02285 AI and MAS, F23
Conflict based search

Todays subjects:

- Multi-agent pathfinding (MAPF)
- Baseline algorithm: Multibody A*
- Explore three (and a half) other algorithms
- In particular, conflict based search

MAPF problem

Multi-agent pathfinding (MAPF) problem

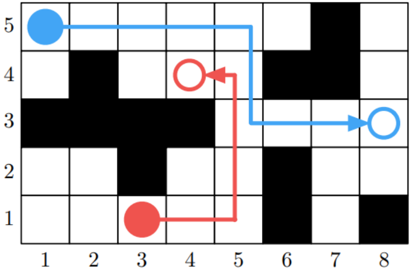
Given a problem instance

- a graph $G = (V, E)$, where vertices V of the graph are possible locations for the agents, and the edges E are the possible transitions between locations;
- a set $a_1, a_2, ..., a_k$ of k agents;
- a set $v_{1,0}, v_{2,0}, ..., v_{k,0}$ of start locations for each agent;
- a set $g_1, g_2, ..., g_k$ of goal locations for each agent;

A solution is a plan $\pi_i = (v_{i,0}, v_{i,1}, ..., g_i)$ for each agent a_i, i.e., a sequence of vertices that lead from the start to the goal.

MAPF problem

- Simpler than the hospital domain
  - No boxes
  - All agents have one goal
  - But considers general graphs, not only grids
- Today we will look at different algorithms for solving this problem



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$$h(s) = \sum_{i=1}^{k} dist(v_{i,s}, g_i)$$

  where

  - $v_{i,s}$ is the vertex where agent a_i is located in state s, and
  - $dist : V \mapsto V$ is the shortest distance from any vertex to any other vertex (can be pre-computed, e.g., using the Floyd-Warshall algorithm in $O(|V|^3)$ time)

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- MBA* is complete (finds a solution if there is one)
- MBA* is optimal (finds the shortest solution if heuristic is admissible)

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- Time/space complexity: $O(b^{kd})$ (upper bound)

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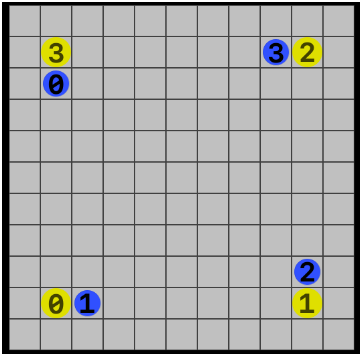

- In a grid domain $b = 5$: Move(N), Move(E), Move(S), Move(W), Noop
- $k = 4$ agent
- Solution depth $d = 7$
- Heuristic guides search perfectly!

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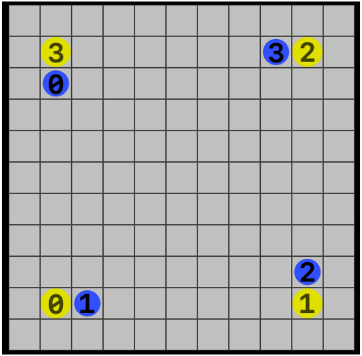

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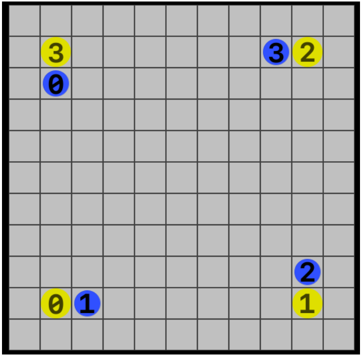

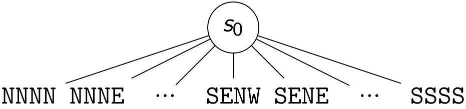

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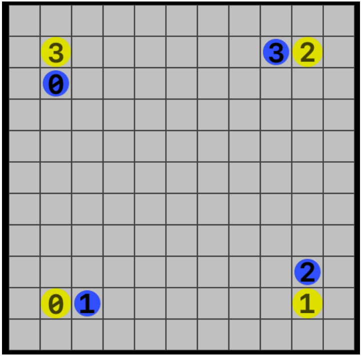

