MATB24 Assignment1
Solutions:

1.

2. 1.2.1 from textbook:

3. (a) We consider $+_R$ and $\times_R$ to be standard operation.

To show that real $n \times n$ matrices form a ring, we need to verify that they satisfy the following axioms of a ring:

1. Addition is closed: For any two $n \times n$ matrices A and B with real entries, their sum $A + B$ is also an $n \times n$ matrix with real entries.

Proof: Let $A_{ij} = (a_{i,j})$ and $B_{ij} = (b_{i,j})$, where $a_{i,j}, b_{i,j} \in \mathbb{R}$ for all $1 \leqslant i, j \leqslant n$

$$(A + B)_{ij} = (a_{i,j} + b_{i,j}) \in \mathbb{R}$$

2. Multiplication is closed: For any two $n \times n$ matrices A and B with real entries, their product AB is also an $n \times n$ matrix with real entries.

Proof: Let $A_{ij} = (a_{i,j})$ and $B_{ij} = (b_{i,j})$, where $a_{i,j}, b_{i,j} \in \mathbb{R}$ for all $1 \leqslant I, j \leqslant n$

$$(A * B)_{ij} = \sum_{k=1}^{n} a_{i,k} b_{k,j}$$

3. The additive identity exists.
There exists an $n \times n$ matrix 0 with real entries such that $A + 0 = A$ for any $n \times n$ matrix A with real entries. Which is the zero matrix.

Proof: Let $A_{ij} = (a_{i,j})$, where $a_{i,j} \in \mathbb{R}$ for all $1 \leqslant i, j \leqslant n$

$$A_{ij} + 0_{i,j} = a_{ij} + 0 = A_{ij}$$

4. The additive inverse exists.
For any $n \times n$ matrix A with real entries, there exists an $n \times n$ matrix $-A$ with real entries such that $A + (-A) = 0$.

Proof. Let $A_{ij} = (a_{i,j})$ and $-A_{ij} = (-a_{i,j})$, where $a_{i,j} \in \mathbb{R}$ for all $1 \leqslant I, j \leqslant n$. Then

$$(A + (-A))_{ij} = a_{i,j} + (-a_{i,j} = 0.$$

5. Associative with addition.
For any three $n \times n$ matrices A, B, and C with real entries, we have $(A+B)+C = A + (B + C)$.

Let $A_{ij} = (a_{i,j})$, $B_{ij} = (b_{i,j})$ and $C_{ij} = (c_{i,j})$, where $a_{i,j}, b_{i,j}, c_{i,j} \in \mathbb{R}$ for all $1 \leqslant i \leqslant n$

$$\begin{aligned}
[A + (B + C)]_{ij} &= a_{i,j} + (b_{i,j} + c_{i,j}) \\
&= (a_{i,j} + b_{i,j}) + c_{i,j} \\
&= [(A + B) + C]_{ij}
\end{aligned}$$

6. Commutative with addition.
For any two $n \times n$ matrices A and B with real entries, we have $A+B = B+A$.

Let $A_{ij} = (a_{i,j})$ and $B_{ij} = (b_{i,j})$, where $a_{i,j}, b_{i,j} \in \mathbb{R}$ for all $1 \leqslant i, j \leqslant n$. Then

$$\begin{aligned}
(A + B)_{ij} &= a_{i,j} + b_{i,j} \\
&= b_{i,j} + a_{i,j} \\
&= (B + A)_{ij}
\end{aligned}$$

7. The multiplication identity exist.
There exists an $n \times n$ matrix I with real entries such that $AI = IA = A$ for any $n \times n$ matrix A with real entries.

Let $A_{ij} = (a_{i,j})$ and $I_{ij} = (I_{i,j})$, where $a_{i,j}, b_{i,j} \in \mathbb{R}$ for all $1 \leqslant i, j \leqslant n$. $I_{i,j} = 0$ if $i \neq j$. Otherwhise, $I_{i,i} = 0$.

$$(AI)_{ij} = \sum_{k=1}^{n} a_{i,k} I_{k,j} = a_{i,j} = A_{ij}$$

8. Associative with multiplication.
For any three $n \times n$ matrices A, B, and C with real entries, we have $(AB)C = A(BC)$.

