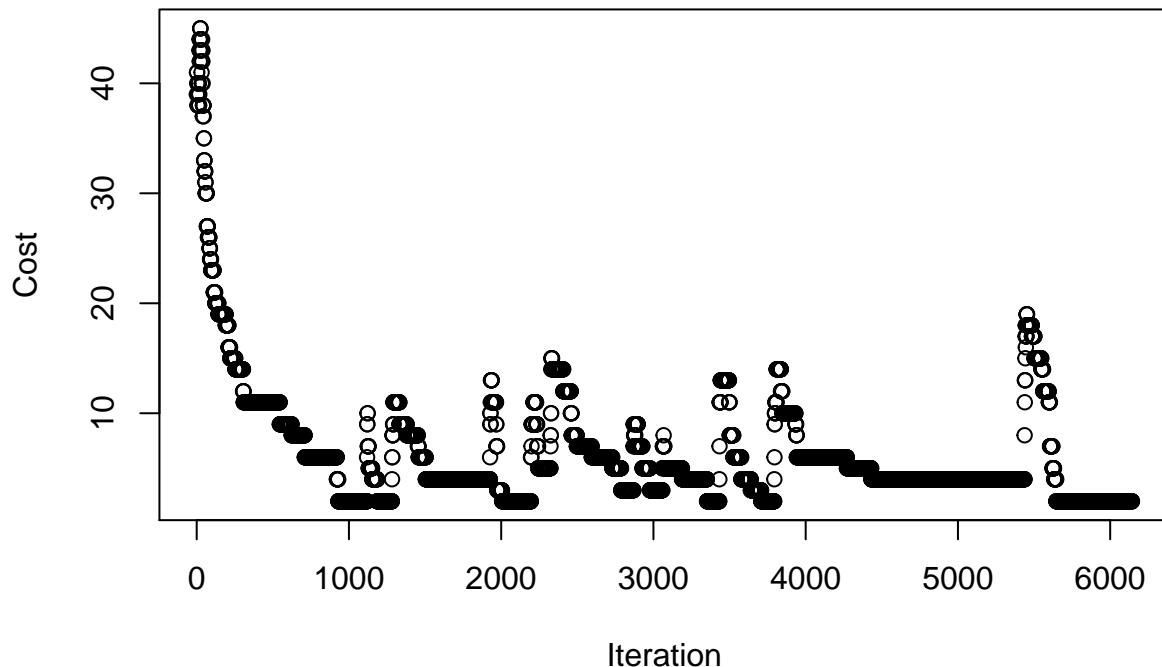
```
## Ran simulated annealing with geometric cooling for 1355 iterations.  
## Solution found .  
## Final temperature is 1.556157e-28  
## Number of reheat in the run 2
```

Cost for Each Iteration



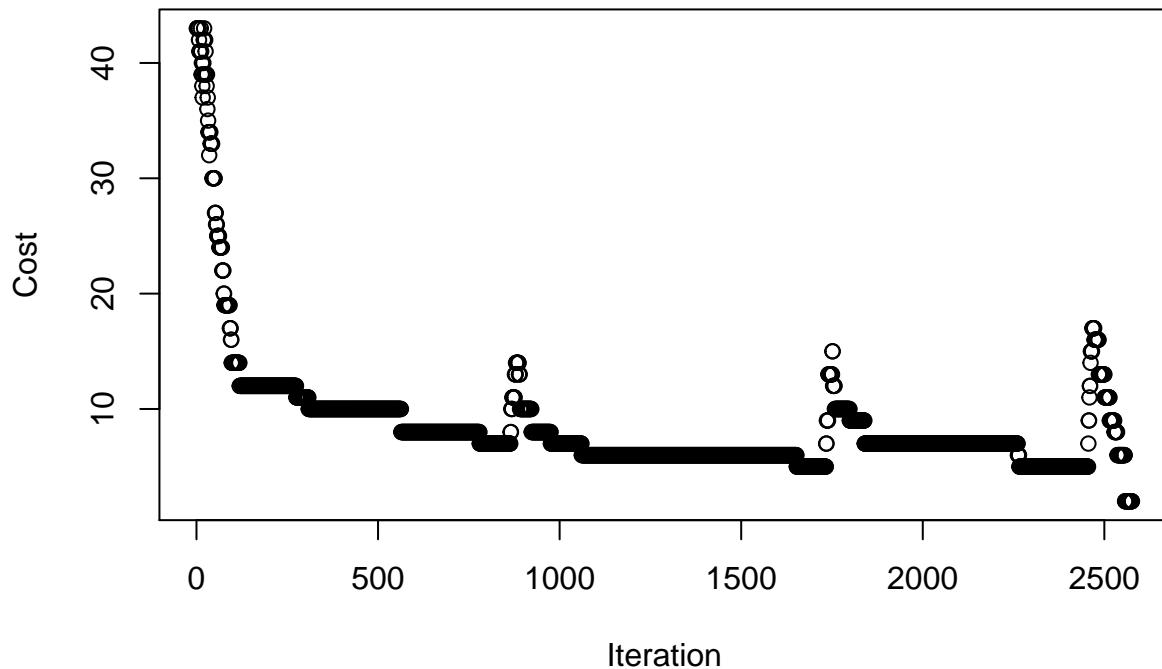
```
## Ran simulated annealing with geometric cooling for 6143 iterations.  
## Solution found .  
## Final temperature is 1.806796e-32  
## Number of reheat in the run 10
```

Cost for Each Iteration



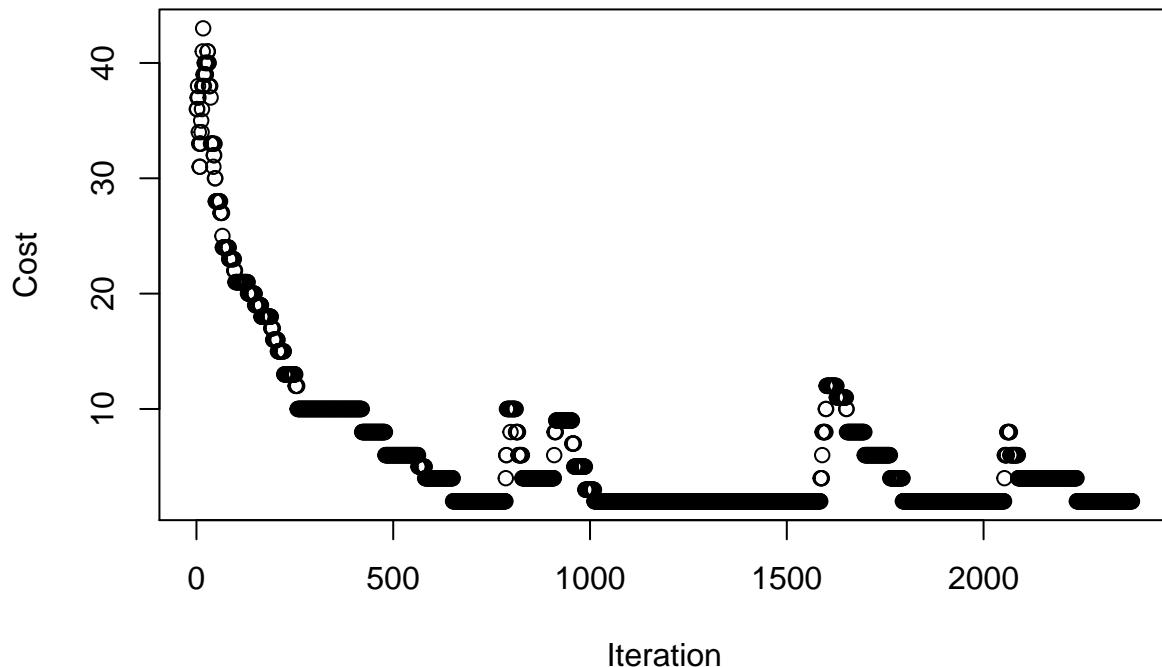
```
## Ran simulated annealing with geometric cooling for 2576 iterations.  
## Solution found .  
## Final temperature is 1.307845e-05  
## Number of reheat in the run 3
```

Cost for Each Iteration



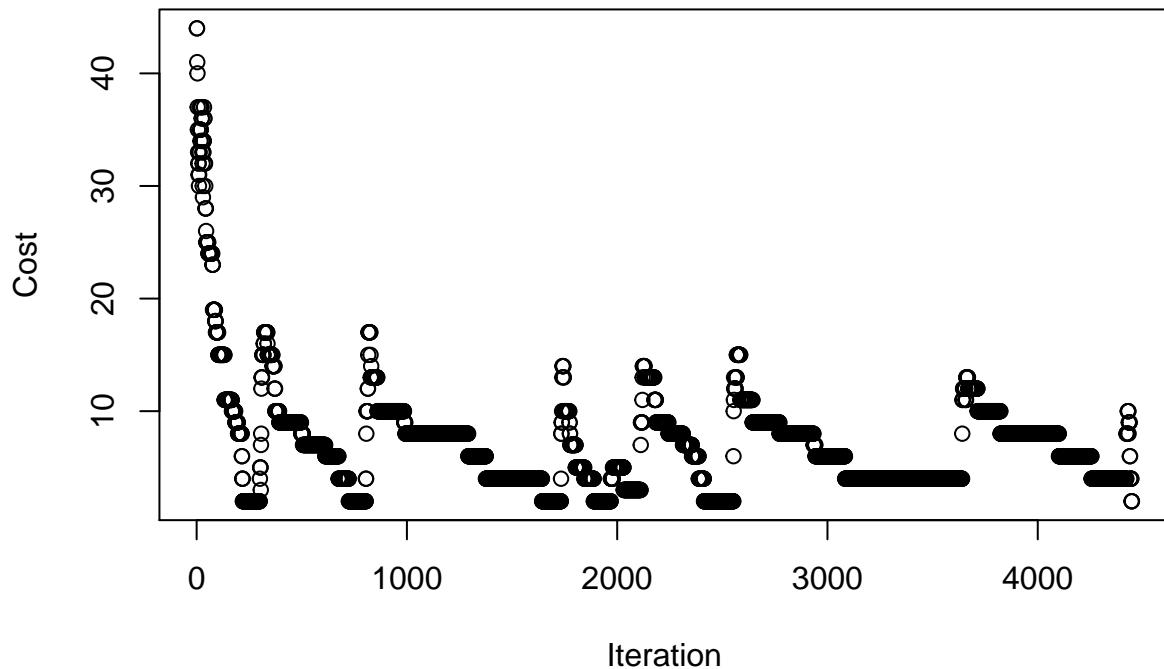
```
## Ran simulated annealing with geometric cooling for 2377 iterations.  
## Solution found .  
## Final temperature is 4.903578e-15  
## Number of reheat in the run 4
```

Cost for Each Iteration



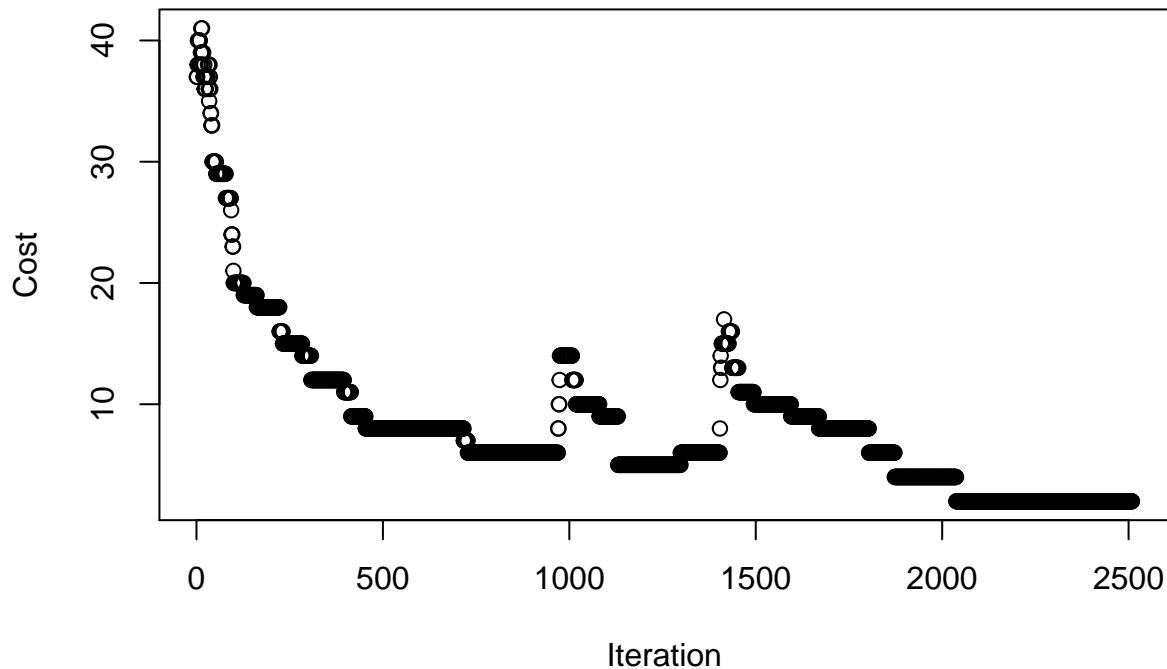
```
## Ran simulated annealing with geometric cooling for 4448 iterations.  
## Solution found .  
## Final temperature is 0.08211602  
## Number of reheat in the run 8
```

Cost for Each Iteration



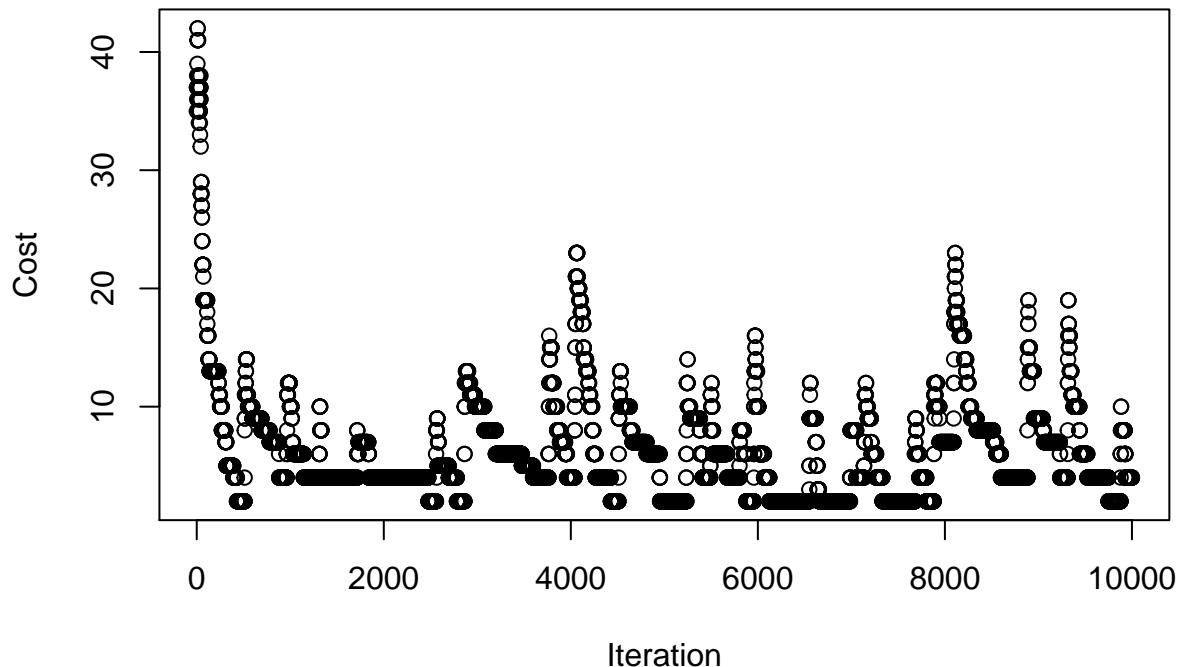
```
## Ran simulated annealing with geometric cooling for 2509 iterations.  
## Solution found .  
## Final temperature is 1.110275e-50  
## Number of reheat in the run 3
```

Cost for Each Iteration



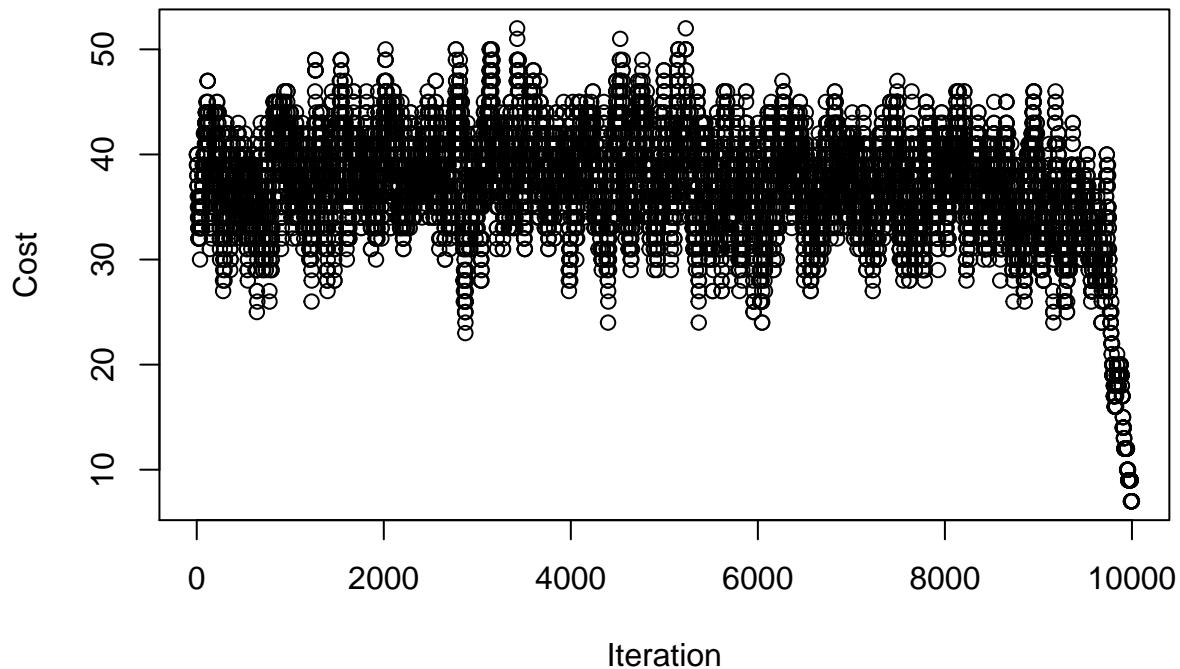
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 1.17706e-05  
## Number of reheat in the run 22
```

Cost for Each Iteration



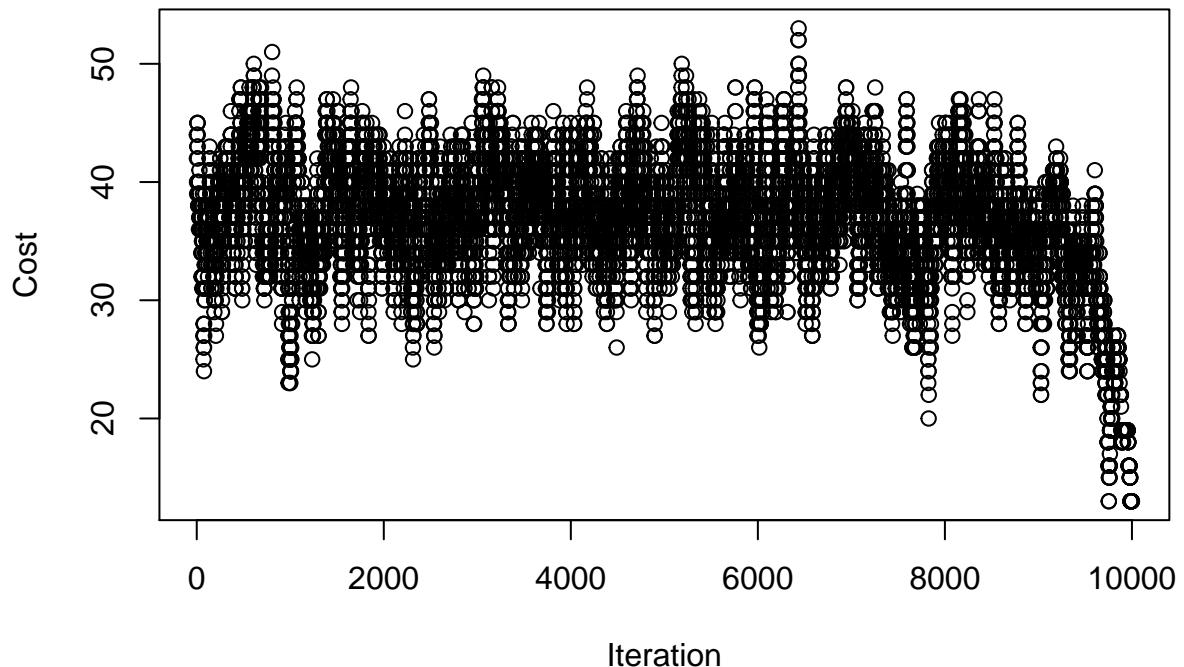
```
for (i in 1:10){  
  solve_sudoku_linear_cooling(10000, 50, ex1)  
}  
  
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 7 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



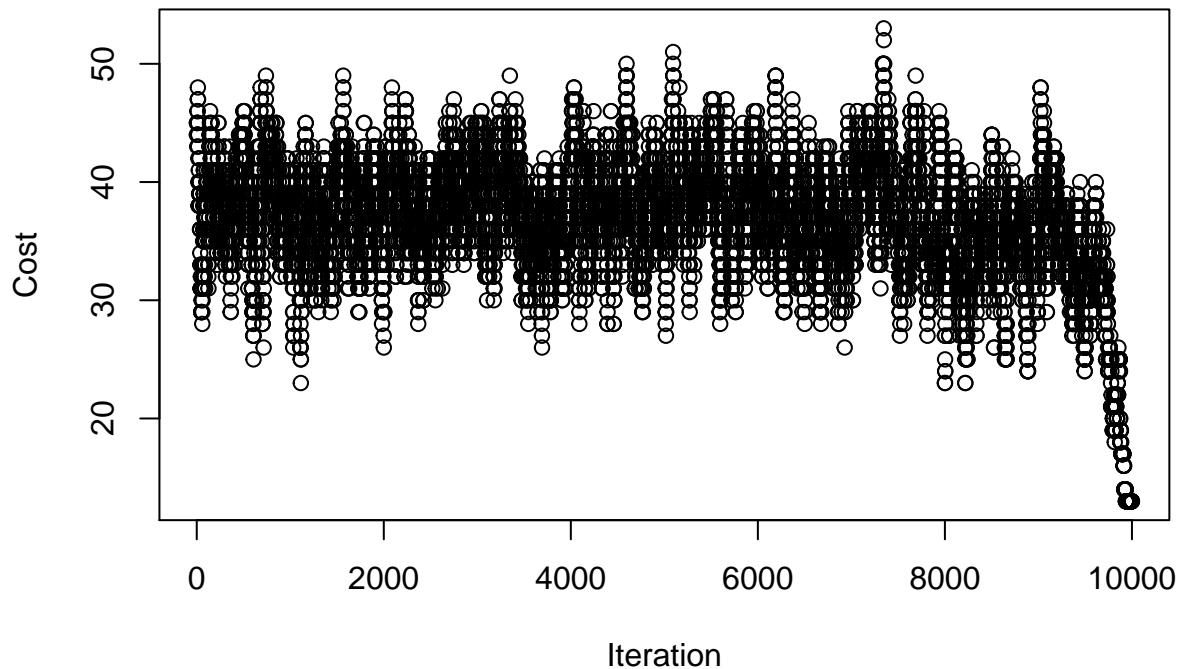
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 13 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



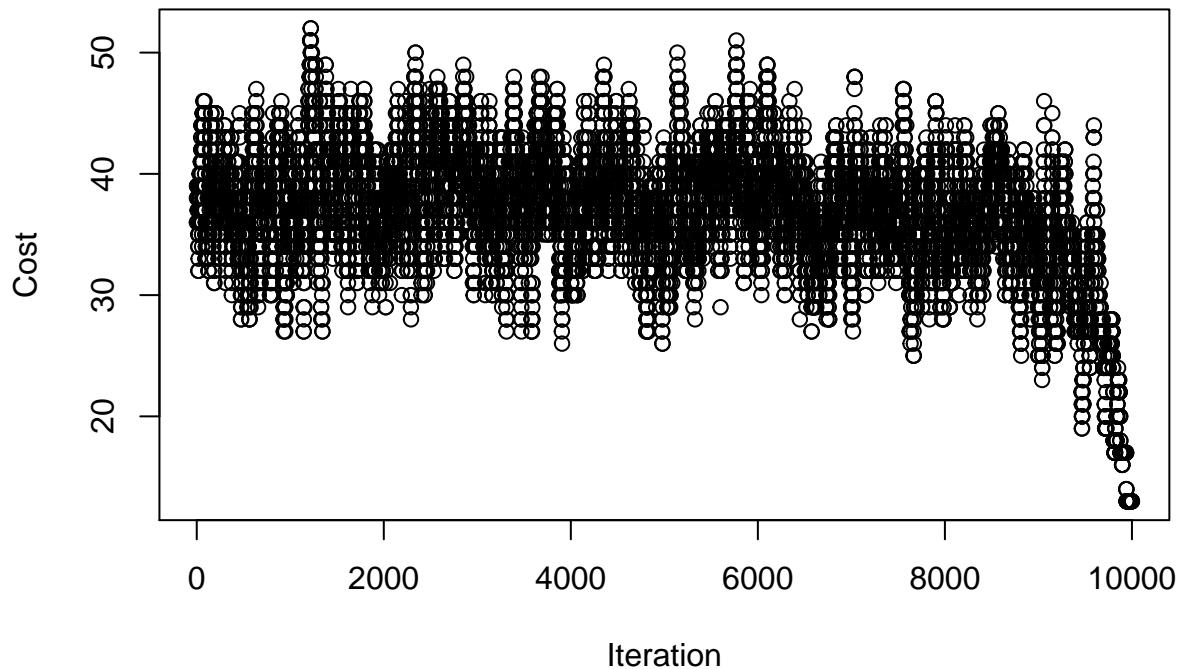
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 13 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



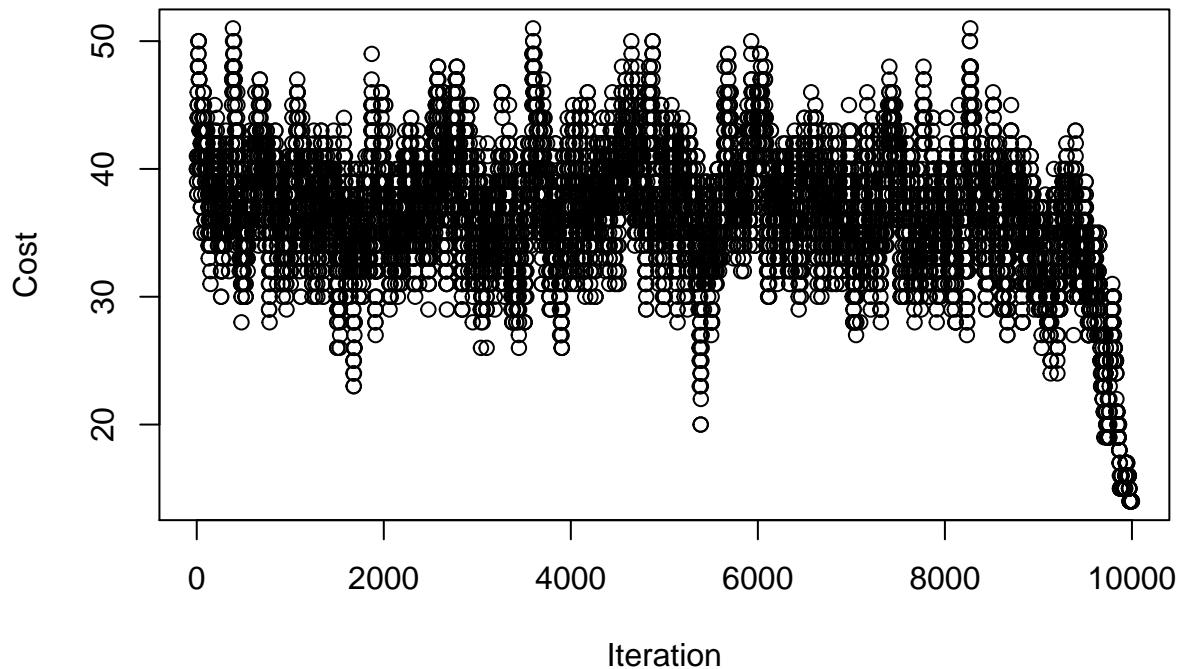
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 13 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



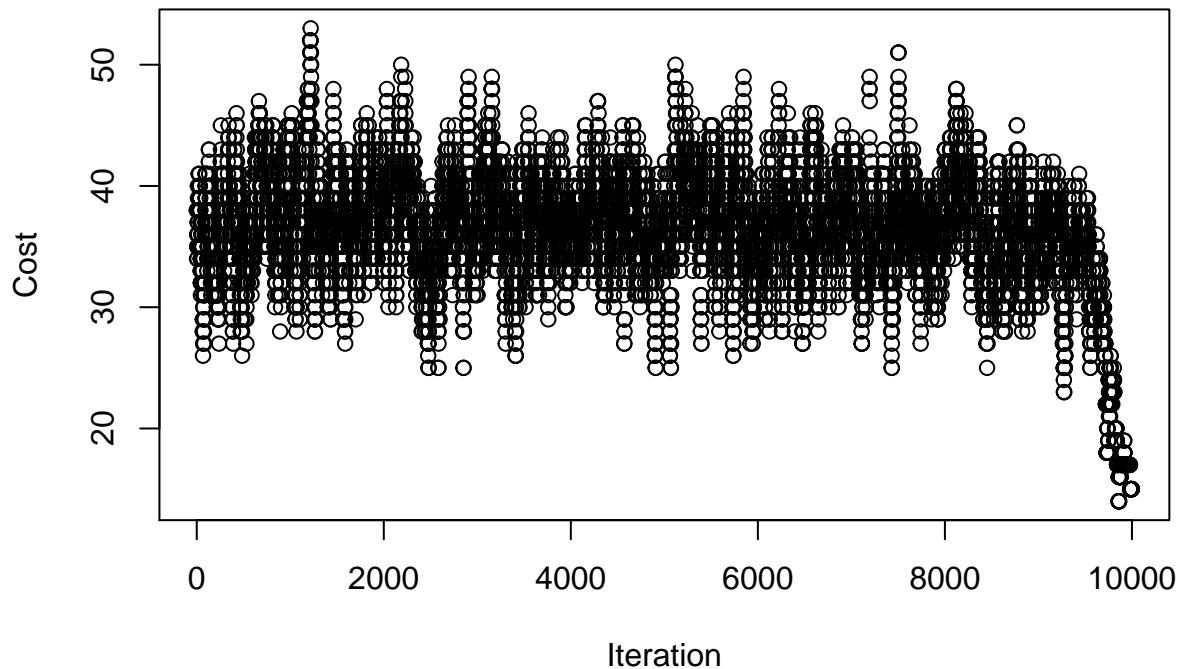
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 14 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



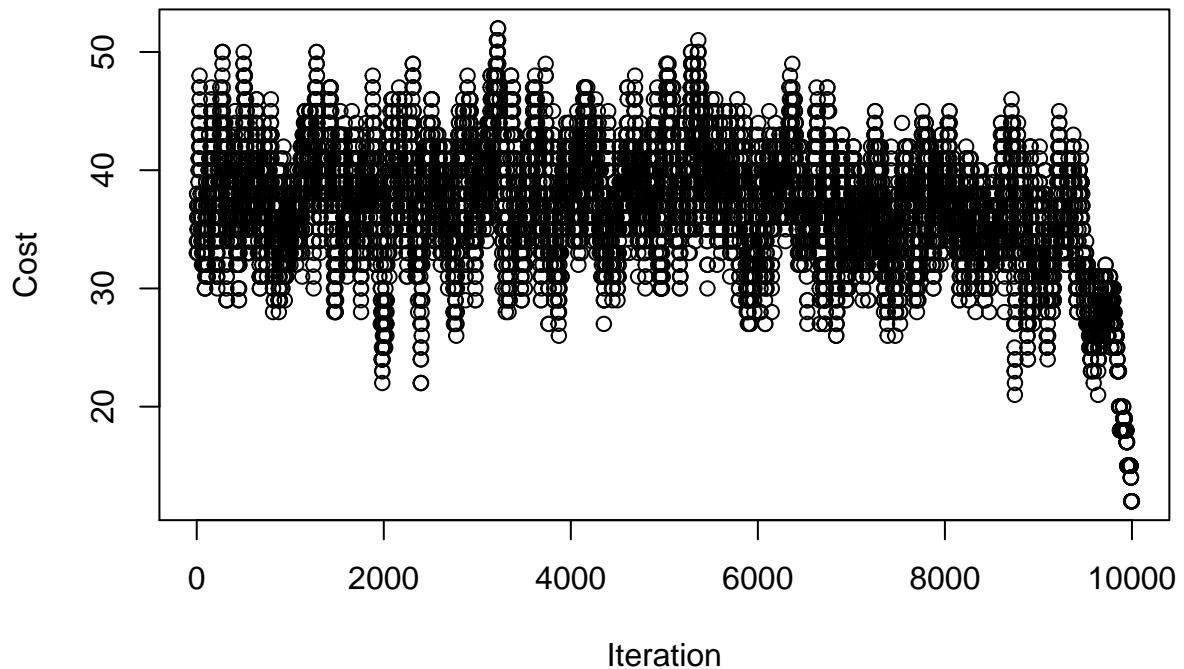
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 14 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



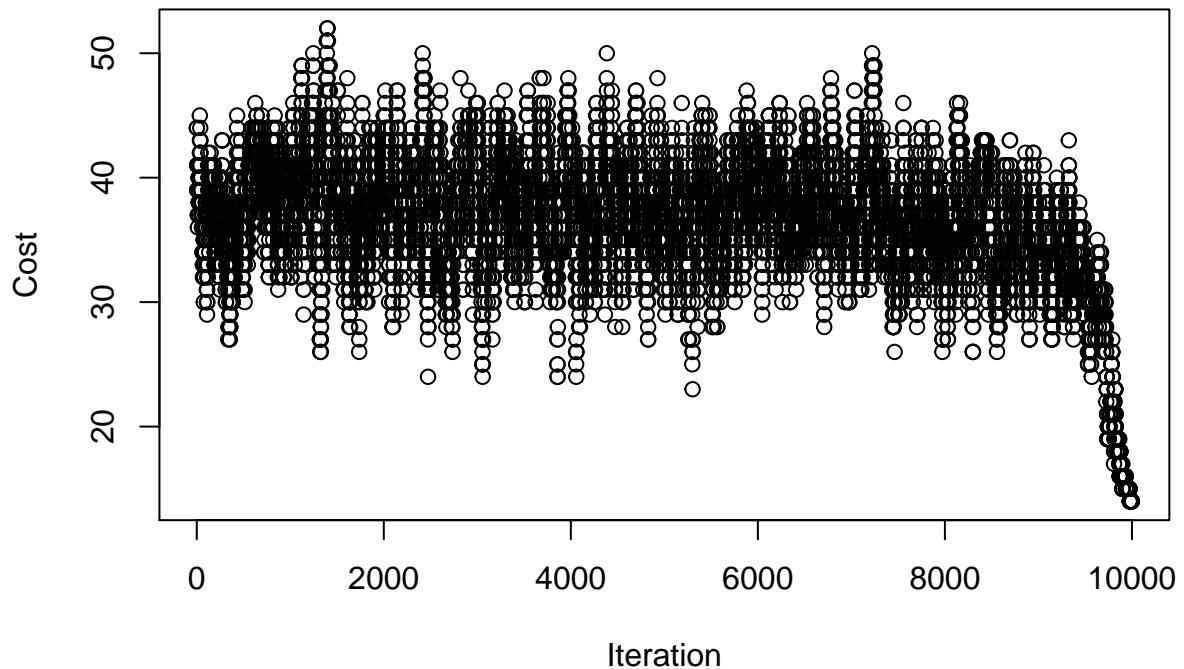
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 12 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



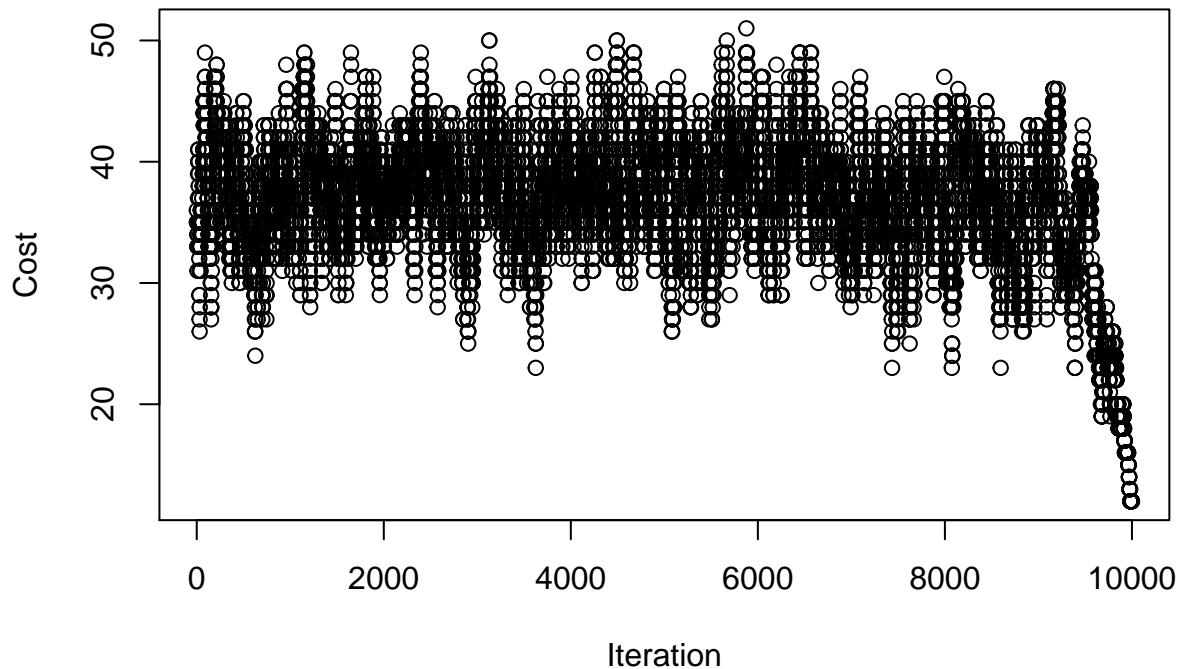
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 14 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



