Heuristics:

**Type one**

Exponential of the height difference

public double getCost(final Point p1, final Point p2)

{

      return Math.exp(getTile(p2) - getTile(p1));

}

h(n) should be considered in different conditions:

(1). Dy =<0; h(n)= e^(Dy)+|Dx|.

(2). Dy>0; In this case there are two types.

(a). Dy-|Dx|=<0; h(n)=|Dx|+Dy\*e;

(b). Dy-|Dx|>0;

If (Dy-|Dx|) mod 2 ==0;

h(n)= Dy\*(1+e);

else

h(n)=Dy\*(1+e)-1;

Prove:

Case (1): This heuristic is for best case of movement, it is the situation that is above the target point. The cost is smaller than one and exponentially decreased. So h(n) is smaller than any other costs. Thus it is an admissible heuristic.

Case (2): For the exponential increasing cost, the smaller the index , the smaller the cost. Thus, when the step of increasing height is equal to 1, we get the best case of movement. Here we disregard the difference of e^2 and (2+2\*e). In both case (a) and (b), the largest cost of one step is e, so these two heuristics are the smallest for all types of movements. Therefore, the heuristics in Case (2) are admissible.

**Type Two:**

New height divided by old height

public double getCost(final Point p1, final Point p2)

{

      return (getTile(p2) / (getTile(p1) + 1));

}

h(n) is related to height.

Let the height of start point be Ys, height of end point be Ye. Ye is always larger than Ys. Make a point on the path to be M, the height to be Ym.

The best case heuristic is h(n)= (Ys+Ye)/(Ys+1);

Prove:

We use the other two types of movements to compare our best case.

Type 1.

H1(n)= Ye/(Ys+1)+Ye/(Ye+1); Because Ye is always larger than Ys, h(n)<h1(n);

Type 2.

H2(n)= Ys/(Ys+1)+Ym/(Ys+1)+Ym/(Ym+1)+Ye/(Ym+1); This is the case that we add some points to change the route. We choose the case of one point to prove it.

Let’s calculate (h(n)-h2(n)). The result is (Ye-Ym)/(Ys+1)-(Ye+Ym)/(Ym+1). We differentiate from the formula and get the maximum value when Ym= sqrt(Ye\*(Ys+1))-1. It never exceeds 0, take Ye=8, Ys=1, Ym=3 as an example. The value is -1/3. Thus h(n) is the minimal cost. In other word, h(n) is an admissible heuristic.