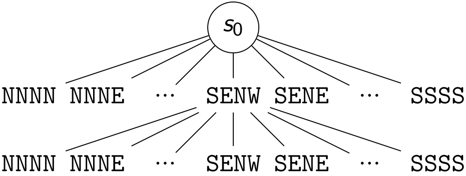

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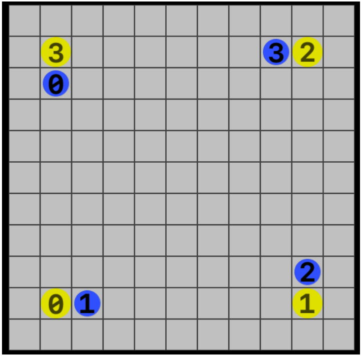

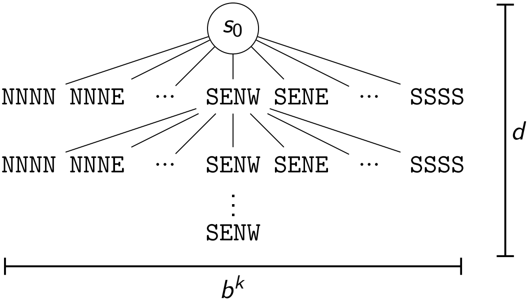

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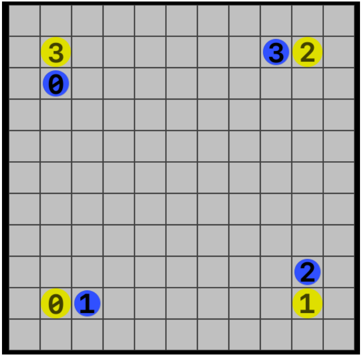

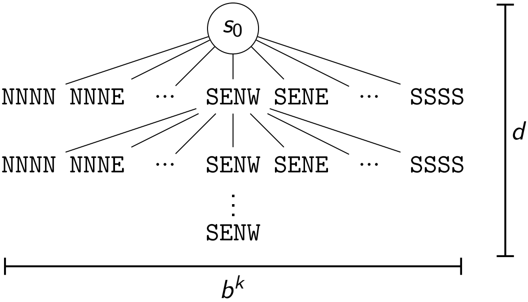

We expand $d = 7$ states, but generate $db^k = 7 \times 5^4 \approx 4300$ states(!)

If each agent could plan for itself, we would generate only $dbk = 7 \times 5 \times 4 = 140$ states

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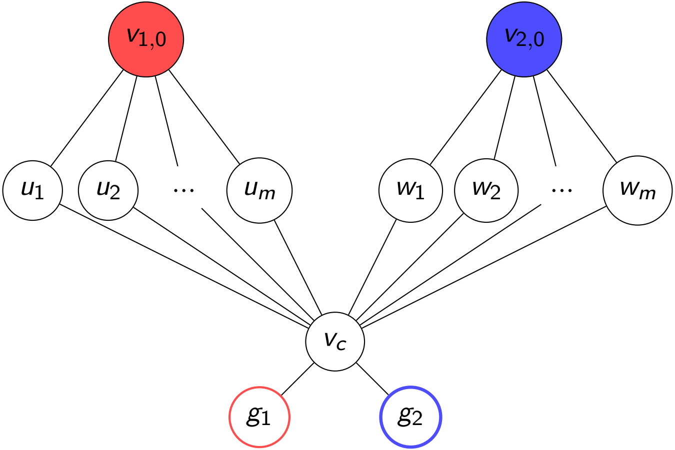