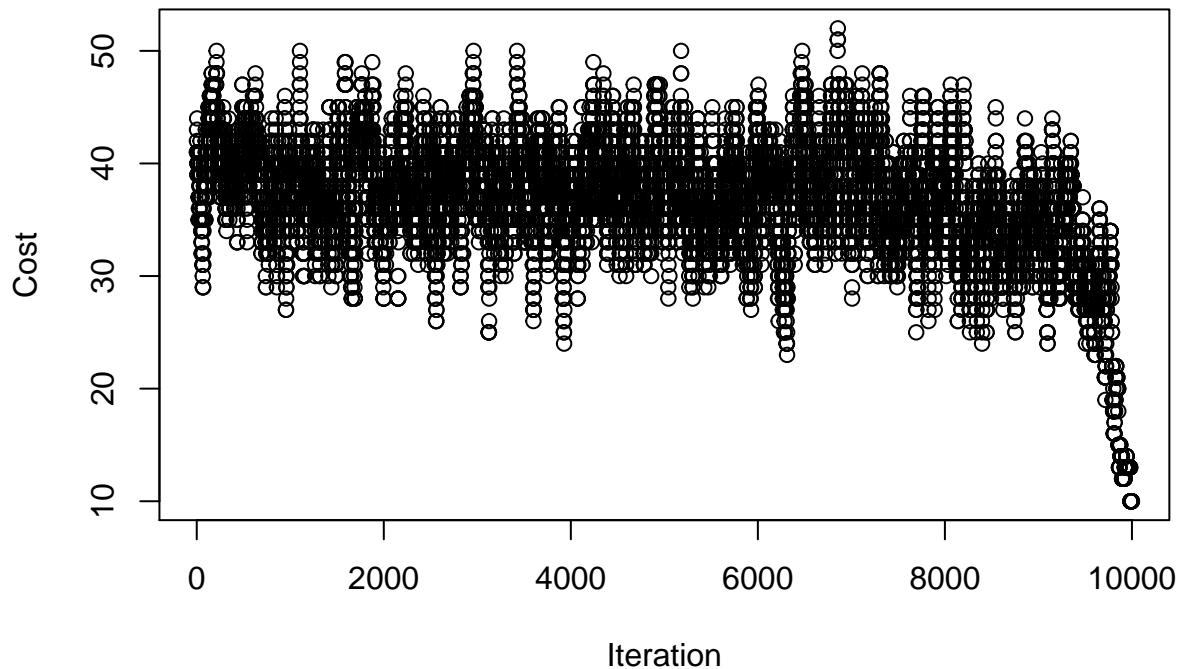
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 12 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



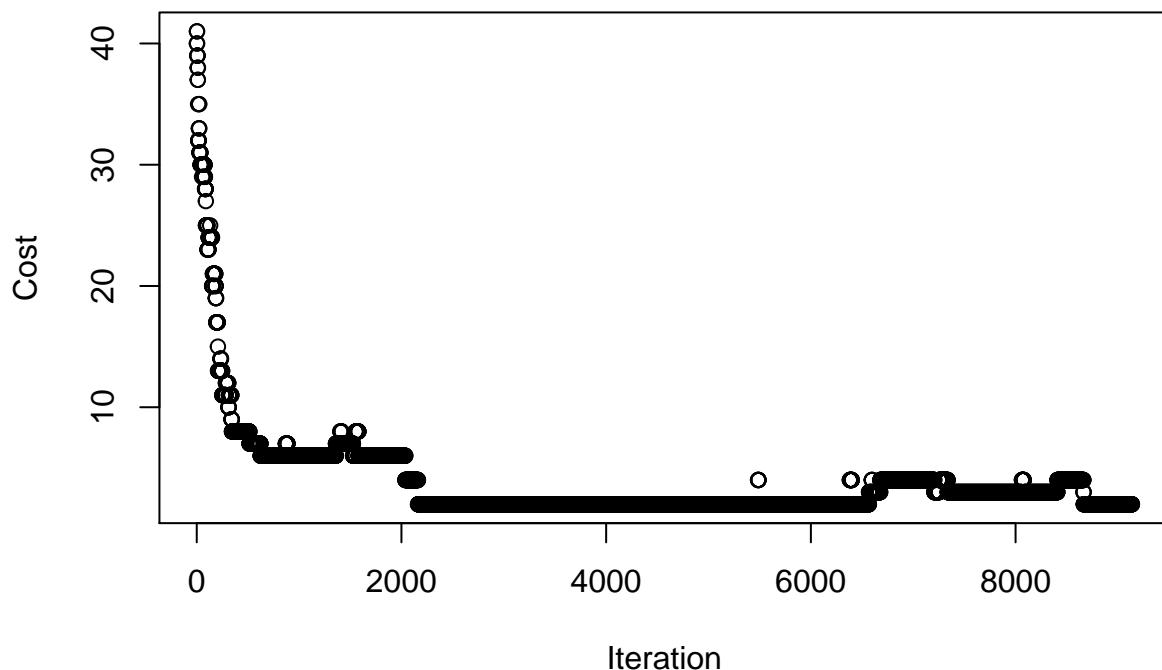
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 10 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



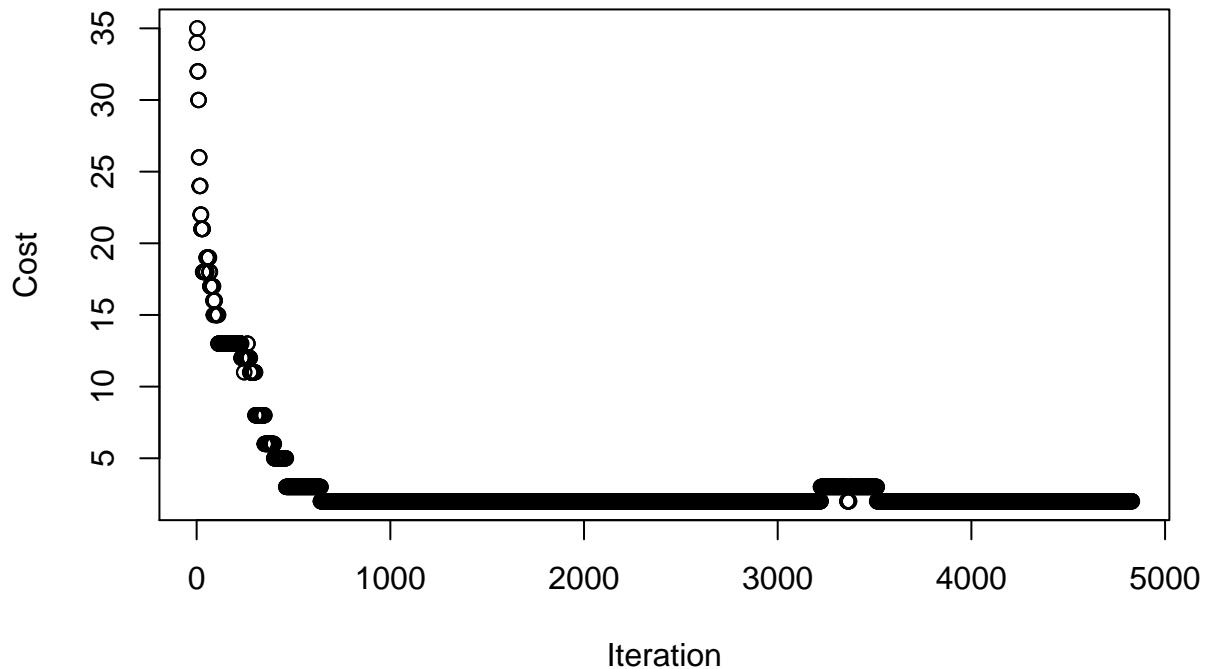
```
for (i in 1:10){  
  solve_sudoku_log_cooling(10000, 50, ex1)  
}  
  
## Ran simulated annealing with logarithmic cooling for 9137 iterations.  
## Solution found .  
## Final temperature is 0.2192935  
## Number of reheat in the run 57
```

Cost for Each Iteration



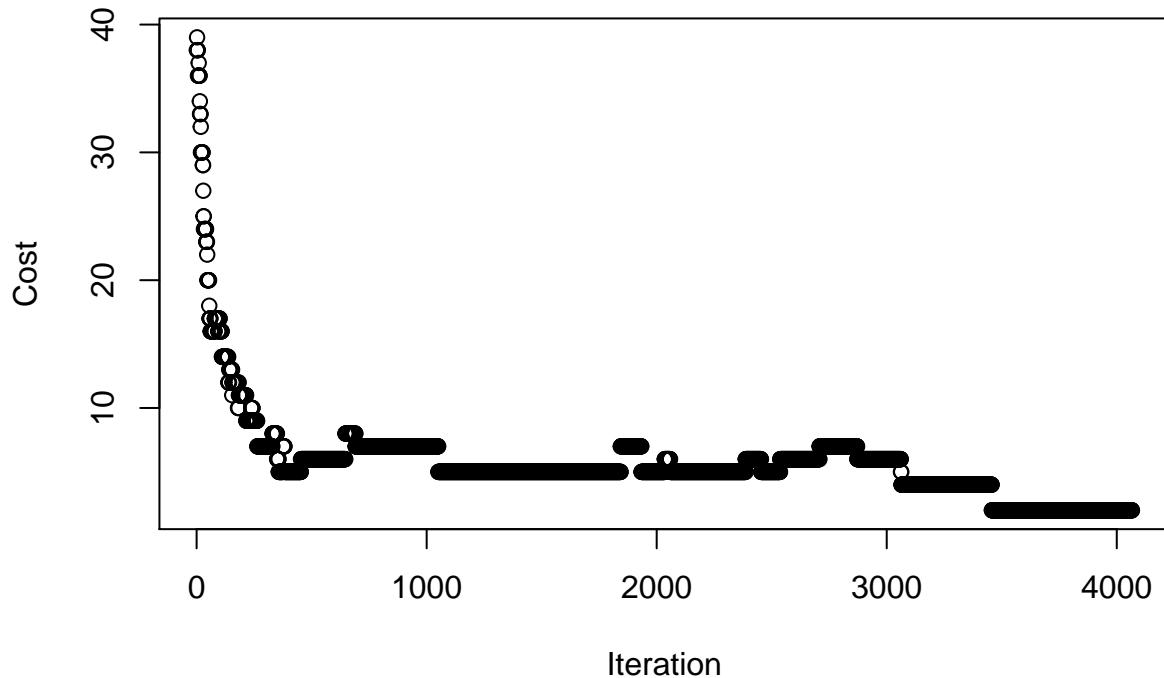
```
## Ran simulated annealing with logarithmic cooling for 4829 iterations.  
## Solution found .  
## Final temperature is 0.2357767  
## Number of reheat in the run 26
```

Cost for Each Iteration



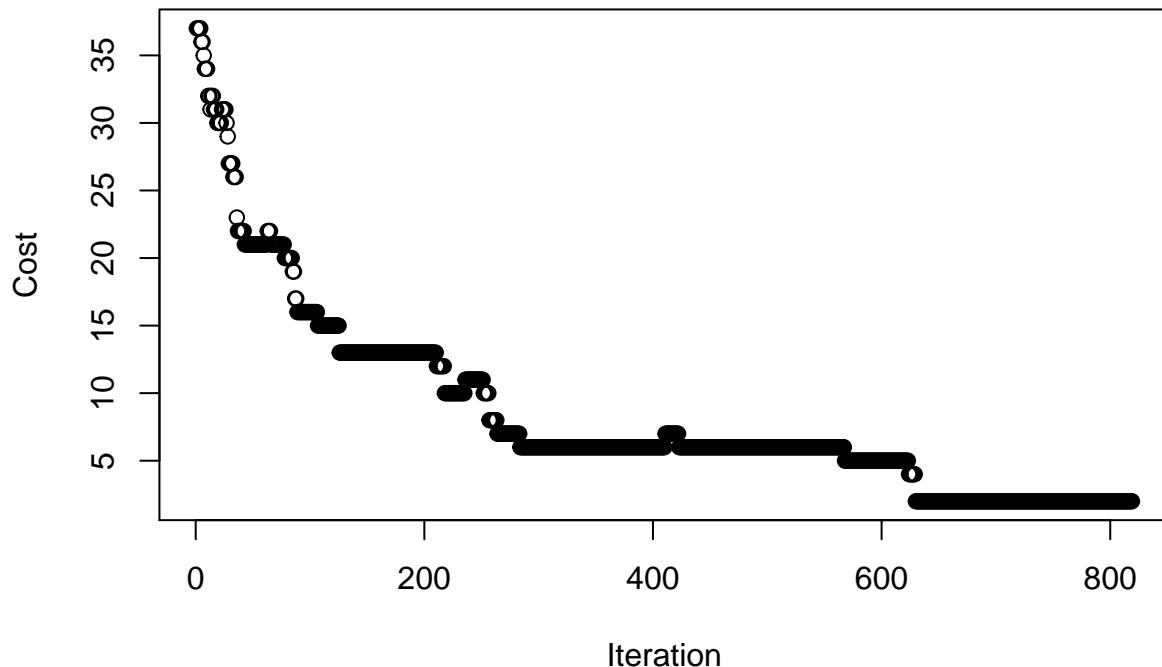
```
## Ran simulated annealing with logarithmic cooling for 4066 iterations.  
## Solution found .  
## Final temperature is 0.2406547  
## Number of reheat in the run 16
```

Cost for Each Iteration



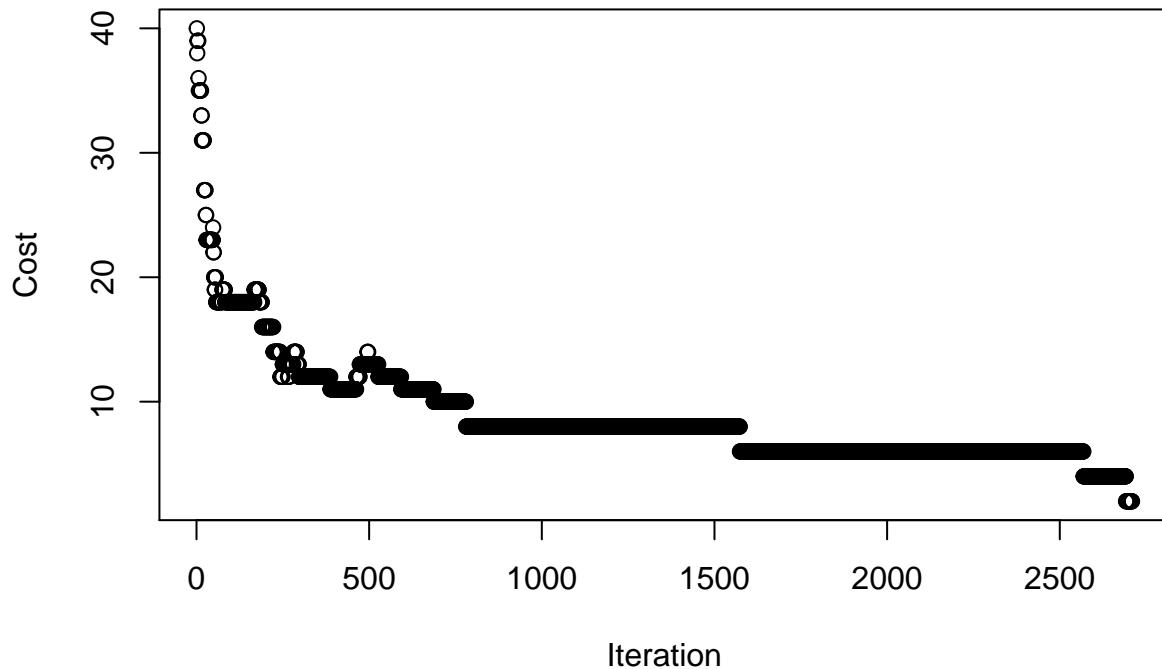
```
## Ran simulated annealing with logarithmic cooling for 819 iterations.  
## Solution found .  
## Final temperature is 0.2980935  
## Number of reheat in the run 1
```

Cost for Each Iteration



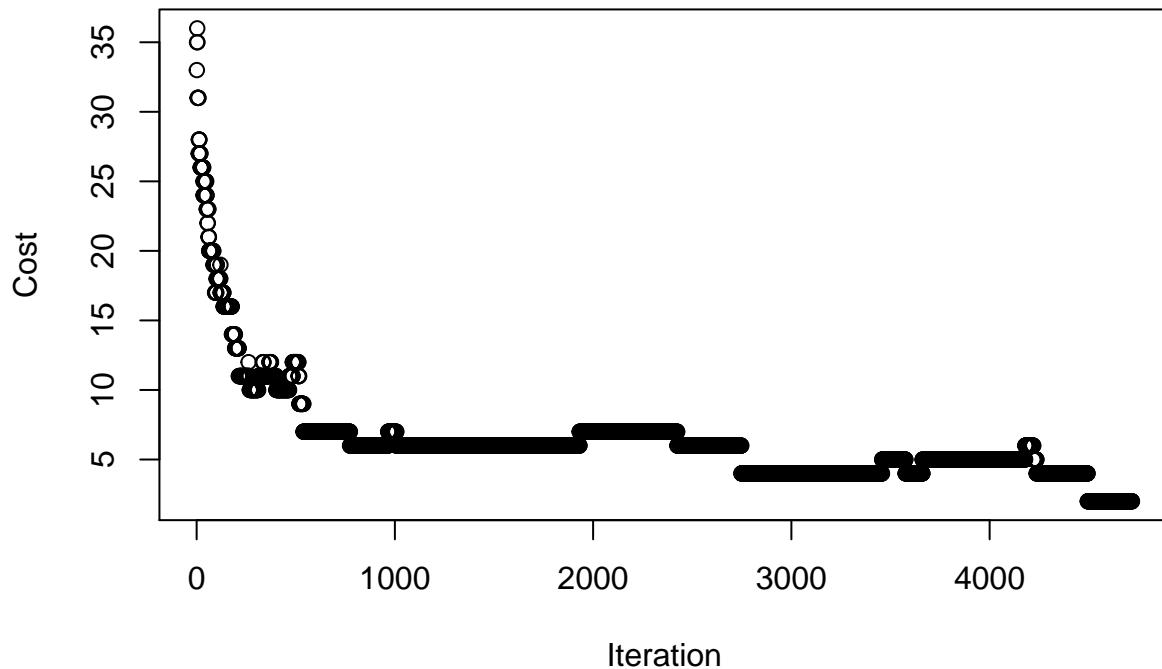
```
## Ran simulated annealing with logarithmic cooling for 2709 iterations.  
## Solution found .  
## Final temperature is 0.2530139  
## Number of reheat in the run 3
```

Cost for Each Iteration



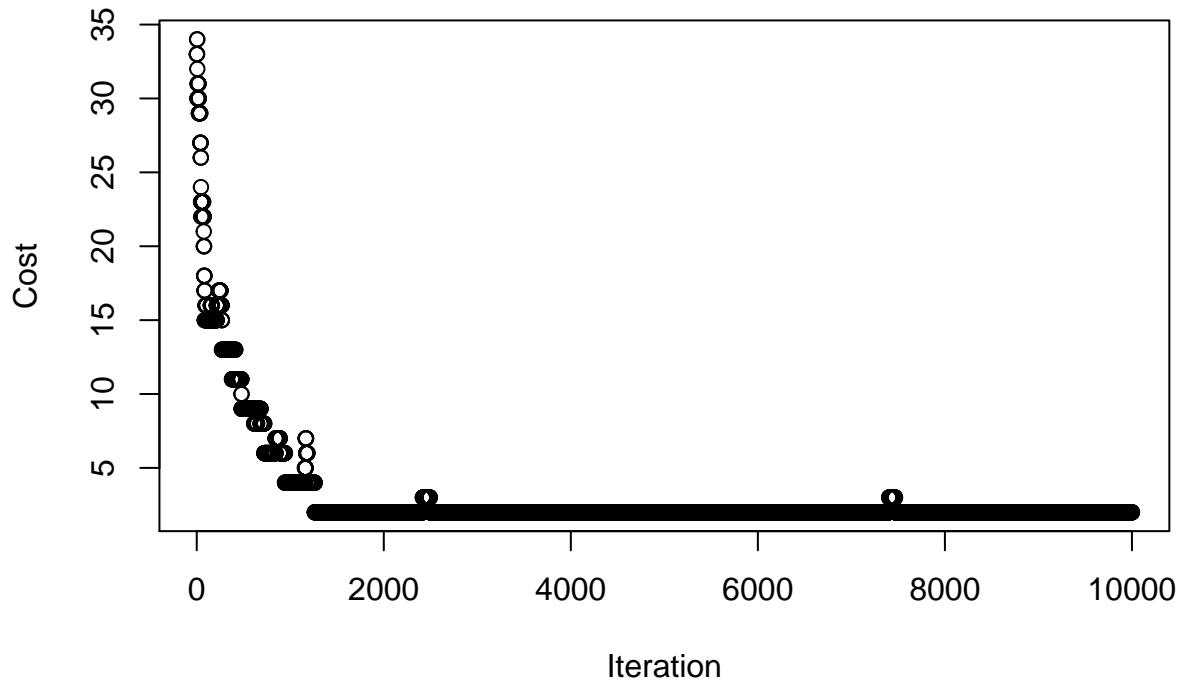
```
## Ran simulated annealing with logarithmic cooling for 4718 iterations.  
## Solution found .  
## Final temperature is 0.2364247  
## Number of reheat in the run 11
```

Cost for Each Iteration



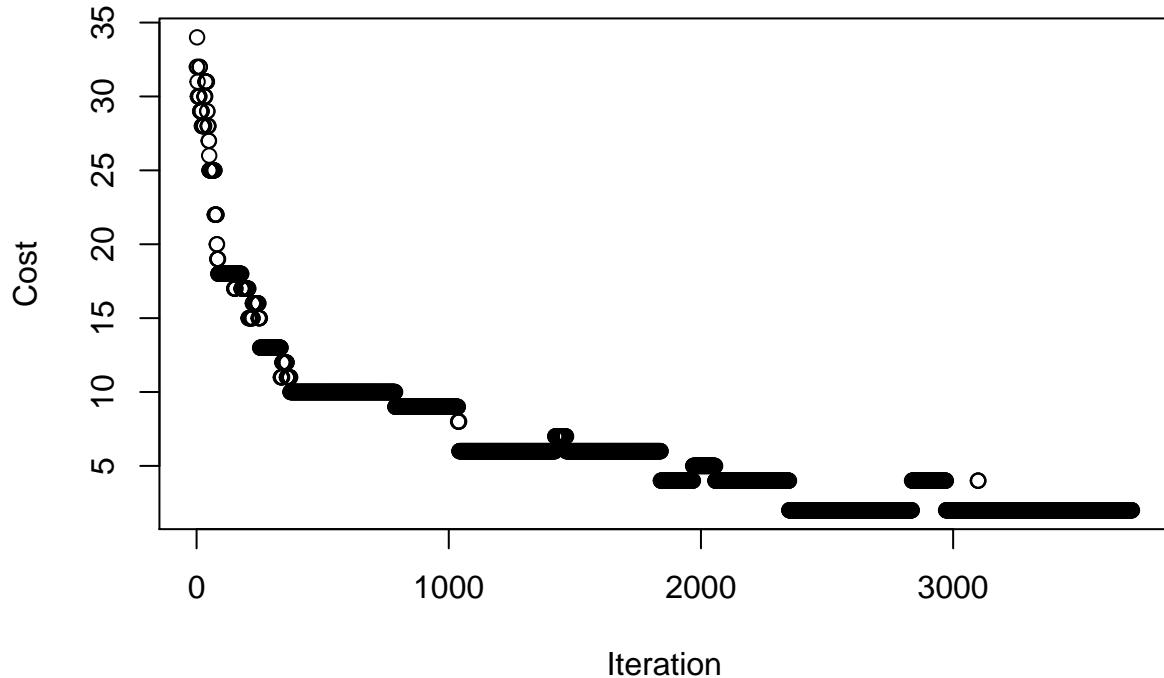
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 107
```

Cost for Each Iteration



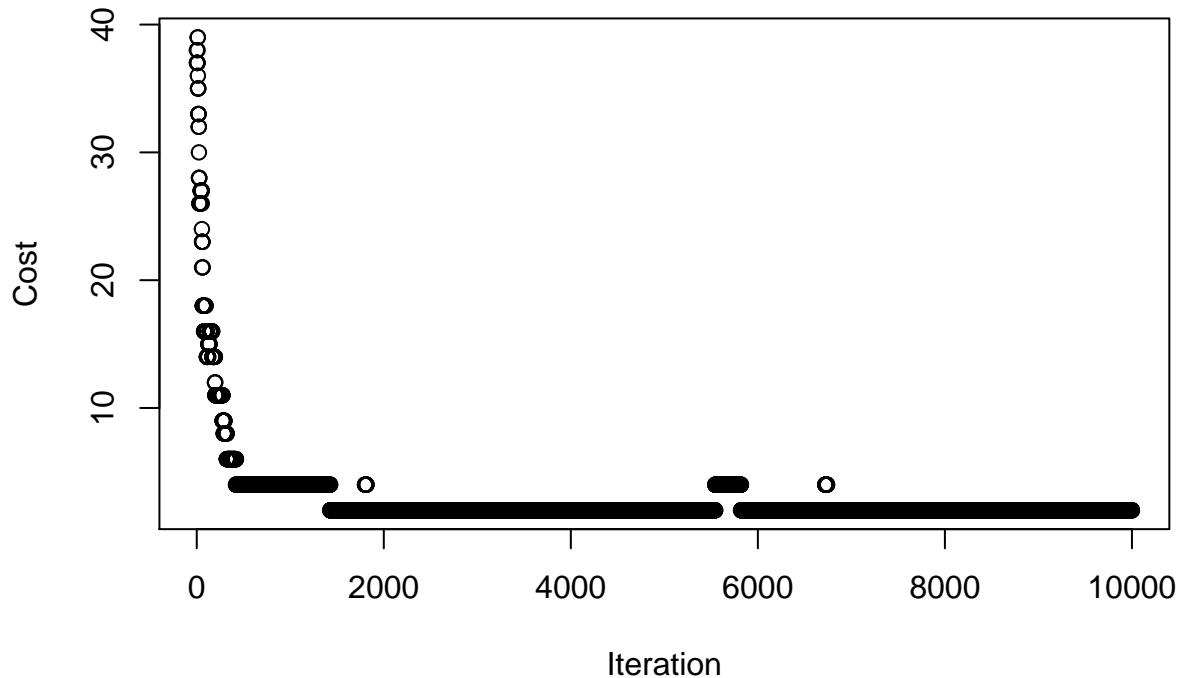
```
## Ran simulated annealing with logarithmic cooling for 3709 iterations.  
## Solution found .  
## Final temperature is 0.2433449  
## Number of reheat in the run 10
```

Cost for Each Iteration



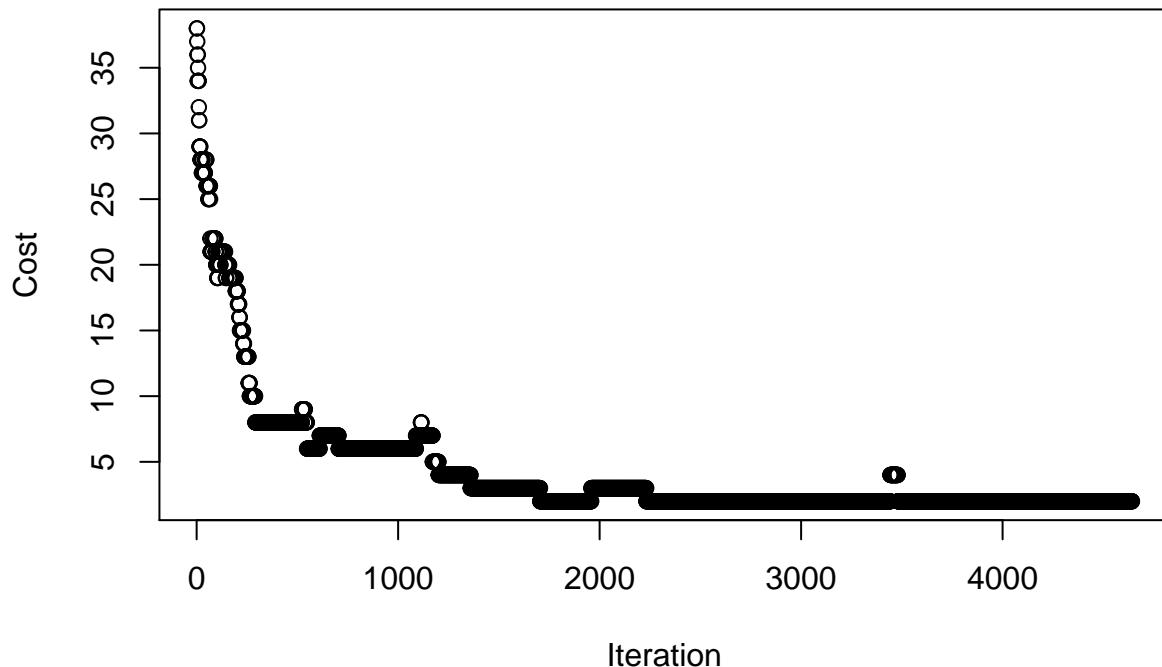
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 63
```

Cost for Each Iteration



```
## Ran simulated annealing with logarithmic cooling for 4642 iterations.  
## Solution found .  
## Final temperature is 0.2368794  
## Number of reheat in the run 19
```

Cost for Each Iteration



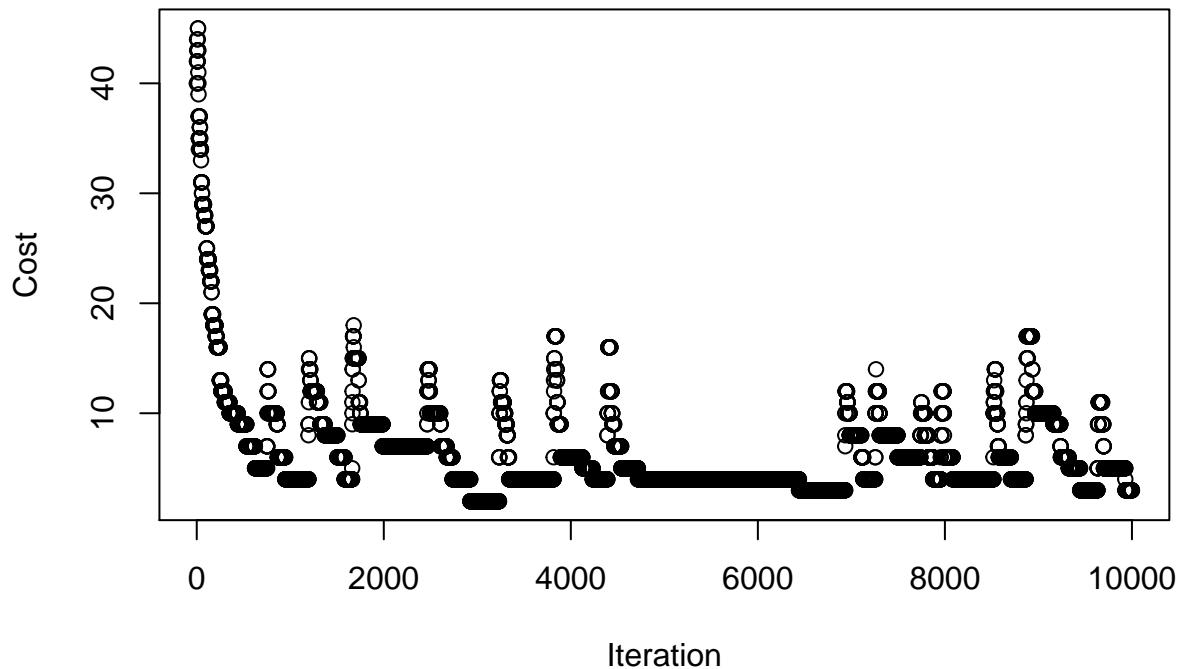
```
ex2 <- read_excel("ex2.xlsx")
ex2

## # A tibble: 9 x 9
##   col1  col2  col3  col4  col5  col6  col7  col8  col9
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     5     3    NA     9    NA    NA    NA    NA    NA
## 2     NA    NA     7     6    NA    NA    NA    NA    NA
## 3     9    NA     6    NA     7    NA    NA    NA     2     4
## 4     NA    NA     4    NA     1     5    NA    NA    NA
## 5     7    NA    NA    NA     8    NA    NA    NA     3
## 6     NA    NA    NA     7     9    NA     6    NA    NA
## 7     8     6    NA    NA     2    NA     3    NA     5
## 8     NA    NA    NA    NA     NA     8     4    NA    NA
## 9     NA    NA    NA    NA     NA     9    NA     7     6

for (i in 1:10){
  solve_sudoku_exp_cooling(10000, 50, ex2, 0.9)
}

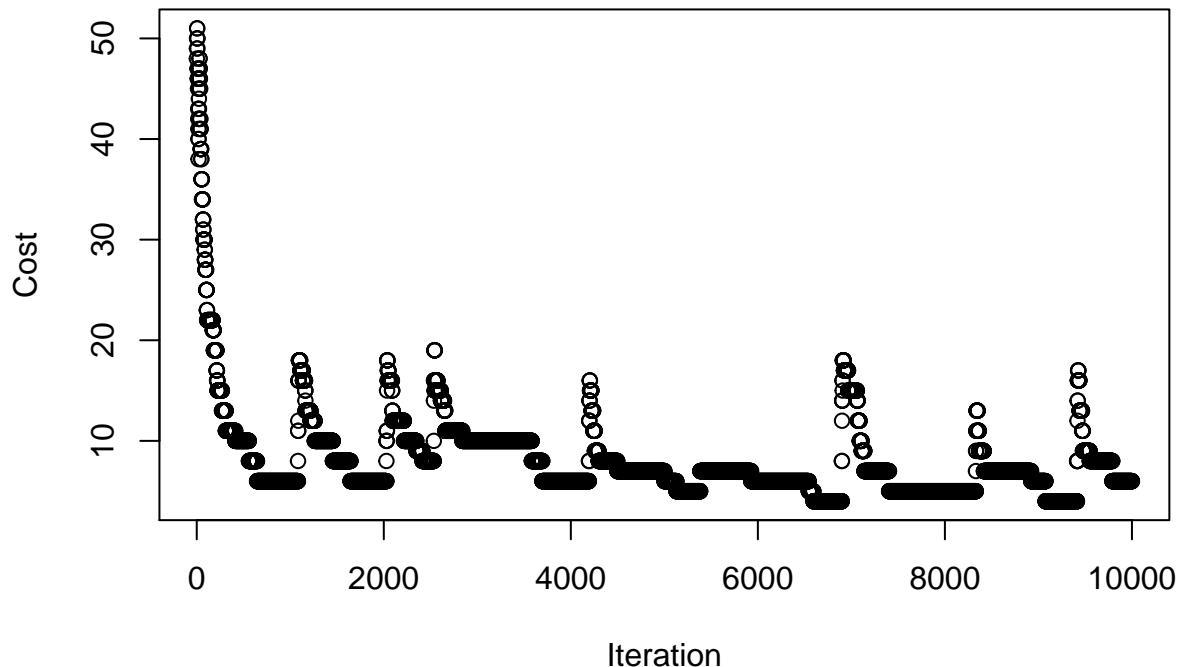
## Ran simulated annealing with geometric cooling for 10000 iterations.
## Solution not found .
## Minimum cost reached 2 .
## Final temperature is 5.870816e-17
## Number of reheat in the run 14
```