(This one is hard) Let $A_{ij} = (a_{i,j})$, $B_{ij} = (b_{i,j})$ and $C_{ij} = (c_{i,j})$, where $a_{i,j}, b_{i,j}, c_{i,j} \in \mathbb{R}$ for all $1 \leqslant I, j \leqslant n$

$$
\begin{aligned}
(AB)C &= \sum_{l=1}^{n}\left(\sum_{k=1}^{n} a_{i,k}b_{k,l}\right)c_{l,j} \\
&= \sum_{l=1}^{n}\sum_{k=1}^{n}(a_{i,k}b_{k,l}c_{l,j}) \\
&= \sum_{k=1}^{n}\sum_{l=1}^{n}(a_{i,k}b_{k,l}c_{l,j}) \\
&= \sum_{k=1}^{n} a_{i,k}\left(\sum_{l=1}^{n} b_{k,l}c_{l,j}\right) \\
&= A(BC)
\end{aligned}
$$

9. Multiplication distributes over addition.
For any three $n \times n$ matrices A, B, and C with real entries, we have $A(B+C) = AB + AC$ and $(A+B)C = AC + BC$.

Let $A_{ij} = (a_{i,j})$, $B_{ij} = (b_{i,j})$ and $C_{ij} = (c_{i,j})$, where $a_{i,j}, b_{i,j}, c_{i,j} \in \mathbb{R}$ for all $1 \leqslant I, j \leqslant n$

$$
\begin{aligned}
A(B+C) &= \sum_{k=1}^{n} a_{i,k}(b_{k,j} + c_{k,j}) \\
&= \sum_{k=1}^{n} a_{i,k}b_{k,j} + a_{i,k}c_{k,j}) \\
&= \sum_{k=1}^{n} a_{i,k}b_{k,j} + \sum_{k=1}^{n} a_{i,k}c_{k,j}) \\
&= AB + AC
\end{aligned}
$$

With these axioms, we can verify that real $n \times n$ matrices form a ring.

4. (a) i. $\mathbb{Z}_8$ is not a field since not every nonzero element has a multiplicative inverse. For example, 2 does not have a multiplicative inverse in $\mathbb{Z}_8$.

   ii. $\{0, 2, 4, 6\}$

   (b) i. $\mathbb{Z}_5$ is a field because 5 is a prime number. In lecture note page 5 we show that $\mathbb{Z}_n$ is a field iff n is a prime number.

   ii. Every nontrivial proper subset of $\mathbb{Z}_5$ is an ideal since $\mathbb{Z}_5$ is a finite field.

5. (a) To show that polynomials form a vector space over $\mathbb{F}^n$, we need to show that they satisfy the following properties:

1. Closure under addition: If $p(x)$ and $q(x)$ are polynomials, then $p(x) + q(x)$ is also a polynomial.

Proof: Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum_{i=0}^{\infty} q_i x^i$, where $p_i, q_i \in \mathbb{F}$

$$\begin{aligned} h(x) = p(x) + q(x) &= \sum_{i=0}^{\infty} p_i x^i + q_i x^i \\ &= \sum_{i=0}^{\infty} (p_i + q_i) x^i \in \mathbb{F}[x] \end{aligned}$$

$(p_i + q_i) \in \mathbb{F}$

2. Closure under scalar multiplication: If $p(x)$ is a polynomial and a is a scalar, then $ap(x)$ is also a polynomial.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$

$$\begin{aligned} ap(x) &= a \sum_{i=0}^{\infty} p_i x^i \\ &= \sum_{i=0}^{\infty} a * p_i x^i \end{aligned}$$

$(a * p_i) \in \mathbb{F}$

3. Associativity of addition: For any polynomials $p(x)$, $q(x)$, and $h(x)$, $h(x) + (p(x) + q(x)) = (h(x) + p(x)) + q(x)$.

Proof: Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, $q(x) = \sum_{i=0}^{\infty} q_i x^i$, and $h(x) = \sum_{i=0}^{\infty} h_i x^i$, where $p_i, q_i, h_i \in \mathbb{F}$

$$\begin{aligned} h(x) + (p(x) + q(x)) &= \sum_{i=0}^{\infty} h_i x^i + (p_i x^i + q_i) x^i \\ &= \sum_{i=0}^{\infty} (h_i + p_i) x_i + q_i x^i \in \mathbb{F}[x] \\ &= (h(x) + p(x)) + q(x) \end{aligned}$$

$(p_i + q_i) \in \mathbb{F}$.