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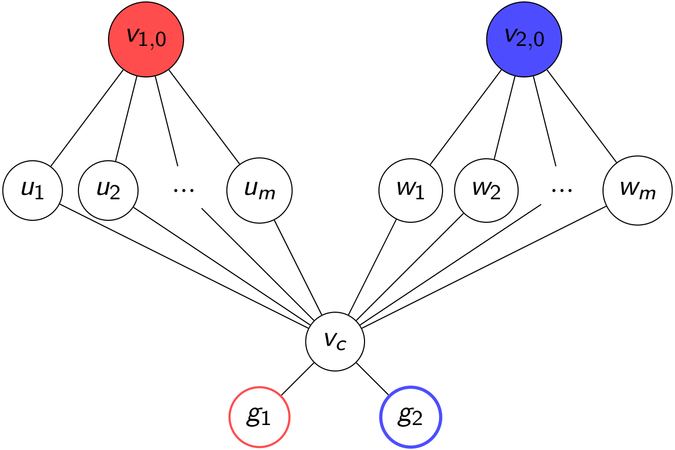

Worst case: All search tree paths branches of length 3 are expanded before we find the solution of length 4 (one agent has to wait), i.e., all $m^2 = b^k$ nodes are expanded.

Solution 2: ???

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- "Best" case (heuritic guides search perfectly):
  - Reduce branching factor
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  - ... instead of one problem of $O\left(db^k\right)$ (140 vs 4300)

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- Can we use CDPS (cooperative distributed problem solving) for MAPF?
  1. Problem decomposition
  2. Subproblem solution
  3. Solution synthesis

Solution 2: ???

Discuss: How do we ensure the plans of each agent are compatible?

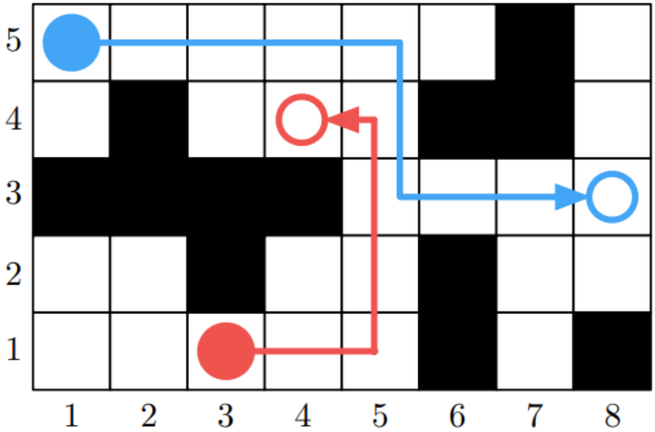


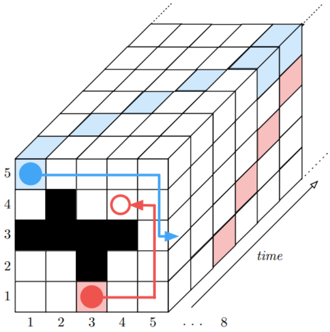
Illustration from August Møbius' thesis *Multi-Agent Pathfinding*

Solution 2: Space-time A*

Space-time A* (STA*): "A* with time constraints"

Definition

A constraint (a, v, t) precludes agent a from being in location $v \in V$ at time t



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Solution 2: Space-time A*

```
def SpaceTimeAStar(problem, agents):
    solution = dict()
    constraints = []
    k = len(agents)
    for i in range(k):
        plan = AStar(problem[agents[i]], constraints)
        constraints += [(agents[j], v, t)
                        for j in range(i+1, k)
                        for t, v in enumerate(plan)]
        solution[agent] = plan
    return solution
```

Assume a sub-routine `AStar`(p, c) with an admissible heuristic that can solve single-agent problem instances given

- a single-agent problem instance, p, and
- a (possibly empty) list of *constraints*, c.