Cost for Each Iteration



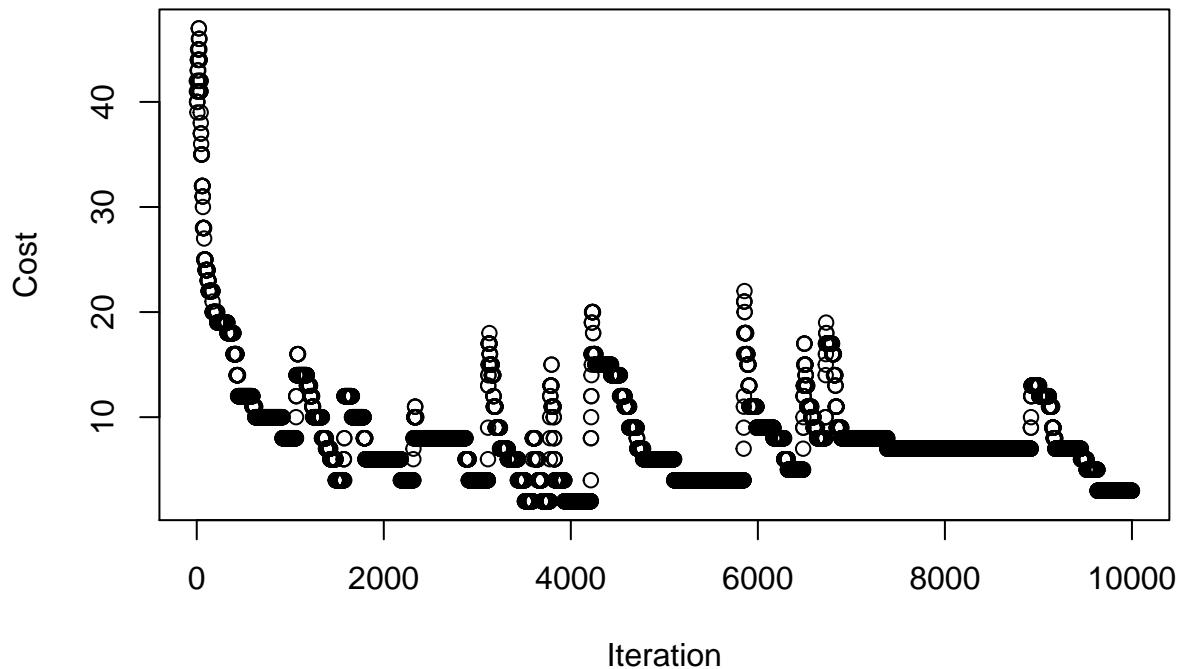
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 5.595103e-27  
## Number of reheat in the run 8
```

Cost for Each Iteration



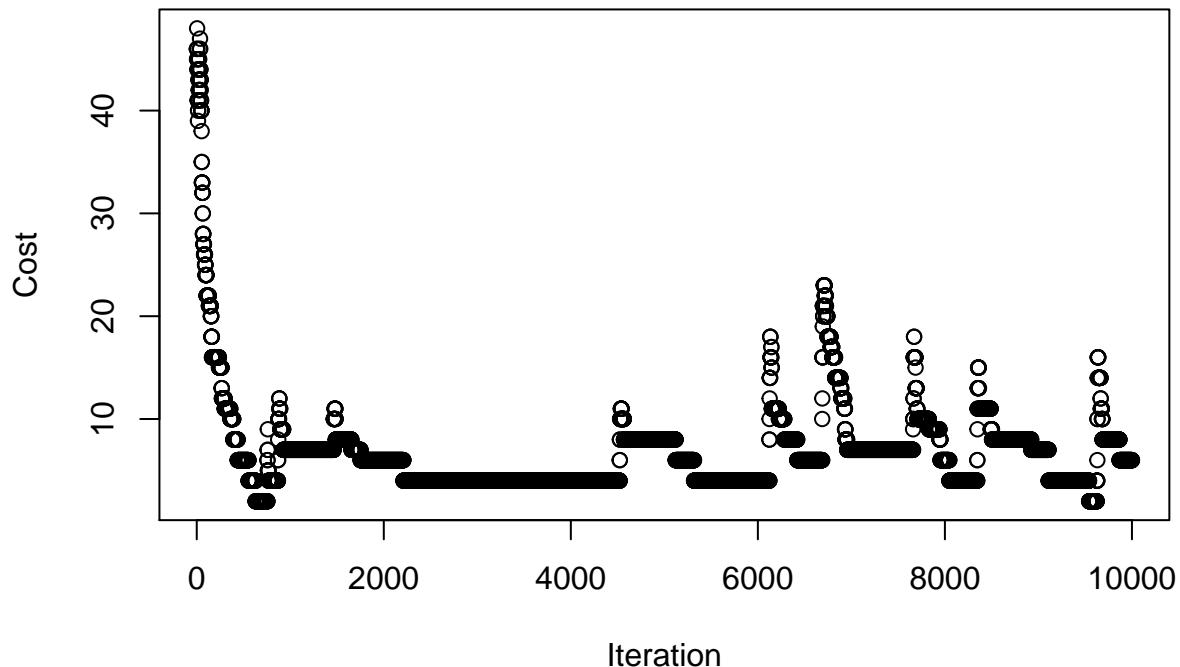
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 1.252709e-49  
## Number of reheat in the run 11
```

Cost for Each Iteration



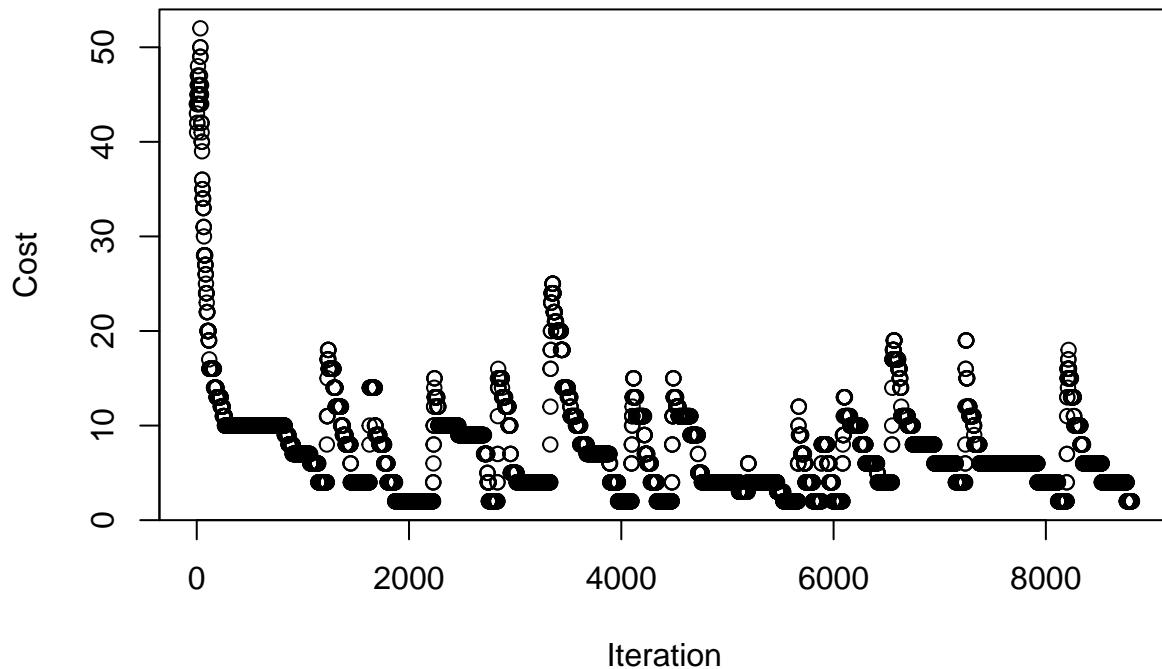
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 2.274475e-17  
## Number of reheat in the run 9
```

Cost for Each Iteration



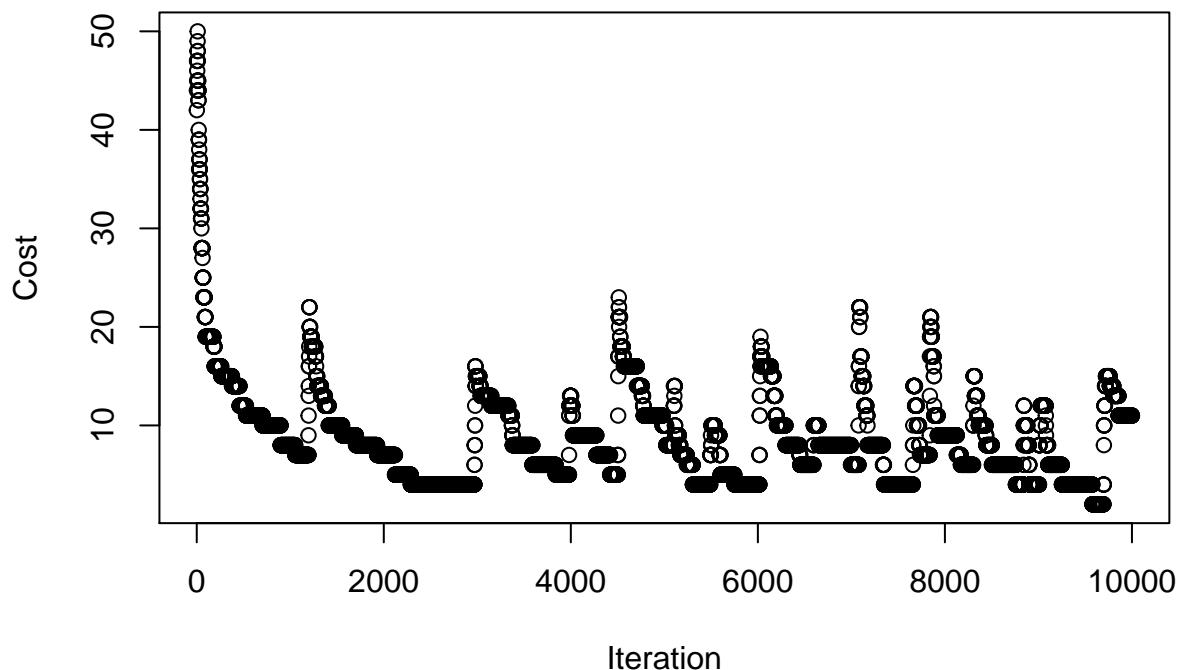
```
## Ran simulated annealing with geometric cooling for 8811 iterations.  
## Solution found .  
## Final temperature is 2.928184e-28  
## Number of reheat in the run 14
```

Cost for Each Iteration



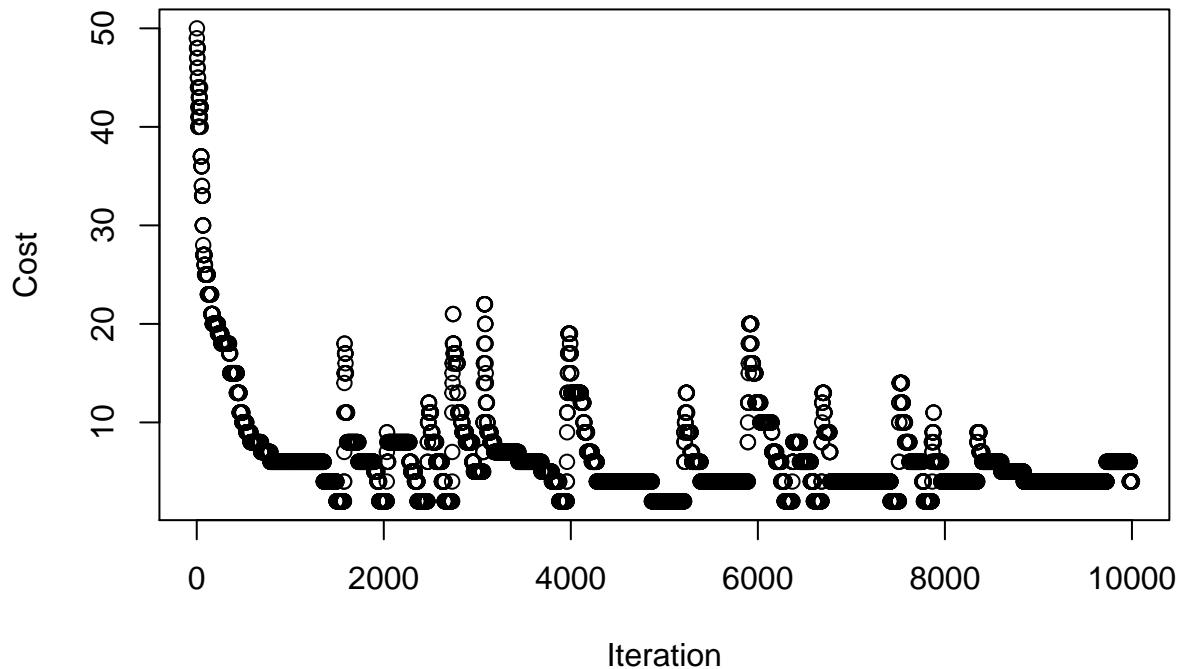
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 4.481469e-14  
## Number of reheat in the run 15
```

Cost for Each Iteration



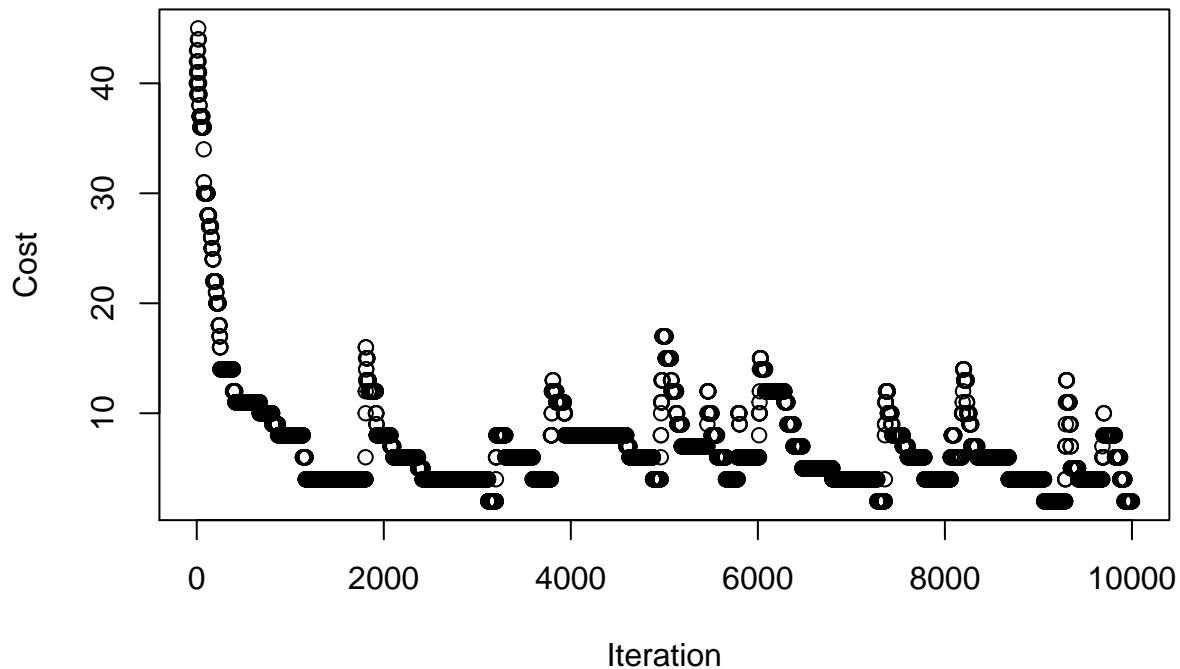
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 1.305149e-12  
## Number of reheat in the run 14
```

Cost for Each Iteration



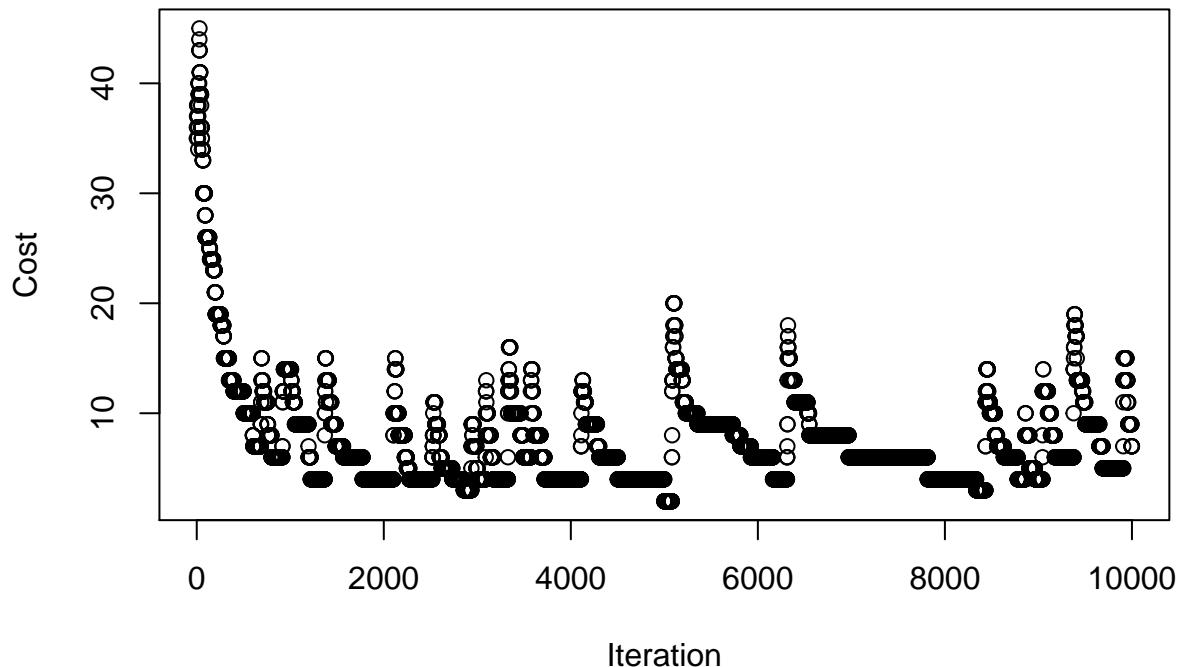
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 1.265699e-14  
## Number of reheat in the run 12
```

Cost for Each Iteration



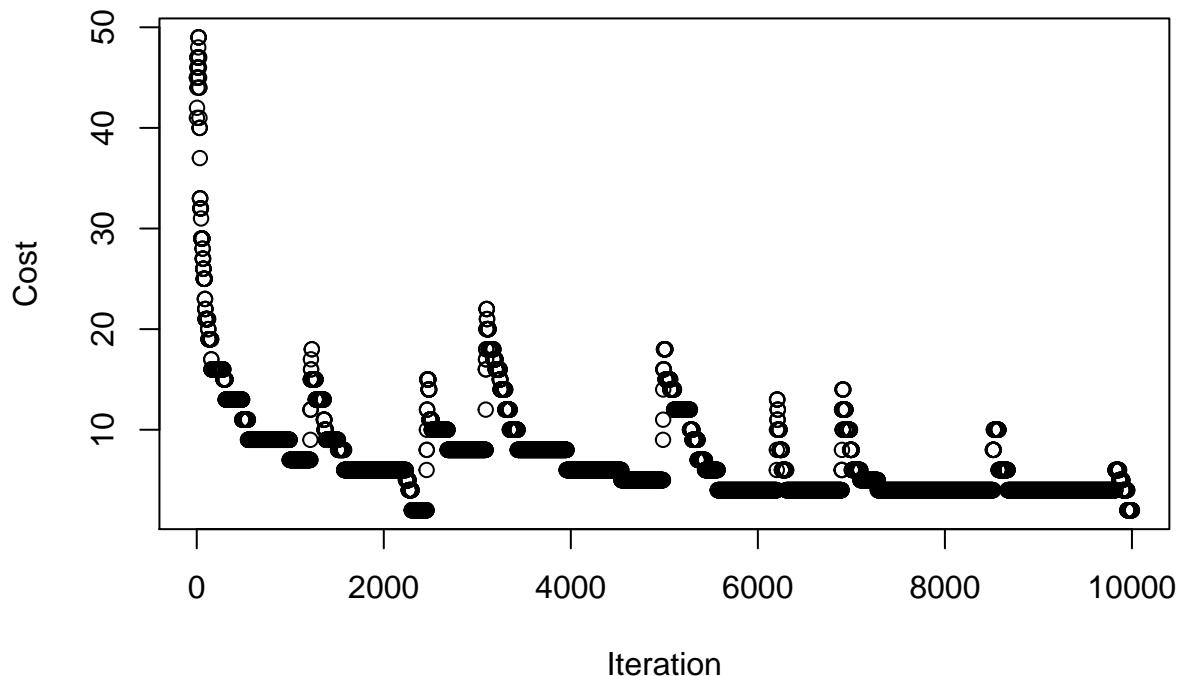
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.0001821769  
## Number of reheat in the run 17
```

Cost for Each Iteration



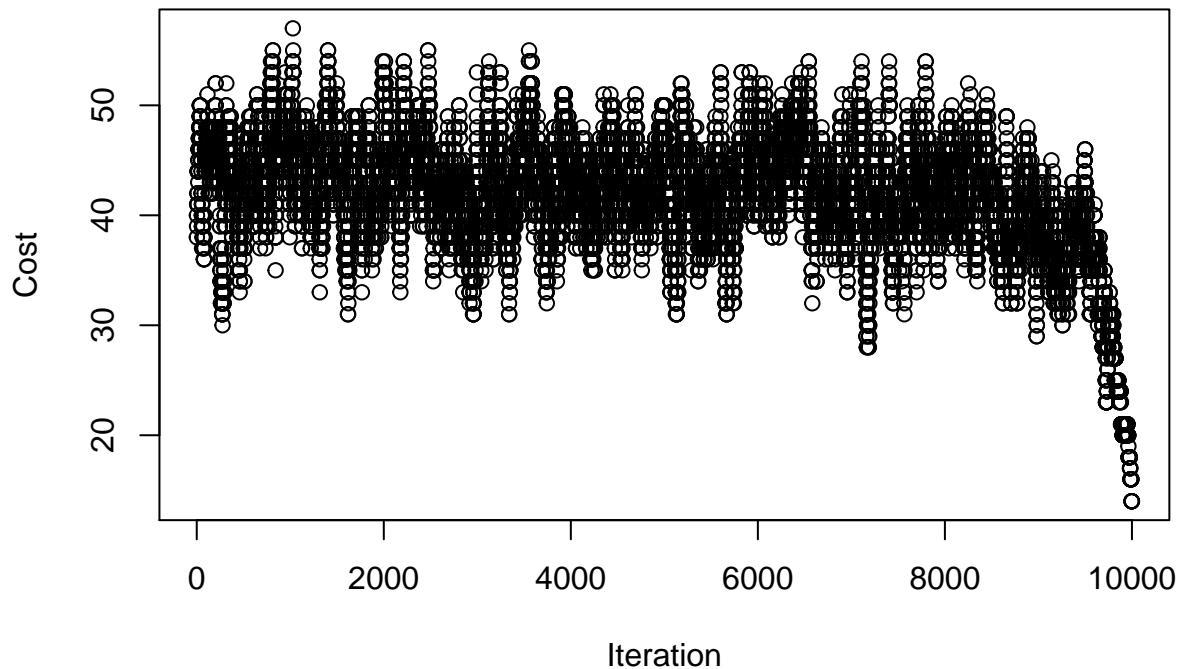
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 1.248998e-08  
## Number of reheat in the run 8
```

Cost for Each Iteration



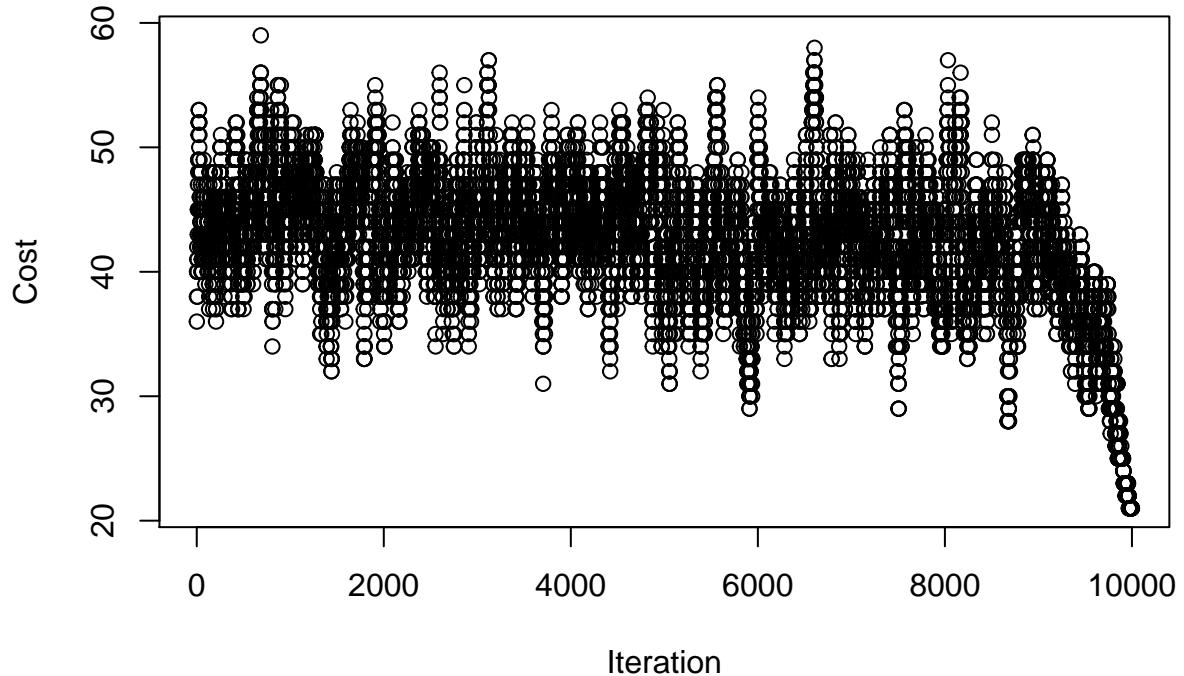
```
for (i in 1:10){  
  solve_sudoku_linear_cooling(10000, 50, ex2)  
}  
  
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 14 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



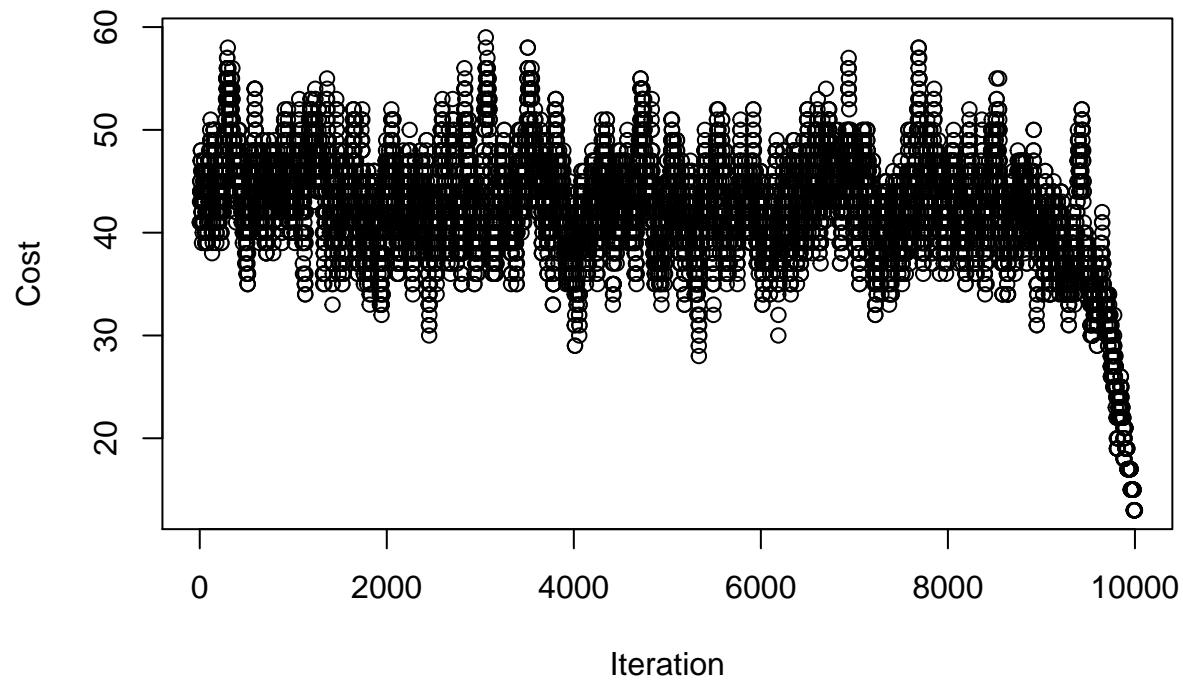
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 21 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



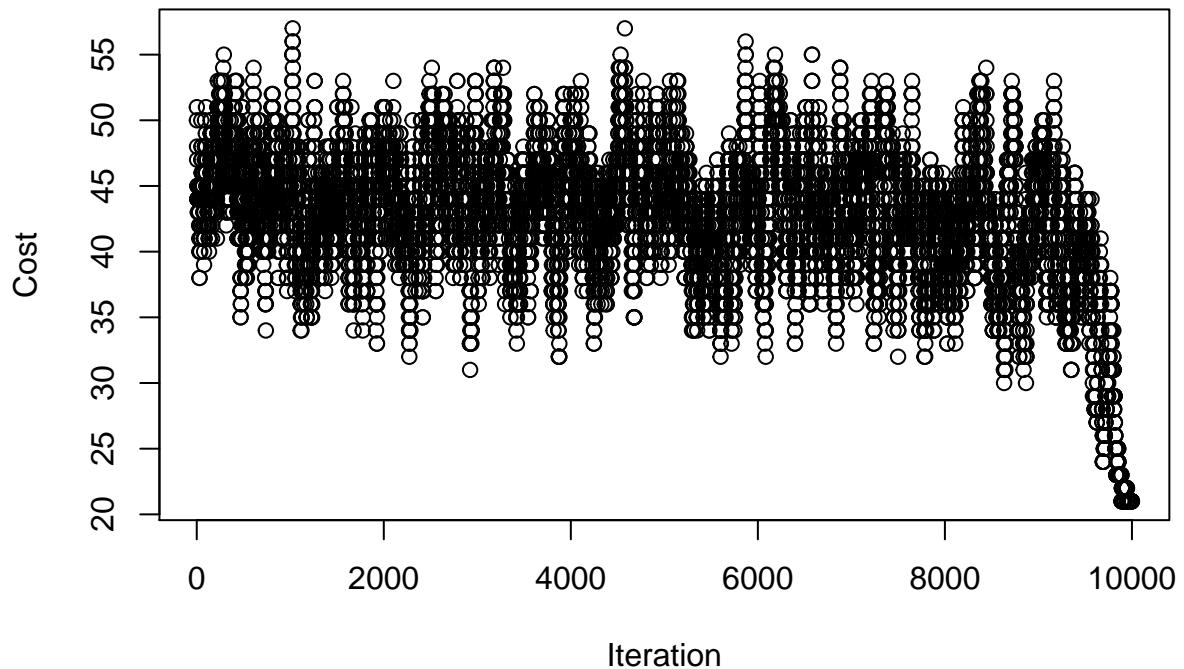
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 13 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



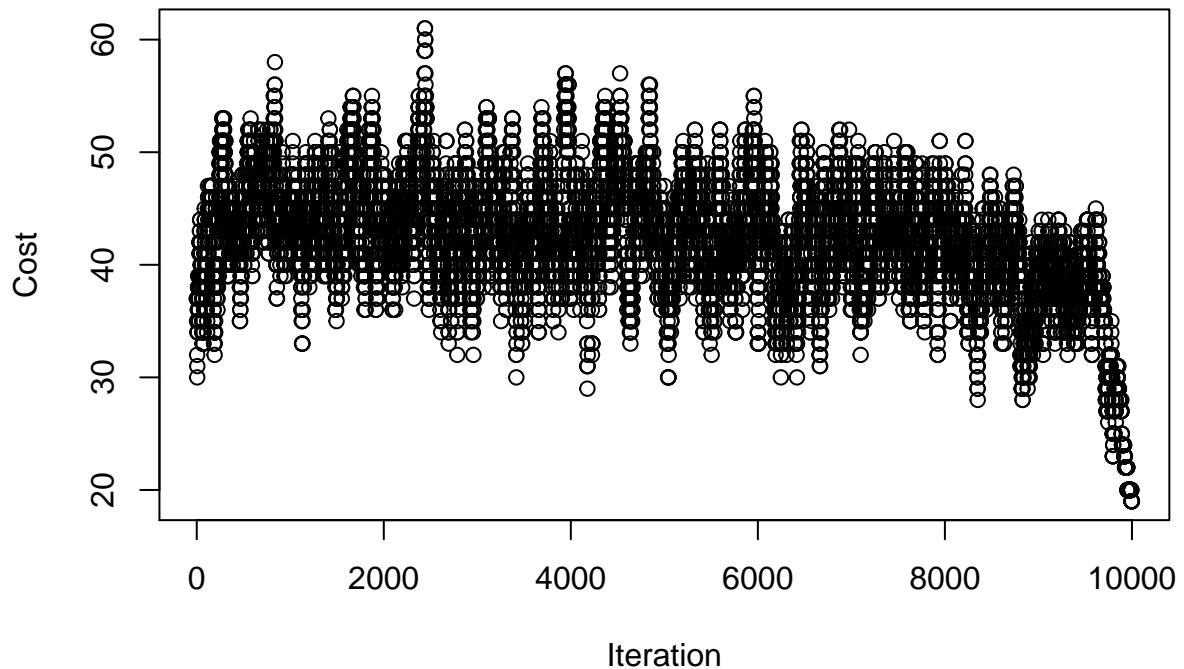
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 21 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



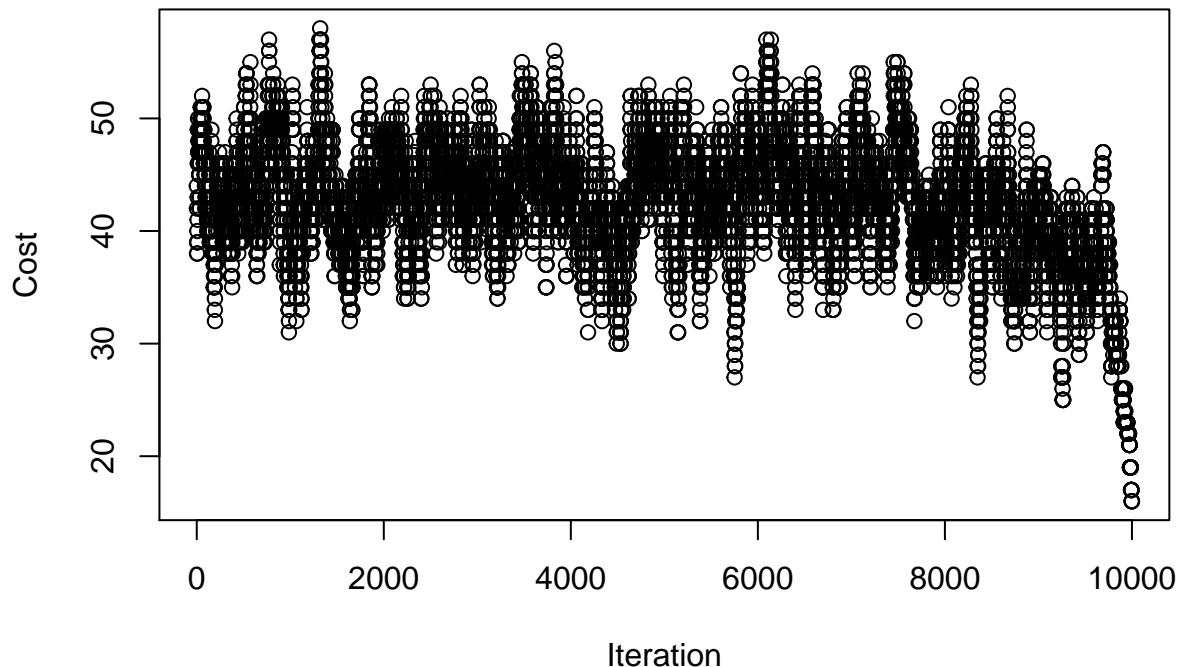
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 19 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



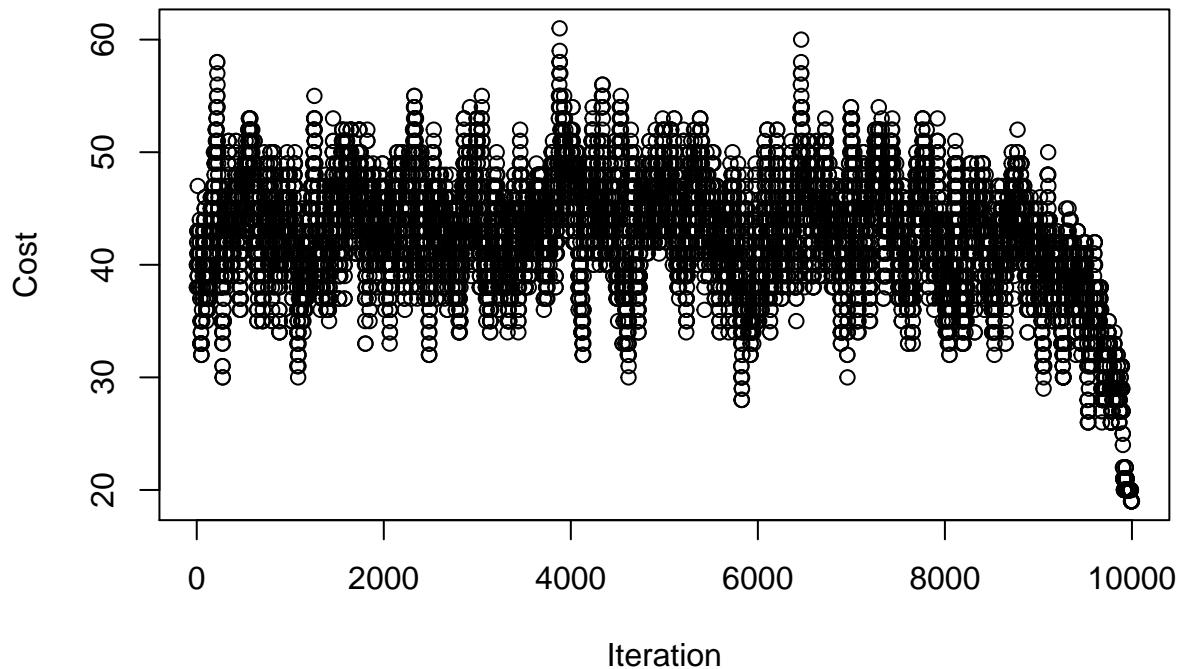
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 16 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



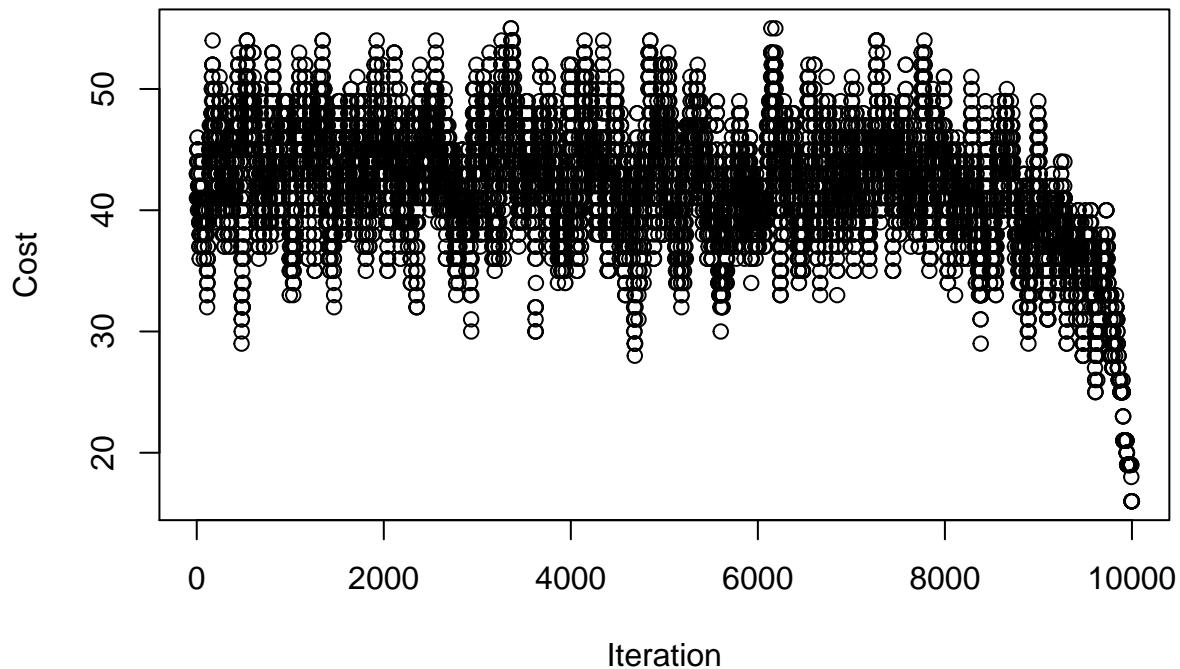
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 19 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



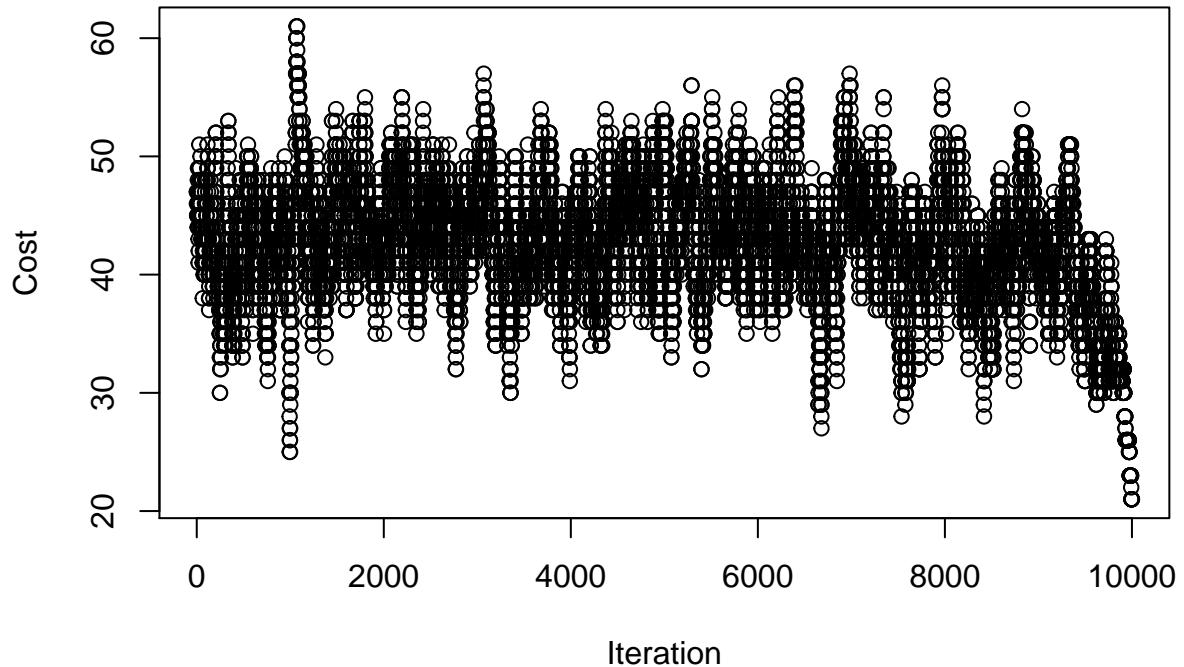
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 16 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



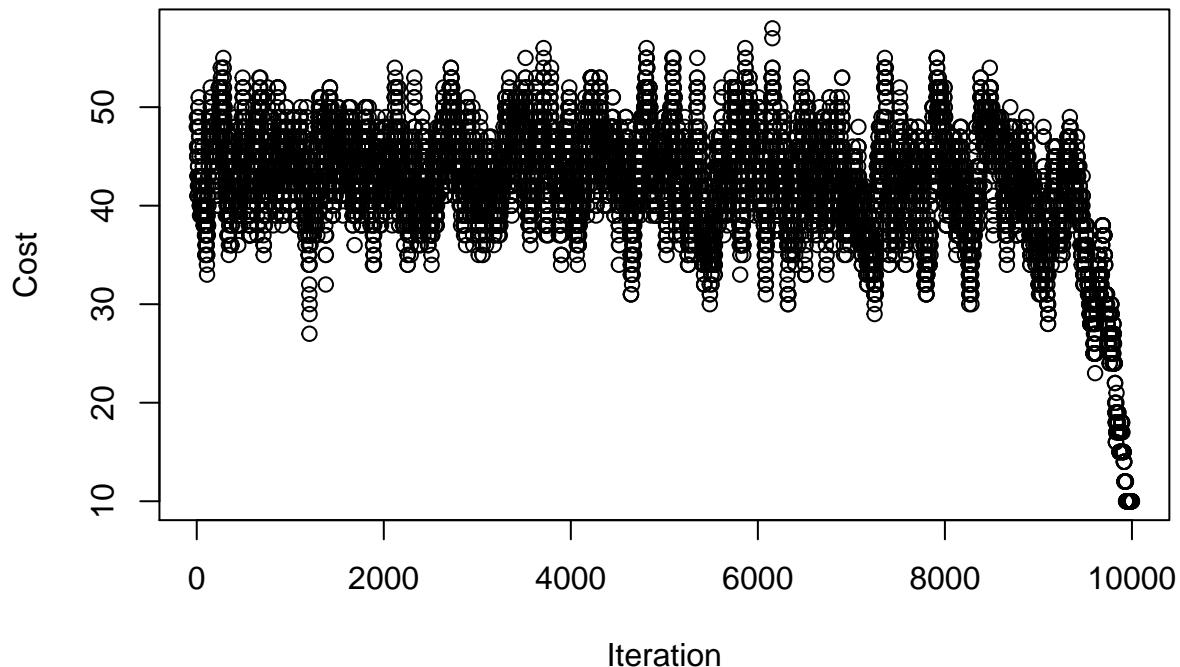
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 21 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



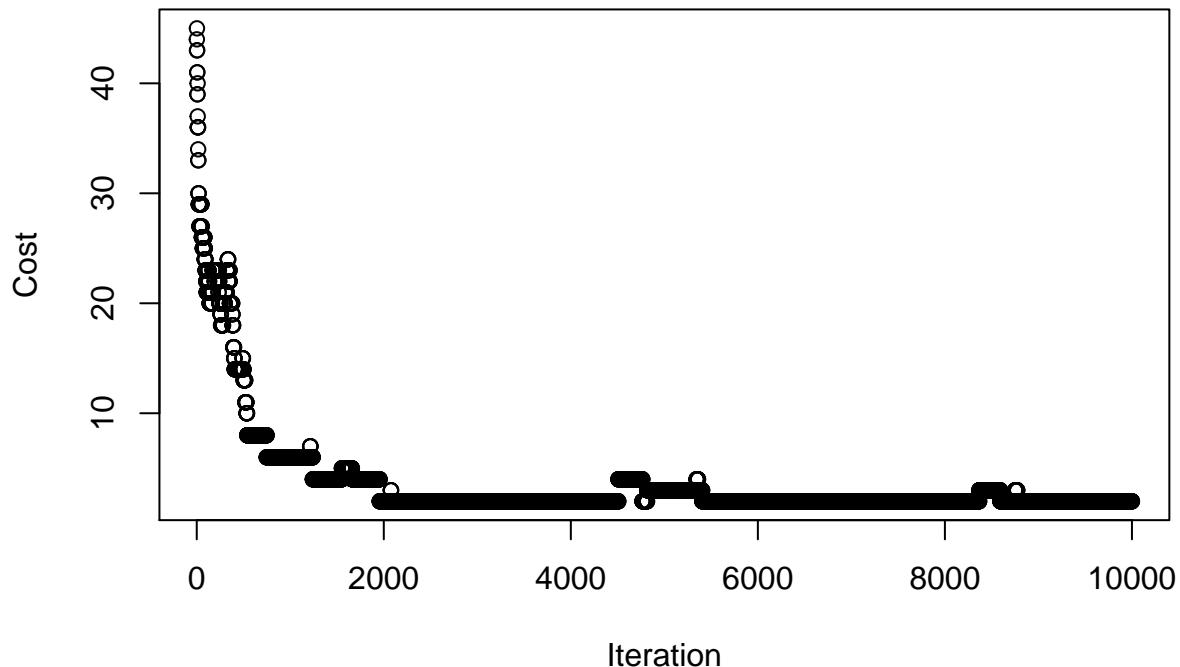
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 10 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



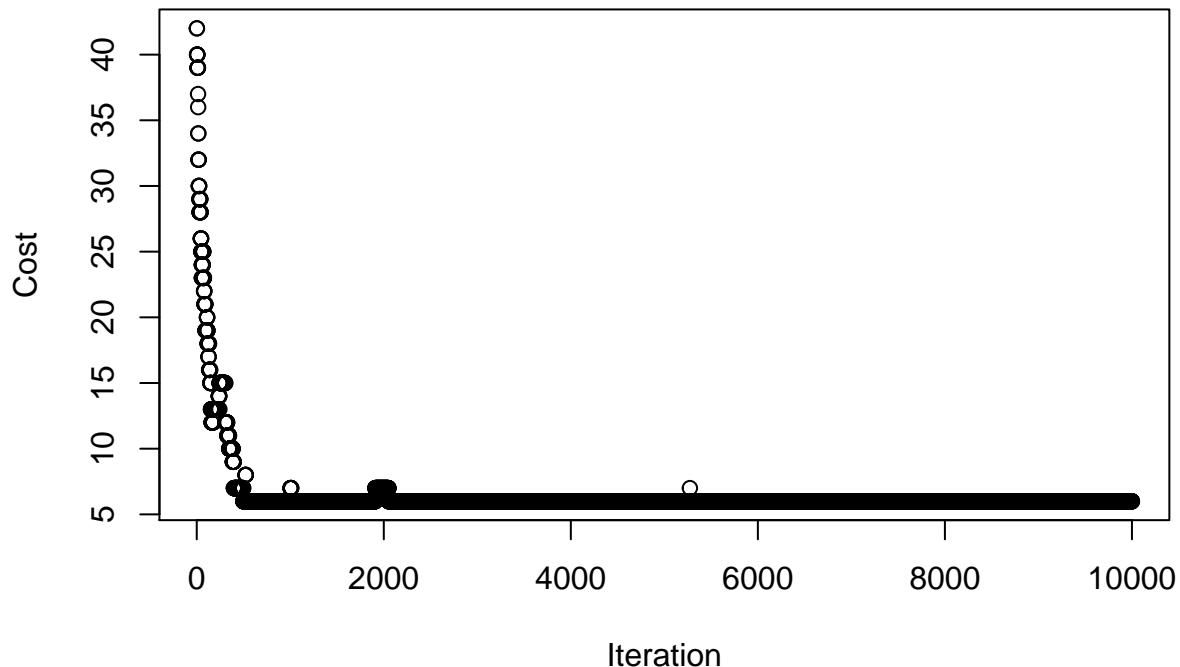
```
for (i in 1:10){  
  solve_sudoku_log_cooling(10000, 50, ex2)  
}  
  