4. Commutativity of addition: For any polynomials $p(x)$ and $q(x)$, $p(x) + q(x) = q(x) + p(x)$.

Proof: Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum_{i=0}^{\infty} q_i x^i$, where $p_i, q_i \in \mathbb{F}$

$$\begin{aligned} p(x) + q(x) &= \sum_{i=0}^{\infty} p_i x^i + q_i x^i \\ &= \sum_{i=0}^{\infty} q_i x^i + p_i x^i \\ &= q(x) + p(x) \end{aligned}$$

5. Existence of zero vector: There exists a polynomial 0 such that for any polynomial $p(x)$, $p(x) + 0 = p(x)$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $0(x) = \sum_{i=0}^{\infty} 0x^i$, where $p_i, \in \mathbb{F}$

$$\begin{aligned} p(x) + 0(x) &= \sum_{i=0}^{\infty} p_i x^i + 0x^i \\ &= \sum_{i=0}^{\infty} (p_i + 0)x^i \\ &= p(x) \end{aligned}$$

6. Existence of additive inverse: For any polynomial $p(x)$, there exists a polynomial -$p(x)$ such that $p(x) + (-p(x)) = 0$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$
Let $-p(x) = \sum_{i=0}^{\infty} -p_i x^i$, where $p_i, \in \mathbb{F}$

$$\begin{aligned} p(x) + (-p(x)) &= \sum_{i=0}^{\infty} p_i x^i + (-p_i x^i) \\ &= \sum_{i=0}^{\infty} (p_i - p_i)x^i \\ &= \sum_{i=0}^{\infty} 0x^i \end{aligned}$$

7. Identity element of scalar multiplication: For any polynomial $p(x)$, $1p(x) = p(x)$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $1 \in \mathbb{F}$

$$
\begin{aligned}
1 * p(x) &= 1 * \sum_{i=0}^{\infty} p_i x^i \\
&= \sum_{i=0}^{\infty} (1 * p_i) x^i \\
&= \sum_{i=0}^{\infty} p_i x^i \\
&= p(x)
\end{aligned}
$$

8. Associativity of scalar multiplication: For any scalars a and b and polynomial $p(x)$, $a(bp(x)) = (ab)p(x)$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $a, b \in \mathbb{F}$

$$
\begin{aligned}
a(bp(x)) &= a(b \sum_{i=0}^{\infty} p_i x^i \\
&= a \sum_{i=0}^{\infty} (b * p_i) x^i \\
&= \sum_{i=0}^{\infty} (a * b) p_i x^i \\
&= (a * b) \sum_{i=0}^{\infty} p_i x^i \\
&= (a * b) p(x)
\end{aligned}
$$

9. Distributivity of scalar multiplication over scalar addition: For any scalars a and b and polynomial $p(x)$, $(a + b)p(x) = ap(x) + bp(x)$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $a, b \in \mathbb{F}$

$$
\begin{aligned}
(a+b)p(x)) &= (a+b)\sum_{i=0}^{\infty} p_i x^i \\
&= a\sum_{i=0}^{\infty}(a+b) * p_i x^i \\
&= \sum_{i=0}^{\infty} a * p_i x^i + b p_i x^i \\
&= \sum_{i=0}^{\infty} a * p_i x^i + \sum_{i=0}^{\infty} b p_i x^i \\
&= ap(x) + bp(x)
\end{aligned}
$$

10. Distributivity of scalar multiplication over vector addition: For any scalar a and polynomials $p(x)$ and $q(x)$, $a(p(x)+q(x)) = ap(x) + aq(x)$.

All of these properties can be verified using the basic properties of addition and multiplication of real numbers. Therefore, polynomials form a vector space over $\mathbb{R}^n$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $q(x) = \sum_{i=0}^{\infty} q_i x^i$, where $q_i, \in \mathbb{F}$ Let $a \in \mathbb{F}$

$$
\begin{aligned}
a(p(x)+q(x)) &= a\left(\sum_{i=0}^{\infty} p_i x^i + \sum_{i=0}^{\infty} p_i x^i\right) \\
&= a\sum_{i=0}^{\infty}(p_i+q_i)x^i \\
&= \sum_{i=0}^{\infty} a * p_i x^i + a q_i x^i \\
&= \sum_{i=0}^{\infty} a * p_i x^i + \sum_{i=0}^{\infty} a q_i x^i \\
&= ap(x) + aq(x)
\end{aligned}
$$

(b) We consider $+_R$ and $\times_R$ to be standard operation.