Here, the constraints c are the solutions of the previous agents

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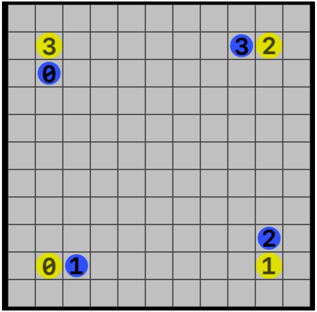

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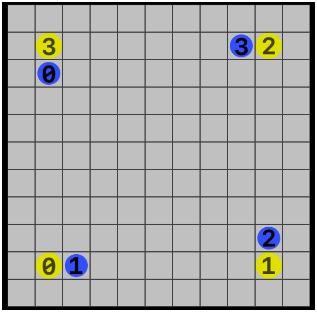

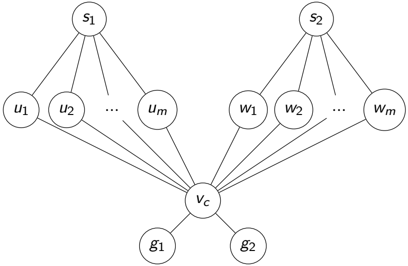

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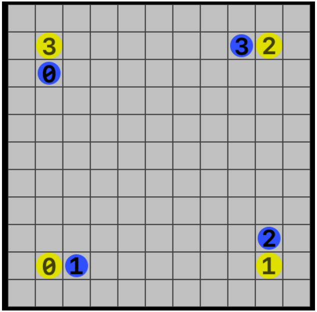

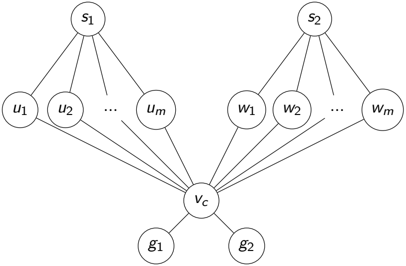

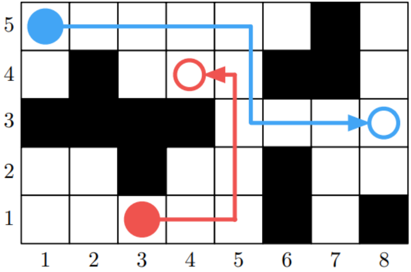

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Questions:

- Is it optimal? Can you give an example where we're not guaranteed to find the shortest solution?
- Is it complete? Can you give an example where we're not guaranteed to find a solution when there is one?

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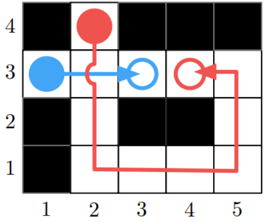
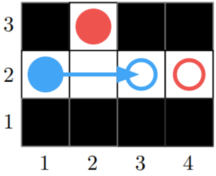
We have serialisable subgoals, but STA* doesn't choose subgoals intelligently





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Algorithm called *Independence detection* (ID)

Solution 3: Independence detection

```
def IndependenceDetection(problem, agents):
    solution = dict()
    groups = [[agent] for agent in agents]
    for group in groups:
       solution[group] = AStar(problem[group], [])
    while not IsValid(solution):
        # Merge confliting groups
        (group1,group2,v,t) = FindFirstConflict(solution)
        del solution[group1]
        del solution[group2]
        merged = [group1, group2]
        groups.append(merged)

        # Plan for the merged group
        solution[merged] = AStar(problem[merged], [])
    return solution
```

$(group_1, group_2, v, t) =$ `FindFirstConflict(solution)` is a conflict of (multibody) agents $group_1$ and $group_1$ because they both plan to be in location v at time t.