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 33
```

Cost for Each Iteration



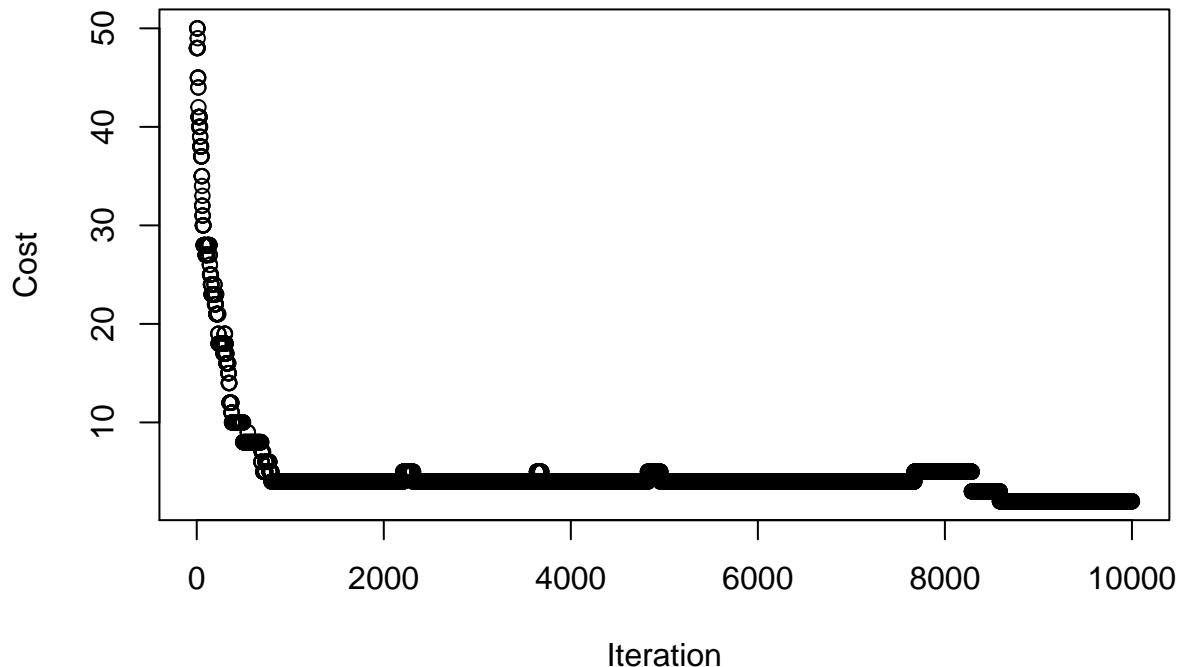
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 6 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 19
```

Cost for Each Iteration



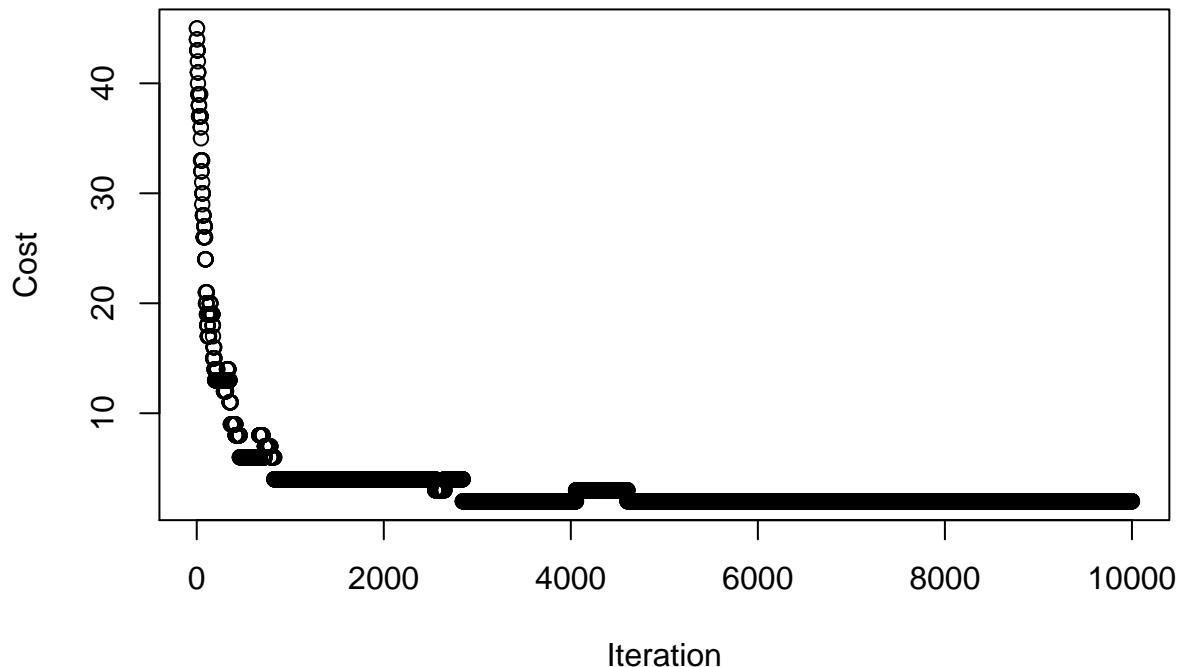
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 40
```

Cost for Each Iteration



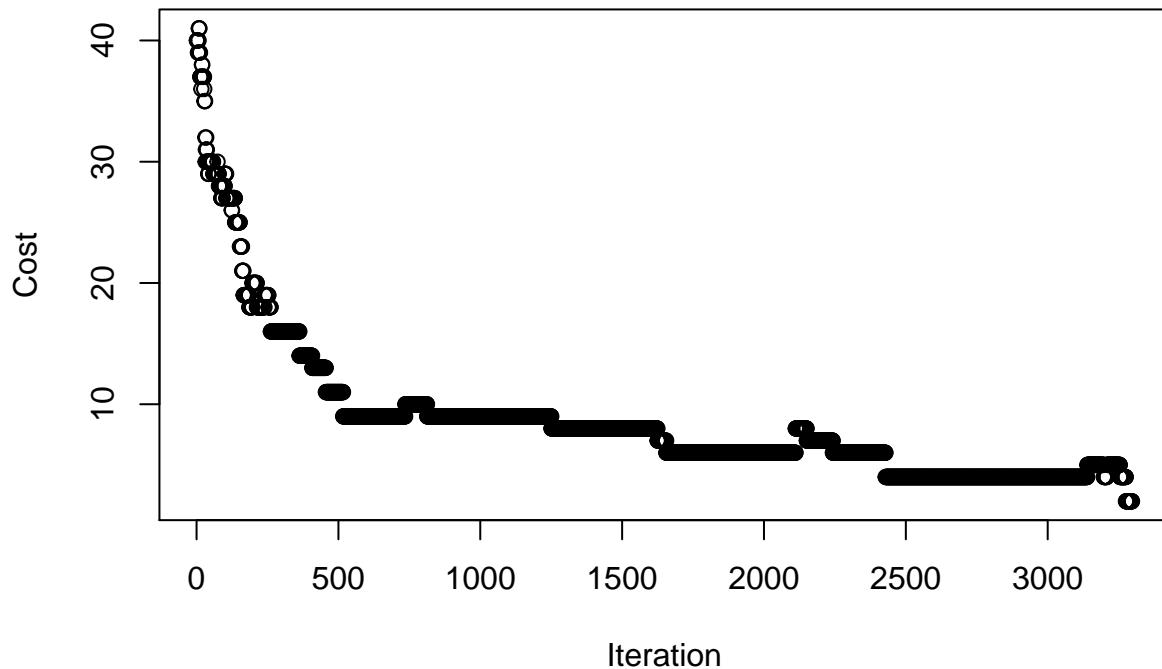
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 34
```

Cost for Each Iteration



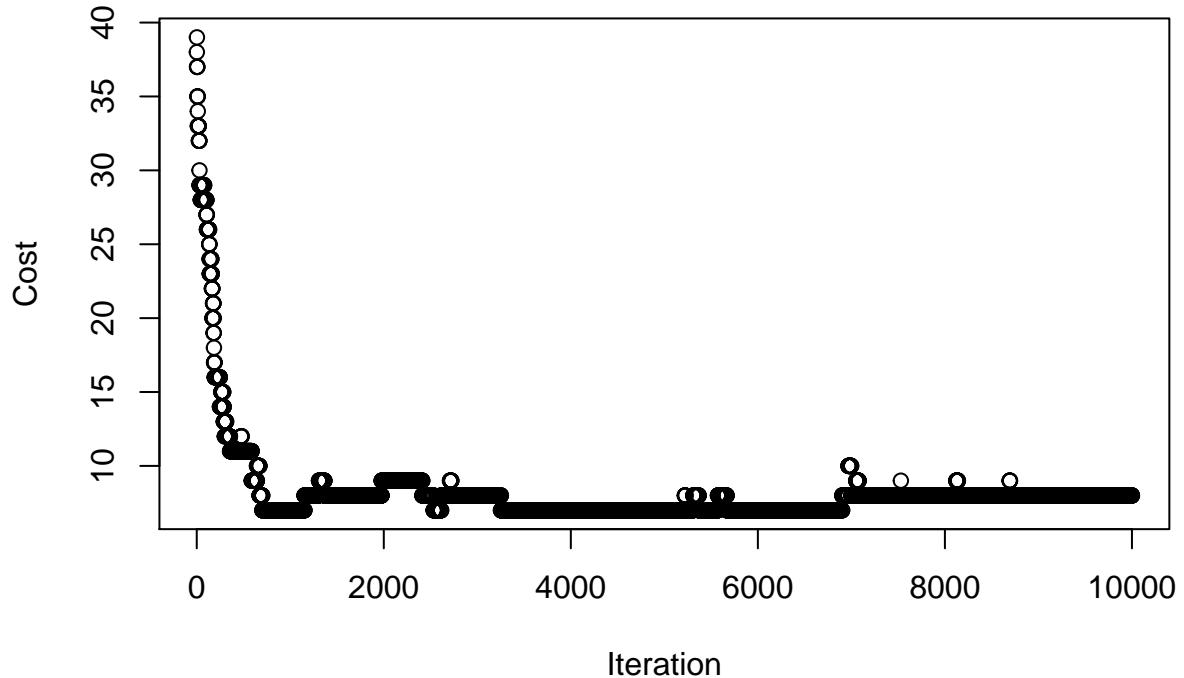
```
## Ran simulated annealing with logarithmic cooling for 3297 iterations.  
## Solution found .  
## Final temperature is 0.2468809  
## Number of reheat in the run 5
```

Cost for Each Iteration



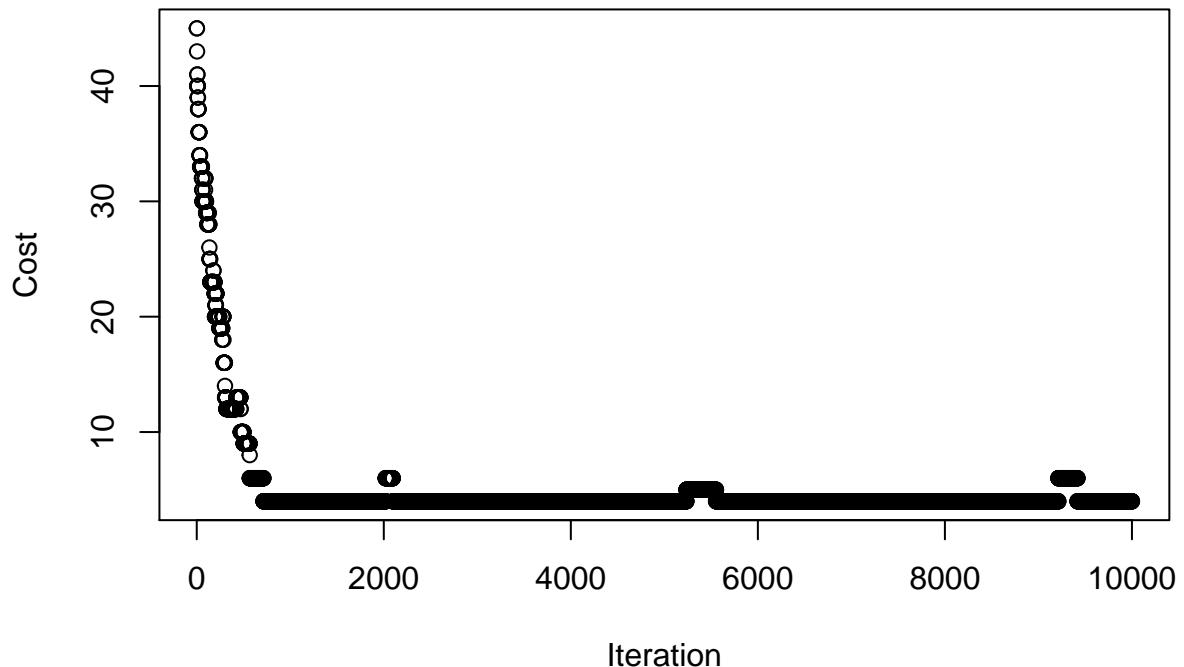
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 7 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 23
```

Cost for Each Iteration



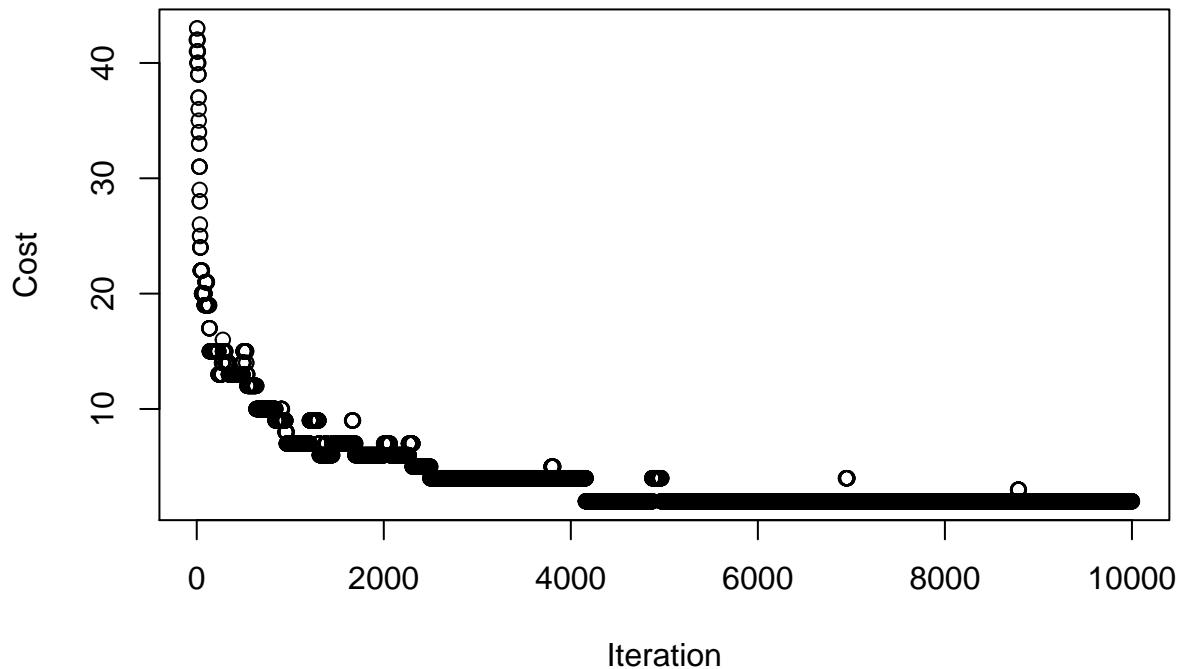
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 29
```

Cost for Each Iteration



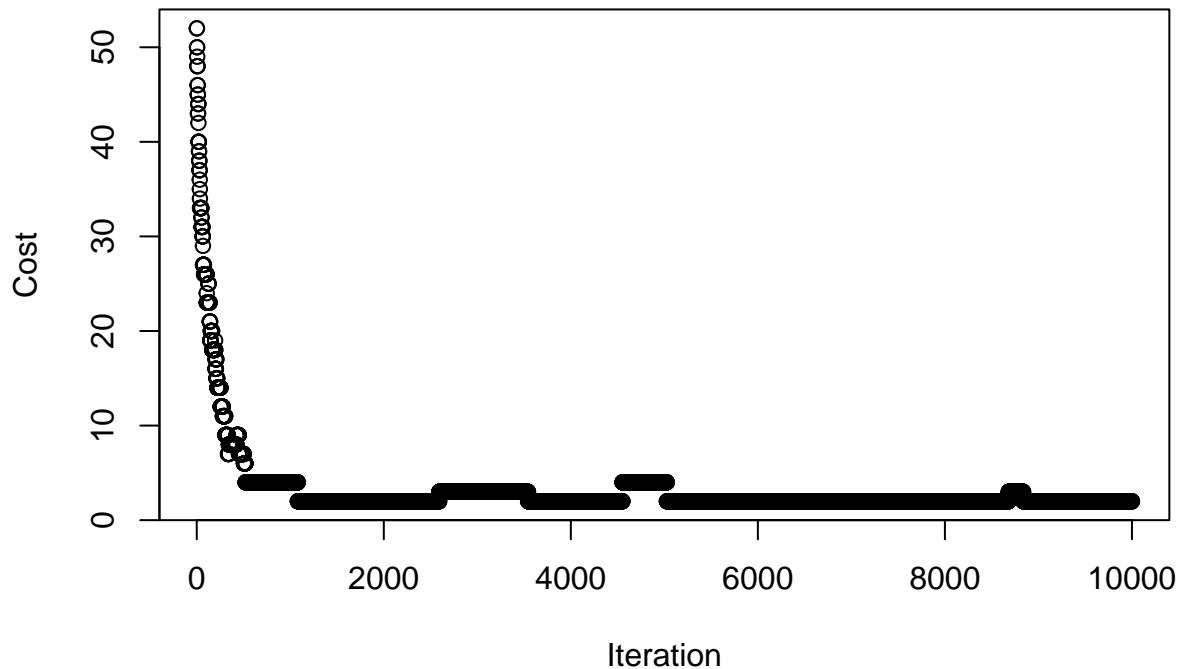
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 44
```

Cost for Each Iteration



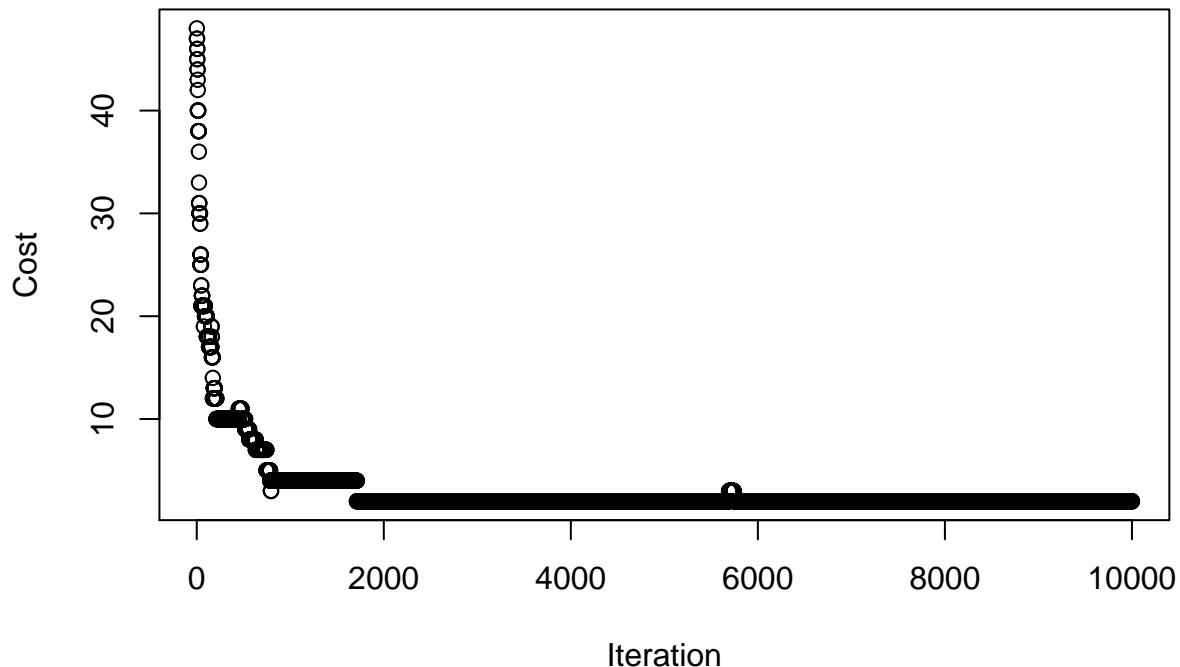
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 91
```

Cost for Each Iteration



```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 44
```

Cost for Each Iteration



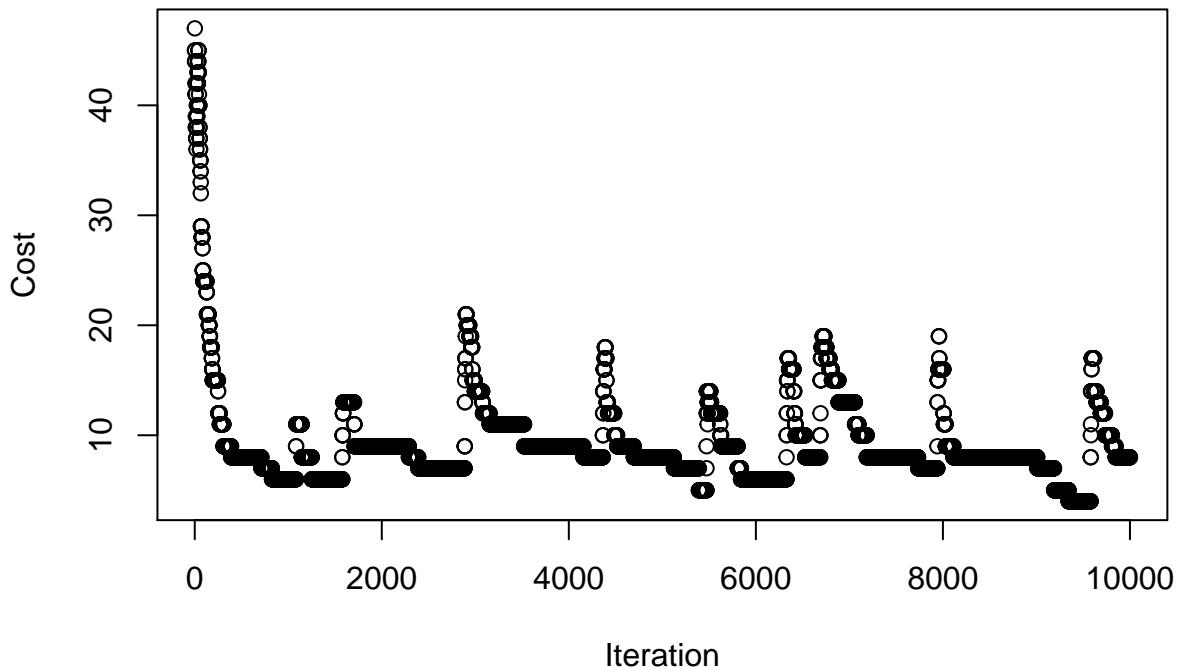
```
ex3 <- read_excel("ex3.xlsx")
ex3

## # A tibble: 9 x 9
##   col1  col2  col3  col4  col5  col6  col7  col8  col9
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     NA     4     NA     NA     NA     6     NA     NA     NA
## 2     1     NA     NA     NA     7     9     NA     NA     NA
## 3     NA     NA     6     5     NA     NA     NA     NA     4
## 4     6     8     7     9     3     NA     NA     NA     NA
## 5     NA     5     NA     NA     NA     NA     NA     9     NA
## 6     NA     NA     NA     NA     5     8     6     7     3
## 7     2     NA     NA     NA     NA     4     5     NA     NA
## 8     NA     NA     NA     3     2     NA     NA     NA     8
## 9     NA     NA     NA     7     NA     NA     NA     3     NA

for (i in 1:10){
  solve_sudoku_exp_cooling(10000, 50, ex3, 0.9)
}

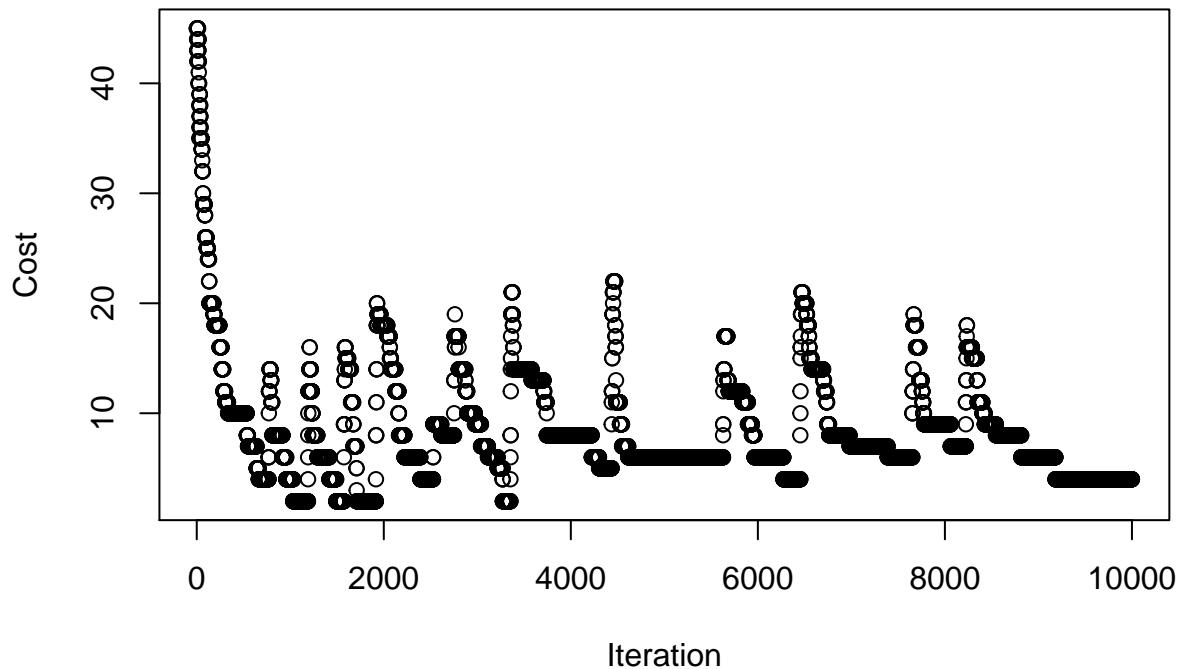
## Ran simulated annealing with geometric cooling for 10000 iterations.
## Solution not found .
## Minimum cost reached 4 .
## Final temperature is 2.450806e-19
## Number of reheat in the run 9
```