To show that polynomials form a ring over $\mathbb{F}$, we need to verify that they satisfy the following axioms of a ring:

1. Addition is closed: If $p(x)$ and $q(x)$ are polynomials, then $p(x)+q(x)$ is also a polynomial.

Proof: Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum_{i=0}^{\infty} q_i x^i$, where $p_i, q_i \in \mathbb{F}$

$$
\begin{aligned}
h(x) = p(x) + q(x) &= \sum_{i=0}^{\infty} p_i x^i + q_i x^i \\
&= \sum_{i=0}^{\infty} (p_i + q_i) x^i \in \mathbb{F}[x]
\end{aligned}
$$

$(p_i + q_i) \in \mathbb{F}$

2. Multiplication is closed: If $p(x)$ and $q(x)$ are polynomials, then $p(x)q(x)$ is also a polynomial.

Proof: Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum_{j=0}^{\infty} q_j x^j$, where $p_i, q_j \in \mathbb{F}$

$$
\begin{aligned}
p(x)q(x) &= \sum_{i=0}^{\infty} p_i x^i \sum_{j=0}^{\infty} q_j x^j \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (p_i q_j) x^{i+j} \in \mathbb{F}[x]
\end{aligned}
$$

$(p_i q_j) \in \mathbb{F}$

3. The additive identity exists.
There exists a polynomial 0 such that for any polynomial $p(x)$, $p(x) + 0 = p(x)$

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $0(x) = \sum_{i=0}^{\infty} 0x^i$, where $p_i, \in \mathbb{F}$

$$
\begin{aligned}
p(x) + 0(x) &= \sum_{i=0}^{\infty} p_i x^i + 0x^i \\
&= \sum_{i=0}^{\infty} (p_i + 0) x^i \\
&= p(x)
\end{aligned}
$$

4. The additive inverse exists.
For any polynomial $p(x)$, there exists a polynomial $-p(x)$ such that $p(x) + (-p(x)) = 0$.

Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$
Let $-p(x) = \sum_{i=0}^{\infty} -p_i x^i$, where $p_i, \in \mathbb{F}$

$$
\begin{aligned}
p(x) + (-p(x)) &= \sum_{i=0}^{\infty} p_i x^i + (-p_i x^i) \\
&= \sum_{i=0}^{\infty} (p_i - p_i) x^i \\
&= \sum_{i=0}^{\infty} 0 x^i
\end{aligned}
$$

5. Associative with addition.
For any polynomials $p(x)$, $q(x)$, and $h(x)$, $h(x) + (p(x) + q(x)) = (h(x) + p(x)) + q(x)$.

Proof: Let $p(x) = \sum\limits_{i=0}^{\infty} p_i x^i$, $q(x) = \sum\limits_{i=0}^{\infty} q_i x^i$, and $h(x) = \sum\limits_{i=0}^{\infty} h_i x^i$, where $p_i, q_i, h_i \in \mathbb{F}$

$$
\begin{aligned}
h(x) + (p(x) + q(x)) &= \sum_{i=0}^{\infty} h_i x^i + (p_i x^i + q_i) x^i \\
&= \sum_{i=0}^{\infty} (h_i + p_i) x_i + q_i x^i \in \mathbb{F}[x] \\
&= (h(x) + p(x)) + q(x)
\end{aligned}
$$

$(p_i + q_i) \in \mathbb{F}$.

6. Commutative with addition.
For any polynomials $p(x)$ and $q(x)$, $p(x) + q(x) = q(x) + p(x)$.

Proof: Let $p(x) = \sum\limits_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum\limits_{i=0}^{\infty} q_i x^i$, where $p_i, q_i \in \mathbb{F}$

$$
\begin{aligned}
p(x) + q(x) &= \sum_{i=0}^{\infty} p_i x^i + q_i x^i \\
&= \sum_{i=0}^{\infty} q_i x^i + p_i x^i \\
&= q(x) + p(x)
\end{aligned}
$$

7. The multiplication identity exist.
There exists a polynomial 1 such that for any polynomial $p(x)$, $1p(x) = p(x)$.

Let $p(x) = \sum\limits_{i=0}^{\infty} p_i x^i$, where $p_i, \in \mathbb{F}$ Let $1 \in \mathbb{F}$

$$\begin{aligned}
1 * p(x) &= 1 * \sum_{i=0}^{\infty} p_i x^i \\
&= \sum_{i=0}^{\infty} (1 * p_i) x^i \\
&= \sum_{i=0}^{\infty} p_i x^i \\
&= p(x)
\end{aligned}$$