Solution 3: Independence detection

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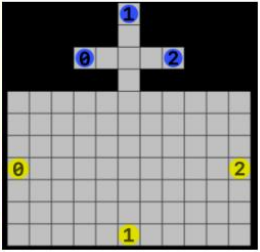

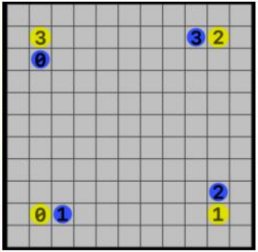

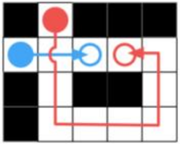
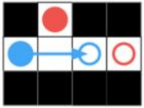

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- Use reservations (like Space-time A*)
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- Conflict based search (CBS)

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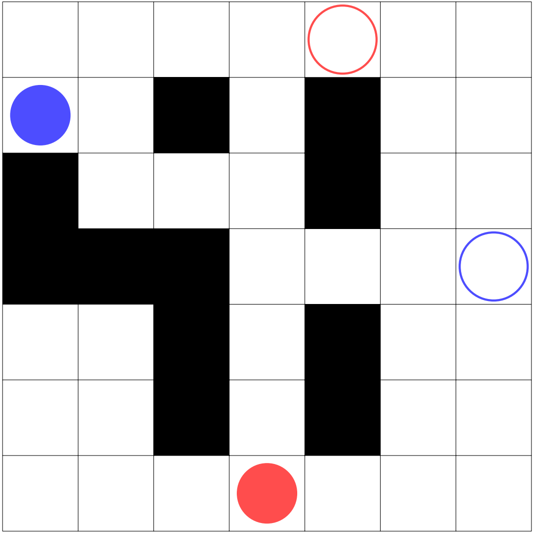

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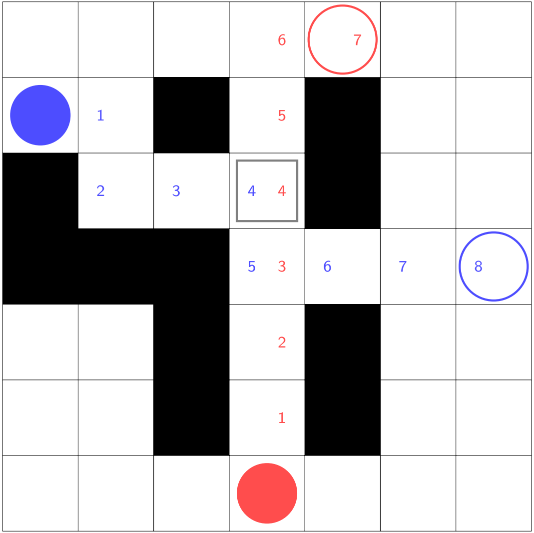

$(\bullet, \bullet, D5, 4)$

Cost: $8 + 7$

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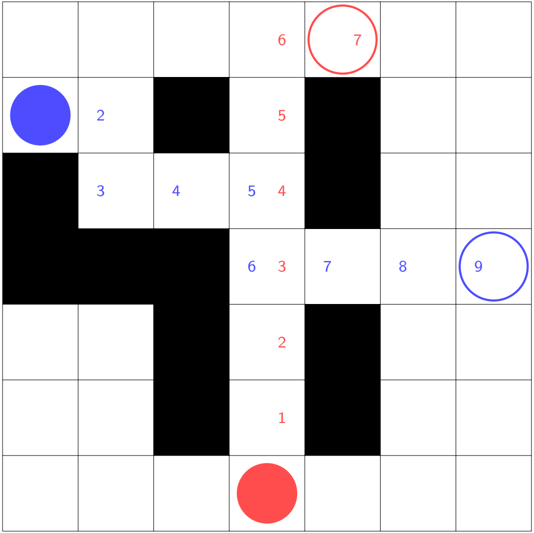