Cost for Each Iteration



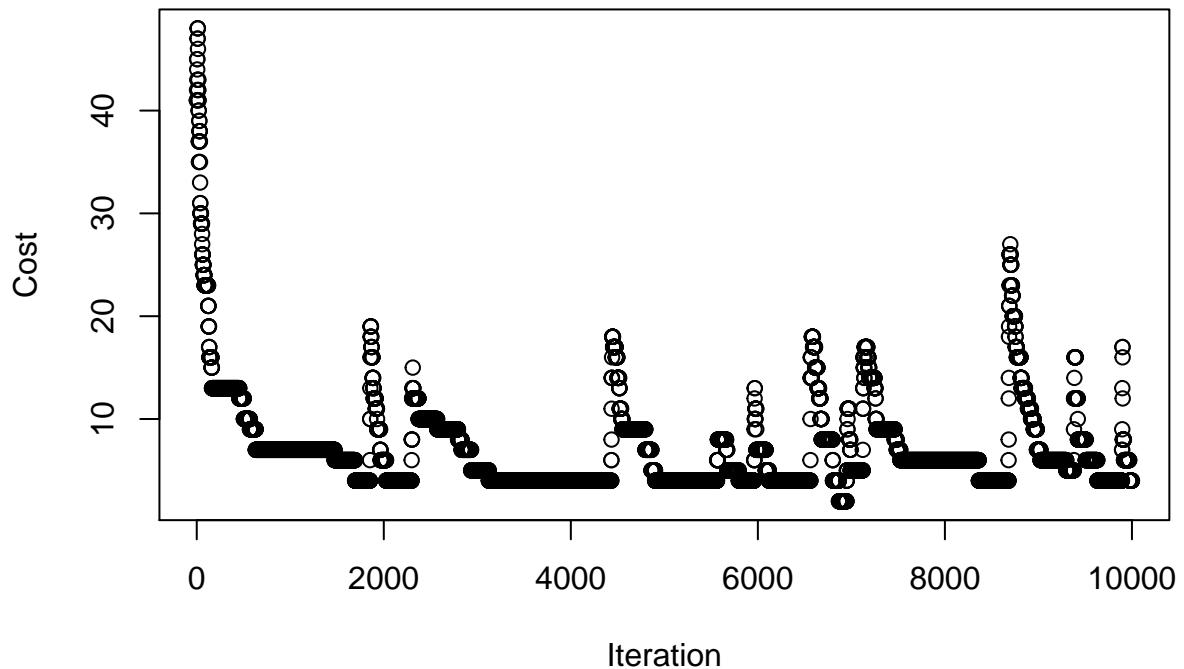
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 2.198628e-81  
## Number of reheat in the run 12
```

Cost for Each Iteration



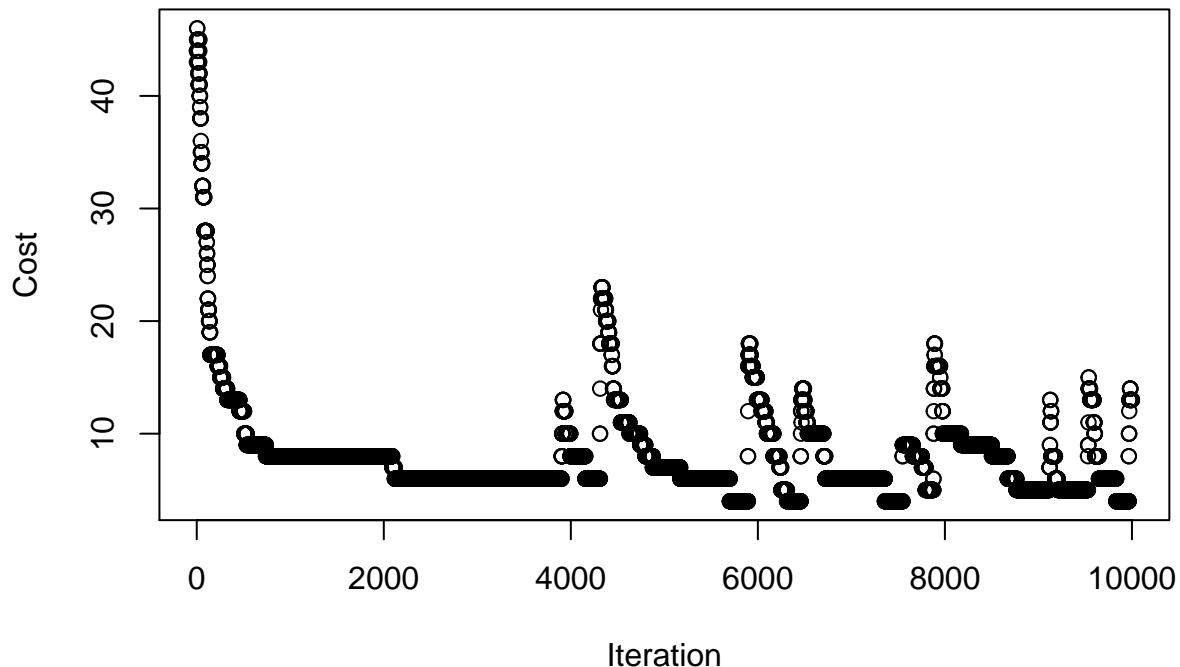
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 5.145215e-05  
## Number of reheat in the run 11
```

Cost for Each Iteration



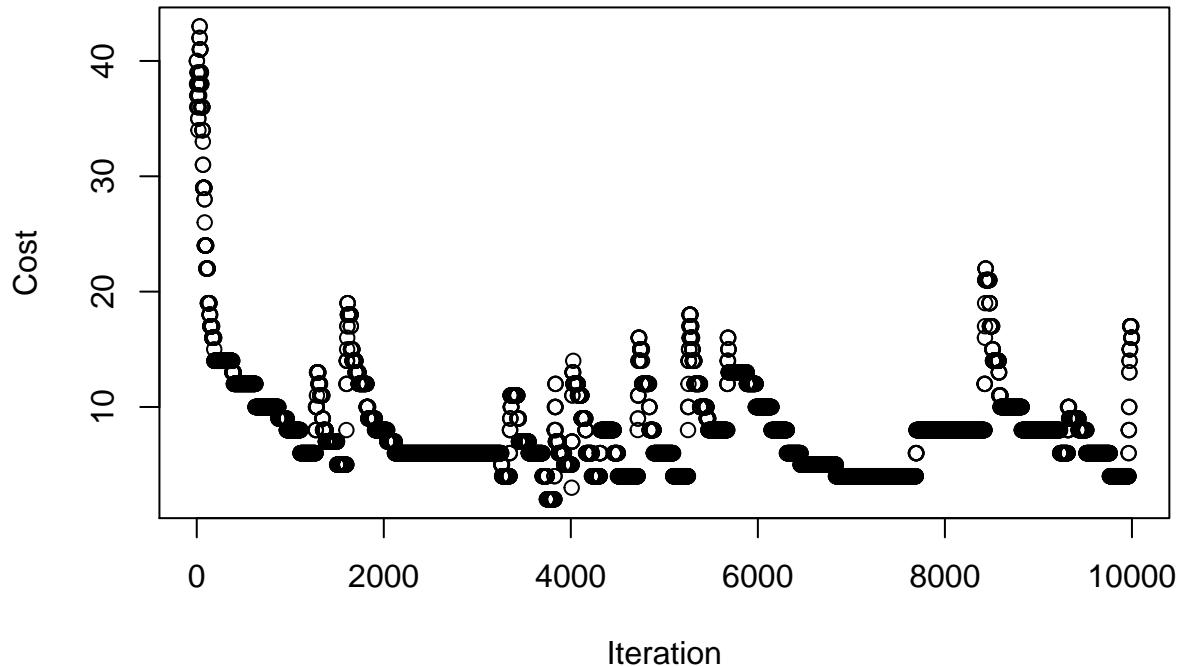
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.1013778  
## Number of reheat in the run 9
```

Cost for Each Iteration



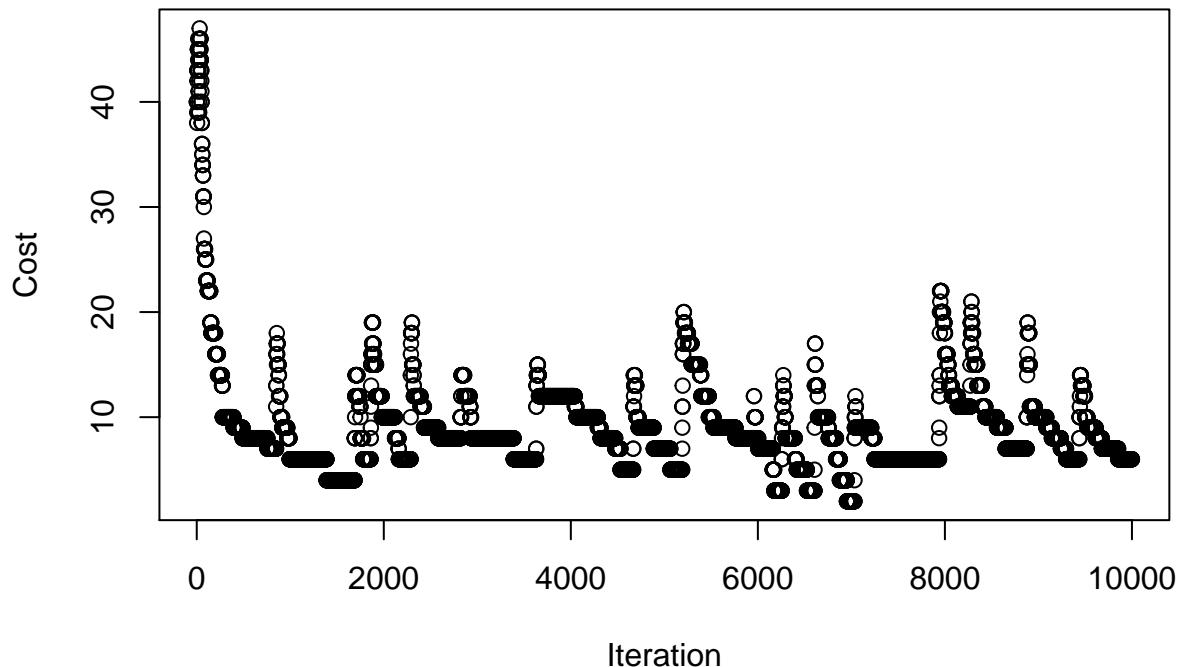
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.06651397  
## Number of reheat in the run 13
```

Cost for Each Iteration



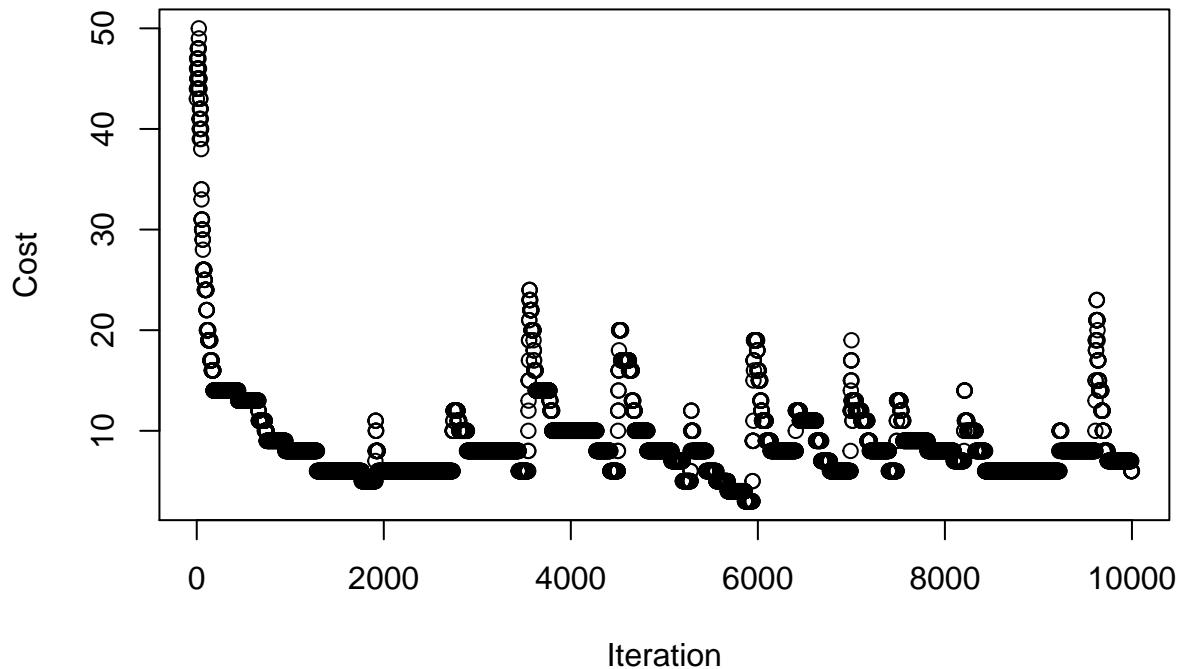
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 7.014357e-26  
## Number of reheat in the run 16
```

Cost for Each Iteration



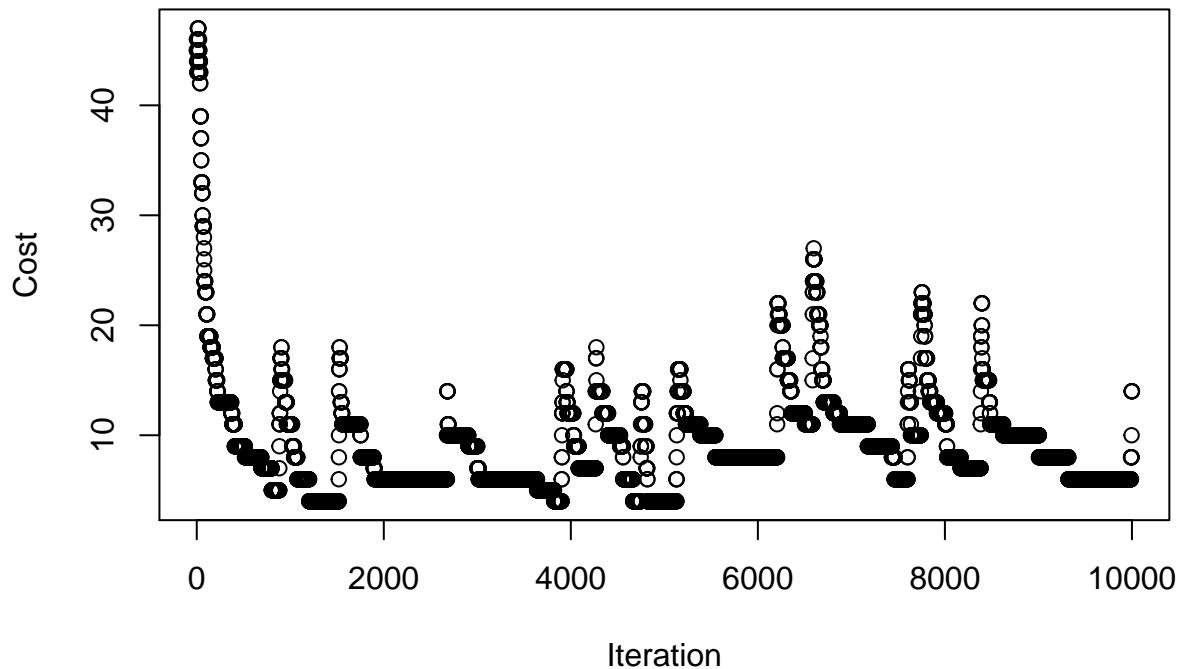
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 3 .  
## Final temperature is 5.203268e-18  
## Number of reheat in the run 12
```

Cost for Each Iteration



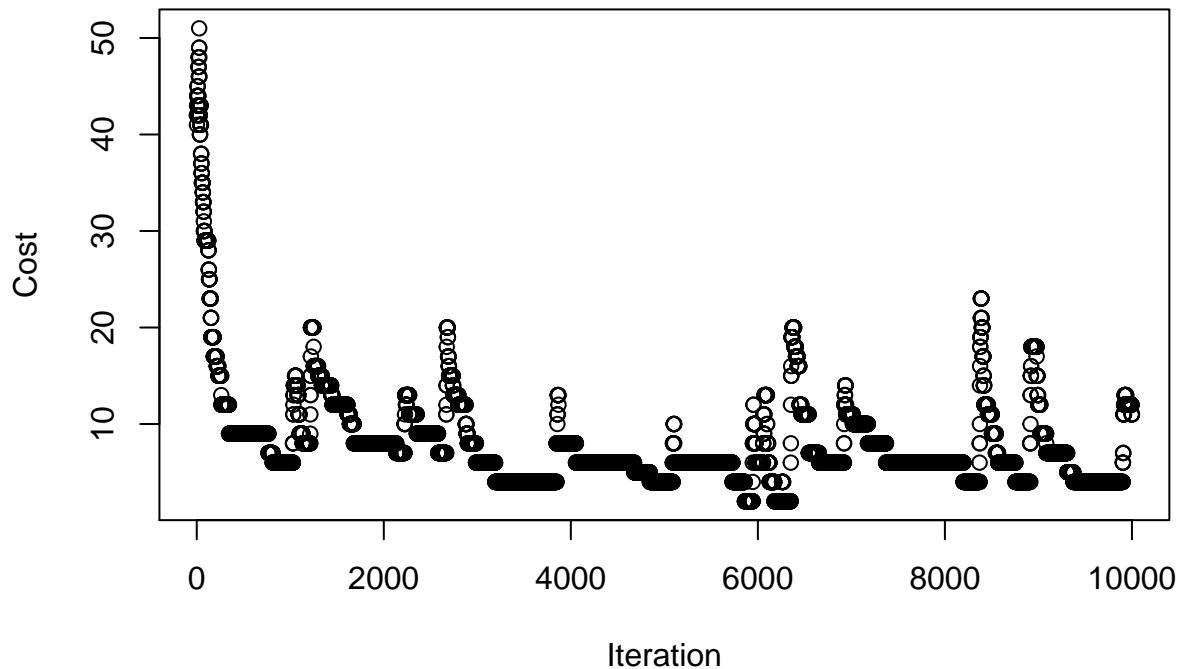
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 1.14384  
## Number of reheat in the run 13
```

Cost for Each Iteration



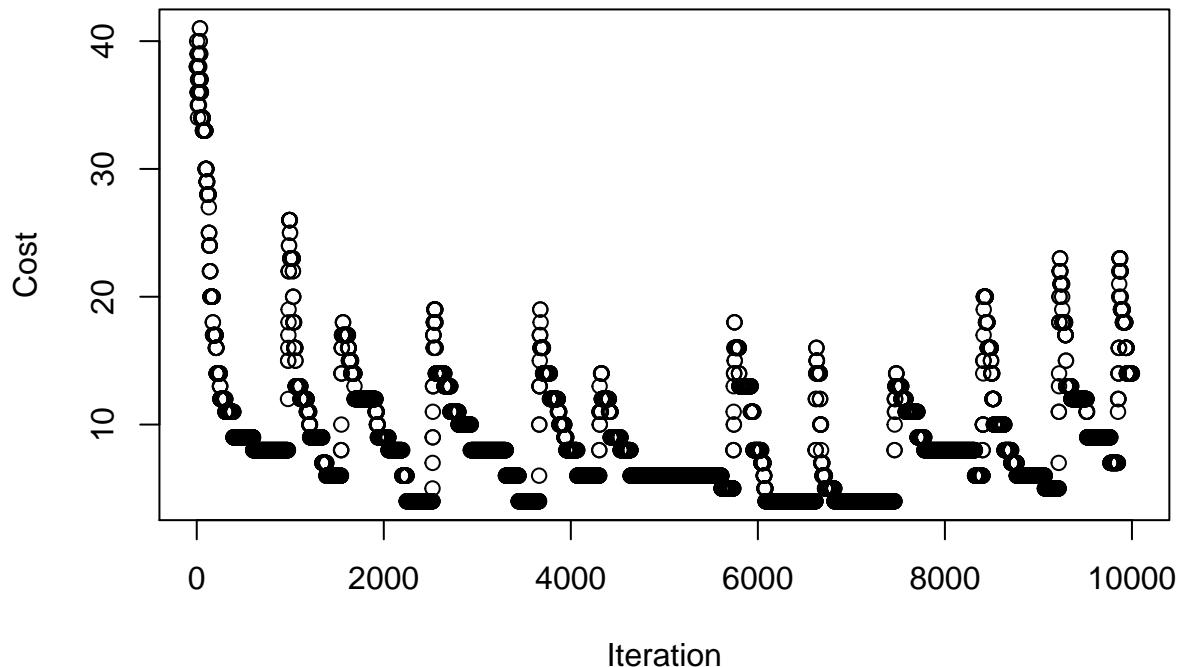
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.0001195263  
## Number of reheat in the run 14
```

Cost for Each Iteration



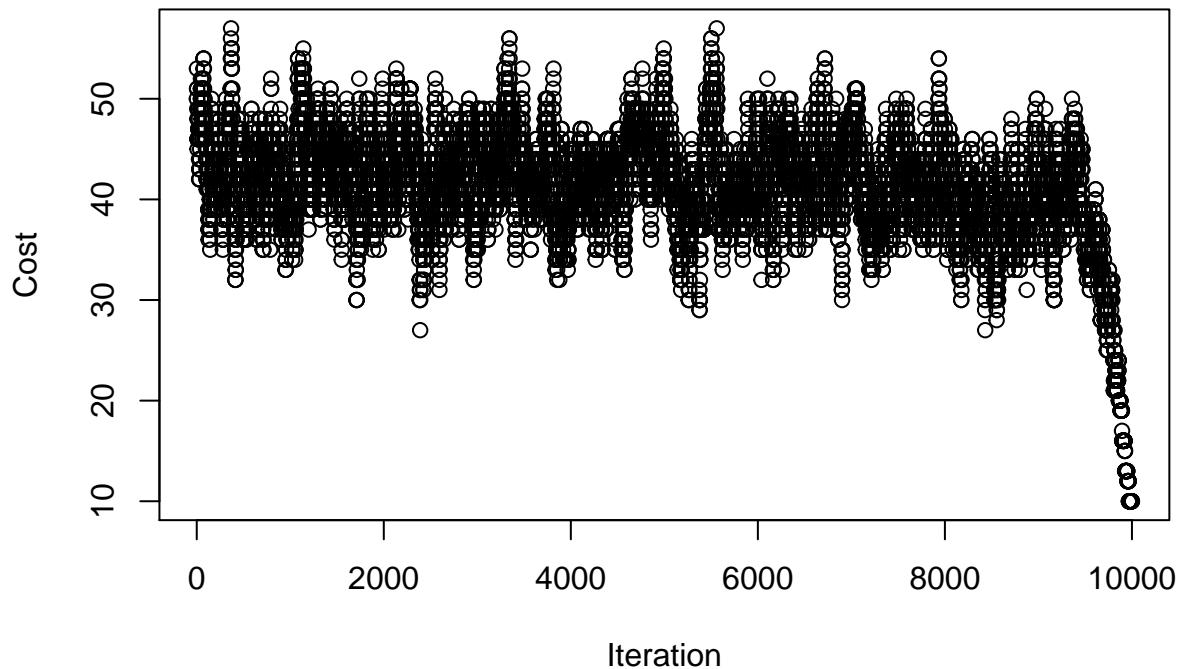
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 5.544105e-07  
## Number of reheat in the run 11
```

Cost for Each Iteration



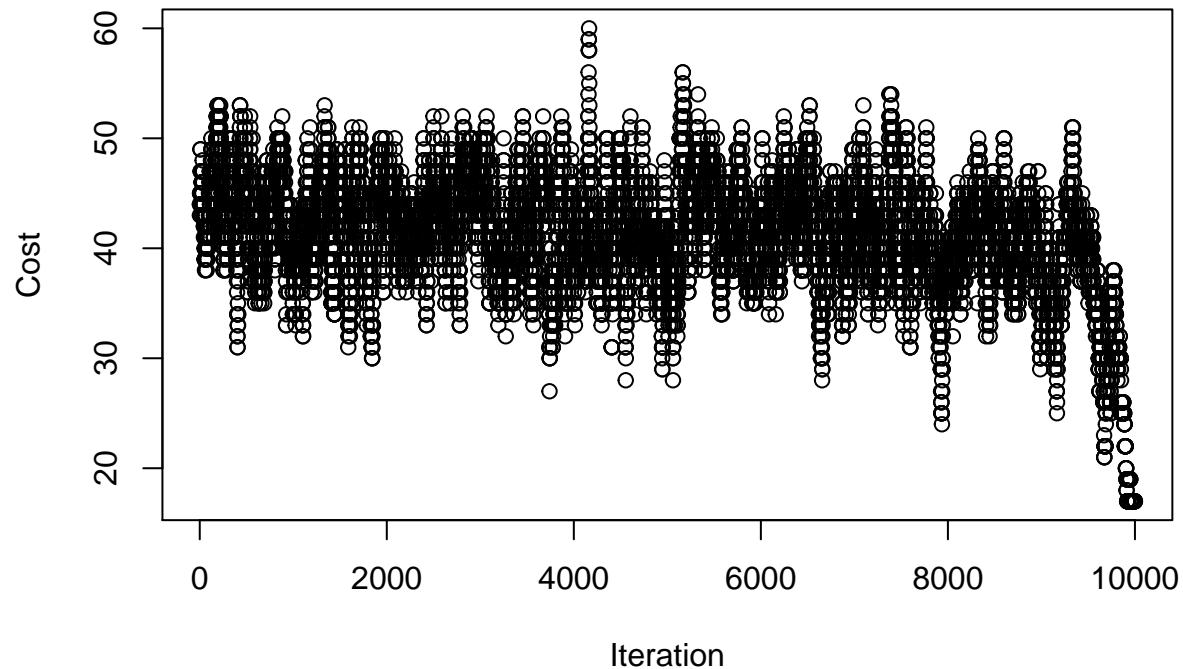
```
for (i in 1:10){  
  solve_sudoku_linear_cooling(10000, 50, ex3)  
}  
  
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 10 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



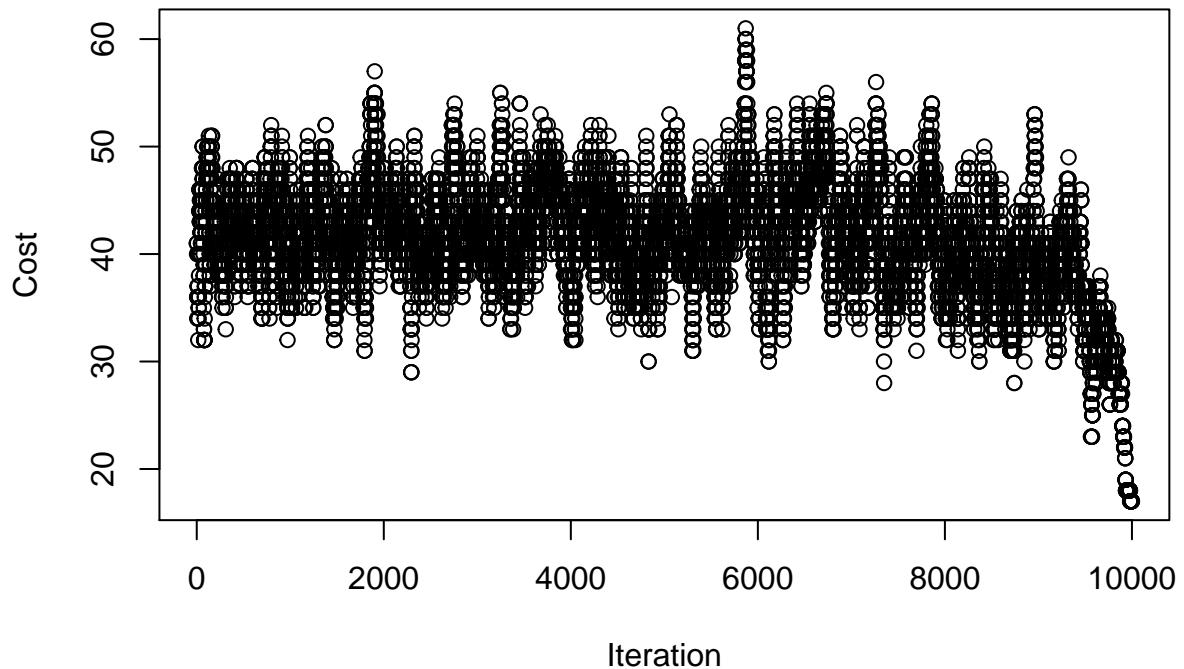
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 17 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



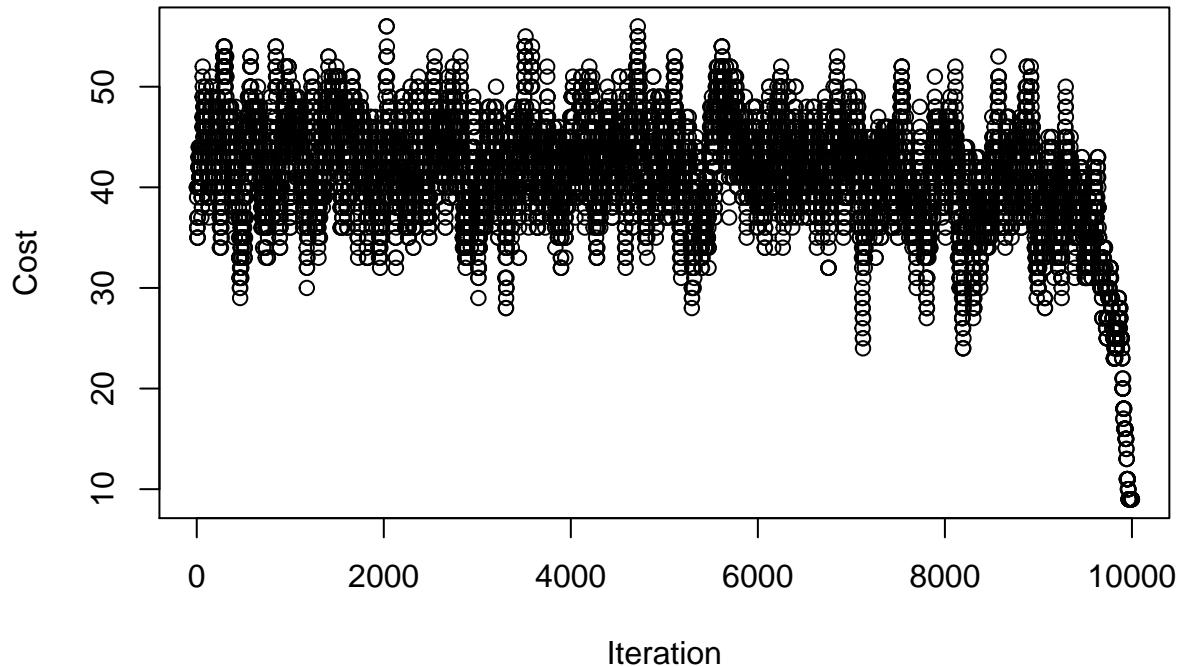
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 17 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



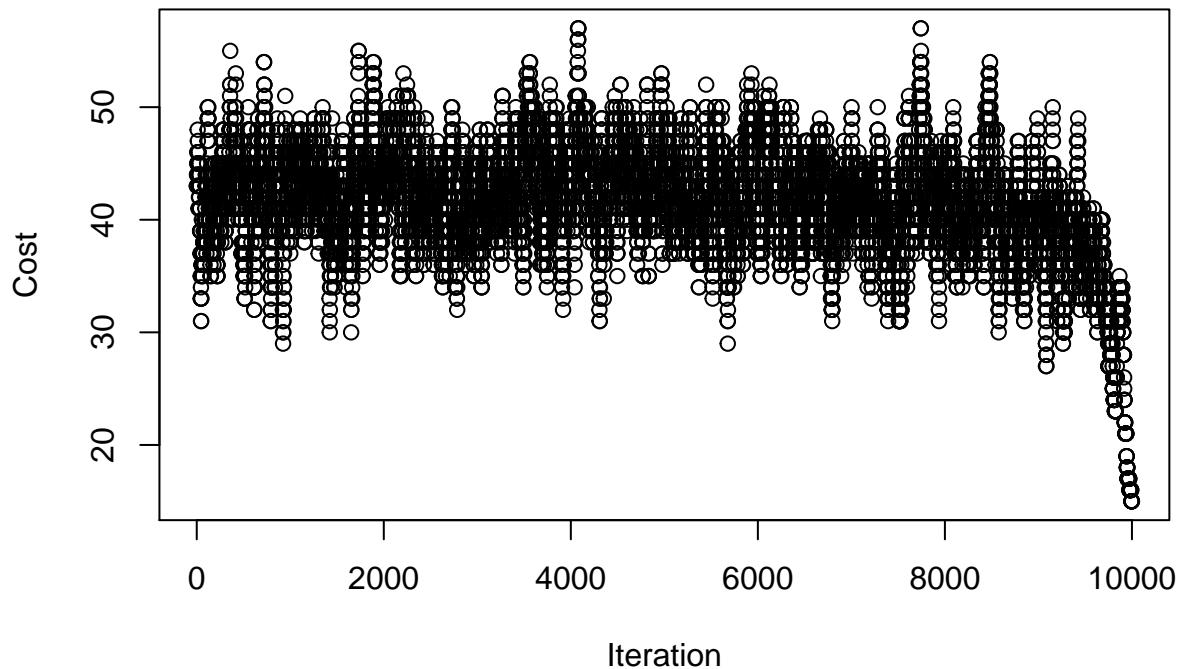
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 9 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



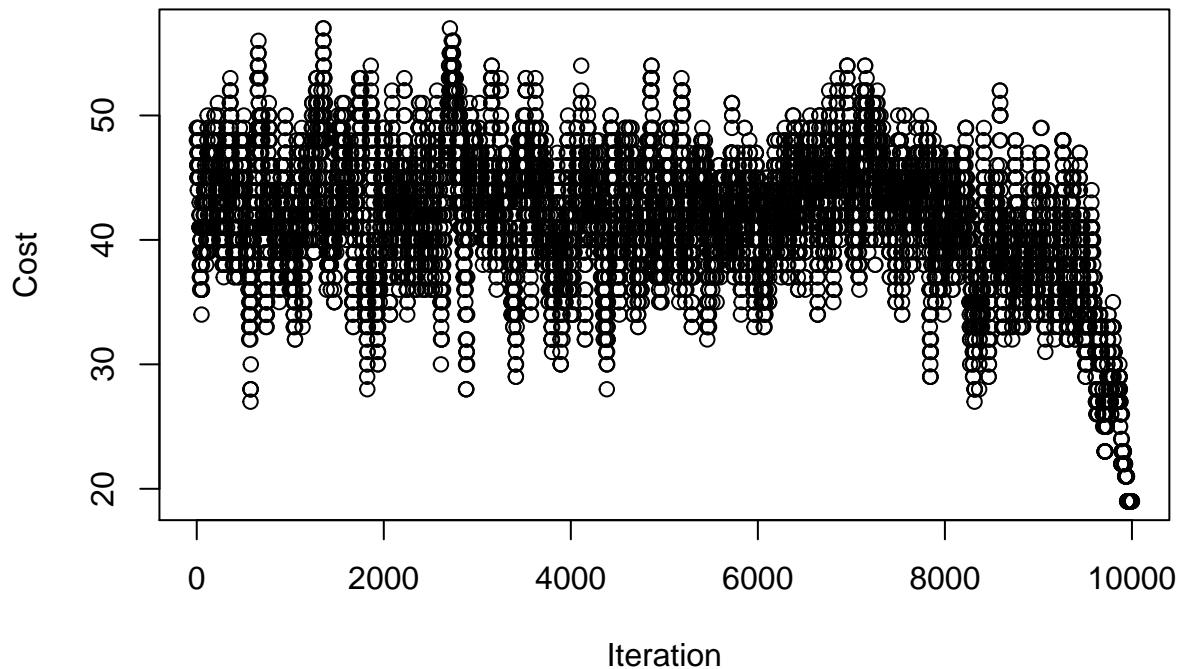
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 15 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