8. Associative with multiplication.
For any polynomials $p(x)$, $q(x)$, and $r(x)$, $(p(x)q(x))r(x) = p(x)(q(x)r(x))$.

Proof: Let $p(x) = \sum\limits_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum\limits_{j=0}^{\infty} q_j x^j$ and $r(x) = \sum\limits_{k=0}^{\infty} r_k x^k$, where $p_i, q_j, r_k \in \mathbb{F}$

$$\begin{aligned}
(p(x)q(x))r(x) &= (\sum_{i=0}^{\infty} p_i x^i \sum_{j=0}^{\infty} q_j x^j) \sum_{k=0}^{\infty} r_k x^k \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (p_i q_j) x^{i+j} \sum_{k=0}^{\infty} r_k x^k \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} (p_i q_j) r_k x^{(i+j)+k} \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} p_i (q_j r_k) x^{i+(j+k)} \\
&= \sum_{i=0}^{\infty} p_i x^i (\sum_{j=0}^{\infty} q_j x^j \sum_{k=0}^{\infty} r_k x^k) \\
&= \sum_{i=0}^{\infty} p_i x^i (q(x)r(x)) \\
&= p(x)(q(x)r(x)) \in \mathbb{F}[x]
\end{aligned}$$

9. Multiplication distributes over addition.
For any polynomials $p(x)$, $q(x)$, and $r(x)$, $p(x)(q(x) + r(x)) = p(x)q(x) + p(x)r(x)$.

Proof: Let $p(x) = \sum\limits_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum\limits_{j=0}^{\infty} q_j x^j$ and $r(x) = \sum\limits_{j=0}^{\infty} r_j x^j$, where $p_i, q_j, r_k \in \mathbb{F}$

$$
\begin{aligned}
p(x)(q(x)+r(x)) &= \sum_{i=0}^{\infty} p_i x^i (\sum_{j=0}^{\infty} q_j x^j + \sum_{j=0}^{\infty} r_j x^j) \\
&= \sum_{i=0}^{\infty} p_i x^i (\sum_{j=0}^{\infty} (q_j + r_j) x^j) \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p_i x^i (q_j + r_j) x^j \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (p_i q_j + p_i r_j) x^{i+j} \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (p_i q_j x^{i+j} + p_i r_j x^{i+j}) \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p_i q_j x^{i+j} + \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} p_i r_j x^{i+j} \\
&= p(x)q(x) + p(x)r(x) \in \mathbb{F}[x]
\end{aligned}
$$

With these axioms, we can verify that polynomials form a ring.

(c) Polynomials satisfy all the properties of a ring, as shown in the part(b). To show that multiplication is commutative, we need to show that for any polynomials $p(x)$ and $q(x)$, $p(x)q(x) = q(x)p(x)$.

Proof: Let $p(x) = \sum_{i=0}^{\infty} p_i x^i$ and $q(x) = \sum_{j=0}^{\infty} q_j x^j$, where $p_i, q_j \in \mathbb{F}$

$$
\begin{aligned}
p(x)q(x) &= \sum_{i=0}^{\infty} p_i x^i \sum_{j=0}^{\infty} q_j x^j \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (p_i q_j) x^{i+j} \\
&= \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (q_j p_i) x^{j+i} \\
&= \sum_{j=0}^{\infty} \sum_{i=0}^{\infty} (q_j p_i) x^{j+i} \in \mathbb{F}[x]
\end{aligned}
$$

Therefore, polynomials form a commutative ring.

6. 2.1.1 from textbook:

7. 2.1.2 from textbook:

8. 2.1.3 from textbook:

9. 2.1.4 from textbook:

   2.1.7 from textbook:

10. 2.1.5 from textbook:

11. 2.2.1 from textbook:

12. 2.2.2 from textbook:

13. 2.2.3 from textbook:

14. 2.3.2 from textbook:

15. 2.3.3 from textbook:

16. 2.3.10 from textbook:

17. 2.3.12 from textbook:

18. 6.6.1 from textbook:

19. 6.6.2 from textbook:

20. A set v_1, v_2, v_3 is a basis of $\mathbb{R}^3$ if v_1, v_2, v_3 is a set of linear independent vectors that generates $\mathbb{R}^3$.

    Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ Let $v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ Let $v_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

In $\mathbb{R}^3$: $\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$ which indicate that v_1, v_2, v_3 are linear independent. Becuase the rank is 3, so v_1, v_2, v_3 spans $\mathbb{R}^3$ which is a basis of $\mathbb{R}^3$.

In $(\mathbb{Z}_2)^3$: $(v_1 + v_2) + v_3 = (\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}) + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ which is linear dependent. So it is not a basis.