$(\bullet, D5, 4)$

Cost: $9 + 7$

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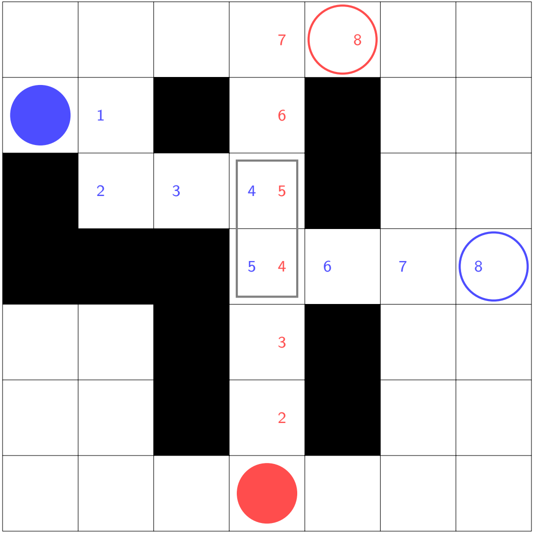
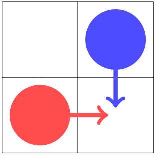


(●, $D5, 4$)

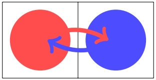
Cost: $8 + 8$

Conflicts

Vertex conflict



Edge conflict



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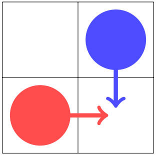

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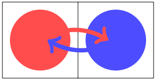
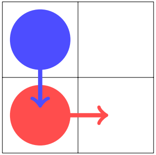


Follow conflict



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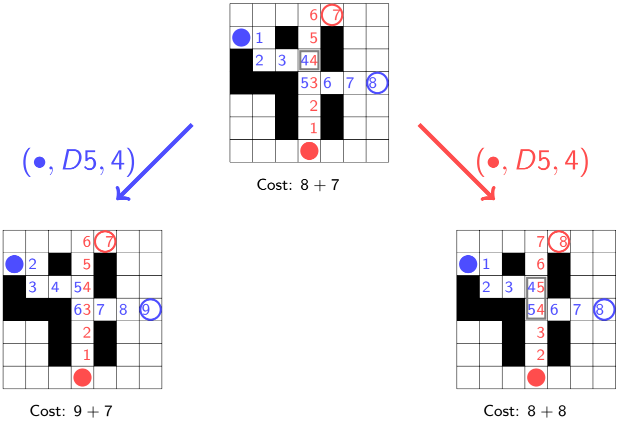

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Cost functions:

$$\textsc{Flowtime}(\pi) = \sum_{i=1}^{k} |\pi_i|$$

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$\textsc{Fuel}$: Like $\textsc{Flowtime}$, but Noop is free

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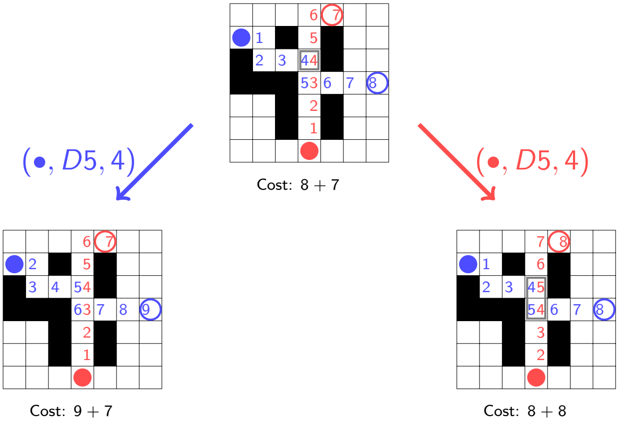