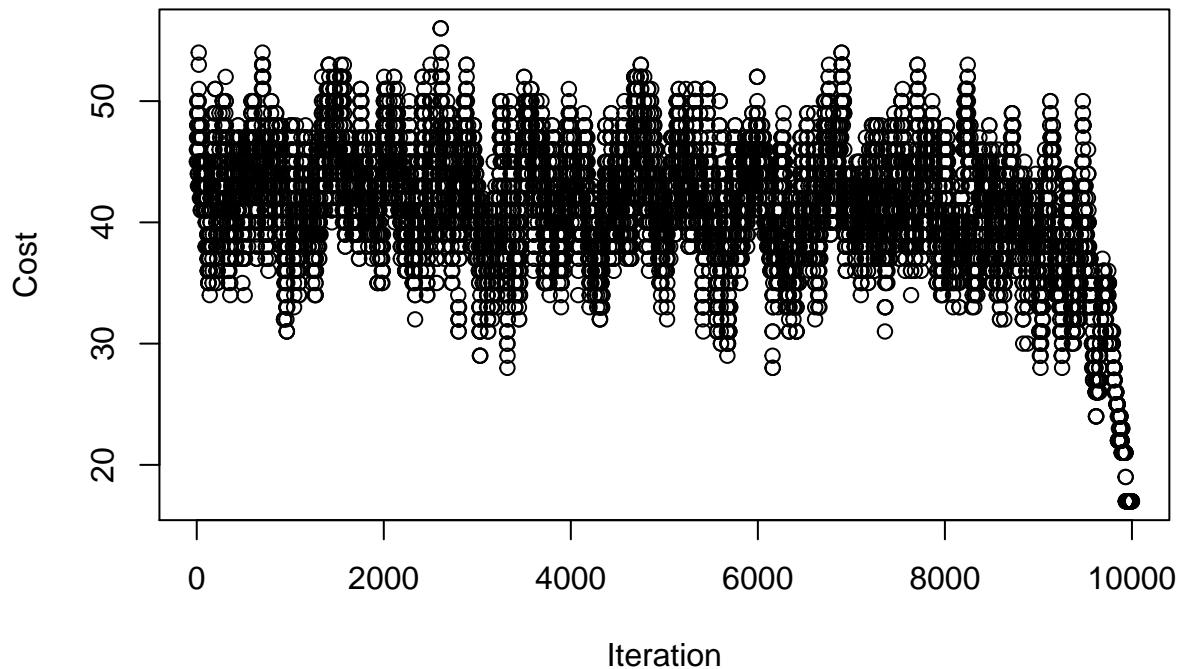
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 19 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



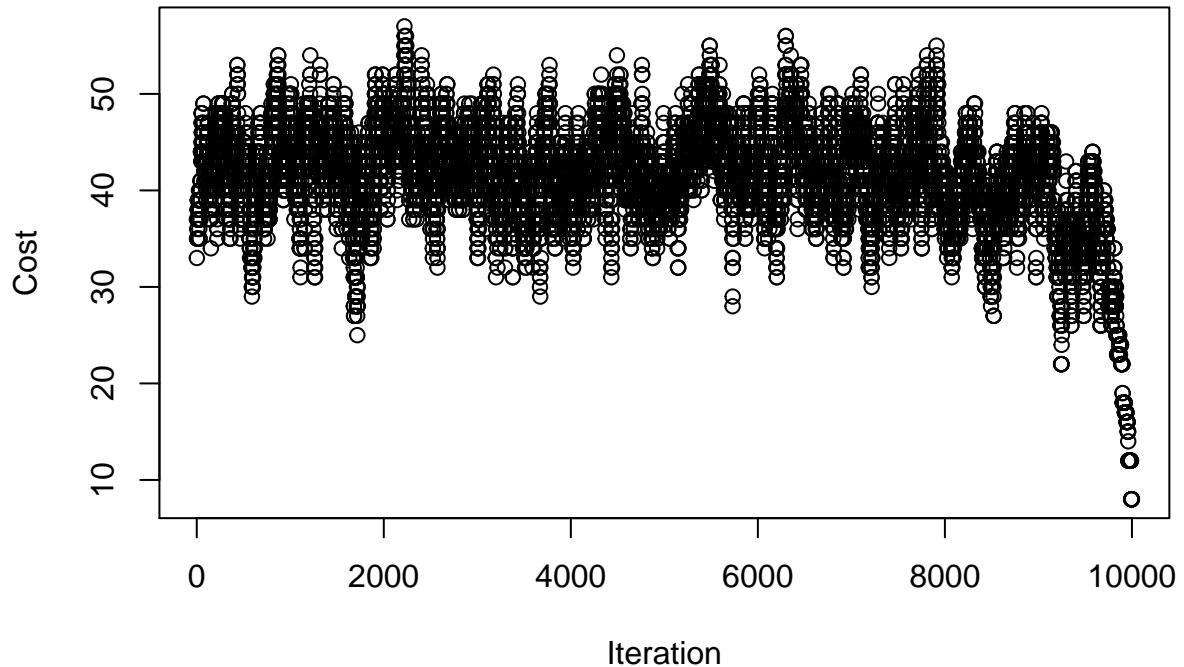
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 17 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



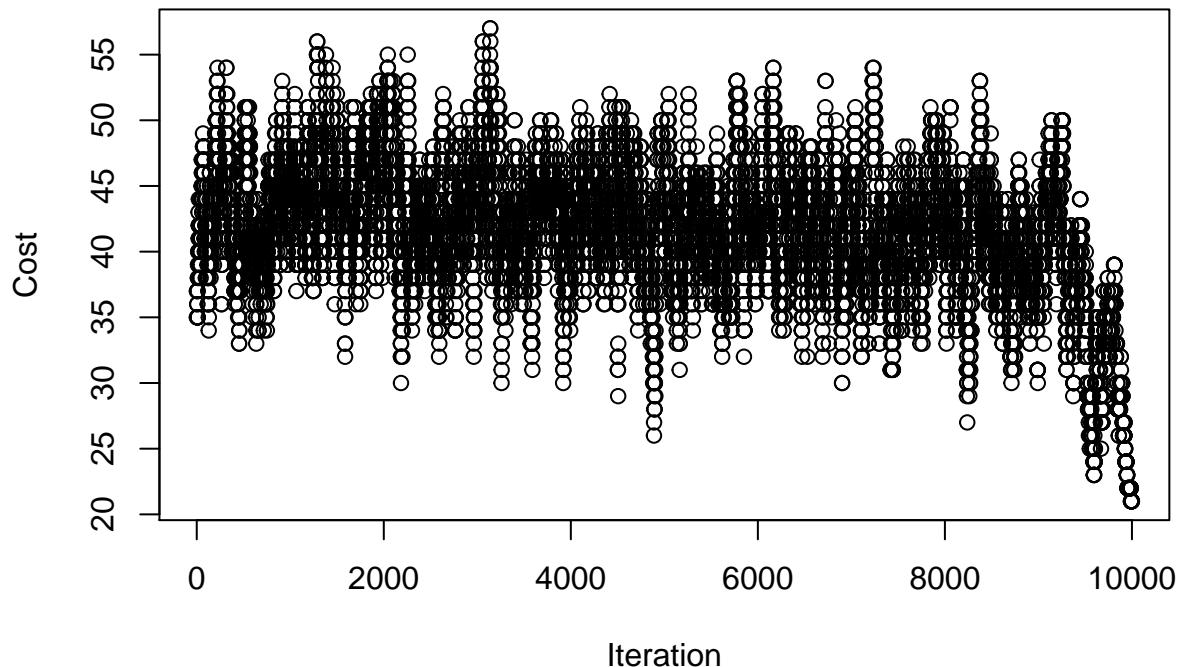
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 8 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



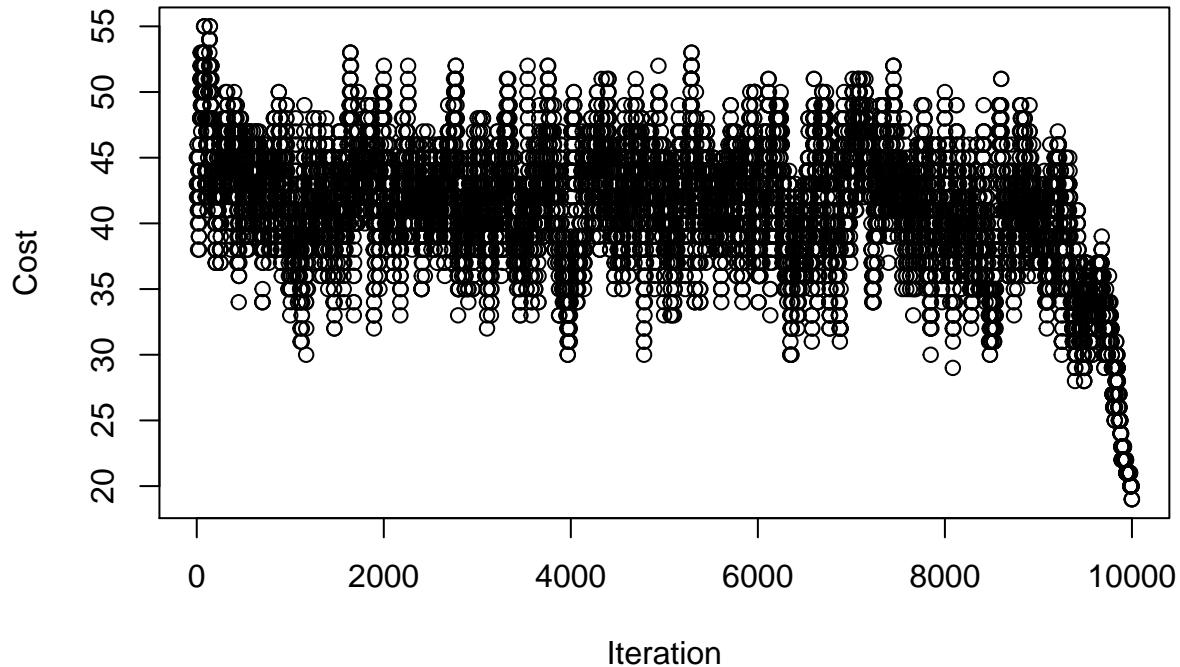
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 21 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



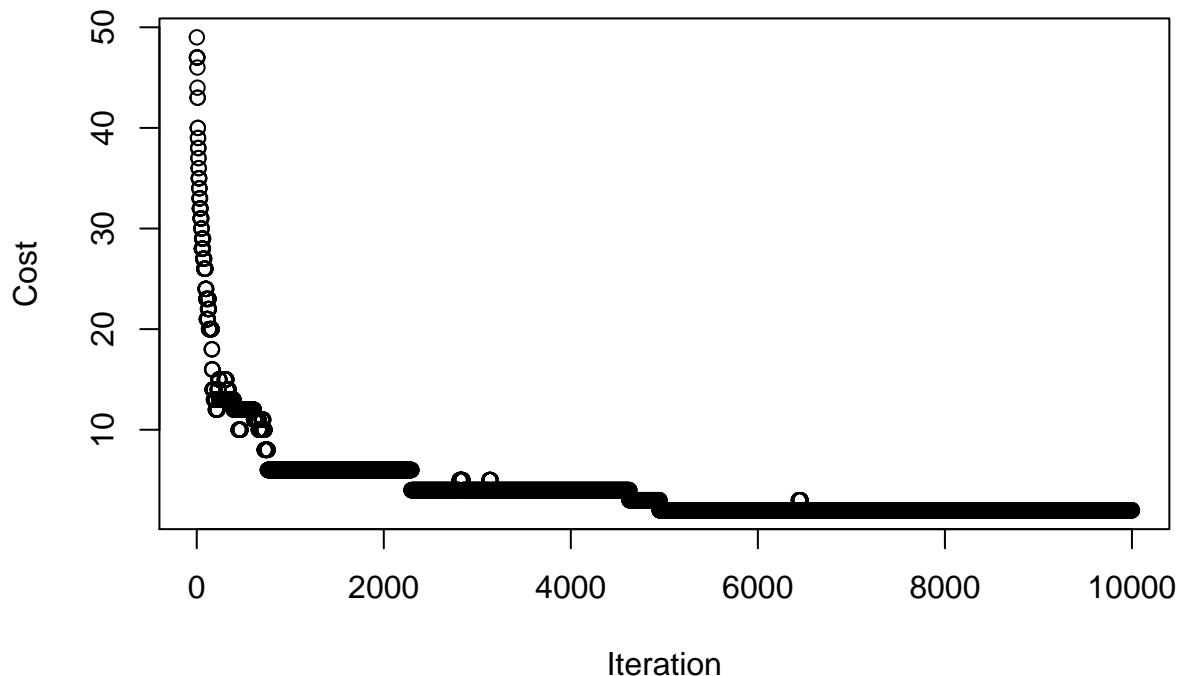
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 19 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



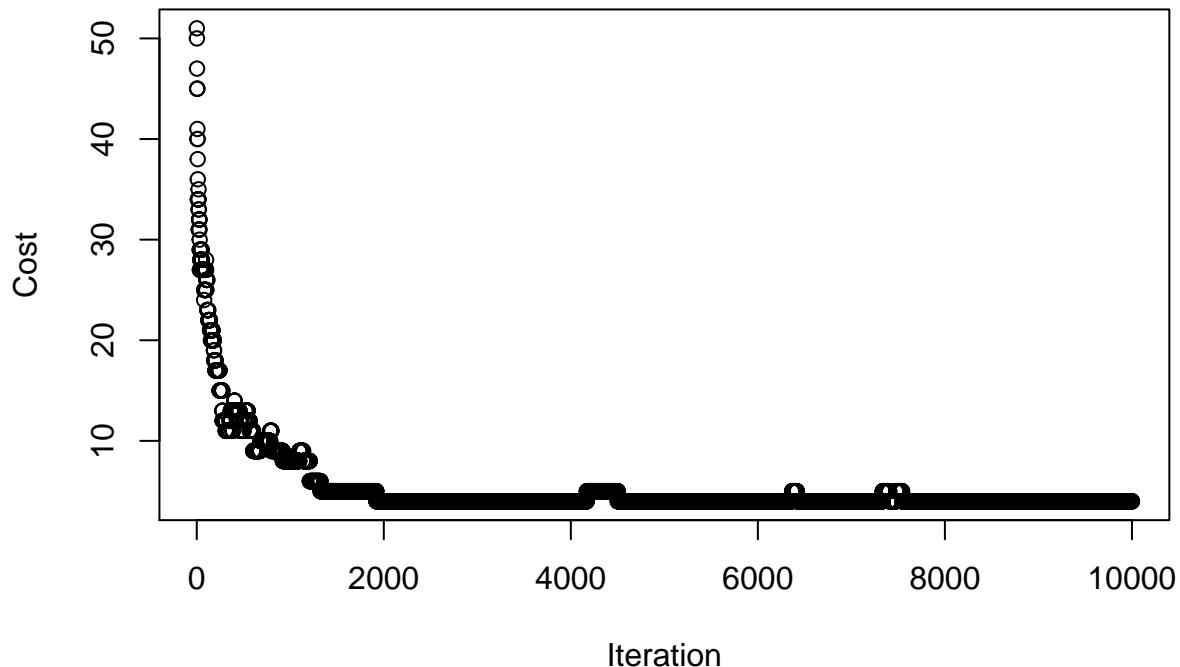
```
for (i in 1:10){  
  solve_sudoku_log_cooling(10000, 50, ex3)  
}  
  
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 55
```

Cost for Each Iteration



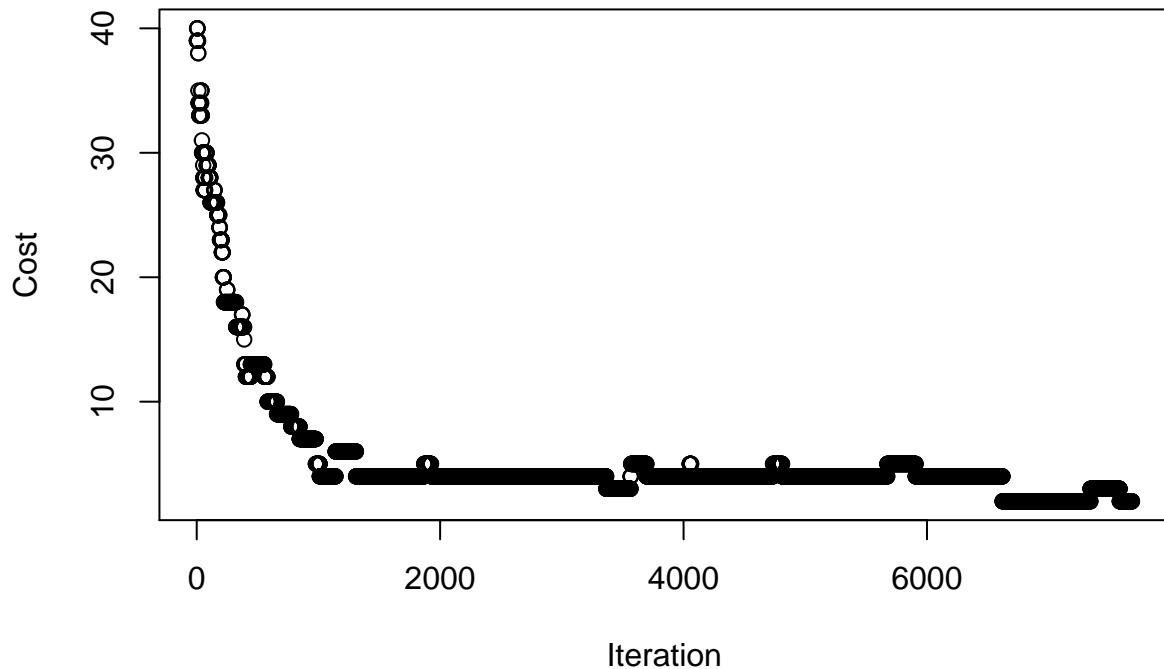
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 21
```

Cost for Each Iteration



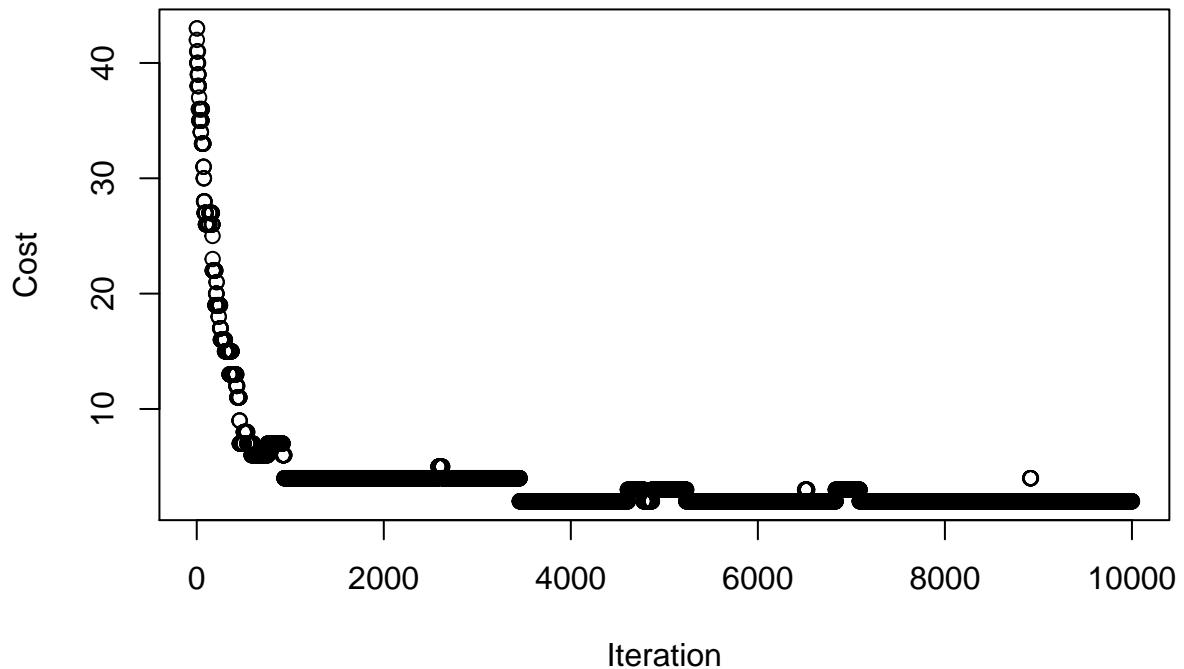
```
## Ran simulated annealing with logarithmic cooling for 7684 iterations.  
## Solution found .  
## Final temperature is 0.223538  
## Number of reheat in the run 25
```

Cost for Each Iteration



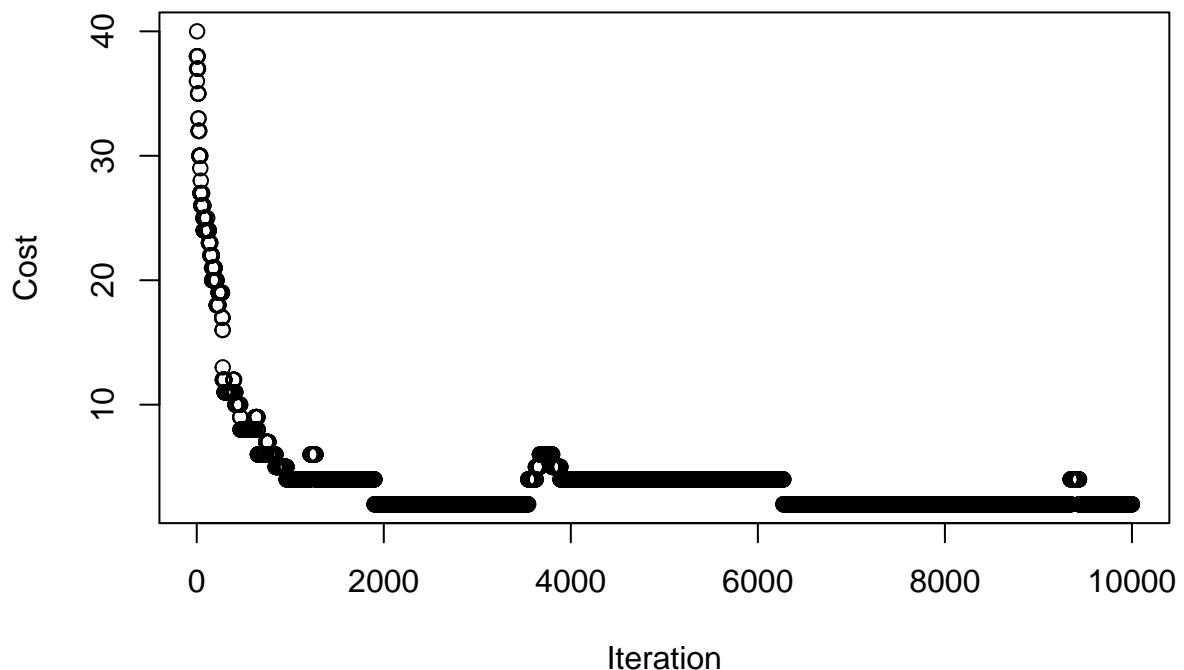
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 66
```

Cost for Each Iteration



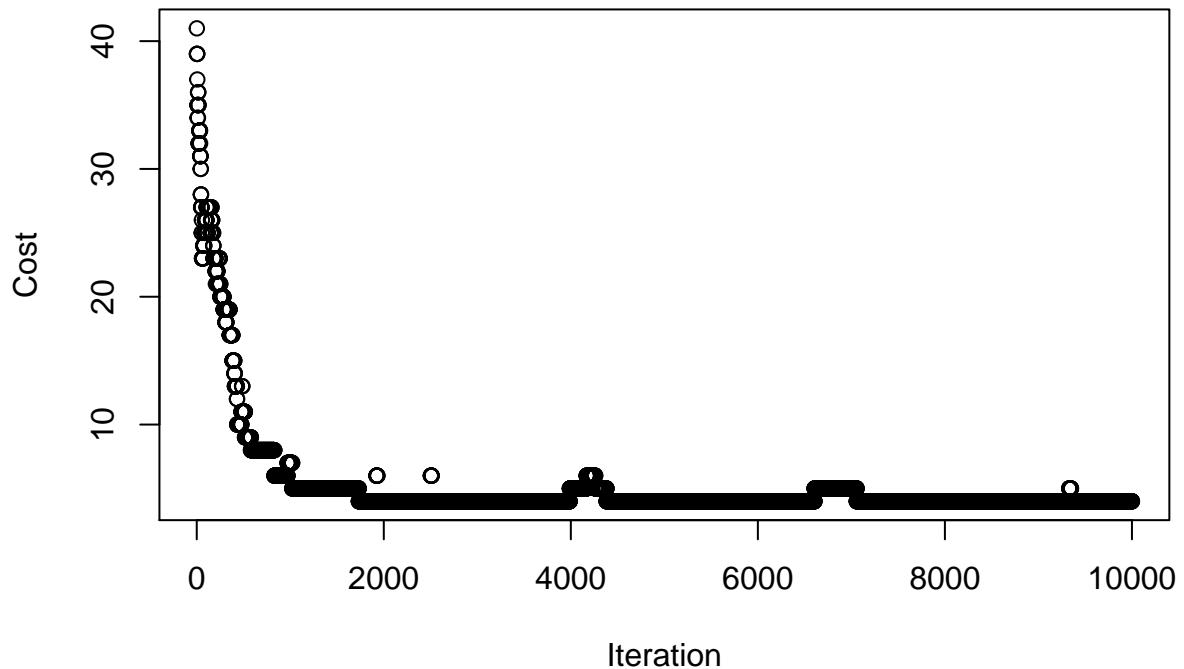
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 39
```

Cost for Each Iteration



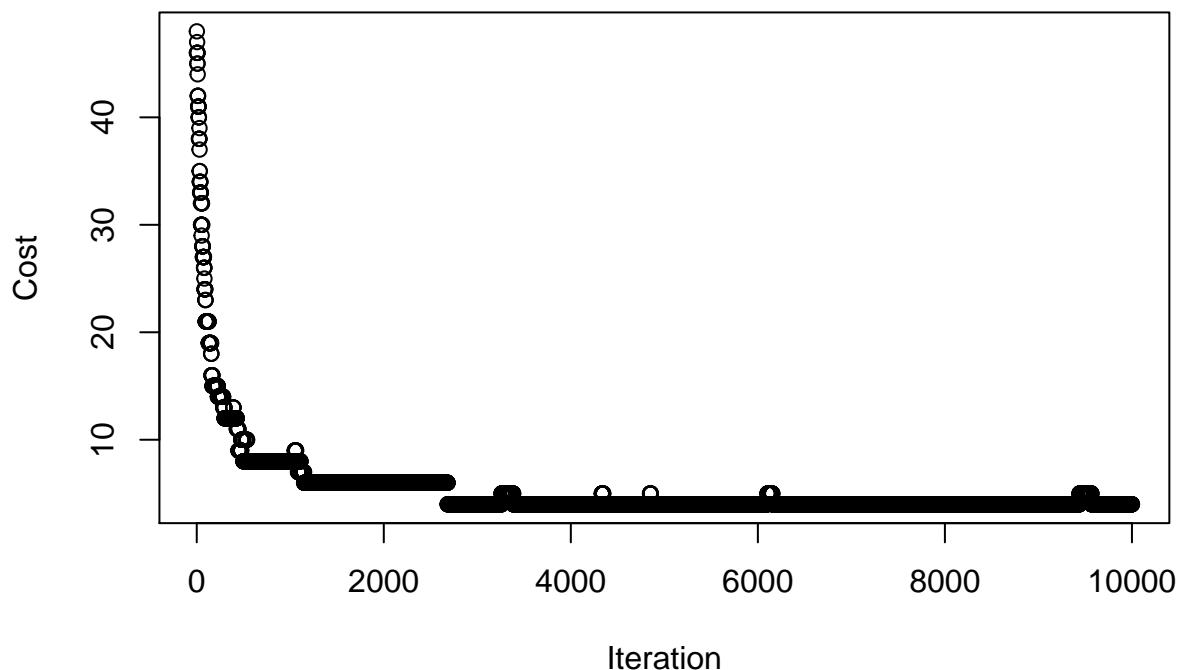
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 54
```

Cost for Each Iteration



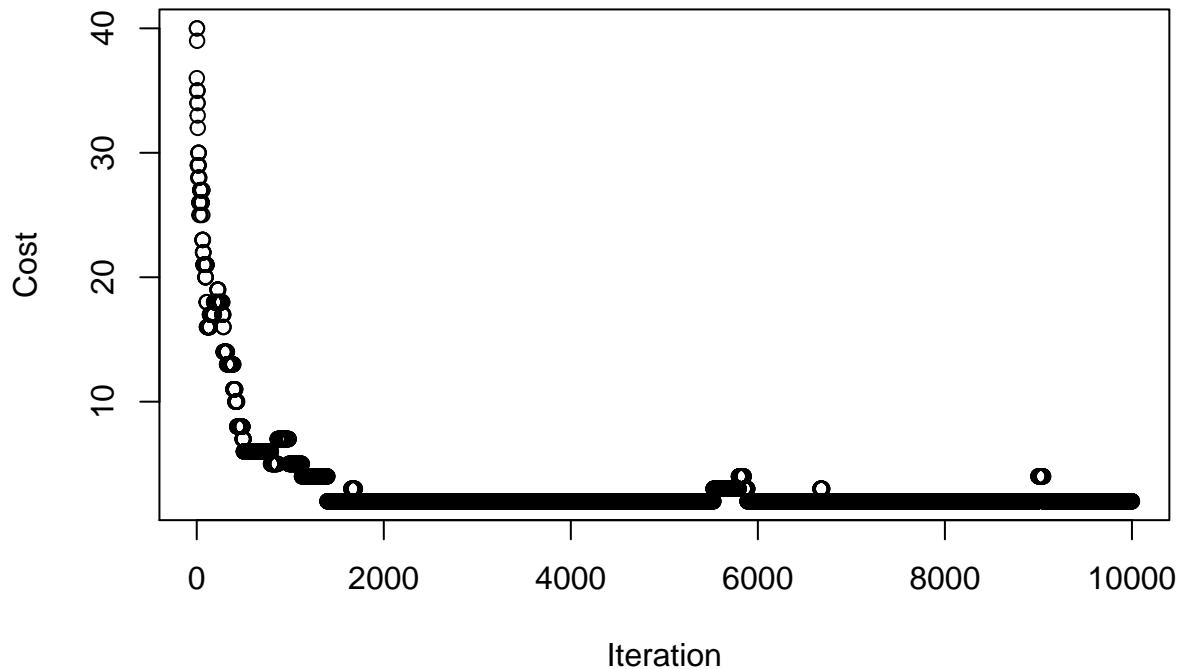
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 26
```

Cost for Each Iteration



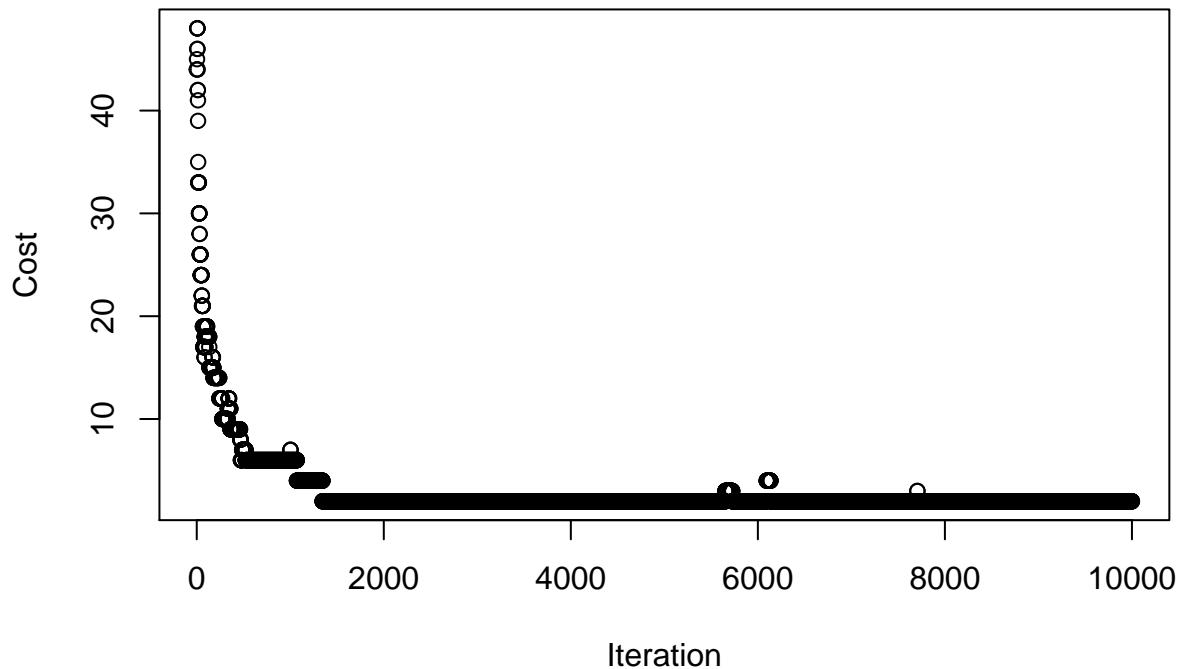
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 67
```

Cost for Each Iteration



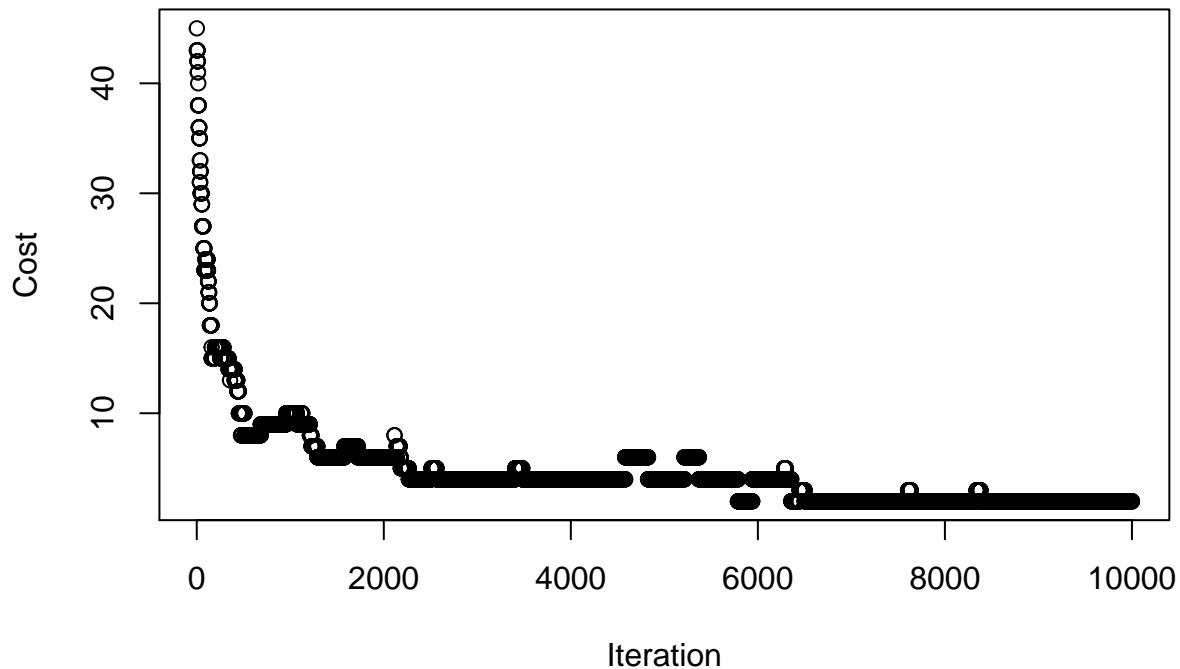
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 48
```

Cost for Each Iteration



