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In-Class Problems

Assume:

- We want to cluster a data set with SOM.
- Data set contains 200 samples
- Each sample has 3 features
- Maximum iterations (k) = 100
- Initial learning rate (L_0) = 0.9

Problems:

1. Remembering that we want a square map ($D \times D$), what is a good value for D ?
2. What is the radius of the map (σ_0)?
3. The first step in the SOM learning process is to find the BMU, which requires calculating the distance between the sample and each neuron. If a sample's features are (0.1,0.3,0.4), what is the distance between that sample and the map neuron with the weights (0.2,0.5,0.6)?
4. What is the radius value for iteration 9, i.e., $\sigma(9)$?
5. What is the learning rate for iteration 9, i.e., $L(9)$?
6. If the BMU is located at (1,1) in the map, what is distBMU for its neighbor located at (2,2)?
7. For the BMU and neighbor in 6 above, what is $\Theta(t)$ for iteration 9 (t)?
8. If the neighbor in 7 is the map neuron in 3 (0.2,0.5,0.6) and the input is the sample in 3 (0.1,0.3,0.4), what are the new weights of the map neuron for iteration 10 ($t+1$)?

1. Remembering that we want a square map ($D \times D$), what is a good value for D ?

$D = \text{round}[\sqrt{M}]$, where $M = 5\sqrt{N}$ and $N = \# \text{ of samples}$.

$N = 200$, so $M = 5\sqrt{200} = 70.71$, so $D = \text{round}[\sqrt{70.71}] = \text{round}(8.408) = 8$.

Hence, $D = 8$

2. What is the radius of the map (σ_0)?

$$\sigma_0 = D - 1 = 7$$

3. The first step in the SOM learning process is to find the BMU, which requires calculating the distance between the sample and each neuron. If a sample's features are (0.1,0.3,0.4), what is the distance between that sample and the map neuron with the weights (0.2,0.5,0.6)?

$$\text{Distance}^2 = \text{SUM from } i=0 \text{ to } n (\text{input}_i - \text{weight})^2$$

$$\text{Distance}^2 = [(0.1-0.2)^2 + (0.3-0.5)^2 + (0.4-0.6)^2] = [0.01 + 0.04 + 0.04] = 0.09 \text{ Distance} = \sqrt{0.09} = 0.3$$

4. What is the radius value for iteration 9, i.e., $\sigma(9)$?

$$\sigma(t) = \sigma_0 e^{-t/\lambda}, \lambda = k/\sigma_0 \Rightarrow \lambda = 100/7$$

$$\sigma(9) = 7 * e^{-9/(100/7)} = 3.73$$

5. What is the learning rate for iteration 9, i.e., $L(9)$?

$$L(t) = L_0 e^{-t/\lambda}$$

$$\lambda = 100/7$$

$$L(9) = 0.9 * e^{-9/(100/7)} = 0.479$$

6. If the BMU is located at (1,1) in the map, what is distBMU for its neighbor located at (2,2)?

$$\sqrt{(x_{\text{BMU}} - x_{\text{neighbor}})^2 + (y_{\text{BMU}} - y_{\text{neighbor}})^2} = \sqrt{(1-2)^2 + (1-2)^2} = \sqrt{1+1} = \sqrt{2} = 1.414$$

7. For the BMU and neighbor in 6 above, what is $\Theta(t)$ for iteration 9 (t)?

$$\Theta(t) = e^{(-\text{distBMU}^2)/(2*[\sigma(t)]^2)} = \Theta(9) = e^{(-2)/(2*[3.73]^2)} = 0.93$$

8. If the neighbor in 7 is the map neuron in 3 (0.2,0.5,0.6) and the input is the sample in 3 (0.1,0.3,0.4), what are the new weights of the map neuron for iteration 10 (t+1)?

$$\begin{aligned} \text{weight}(t+1) &= \text{weight}(t) + \Theta(t)L(t)[\text{input}(t)-\text{weight}(t)] \\ \text{weight}(9+1) &= \text{weight}(9) + \Theta(9)L(9)[\text{input}(9)-\text{weight}(9)] \\ \text{weight}(10) &= (0.2,0.5,0.6) + 0.930*.479[(0.1,0.3,0.4)-(0.2,0.5,0.6)] \\ \text{weight}(10) &= (0.2,0.5,0.6) + 0.930*.479[(-0.1,-0.2,-0.2)] \\ \text{weight}(10) &= (0.2,0.5,0.6) + 0.930*.479[(-0.1,-0.2,-0.2)] \\ \text{weight}(10) &= (0.2,0.5,0.6) + [(-0.045,-0.089,-0.089)] \text{ weight}(10) \\ &= (0.155, 0.411, 0.511) \end{aligned}$$