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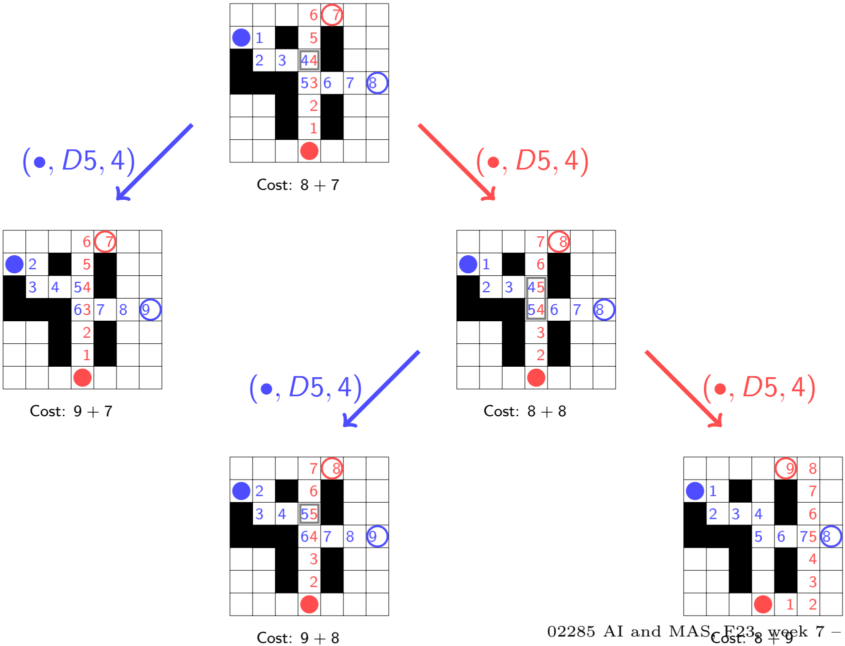

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def ConflictBasedSearch(problem, agents, cost):
    root.constraints = []
    for a in agents:
       root.solution[a] = AStar(problem[a])
    root.cost = cost(root.solution)
    frontier = PriorityQueue(root, lambda n: n.cost)
    while not frontier.empty():
        node = frontier.get()  # get lowest cost
        if IsValid(node.solution):
            return node.solution
        (a1, a2, v, t) = FindFirstConflict(node.solution)
        for a in [a1, a2]:
            m = n.copy()
            m.constraints.append((a, v, t))
            m.solution[a] = AStar(problem[a],
                                  m.constraints)
            m.cost = cost(m.solution)
            if m.cost < infinity:  # solution is found
                frontier.put(m)
```

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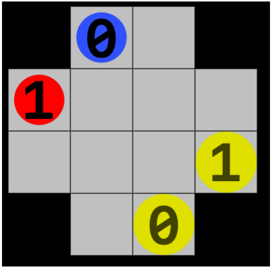
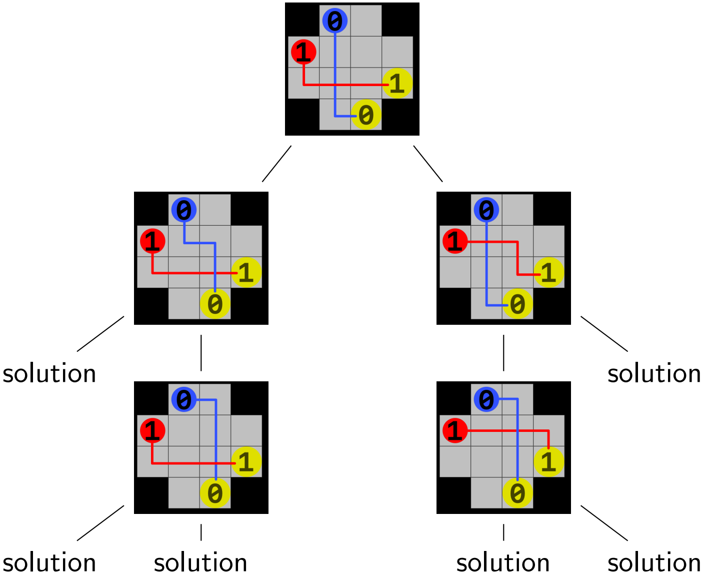
- Is CBS always better than MBA*, ID etc.?
- No, not if two agents are *strongly coupled*, i.e., have many conflicts among them
- In an open $n \times n$ grid there are $2^{n/2}$ optimal paths between opposite corners



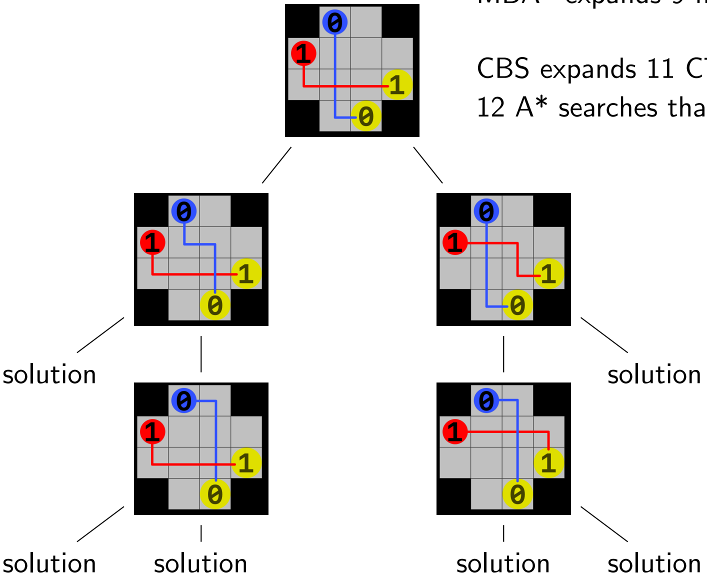
Solution 4: Conflict based search



Solution 4: Conflict based search

MBA* expands 9 nodes

CBS expands 11 CT nodes:
12 A* searches that expand 7 nodes



Solution 4.5: Meta-agent CBS

- Improvement: Meta-agent CBS (MA-CBS)
- If there are more than x conflicts between a_i and a_j, merge them into one *meta-agent*
- Same idea as ID and the Martha project

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- Improvement: Meta-agent CBS (MA-CBS)
- If there are more than x conflicts between a_i and a_j, merge them into one *meta-agent*
- Same idea as ID and the Martha project
- We can tune the parameter x:
  - MA-CBS(∞) $=$ CBS
  - MA-CBS(0) $=$ ID

Solution 4.5: Meta-agent CBS