```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 34
```

Cost for Each Iteration



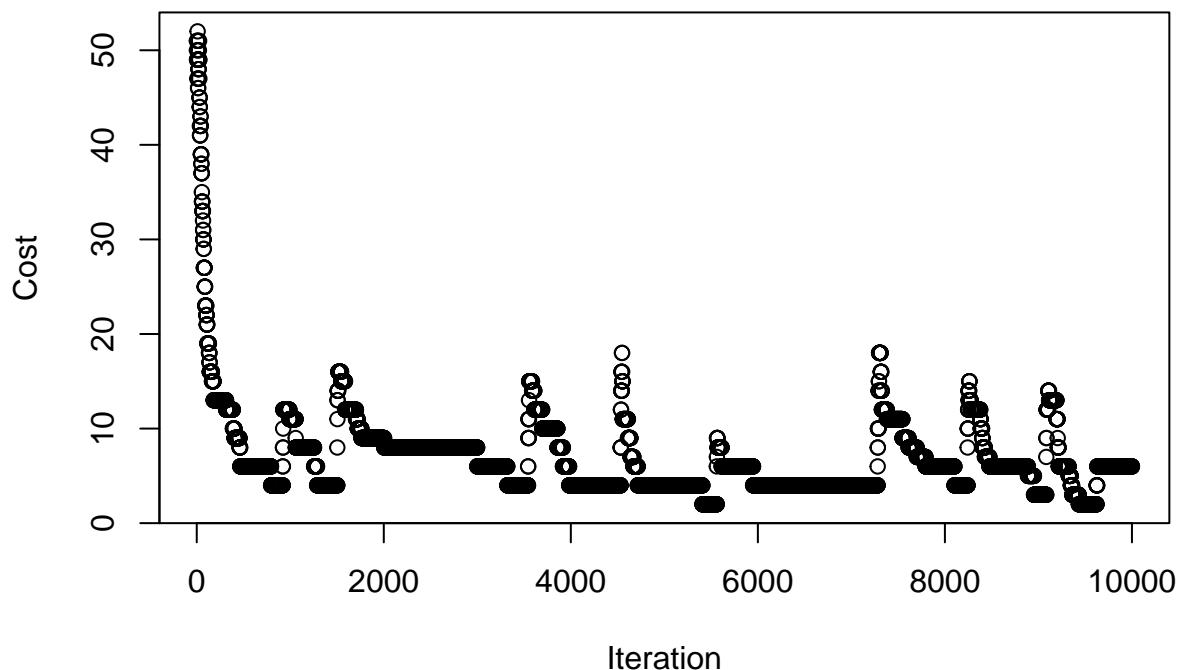
```
ex4 <- read_excel("ex4.xlsx")
ex4

## # A tibble: 9 x 9
##   col1  col2  col3  col4  col5  col6  col7  col8  col9
##   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl>
## 1     1    NA     5    NA    NA    NA    NA    NA    NA
## 2    NA    NA    NA    NA     8     9    NA     5    NA
## 3     2    NA     9    NA     7    NA    NA    NA     3
## 4    NA     2    NA     3    NA    NA    NA     7    NA
## 5     4    NA    NA    NA    NA    NA    NA    NA     5
## 6    NA     5    NA    NA    NA     1    NA     8    NA
## 7     8    NA    NA    NA     2    NA     3    NA     4
## 8    NA     3    NA     4     1    NA    NA    NA    NA
## 9    NA    NA    NA    NA    NA    NA     6    NA     8

for (i in 1:10){
  solve_sudoku_exp_cooling(10000, 50, ex4, 0.9)
}

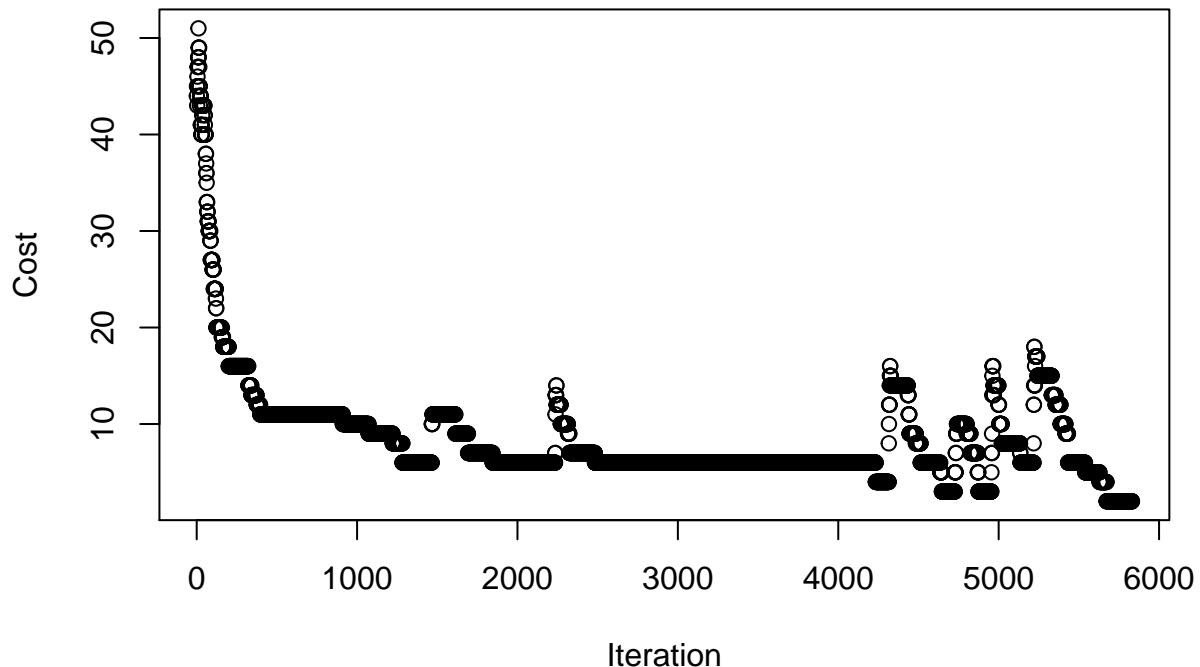
## Ran simulated annealing with geometric cooling for 10000 iterations.
## Solution not found .
## Minimum cost reached 2 .
## Final temperature is 1.343054e-17
## Number of reheat in the run 9
```

Cost for Each Iteration



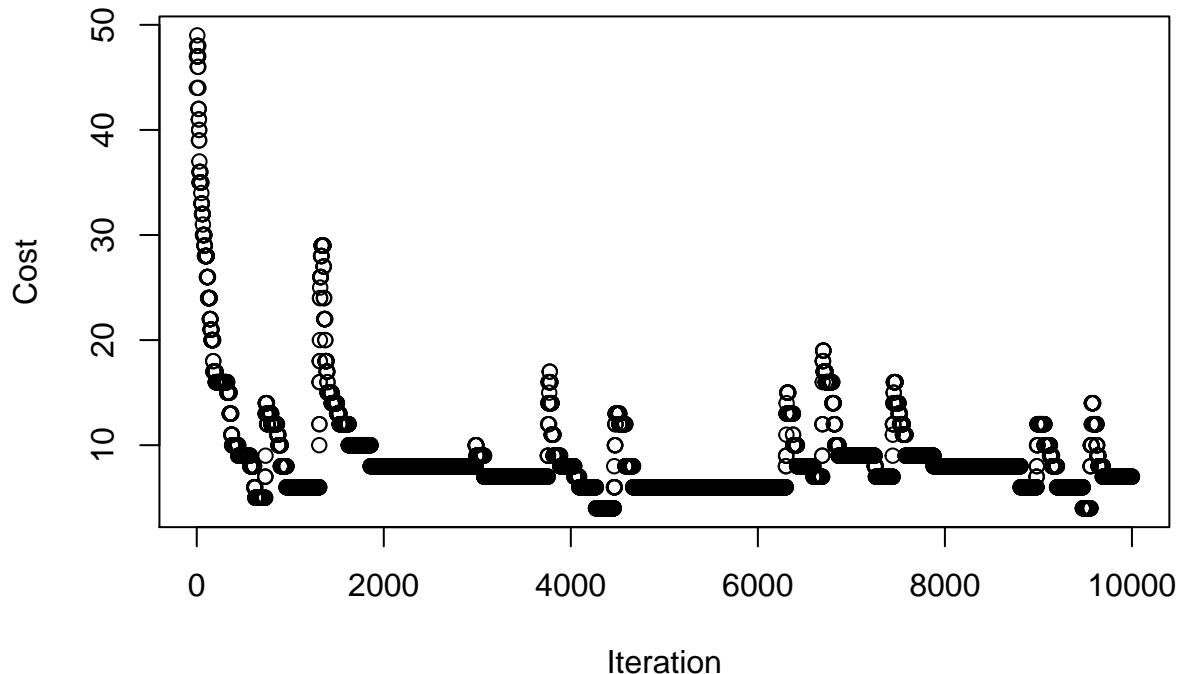
```
## Ran simulated annealing with geometric cooling for 5831 iterations.  
## Solution found .  
## Final temperature is 3.253538e-28  
## Number of reheat in the run 6
```

Cost for Each Iteration



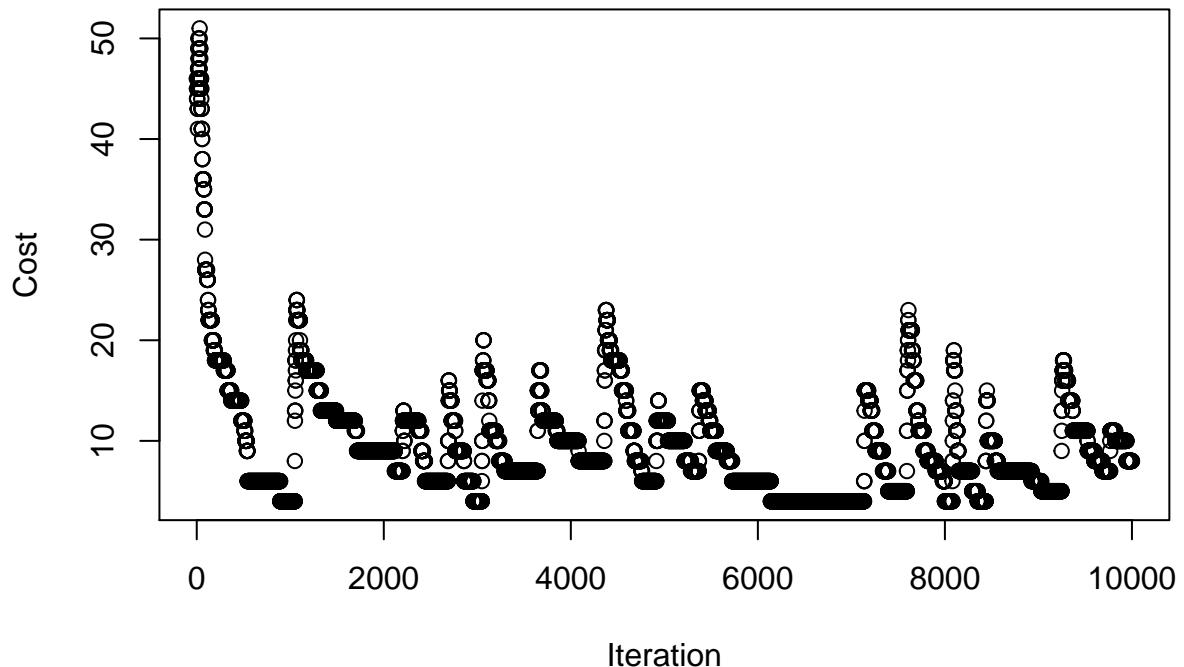
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 1.954921e-20  
## Number of reheat in the run 10
```

Cost for Each Iteration



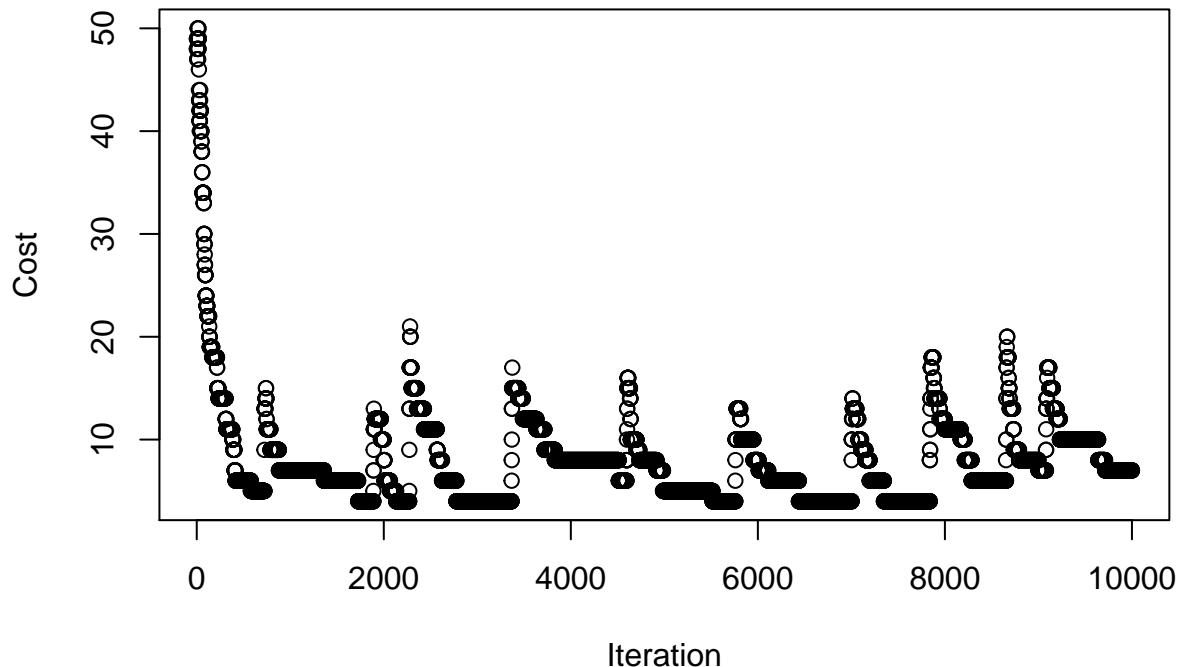
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 7.152284e-11  
## Number of reheat in the run 14
```

Cost for Each Iteration



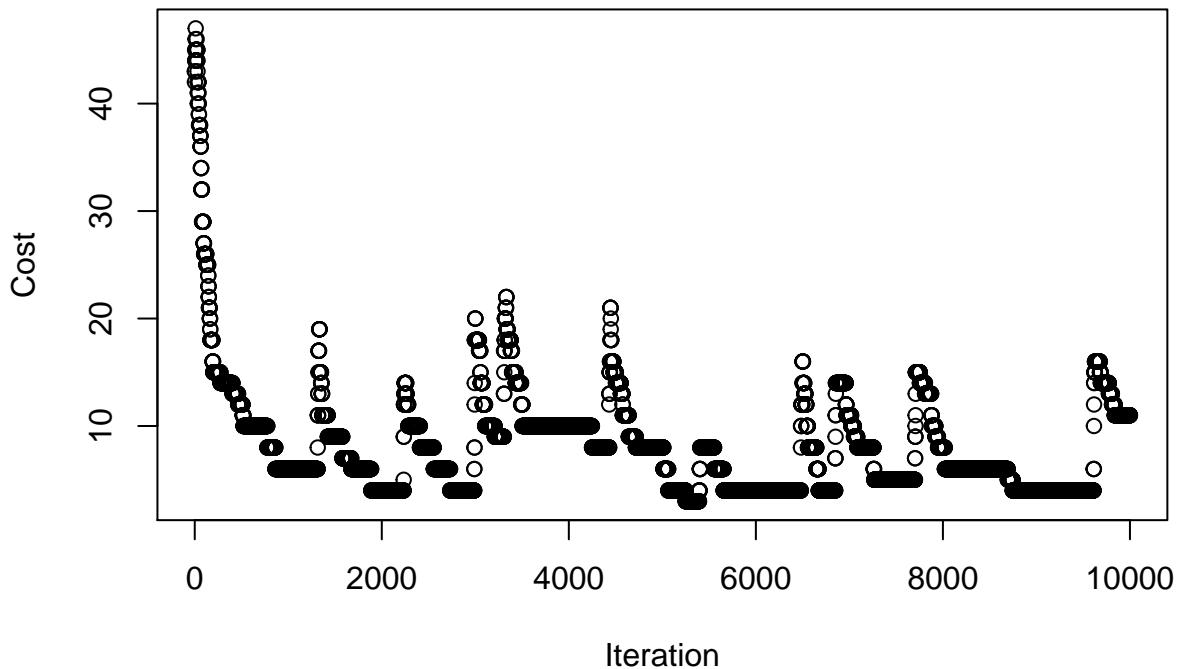
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 3.600155e-42  
## Number of reheat in the run 10
```

Cost for Each Iteration



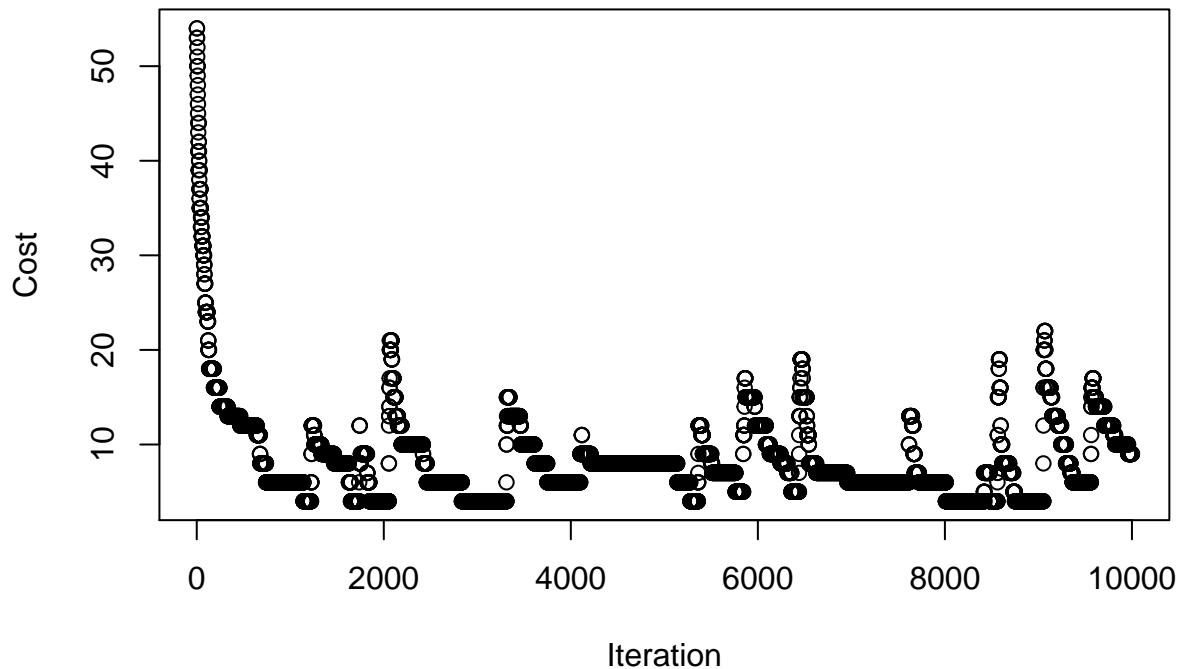
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 3 .  
## Final temperature is 7.137542e-18  
## Number of reheat in the run 10
```

Cost for Each Iteration



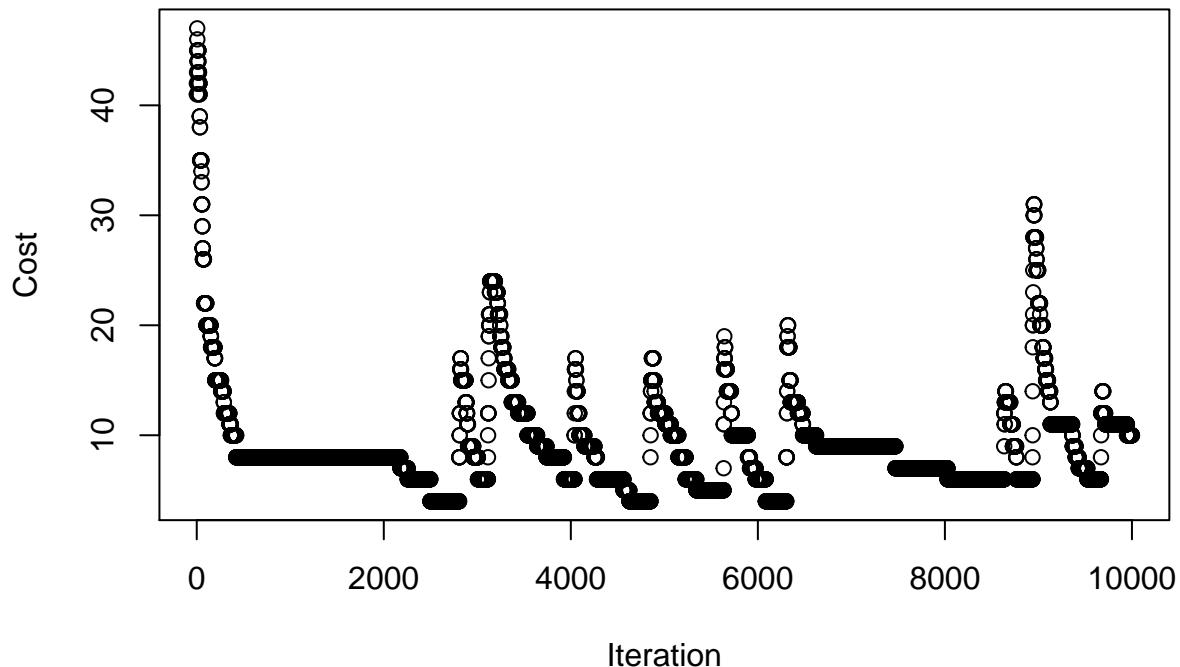
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 3.678529e-20  
## Number of reheat in the run 13
```

Cost for Each Iteration



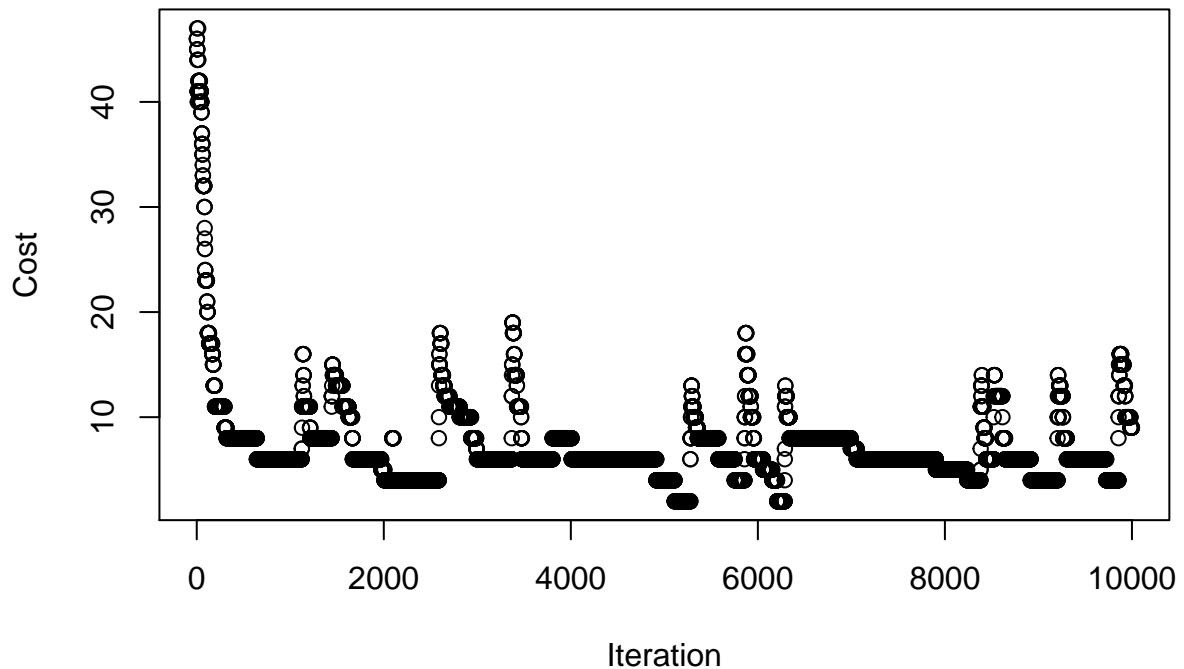
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 2.605962e-15  
## Number of reheat in the run 9
```

Cost for Each Iteration



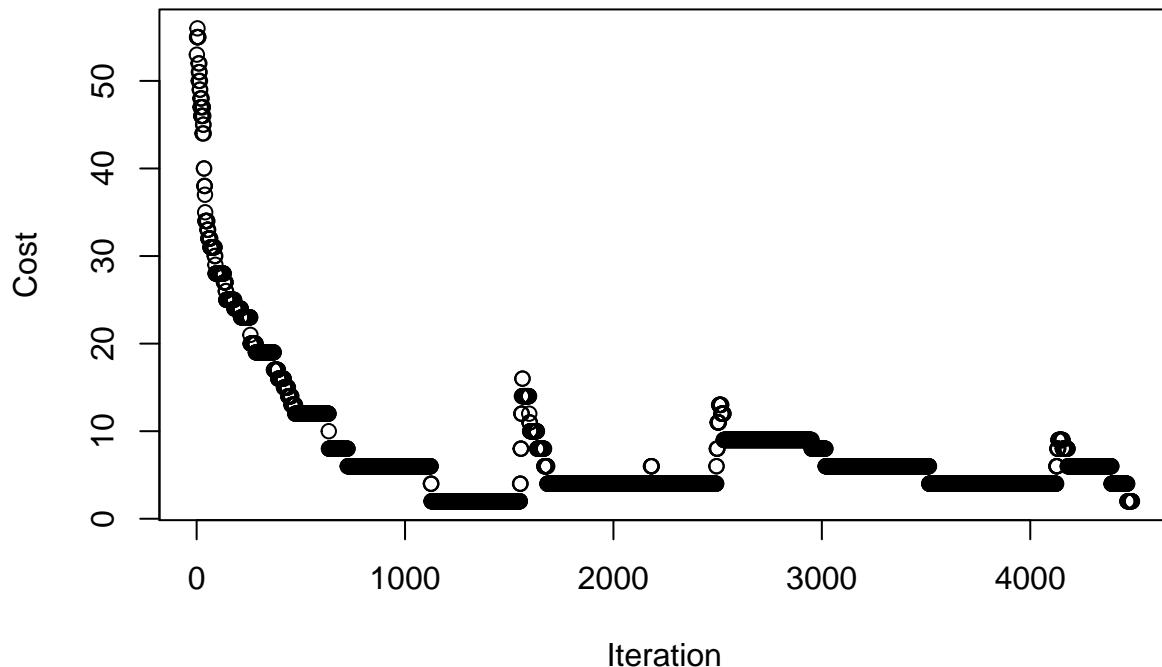
```
## Ran simulated annealing with geometric cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 7.605082e-07  
## Number of reheat in the run 13
```

Cost for Each Iteration



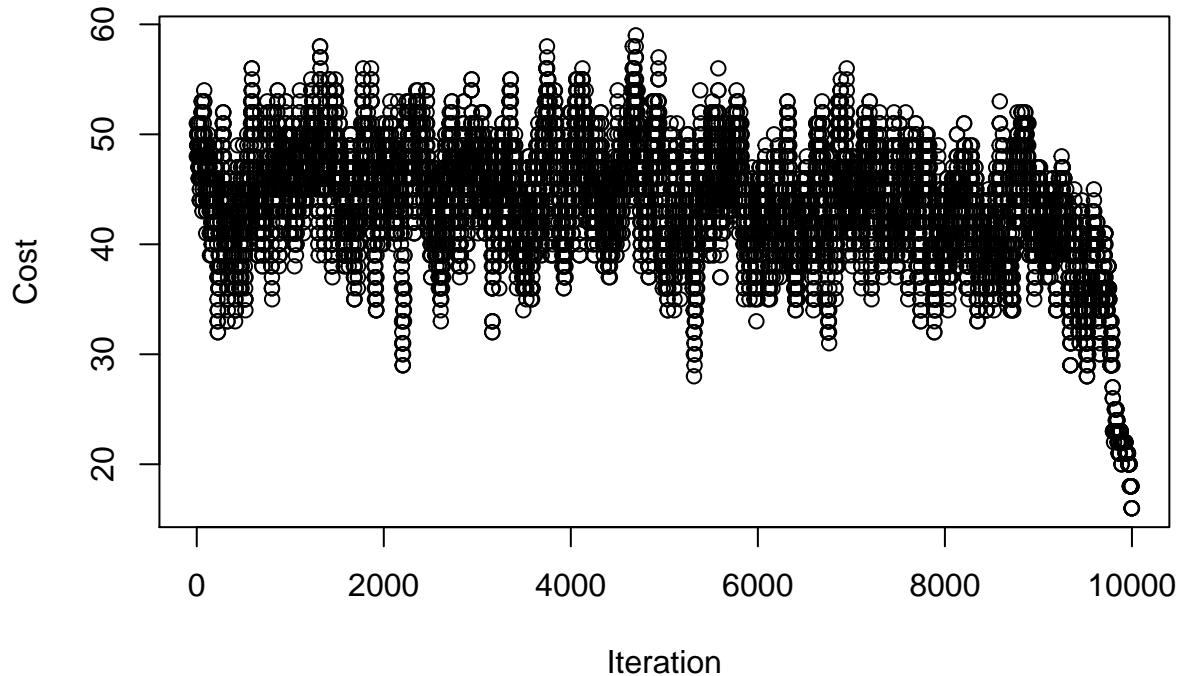
```
## Ran simulated annealing with geometric cooling for 4488 iterations.  
## Solution found .  
## Final temperature is 8.948051e-17  
## Number of reheat in the run 4
```

Cost for Each Iteration



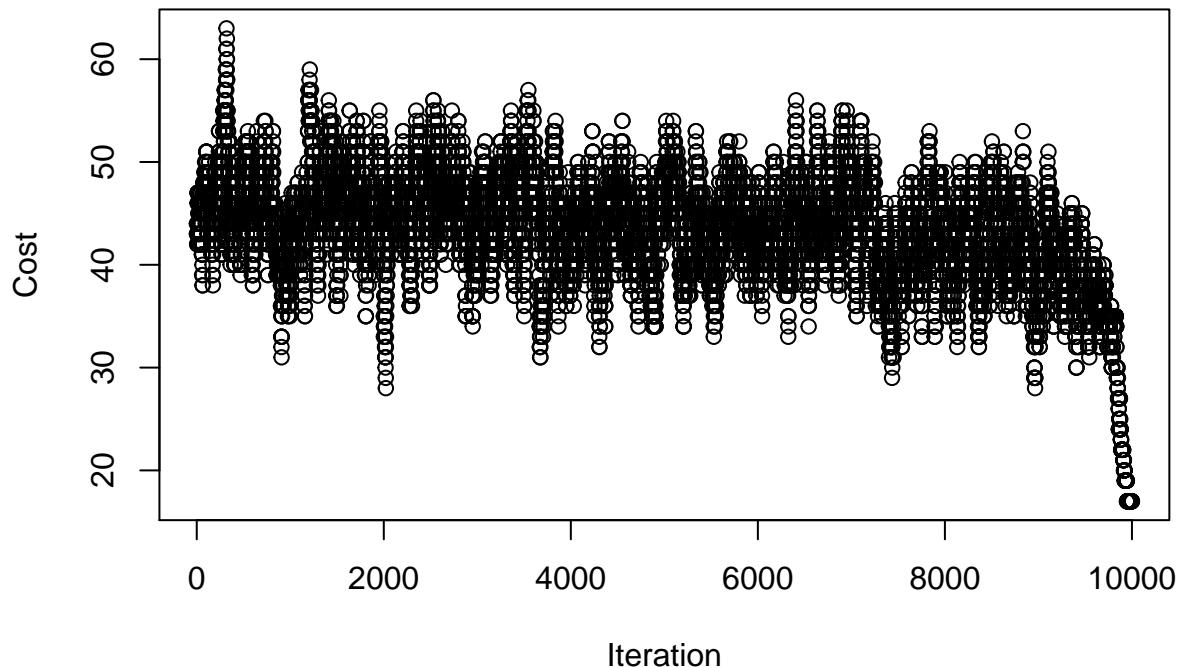
```
for (i in 1:10){  
  solve_sudoku_linear_cooling(10000, 50, ex4)  
}  
  
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 16 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



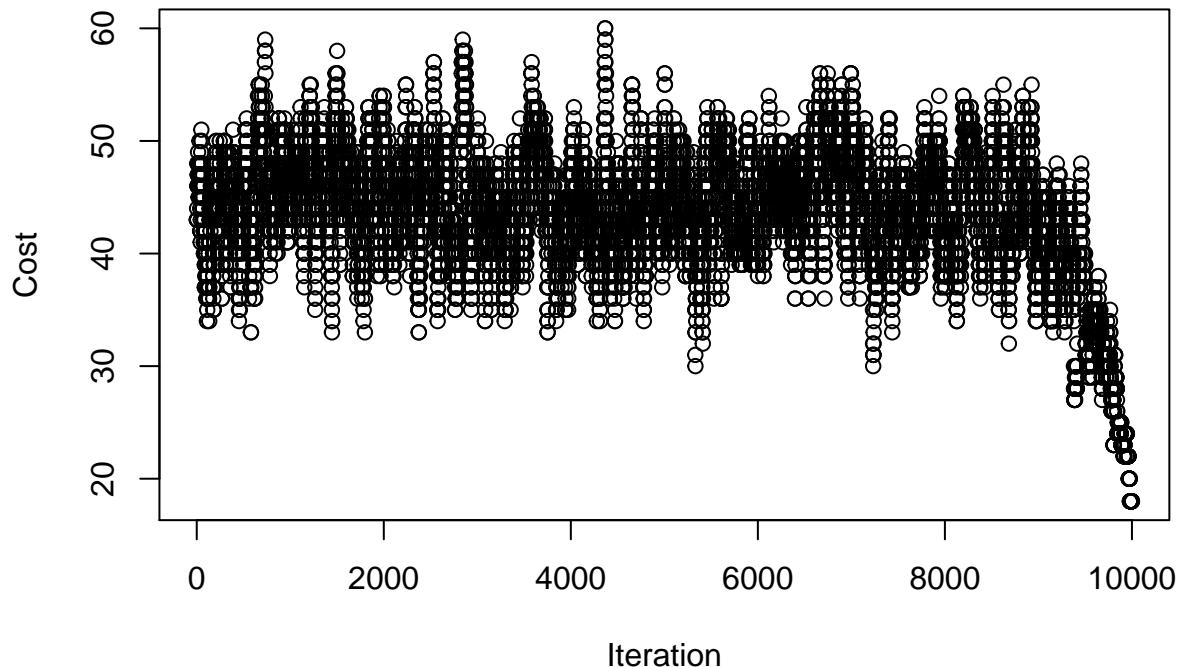
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 17 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



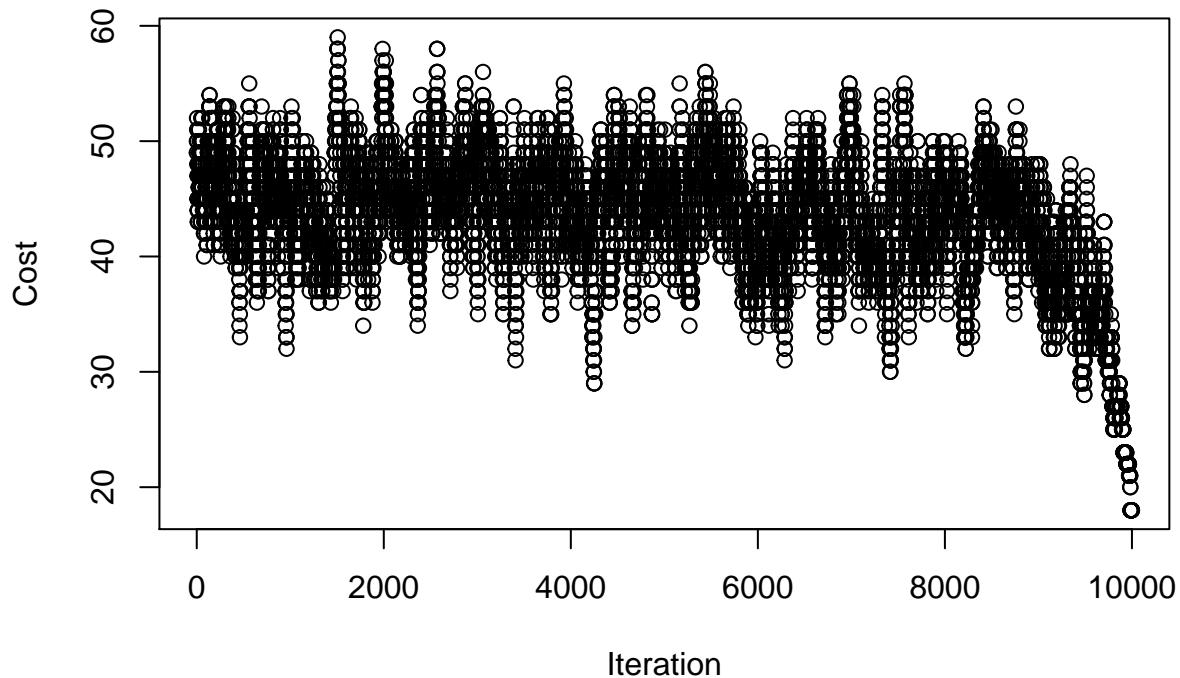
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 18 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



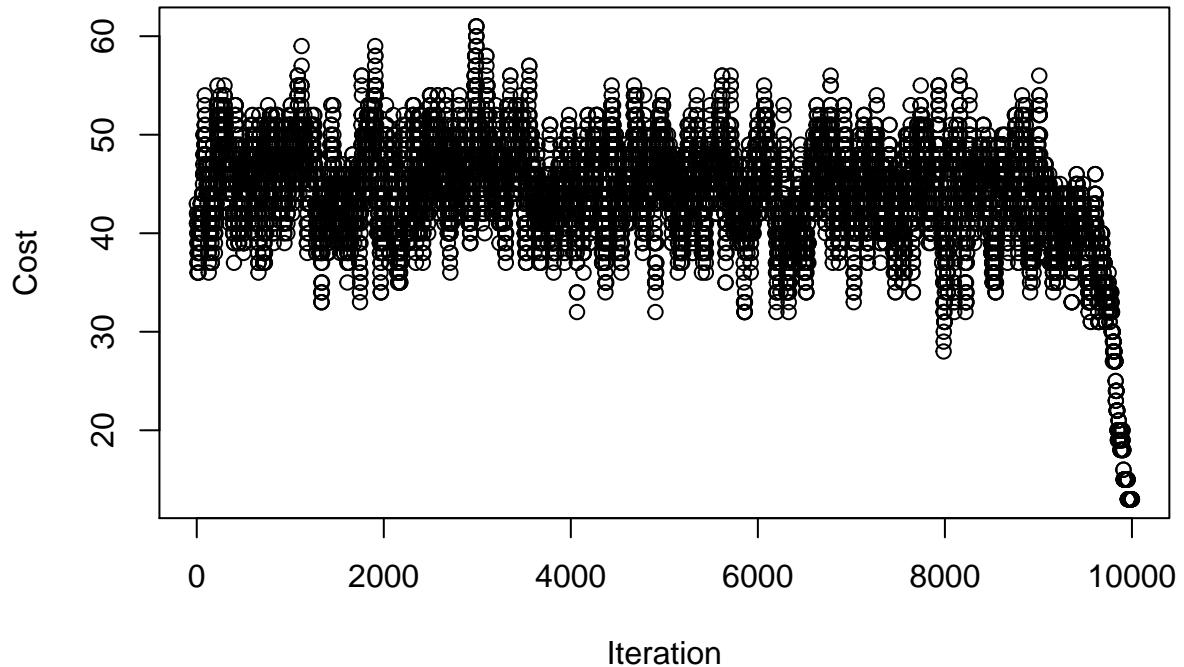
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 18 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



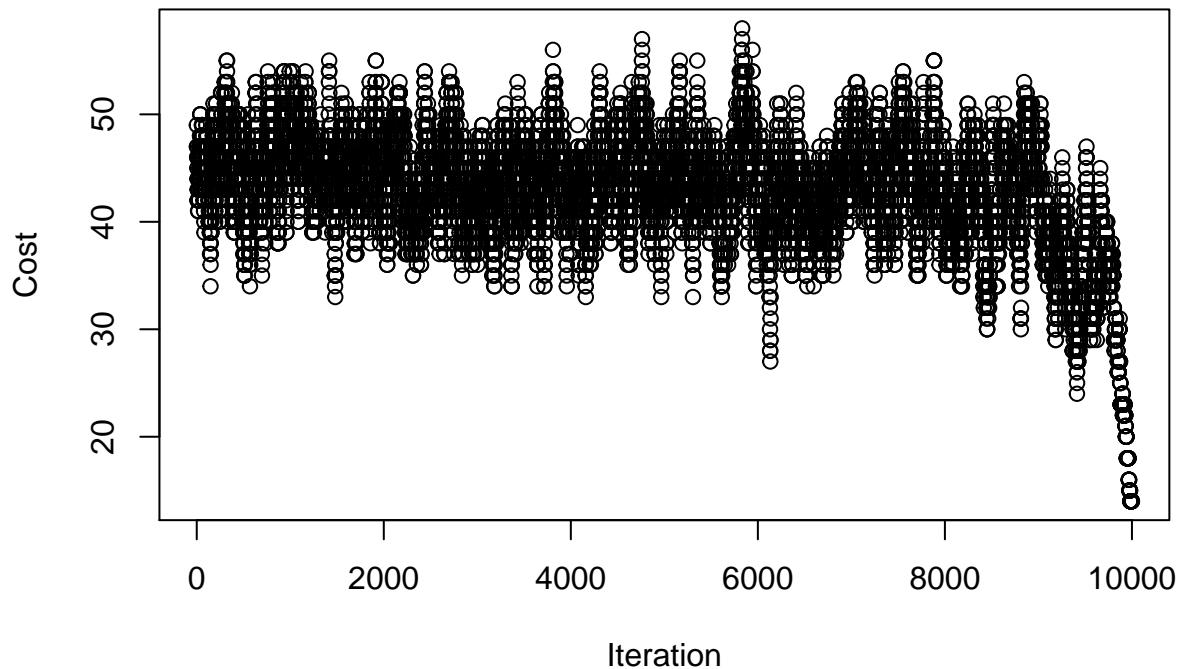
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 13 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



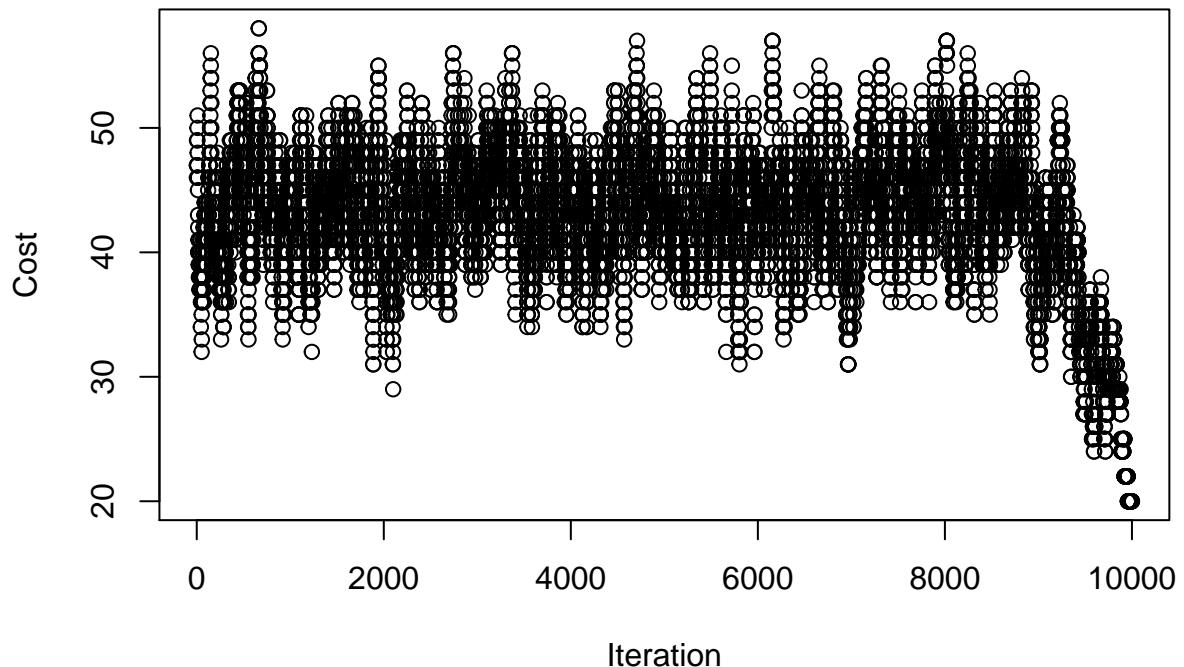
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 14 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



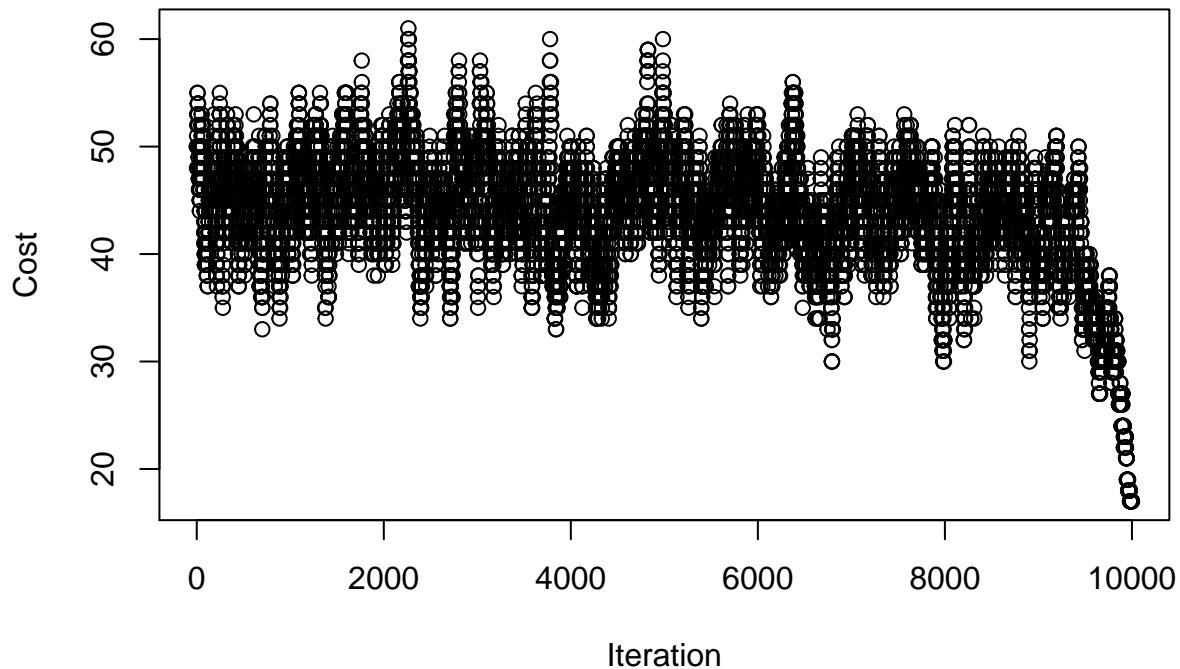
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 20 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



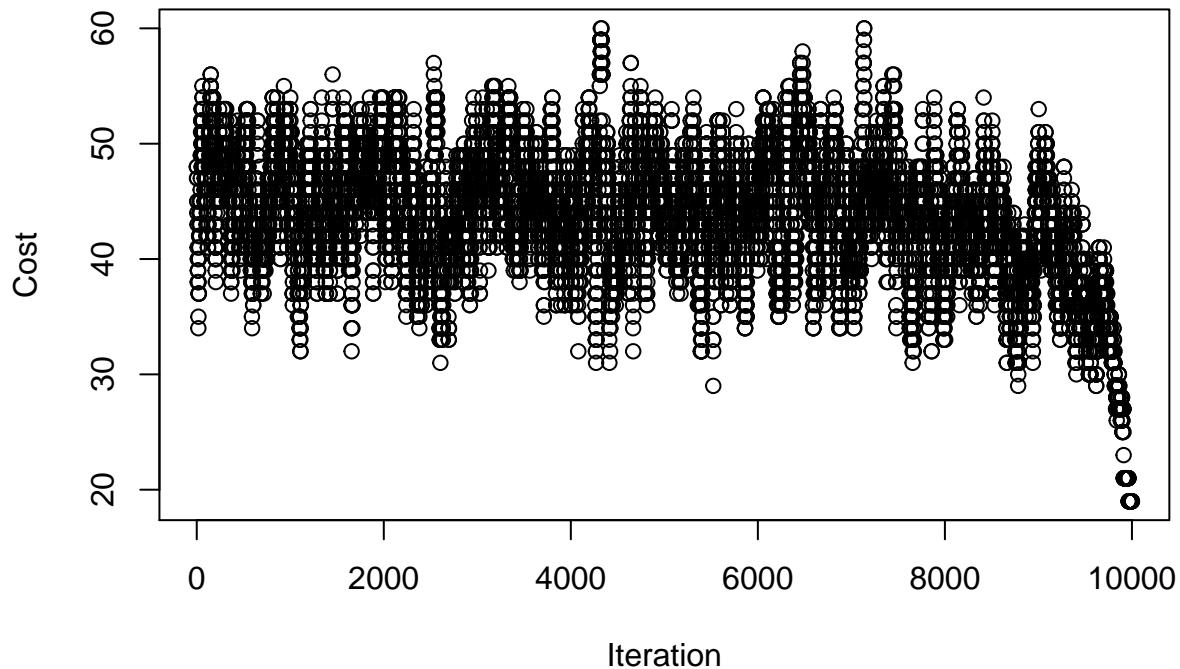
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 17 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



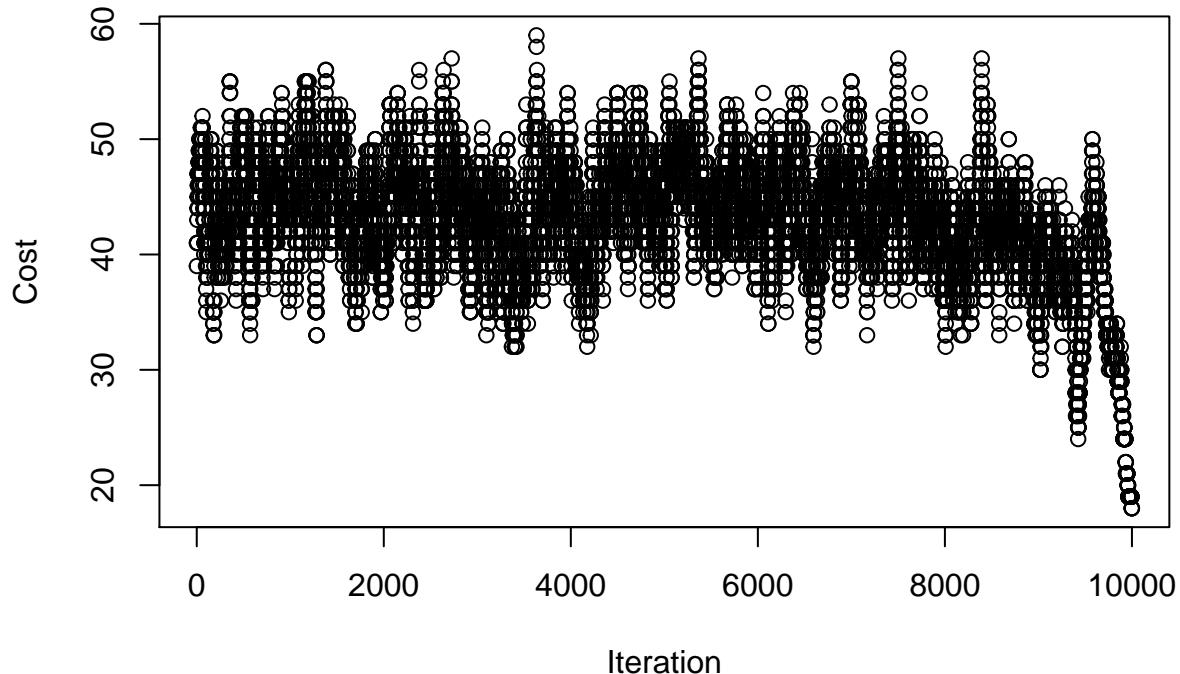
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 19 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



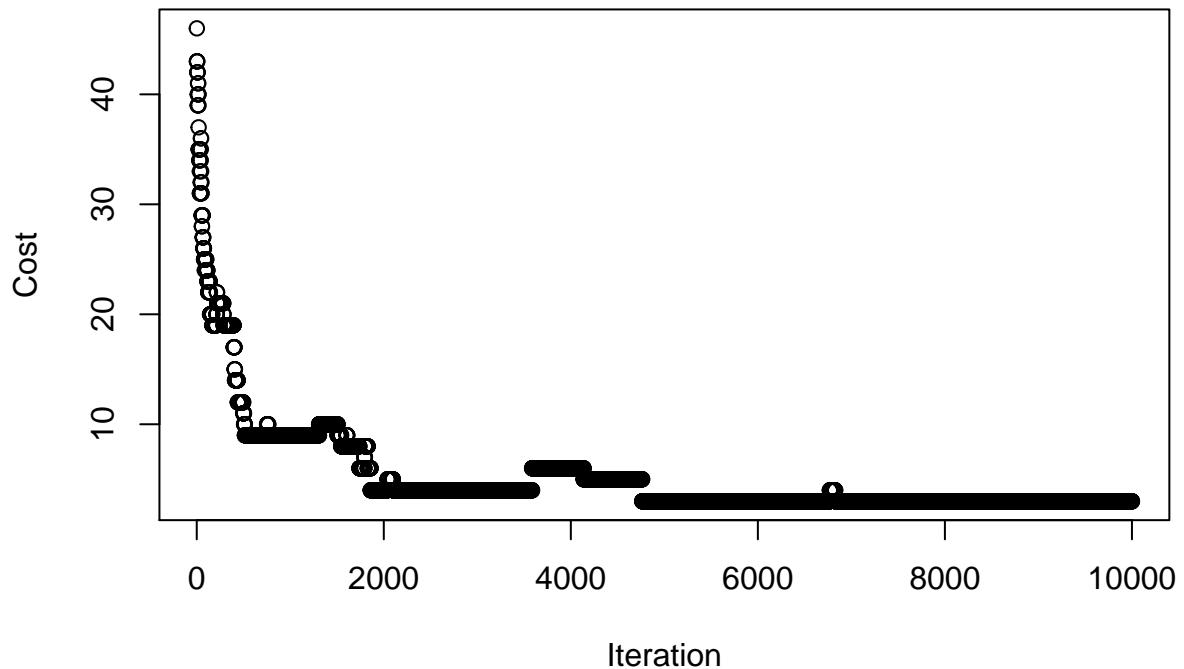
```
## Ran simulated annealing with linear cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 18 .  
## Final temperature is 0.01  
## Number of reheat in the run 0
```

Cost for Each Iteration



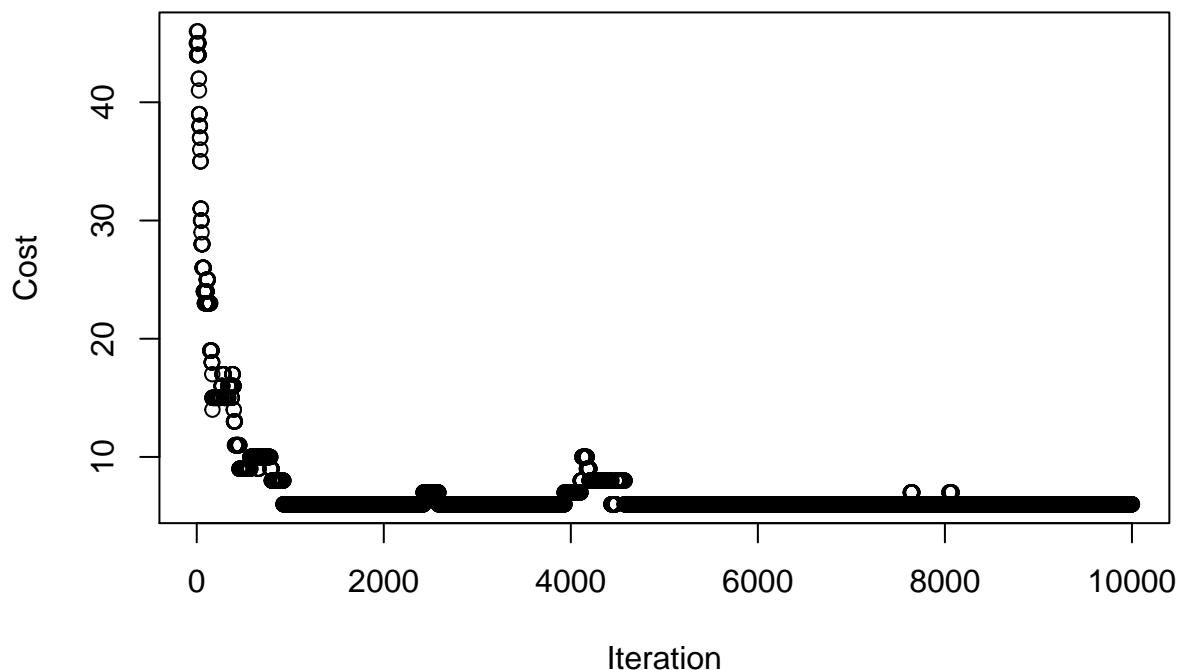
```
for (i in 1:10){  
  solve_sudoku_log_cooling(10000, 50, ex4)  
}  
  
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 3 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 34
```

Cost for Each Iteration



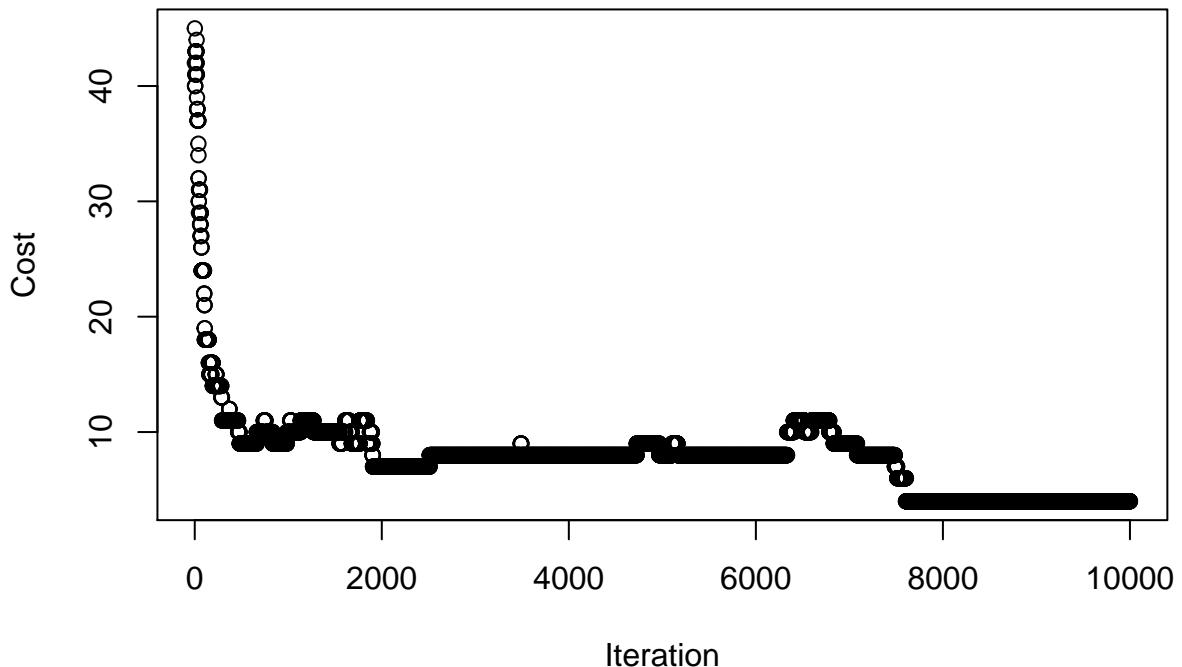
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 6 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 10
```

Cost for Each Iteration



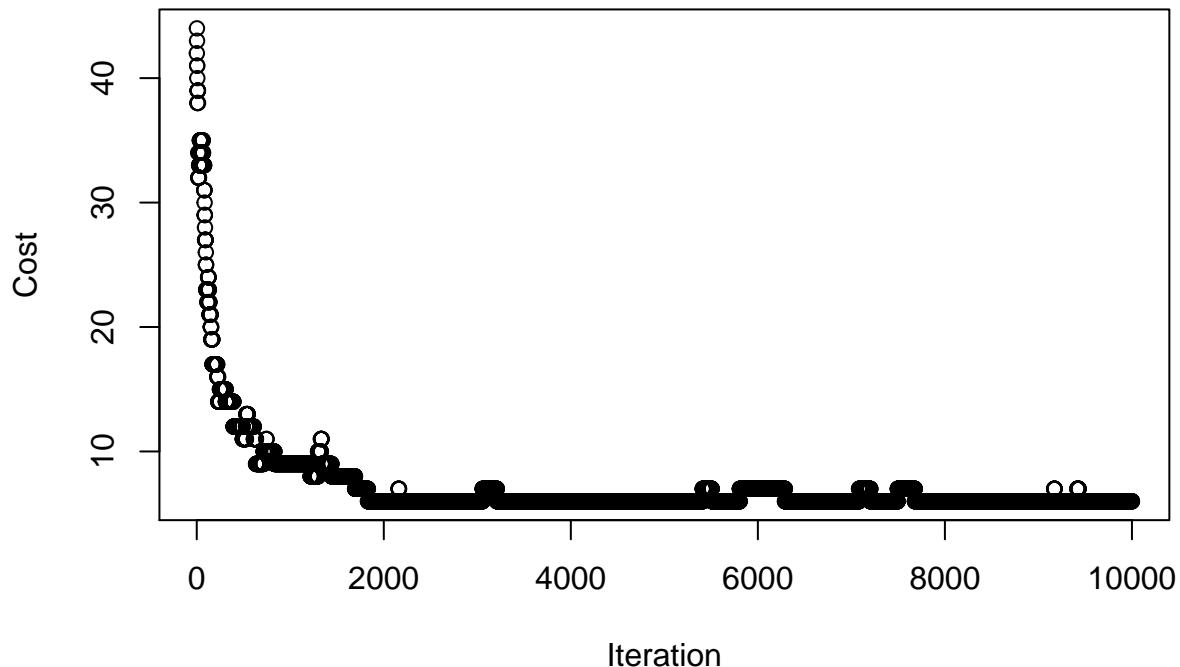
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 24
```

Cost for Each Iteration



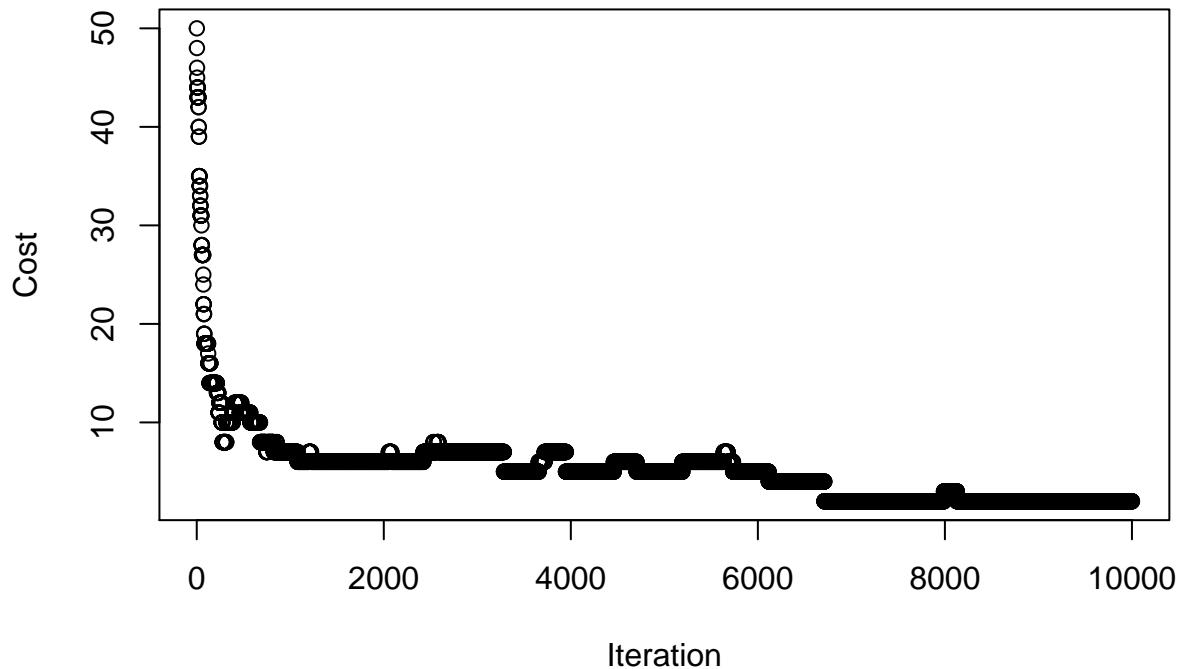
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 6 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 13
```

Cost for Each Iteration



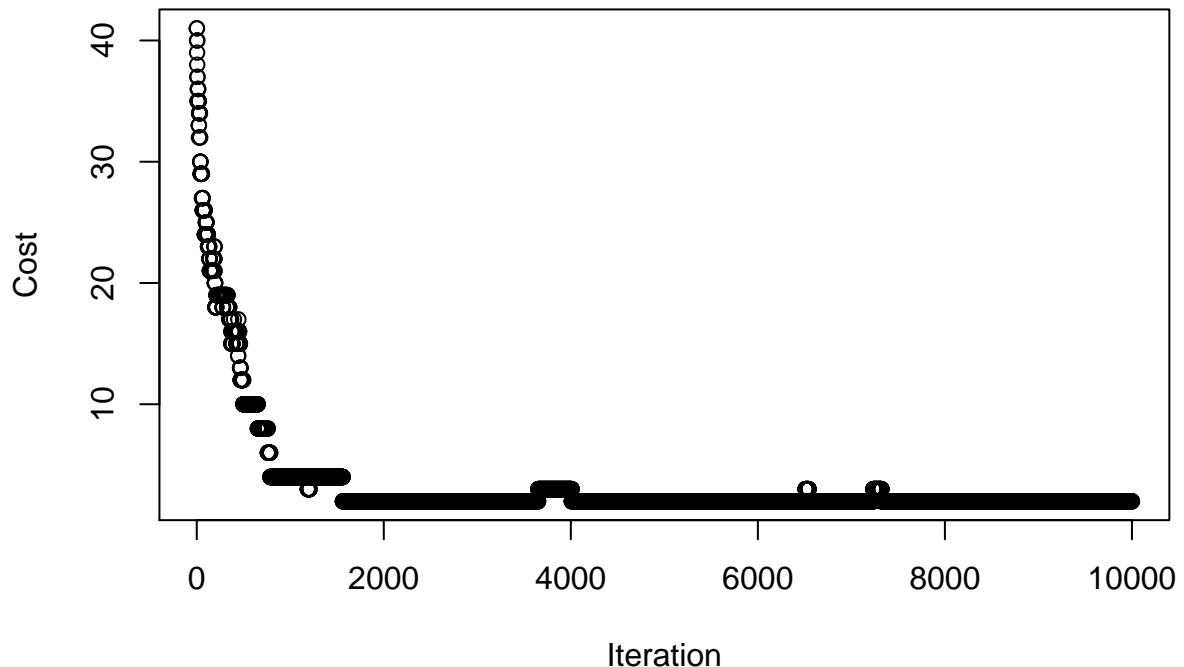
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 63
```

Cost for Each Iteration



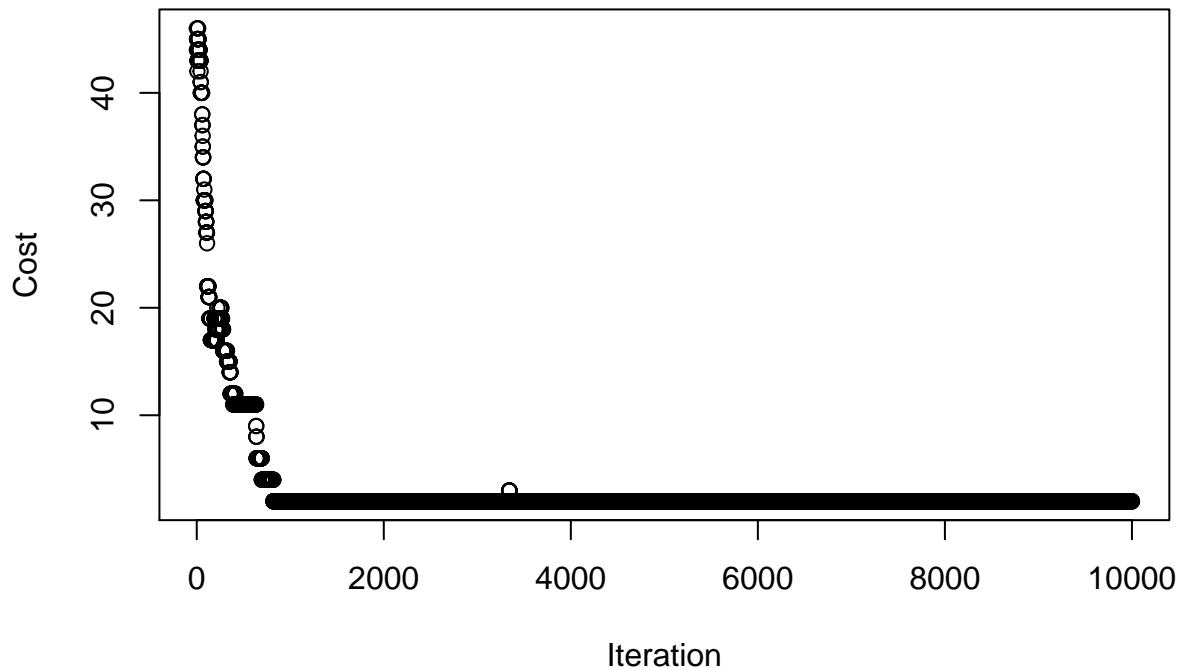
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 48
```

Cost for Each Iteration



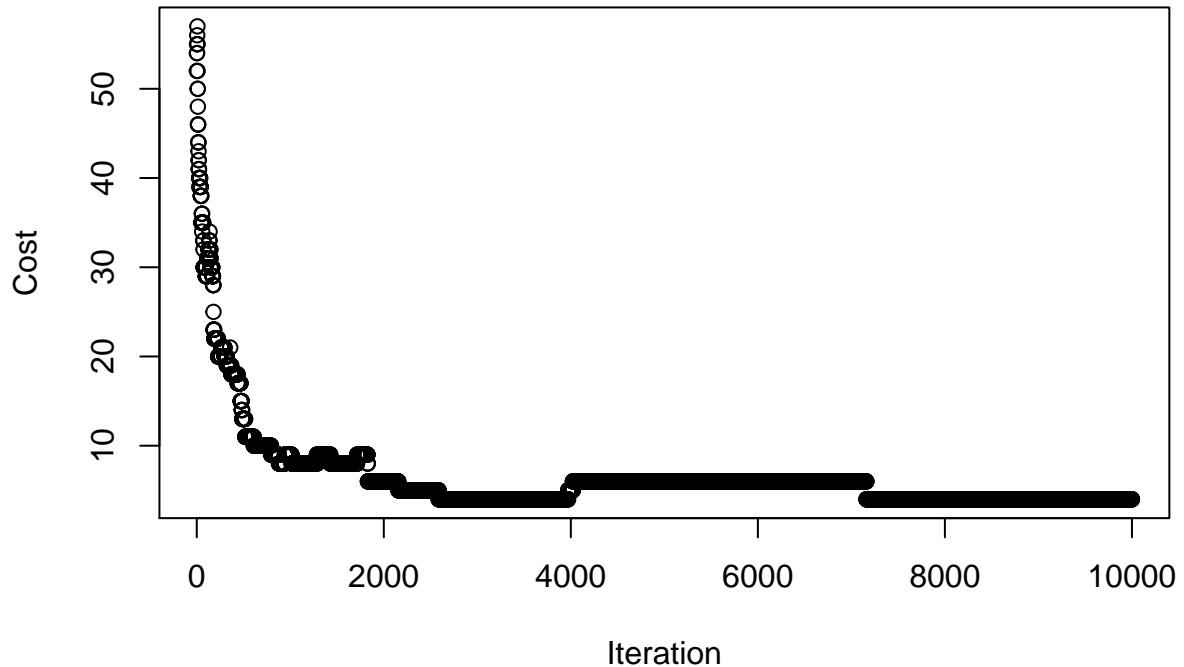
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 2 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 51
```

Cost for Each Iteration



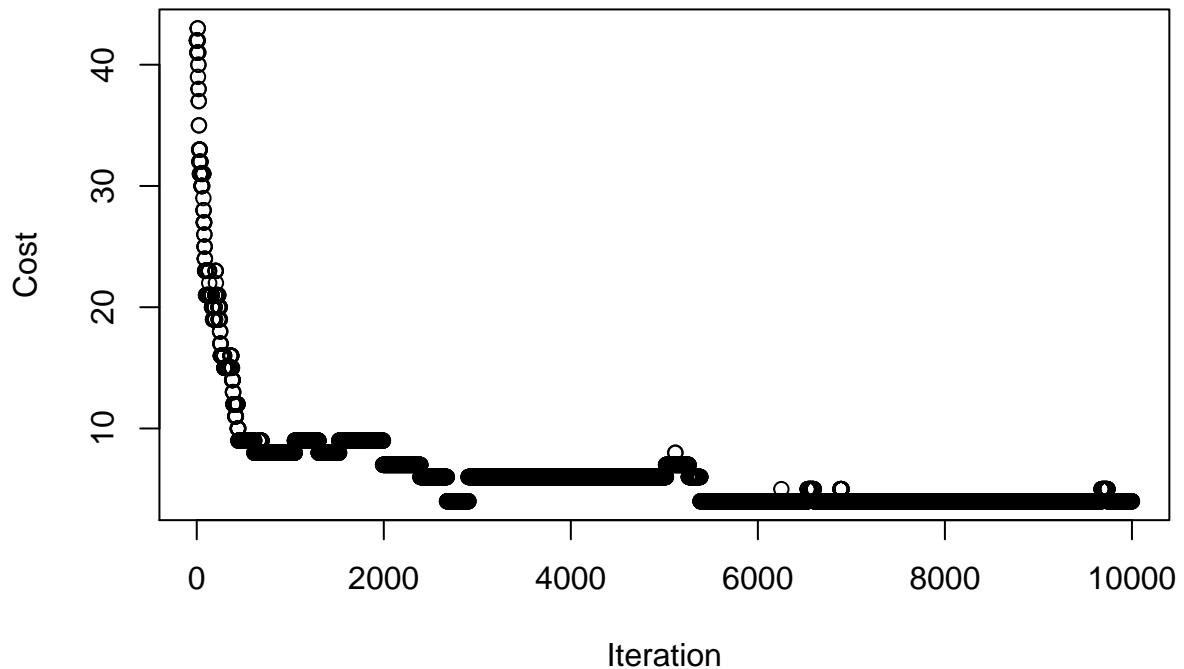
```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 52
```

Cost for Each Iteration



```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 10.21714  
## Number of reheat in the run 24
```

Cost for Each Iteration



```
## Ran simulated annealing with logarithmic cooling for 10000 iterations.  
## Solution not found .  
## Minimum cost reached 4 .  
## Final temperature is 0.2171449  
## Number of reheat in the run 33
```

Cost for Each Iteration