```
def ConflictBasedSearch(problem, agents, cost, x):
    conflicts = dict()
    root.constraints = []
    for a in agents:
       root.solution[a] = AStar(problem[a])
    root.cost = cost(root.solution)
    frontier = PriorityQueue(root, lambda n: n.cost)
    while not frontier.empty():
        node = frontier.get()  # get lowest cost
        if IsValid(node.solution):
            return node.solution
        (a1, a2, v, t) = FindFirstConflict(node.solution)
        if conflicts[(a1,a2)] > x:
            m = merge(a1, a2, constraints)
            m.solution[a] = AStar(problem[(a1,a2)]
                                      m.constraints)
            m.cost = cost(m.solution)
            if m.cost < infinity:  # solution is found
                frontier.put(m)
        else:
          conflicts[(a1,a2)] += 1  # then do as before
```

Summary

	Pros	**Cons**
MBA*	It works!	But inefficient

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Summary

	Pros	**Cons**
MBA*	It works!	But inefficient
STA*	It's efficient!	But not complete, not optimal
ID	Complete and optimal!	But corridors degrade performance
CBS	Corridors, no problem!	But some agent merging again plz?
MA-CBS	Have some merge!	But what's a good x?

Now what?

- See (Silver, 2005) for STA*
- See (Standley, 2012) for ID
- CBS was introduced in (Sharon et al., 2015) and compares it to other state-of-the-art algorithms
    - Benchmark against other algorithms
    - Focus on completeness and optimality

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- See (Silver, 2005) for STA*
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- CBS was introduced in (Sharon et al., 2015) and compares it to other state-of-the-art algorithms
  - Benchmark against other algorithms
  - Focus on completeness and optimality
- This won't solve the hospital domain, but hope you can take inspiration from the analysis and comparisons of algorithms

References

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